Scaling up MAPF

Daniel D. Harabor

AAMAS 2022 Tutorial

Recap

Things we want from a MAPF solver:

- Compute collision-free plans
- For as many agents as possible
- Computed as **fast** as possible
- While maximising throughput (i.e., optimise the efficiency of each individual agent).

Optimal algorithms can reliably solve MAPF problems with 150+ agents.
But it's not enough.

Application examples

An amazon parcel sortation centre.

Application examples

Robots drop parcels into sorted bins, for onward delivery.

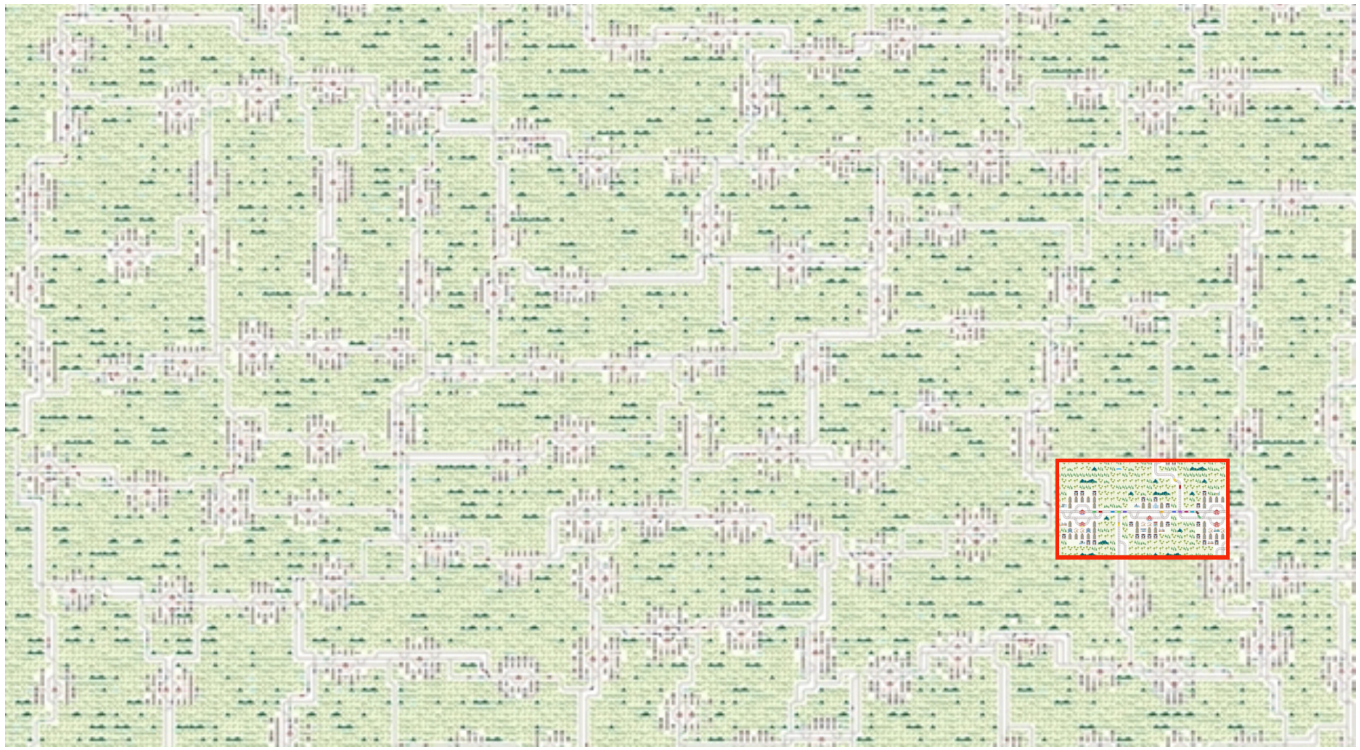
Application examples

Up to 800 agents can be in operation at the same time.

Application examples

Flatland Challenge is a industry sponsored rail planning competition.

Application examples



Several thousand agents can be on the map at the same time.

Tradeoffs

We can scale up (and speed up) existing MAPF algorithms by relaxing some strict requirements.

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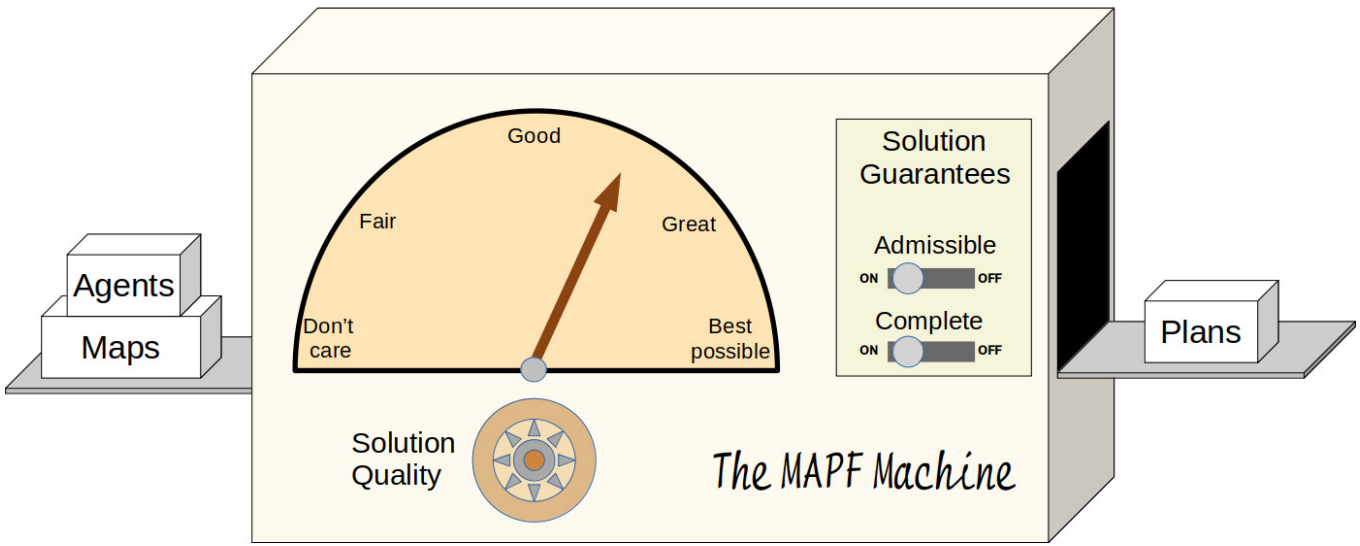
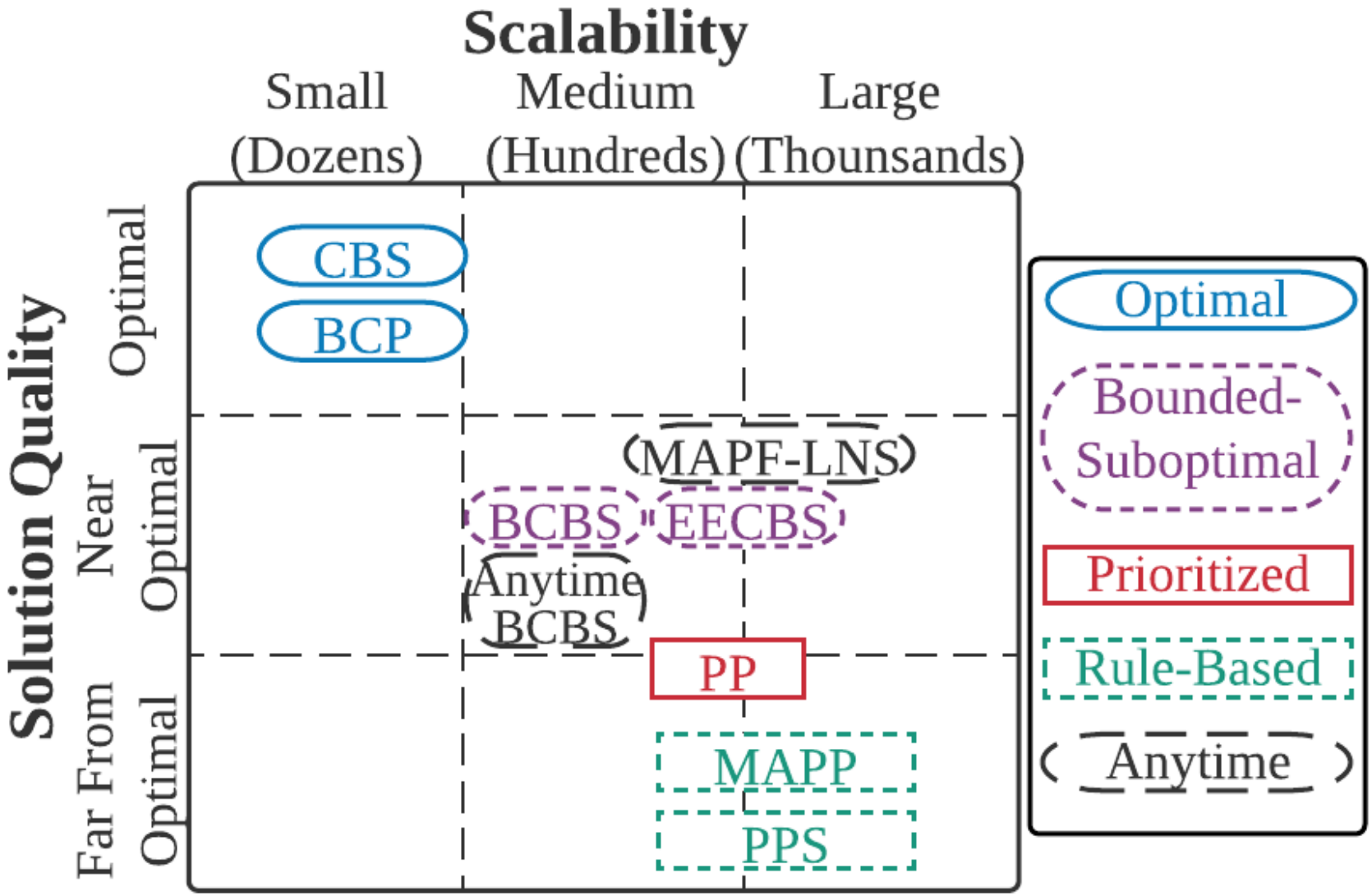


Approaches that have been considered



Bounded Suboptimal MAPF

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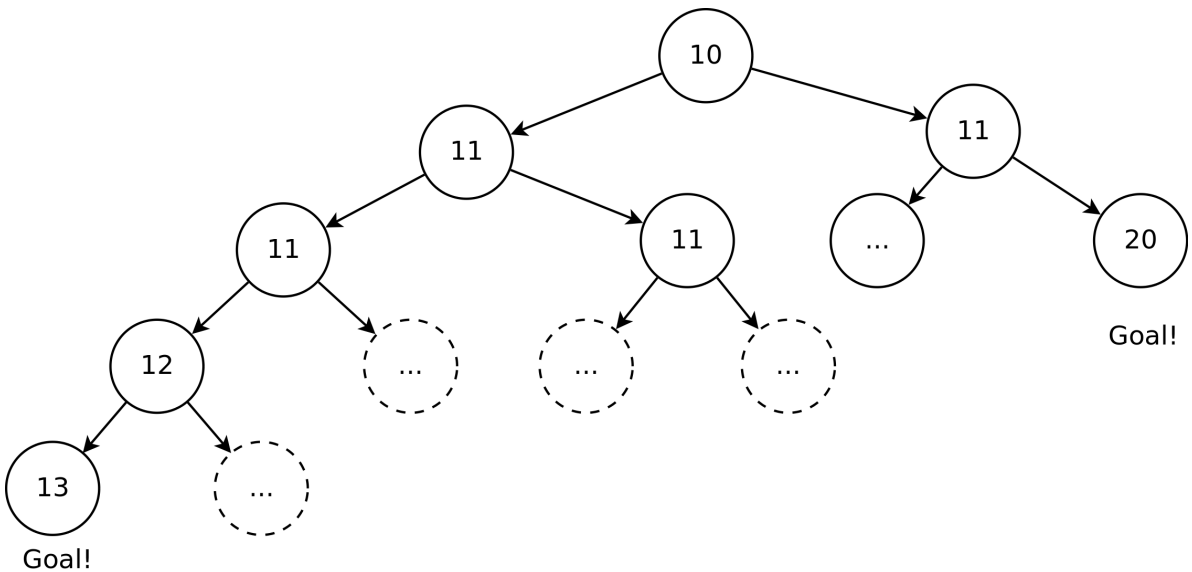
Bounded Suboptimal Search

Idea: Accept any plan whose cost C is not more than some fixed amount larger than C^*, the cost of the optimal plan.

- $C \leq w \times C^*$ where $w \geq 1$ (w-admissible)
- $C \leq \epsilon + C^*$ where $\epsilon \geq 0$ (ϵ-admissible)



In this CT a $w = 2$ suboptimal plan is available but not within reach for CBS.

How do we compute bounded suboptimal plans?

We use a variant of FOCAL Search [Pearl and Kim, 1982] for the high- and low- level of CBS.

In FOCAL search the frontier comprises two lists:

- OPEN, sorted by $f_1(n) = g(n) + h(n)$ (strict admissibility).
- FOCAL $\subseteq$ OPEN, sorted by $f_2(n)$ (possibly inadmissible).

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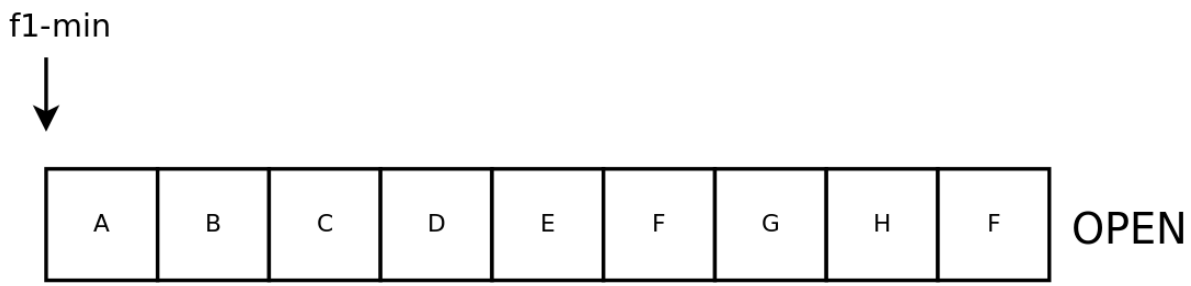

With a conventional OPEN list we always expand the node with minimum f_1

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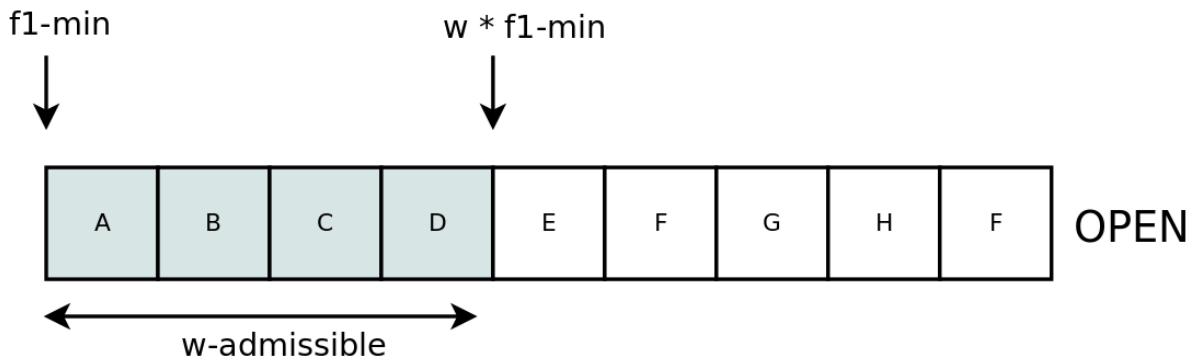

Notice however that many nodes may satisfy the bounded-suboptimal criteria

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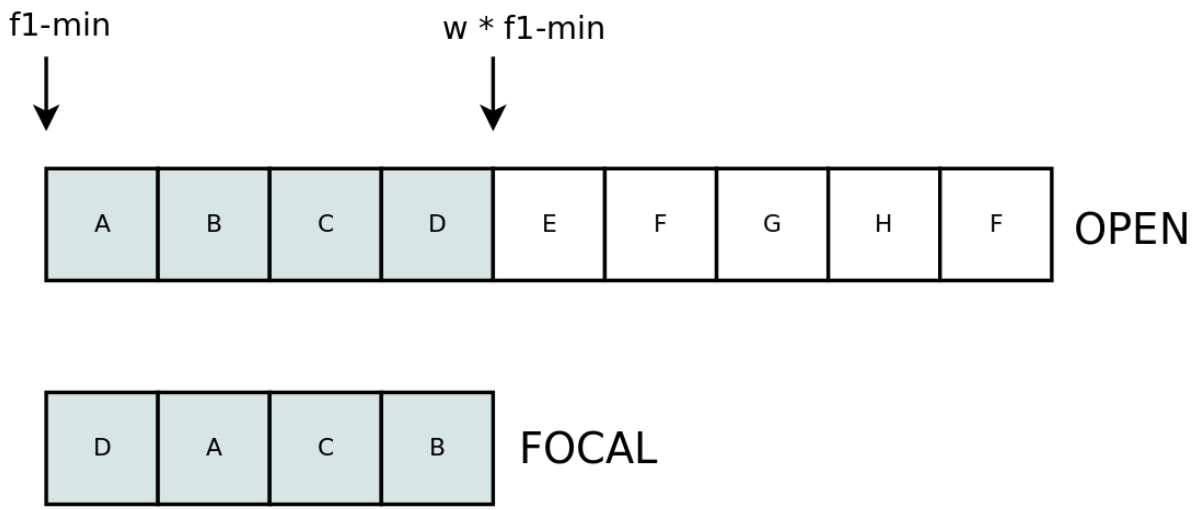

FOCAL allows us to expand the bounded suboptimal nodes in any order, f_2

How do we compute bounded suboptimal plans?

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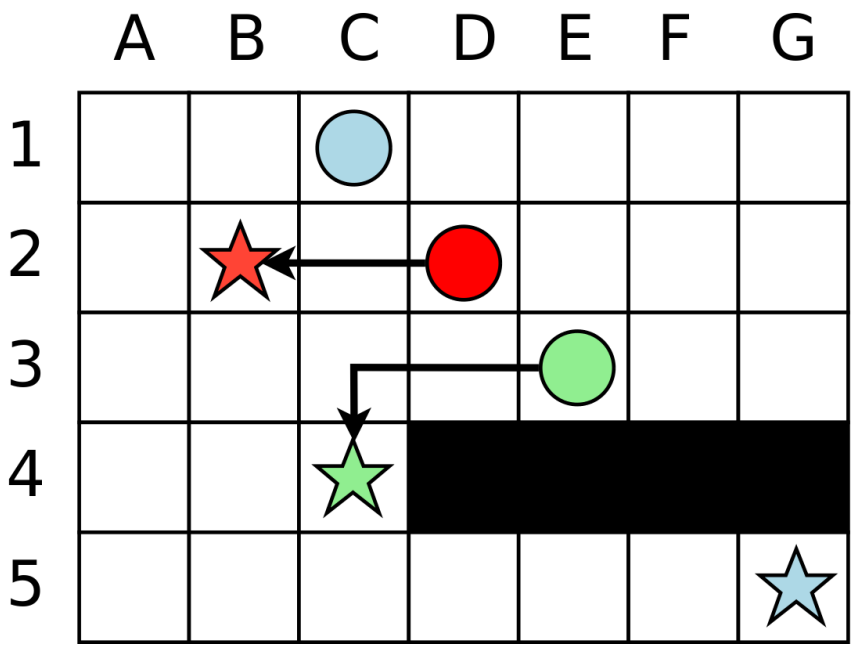
On termination, FOCAL search can return:

- failure (no plan exists) **or**
- a bounded-suboptimal plan, π, and
- (optionally) a best-known lower-bound $LB = \min f_1 \leq C^*$

Modified low-level search

We use FOCAL Search plan single-agent paths. This helps to reduce the number of collisions in a CBS CT node. We have:

- $f_1 = g + h$ (with $h =$ manhattan distance) and $w = 1.5$
- $f_2 =$ minimum number of conflicts.



We need to re-plan the blue agent.

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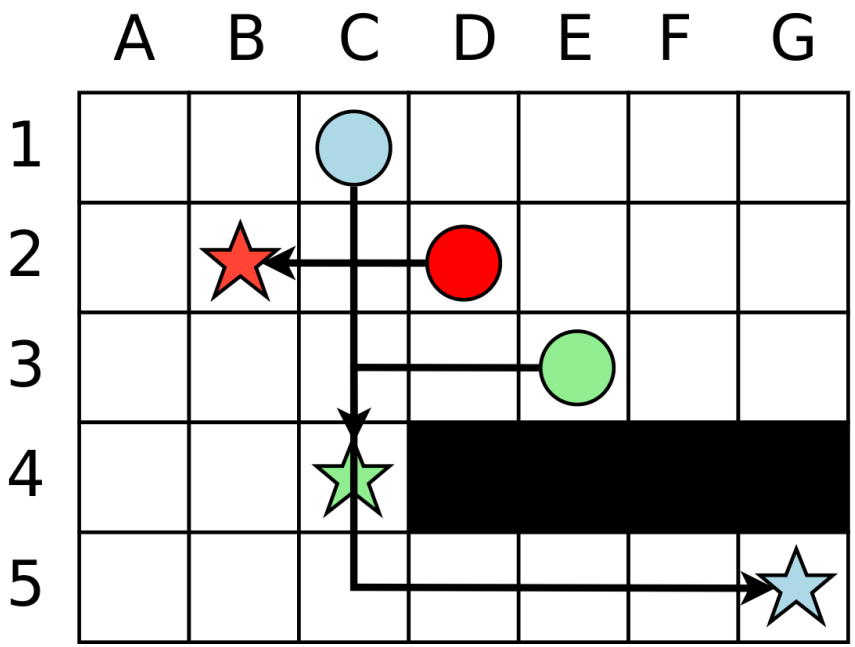

This plan is f_1-optimal but blue is incompatible with red and green

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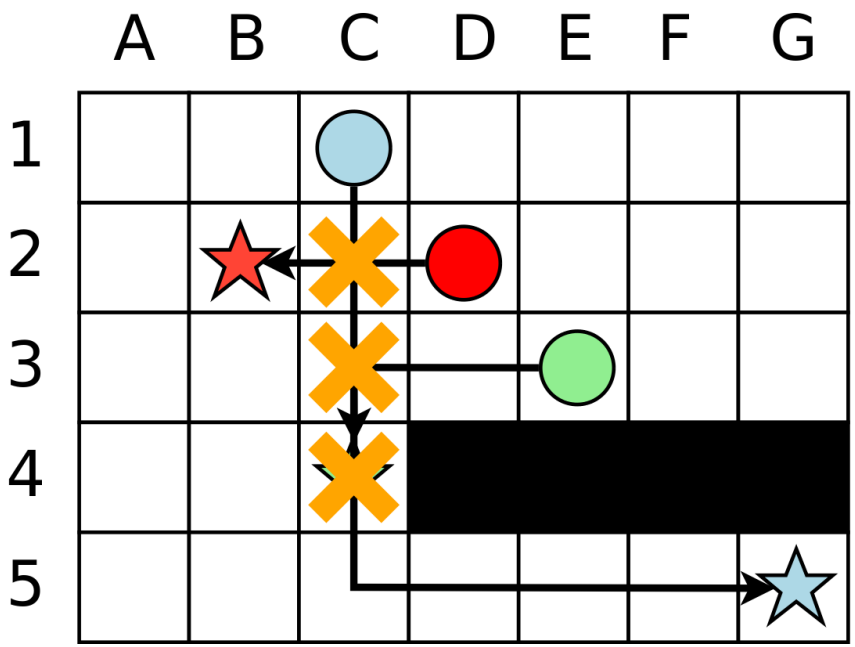

High-level CBS will need to split to resolve these problems.

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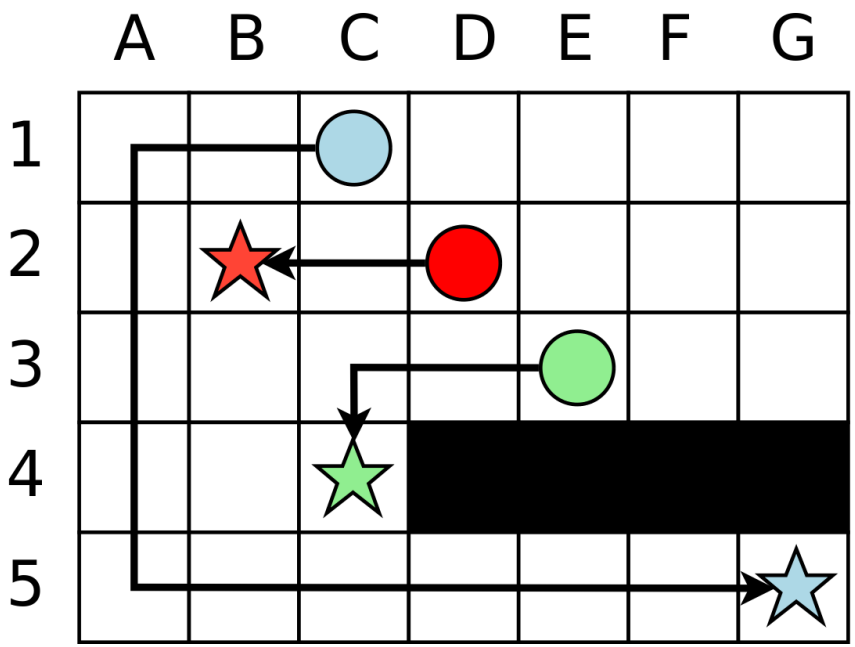

FOCAL search will return this path which w-suboptimal and conflict-free.

The situation so far...

We use FOCAL search inside CBS: to compute a path π_i for each individual agent $a_i \in A$. Thus, for each CT node we have:

- $c(\pi_i) \leq w \times \pi_i^*$
- $\sum_{a_i \in A} c(\pi_i) \leq w \times C^*$

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This approach is indeed bounded suboptimal, but we can do better...

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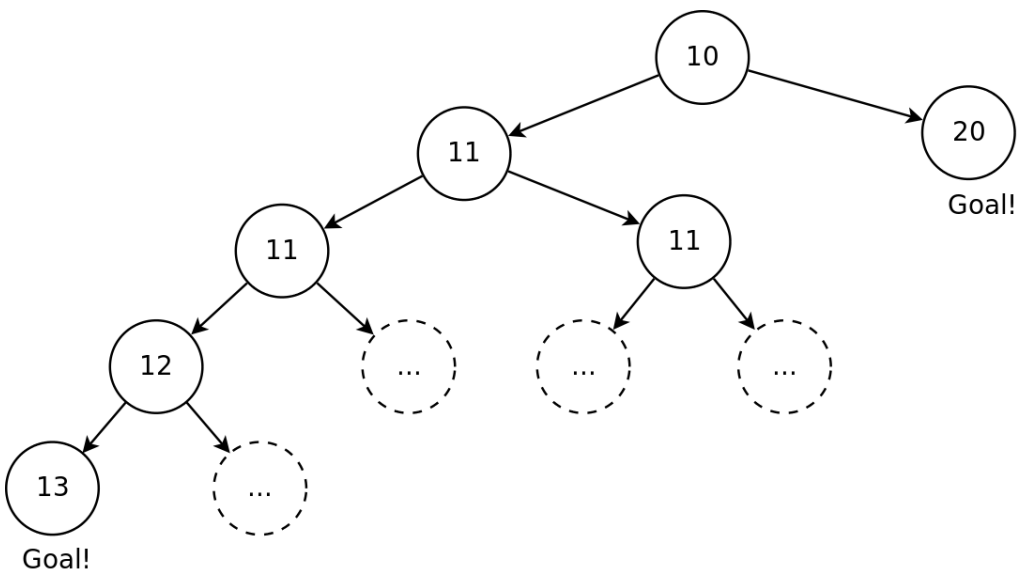

In this CT a $w = 2$ suboptimal plan is available immediately.

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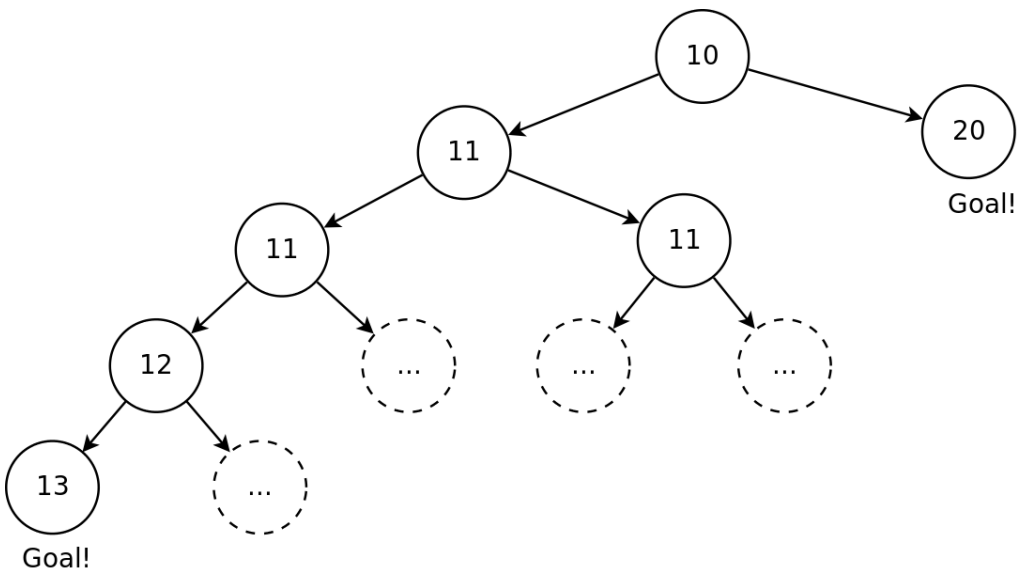

Yet the high-level CBS expands nodes in f-min order!

Modified high-level search

We use FOCAL Search to explore the CBS Conflict Tree. We have:

- $f_1 = \sum_{a_i \in A} LB_i(a_i)$
- $f_2 =$ minimum number of pairwise conflicts

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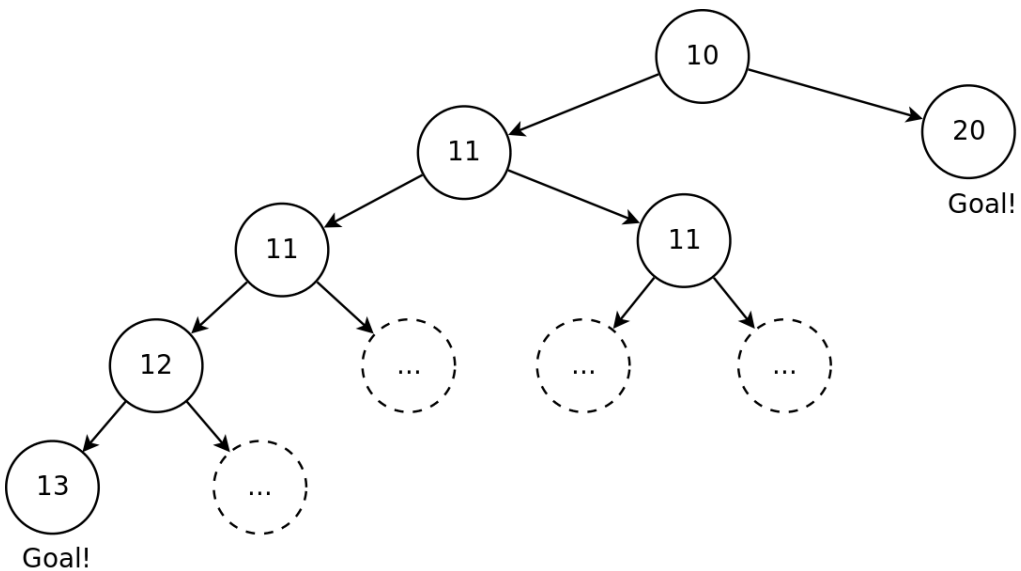

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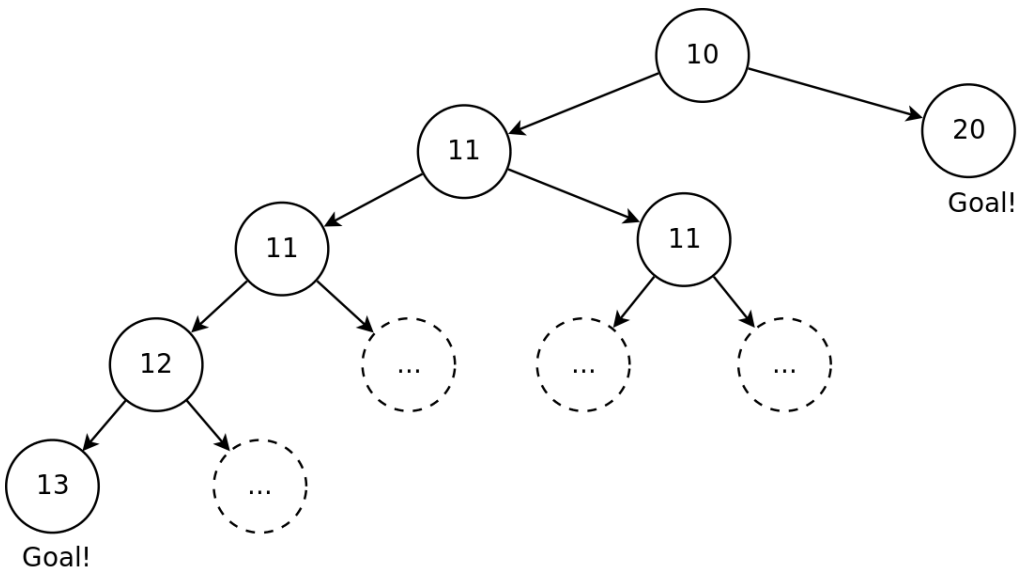

Here $\min f_1 = 10$ and $w(=2) \times \min f_1 = 20$

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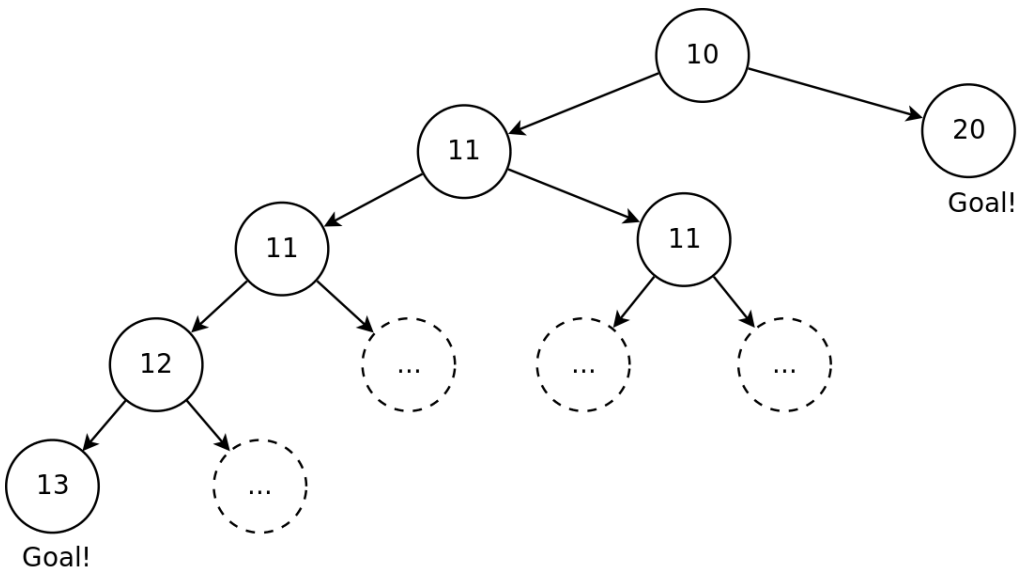

CBS is now free to expand the shallow solution!

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CBS with high-level FOCAL search is itself bounded suboptimal. But for best results, we combine it with low-level FOCAL search.

Enhanced CBS

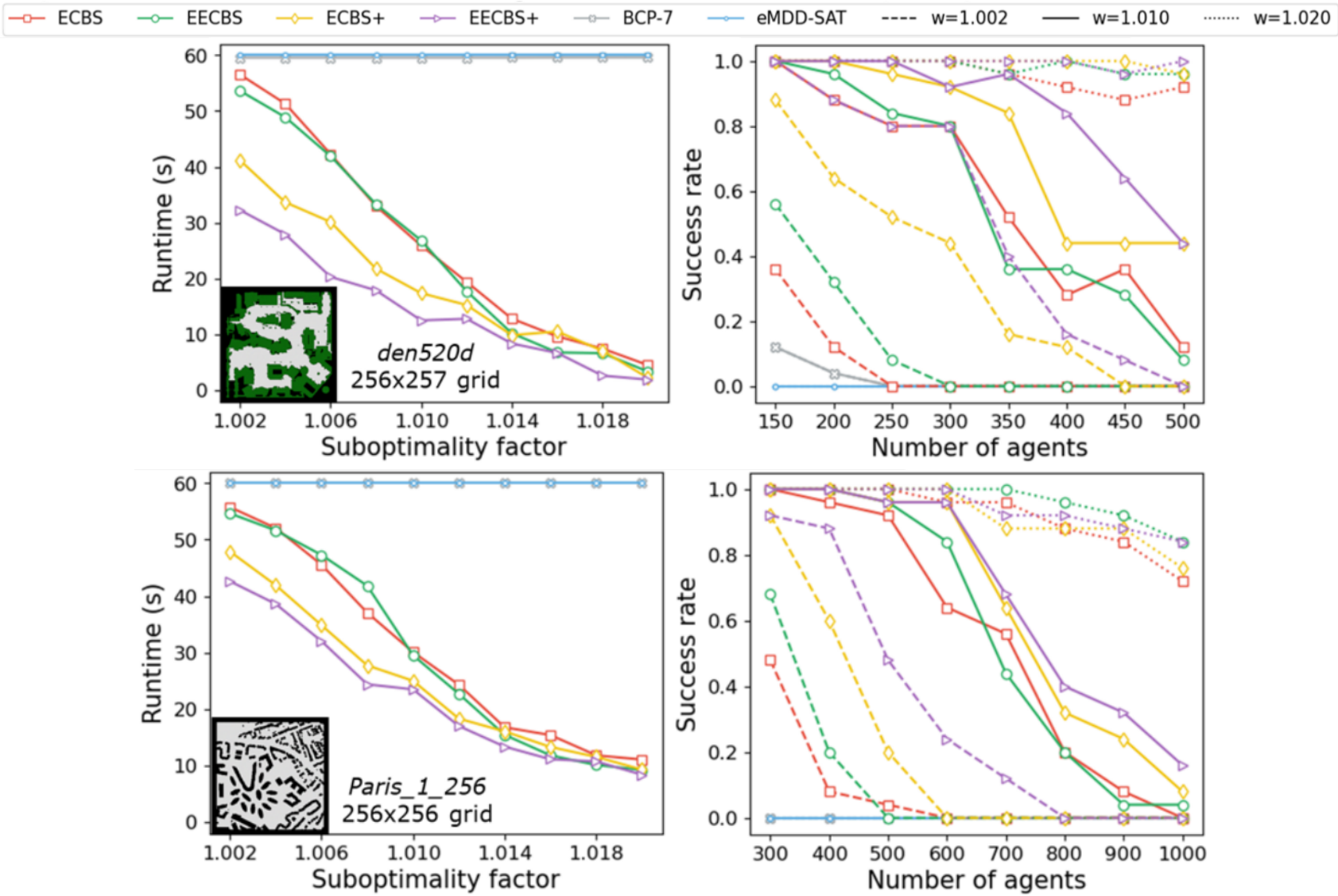
The combination of FOCAL search, at the high- and low- level, is known as **Enhanced CBS** [Barer *et al.*, 2014].

- This an influential workhorse algorithm for MAPF.
- Easily handles hundreds of agents with only small suboptimality.
- Further improved with heuristics and constraints from optimal CBS

Recent developments in this area:

- Explicit Estimation CBS (EECBS) [Li *et al.*, 2021c] investigates new high-level search strategies for ECBS (the frontier is now comprised of three lists instead of two).
- Flex distribution [Chan *et al.*, 2022] for ECBS (FEECBS) investigates how the available suboptimality ("flex") can be divided up amongst the different agents.

Some results for ECBS [Li *et al.*, 2021c]



Rule-based MAPF

Daniel D. Harabor

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AAMAS 2022 Tutorial

Rule-based Search Algorithms

Idea: Develop a policy that decides how collisions between agents will be resolved. i.e., who has priority where, and when.

Rule-based solvers are generally iterative algorithms:

- Agents are processed sequentially and in order
- Each agent plans one or more steps towards its target
- Later agents cannot derail the plans of earlier agents
- After every agent is planned, the current state is updated
- The process continues until success (unless terminated sooner)

The main ideas

Two types of approaches exist in this space:

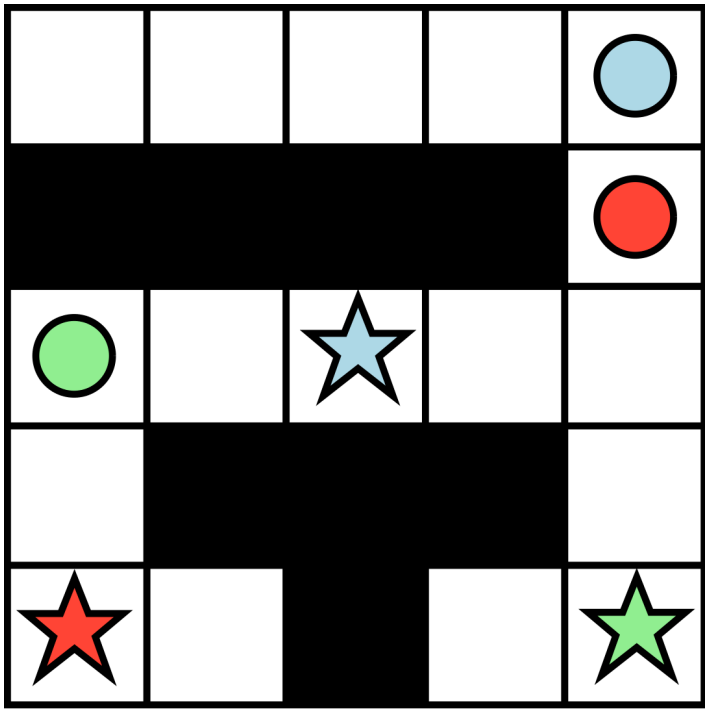
- Move operators
- Temporal reservation methods

In this section we look at examples of each type:

- The PUSH move operator (Okumura variant [Okumura *et al.*, 2019])
- Fixed-order prioritised planning

The PUSH move operator

Idea: Resolve collisions by recursively PUSHING other agents aside



In this problem the agents must traverse a narrow passage.

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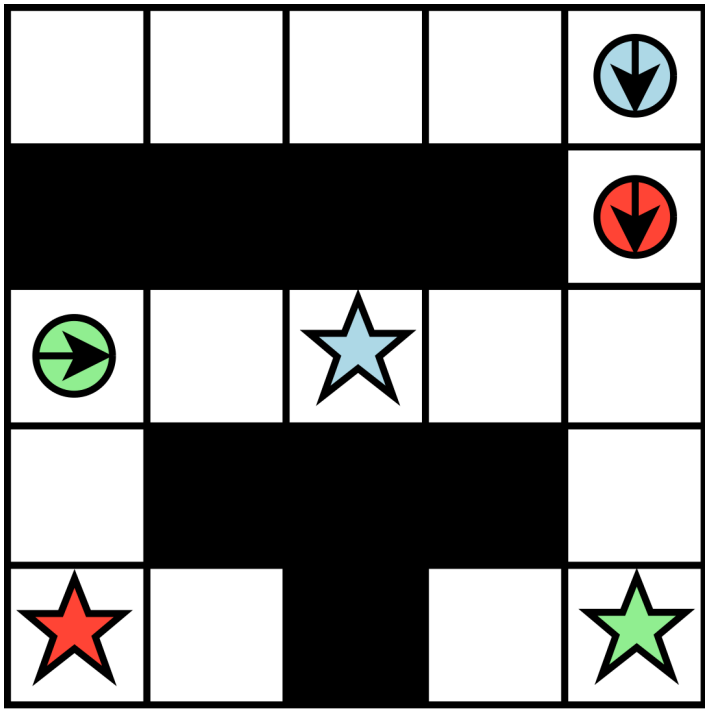

Each iteration every agent plans one step towards its target.

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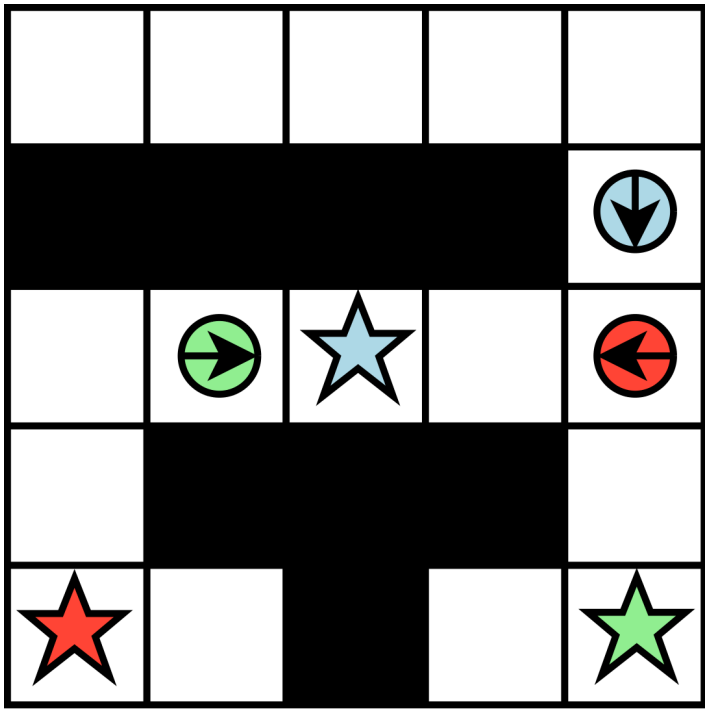

The agents are planned sequentially.

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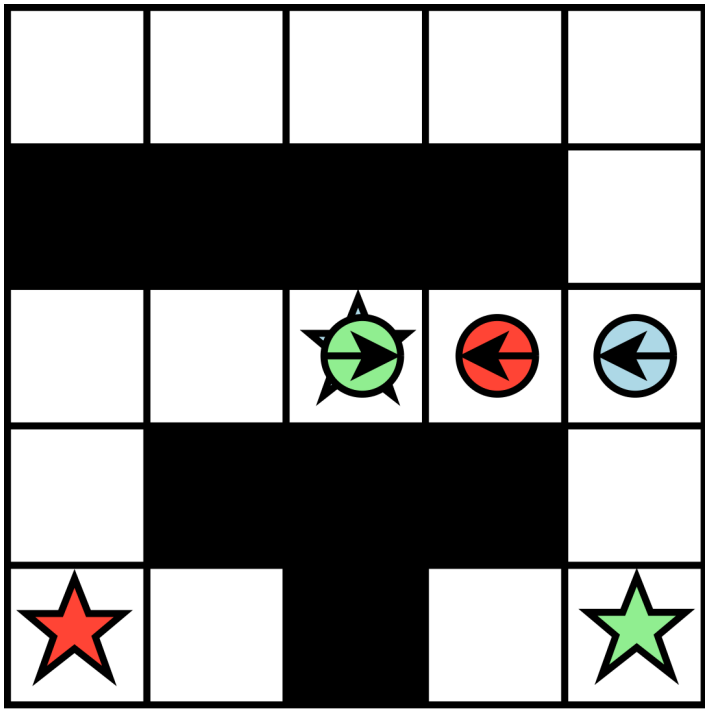

Each can move to any adjacent location that is not already claimed.

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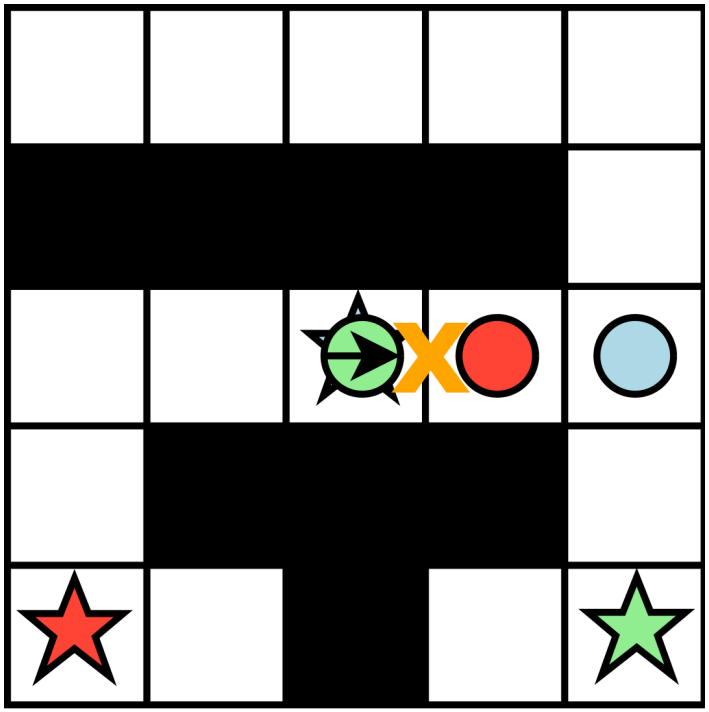

Here we plan Green first. But the tile ahead is occupied by Red.

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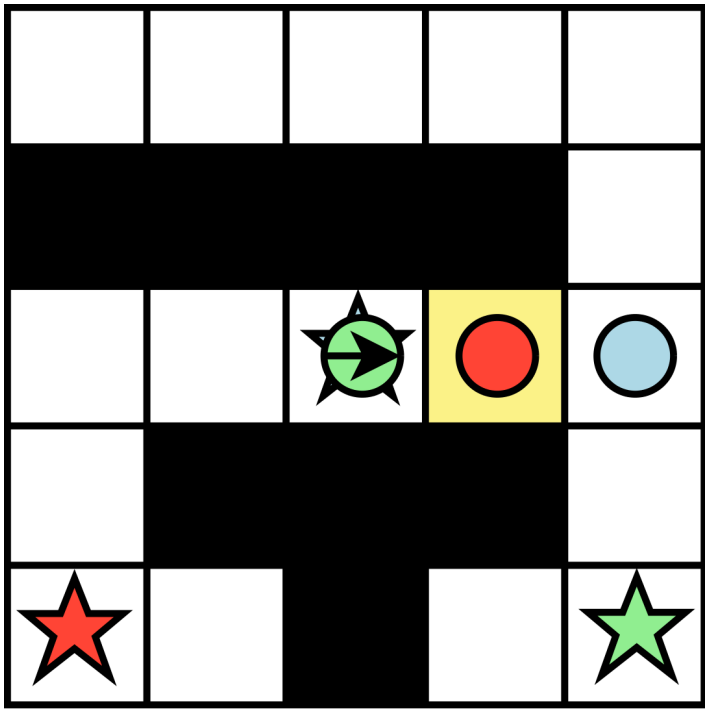

Since Green has priority, it claims the tile.

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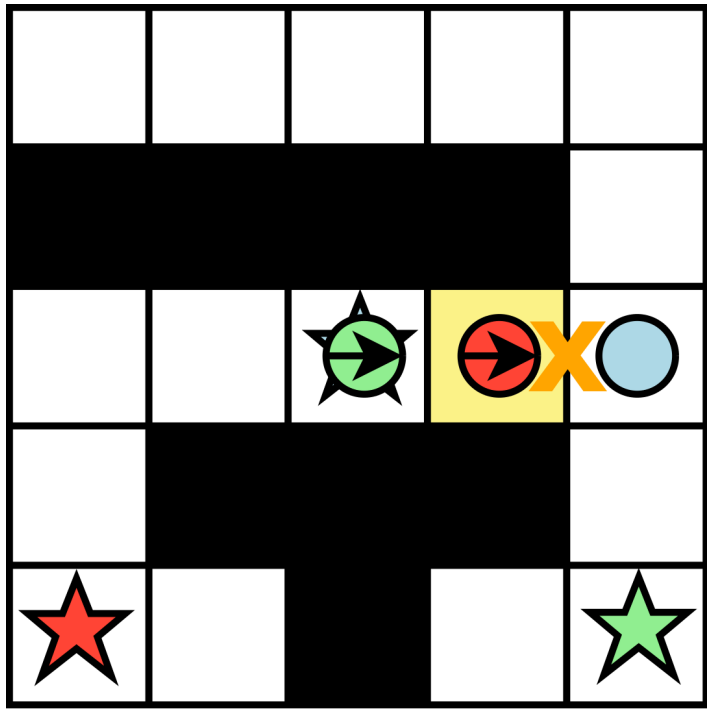

(Recurse) Red plans next. It tries to step aisde, but the way is blocked by Blue.

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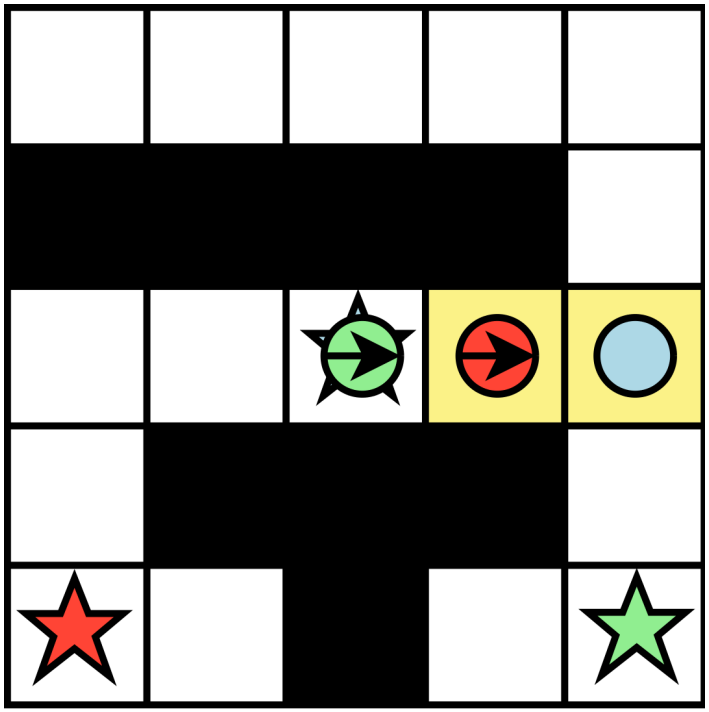

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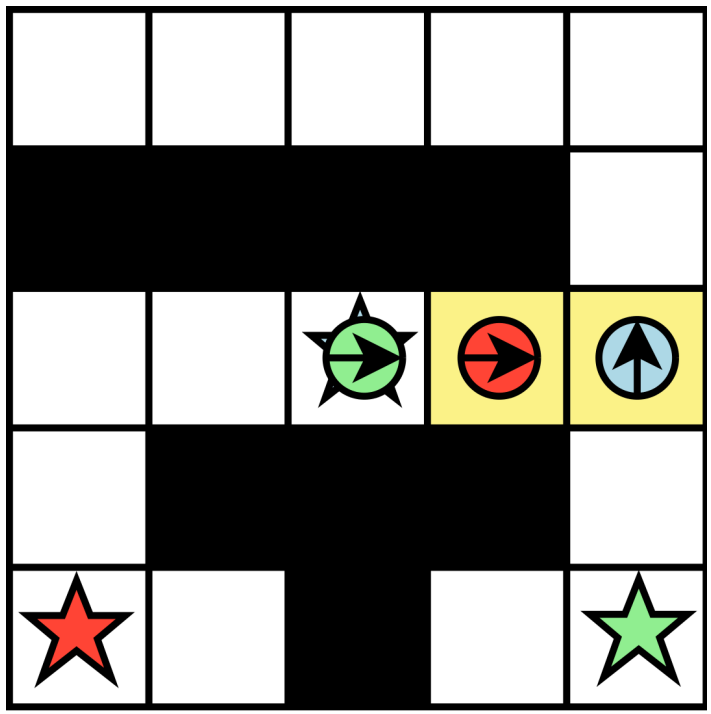

(Recurse) Blue plans next and is forced to step aside for Red.

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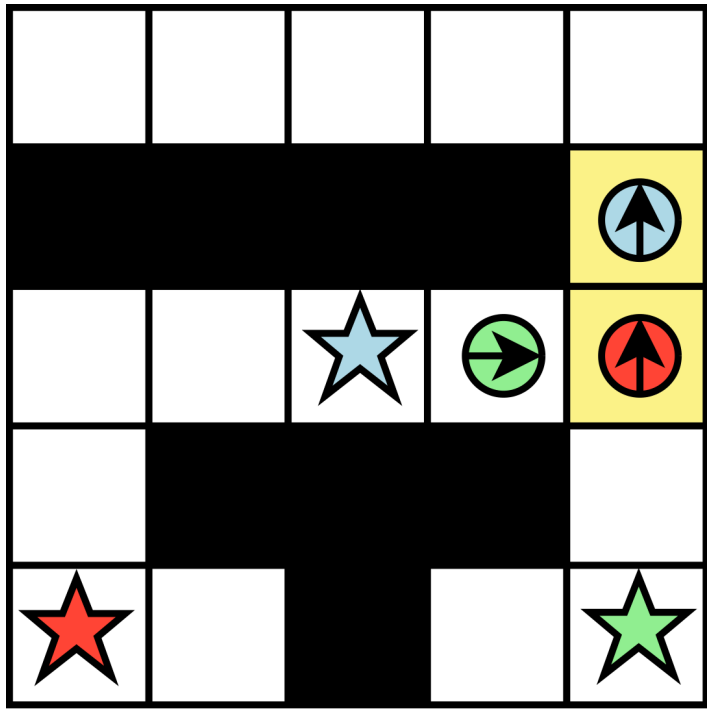

In the next iteration Green once again pushes Red (pushes Blue).

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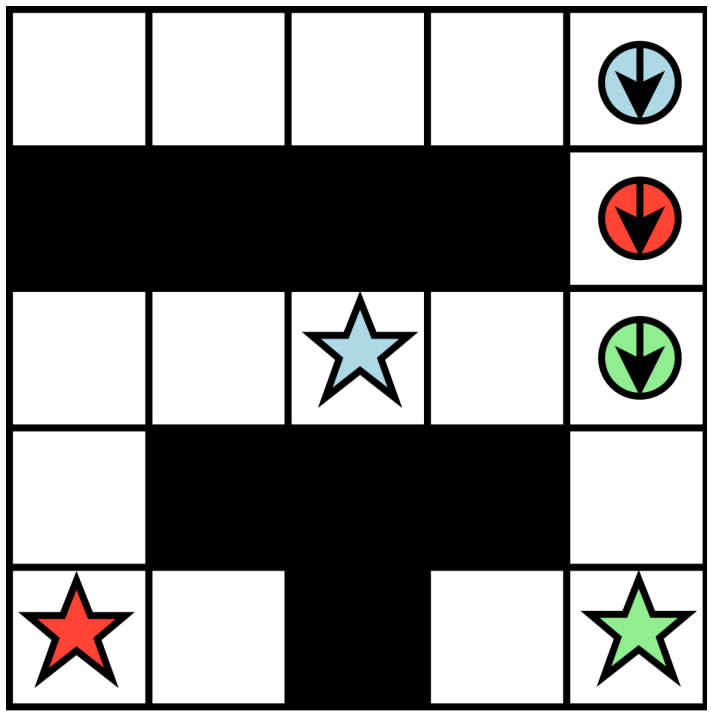

Once Green passes, all agents can resume toward their targets.

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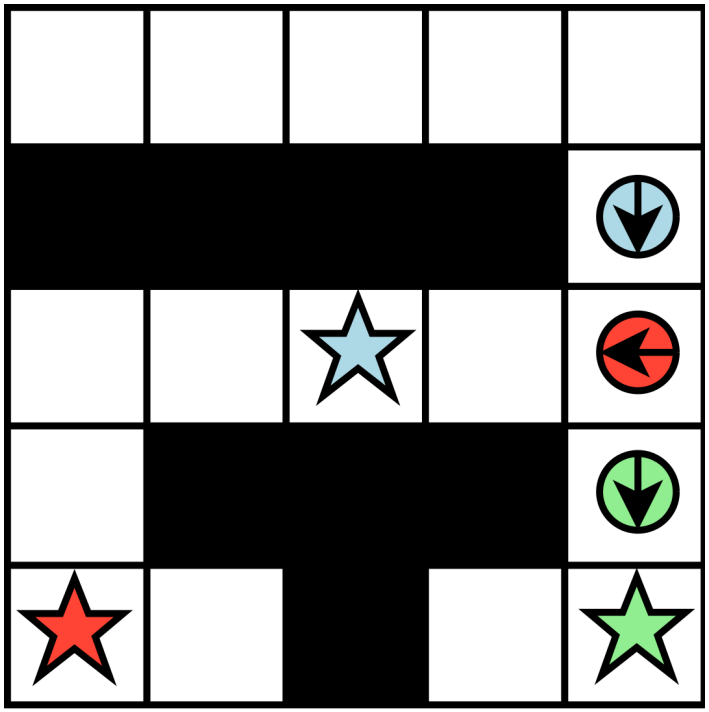

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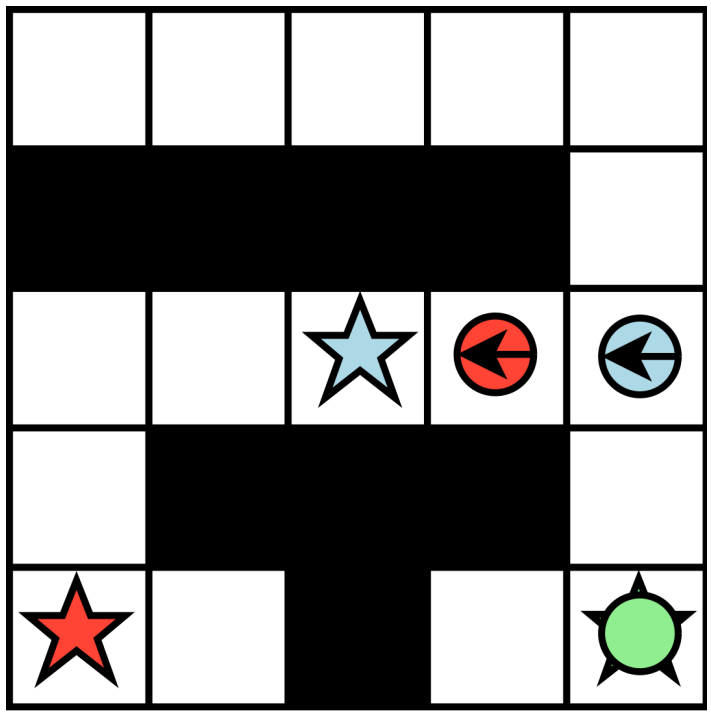

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Analysing PUSH

Pros:

- Simple
- Extremely fast (linear time per iteration)
- Scales to thousands of agents
- Can sometimes produce good quality plans

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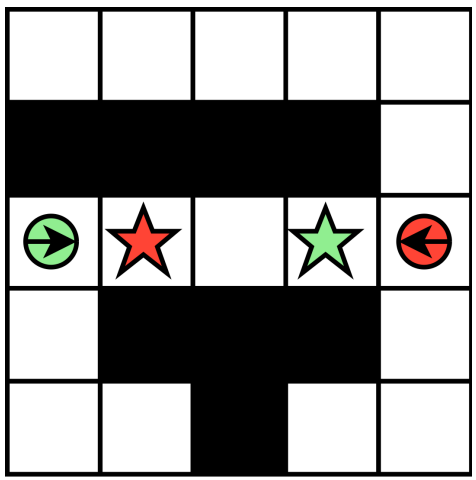

PUSH fails here due to livelock

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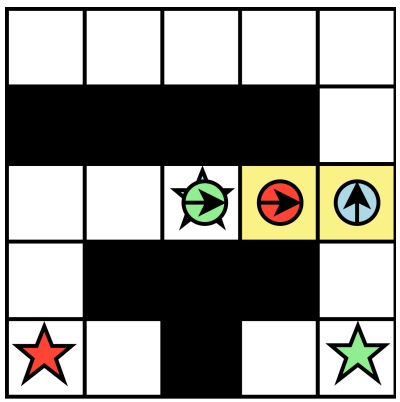
- Incomplete
- Only somewhat aware of the objective function



Here Blue could move up or down. But one is much worse than the other.

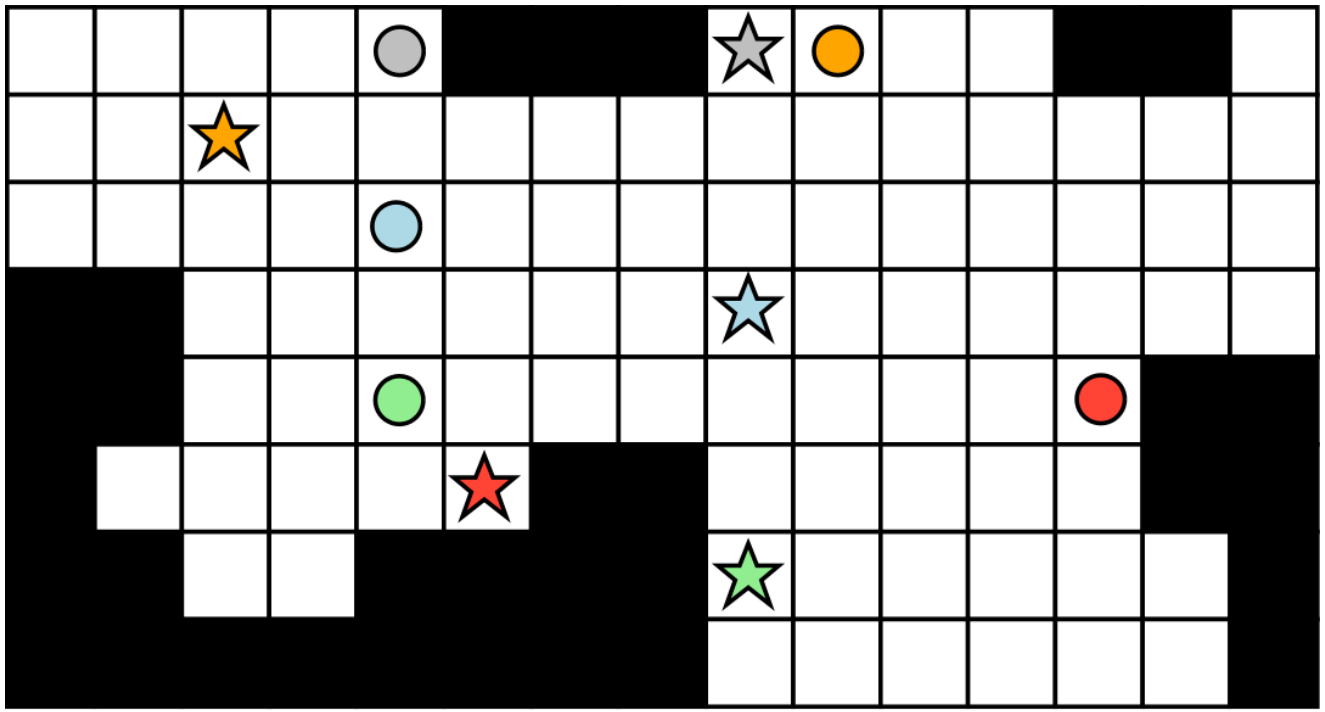
Recent development for move operators

PUSH forms the basis for a very scalable MAPF algorithm known as Priority Inheritance with Backtracking (PIBT) [Okumura *et al.*, 2019]. This is a very capable solver. Recent versions can compute good quality solutions for up to thousands of agents.

PUSH can also be combined with other similar move operators, such as SWAP, to derive more powerful and complete MAPF algorithms. The state of the art is a called PUSH-AND-ROTATE [de Wilde *et al.*, 2014].

Prioritised Planning [Erdmann and Lozano-Pérez, 1987]

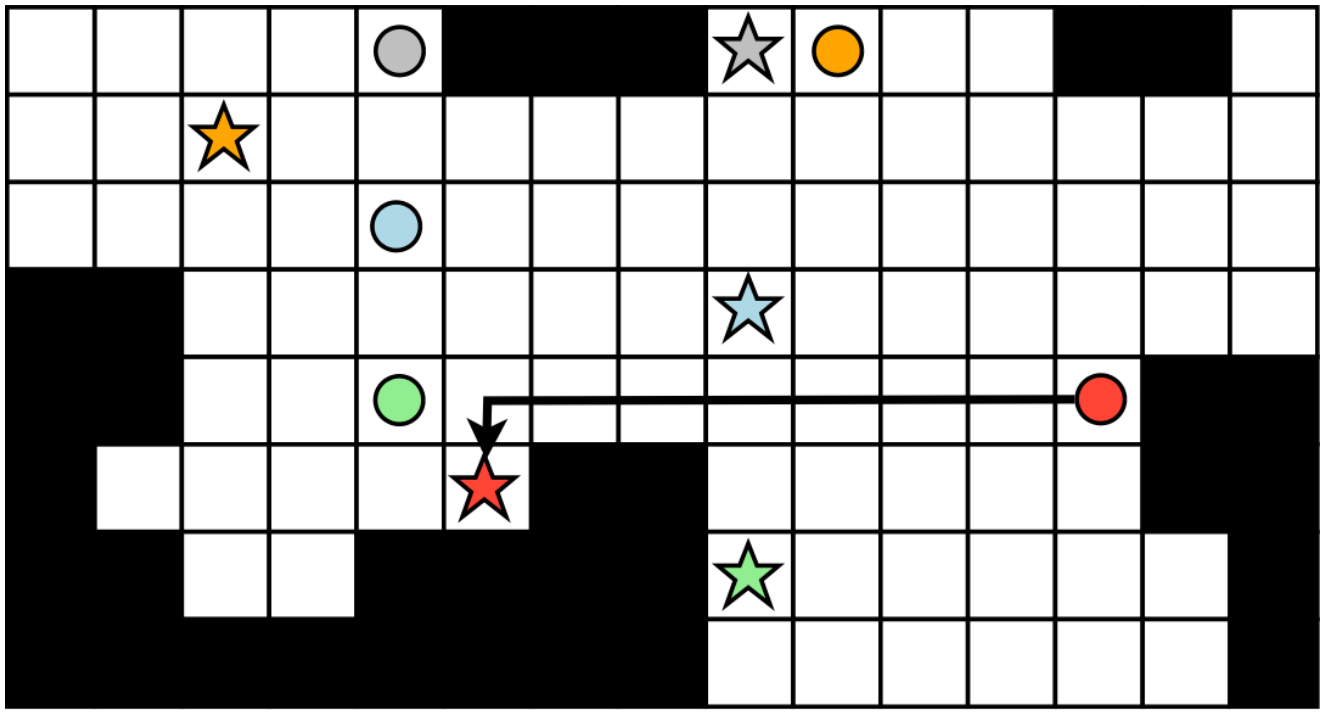
Idea: Reserve the (spatio-temporal) shortest path of each agent.



We plan the agents one by one.

Prioritised Planning [Erdmann and Lozano-Pérez, 1987]

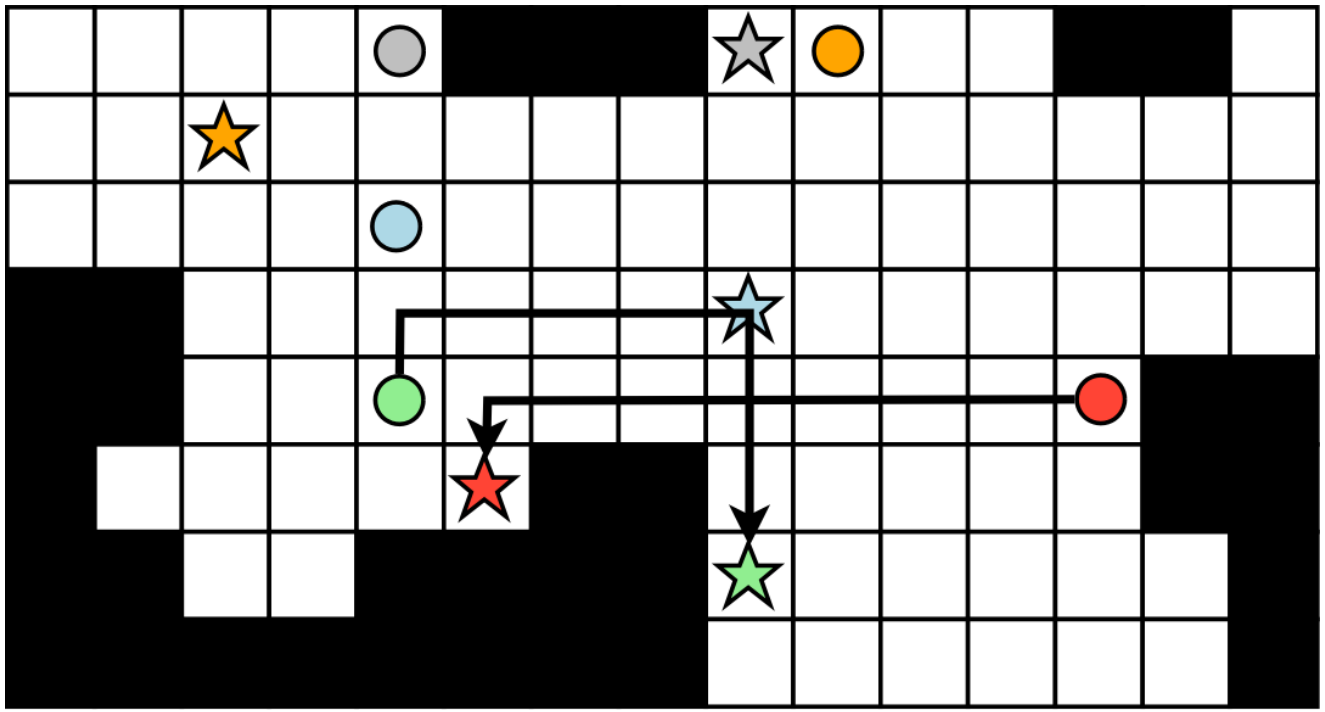
Idea: Reserve the (spatio-temporal) shortest path of each agent.



Red goes first and reserves its individually optimal path.

Prioritised Planning [Erdmann and Lozano-Pérez, 1987]

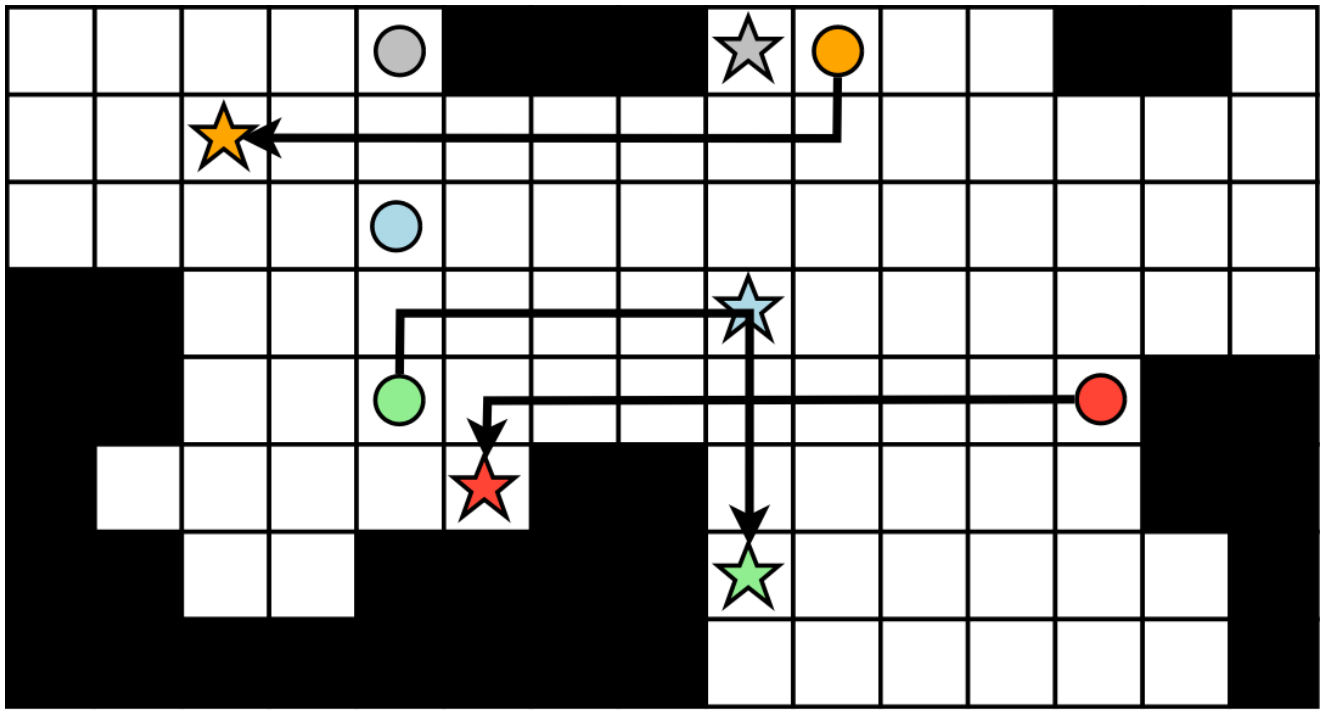
Idea: Reserve the (spatio-temporal) shortest path of each agent.



Green is next. It must avoid the path of the higher-priority Red agent.

Prioritised Planning [Erdmann and Lozano-Pérez, 1987]

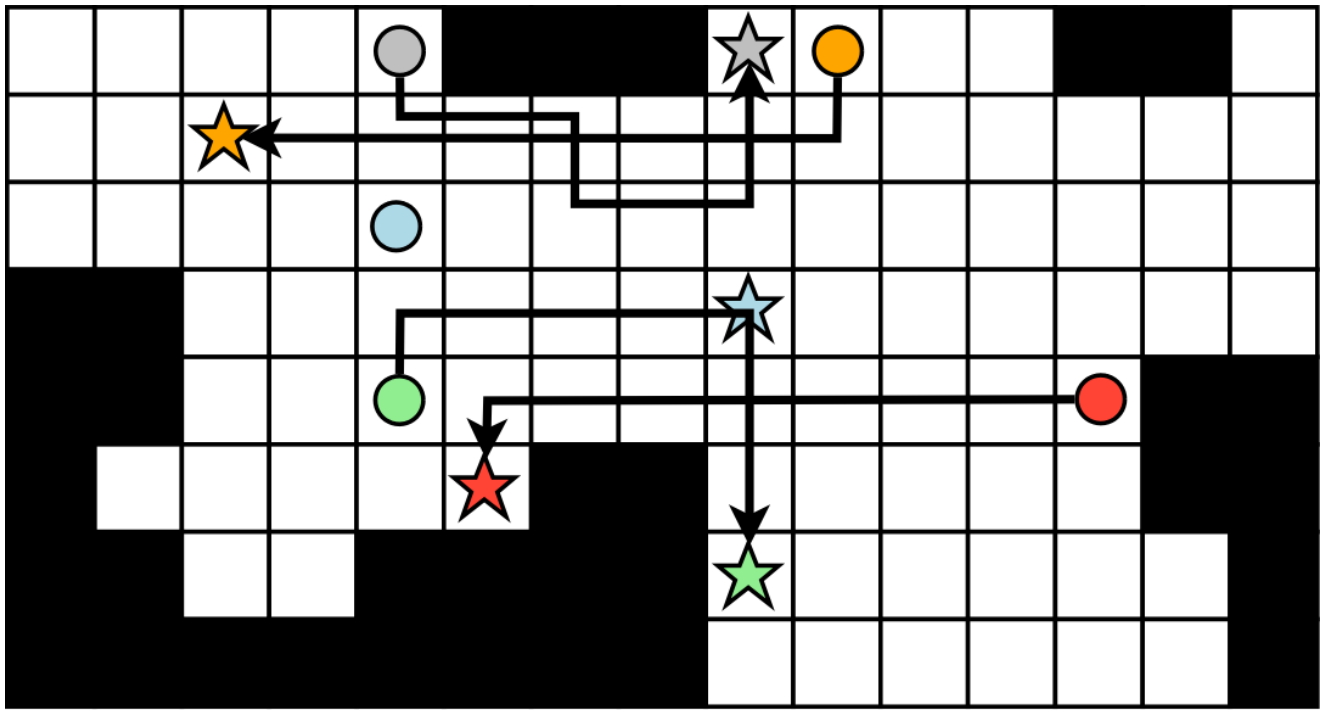
Idea: Reserve the (spatio-temporal) shortest path of each agent.



Each subsequent agent avoids all those previously planned.

Prioritised Planning [Erdmann and Lozano-Pérez, 1987]

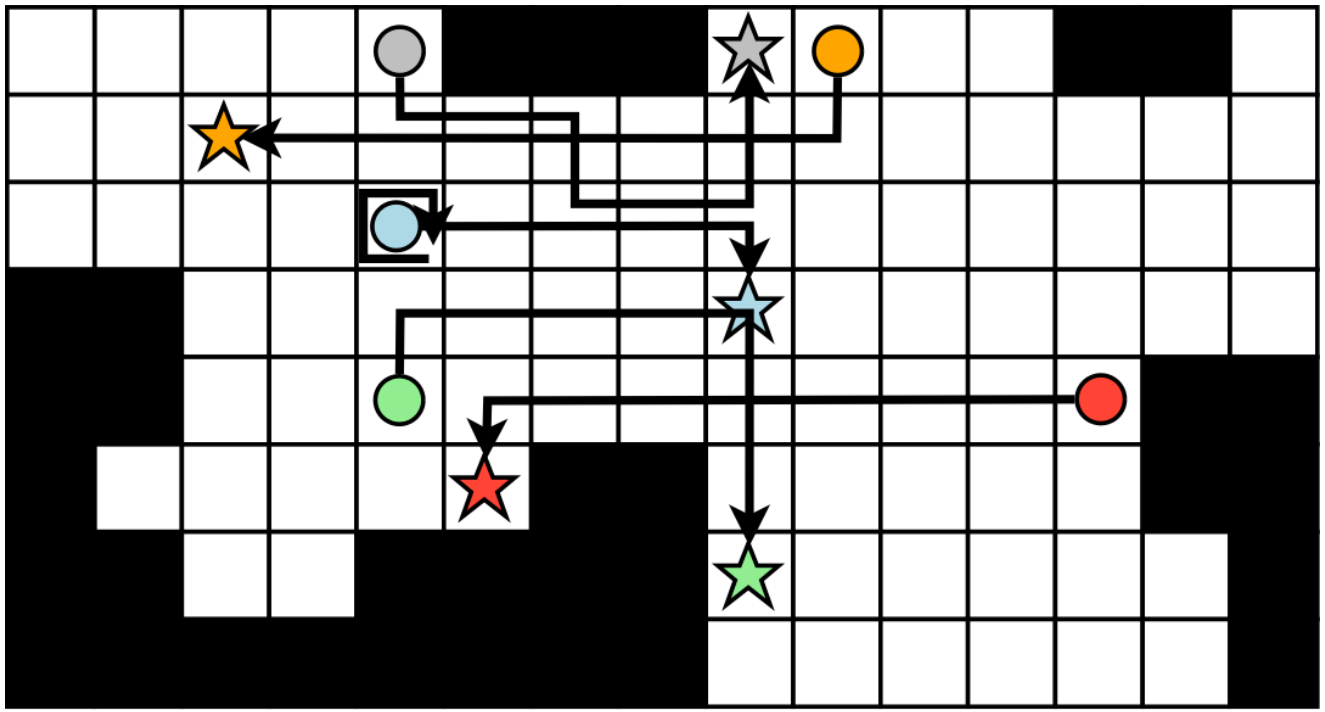
Idea: Reserve the (spatio-temporal) shortest path of each agent.



Notice high priority agents never wait for any low priority agents.

Prioritised Planning [Erdmann and Lozano-Pérez, 1987]

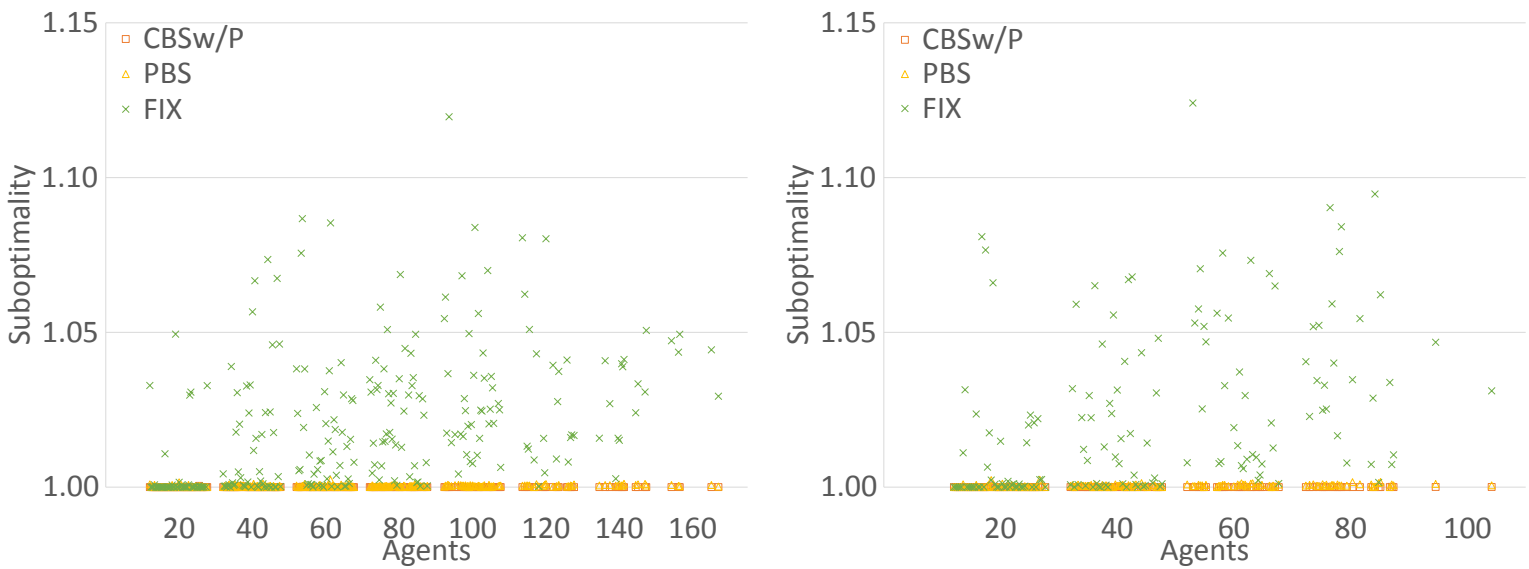
Idea: Reserve the (spatio-temporal) shortest path of each agent.



If all agents have a feasible path, the problem is solved.

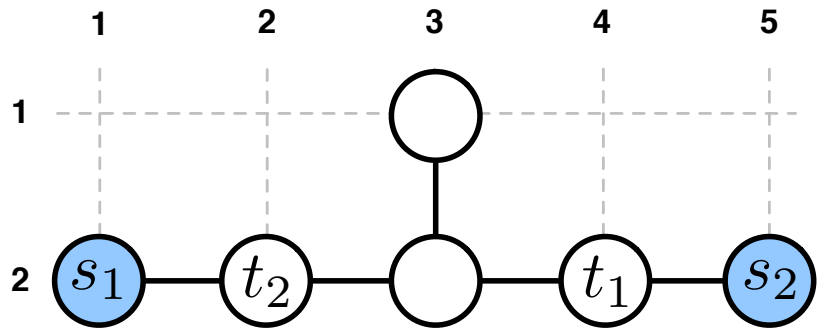
Is it any good?

Prioritised planning is simple and fast. It offers no guarantees yet often produces very high quality plans. It is an extremely popular algorithm!



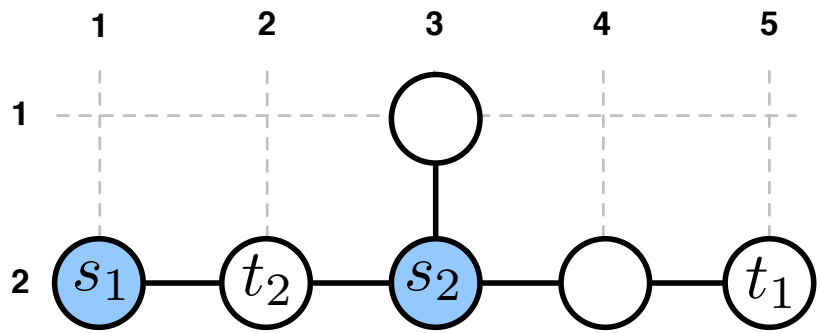
Solution quality results (on game grids) for several variants of prioritised planning. As reported in [Ma *et al.*, 2019]

How Prioritised Planning Fails



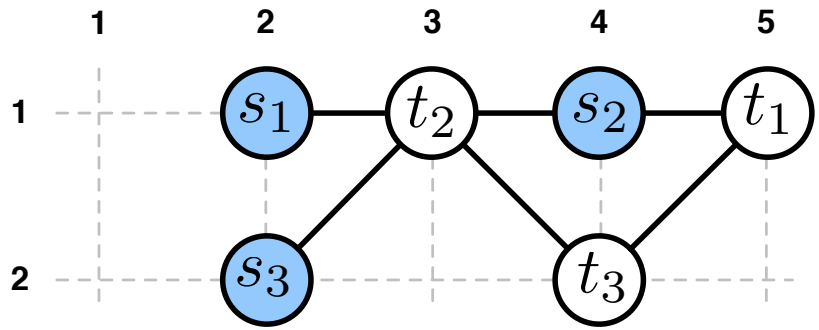
Some instances are outside the reach of prioritised planning.

How Prioritised Planning Fails



Some solvable problems only admit certain orderings

How Prioritised Planning Fails



Even if an optimal order exists, and we know it, planning can still fail

Limits of prioritised planning

Some strong restrictions:

- Not every problem is priority solvable
- Among those that are solvable, not all can be solved optimally with prioritised planning
- Even if we know an optimal plan is computable, and even if we have the optimal ordering, we might still fail due to tie-breaking

On the other hand!

- Feasible solutions are often plentiful
- Prioritised plans can be near-optimal in many practical settings
- Some prioritised planners are better than others!

Recent developments in this area

Priority-Based Search (PBS) [Ma *et al.*, 2019] is CBS-like planner that fixes priorities between two agents only in the event of a conflict. This strategy is much more effective than conventional ordering strategies.

Rolling Horizon Collision Resolution (RHCR) [Li *et al.*, 2020] is an iterated priority planner that reserves only k steps in advance (cf. the entire path). Authors report large performance gains for teams of up to 1000 agents.

Large Neighbourhood Search

Daniel D. Harabor

AAMAS 2022 Tutorial

The story so far

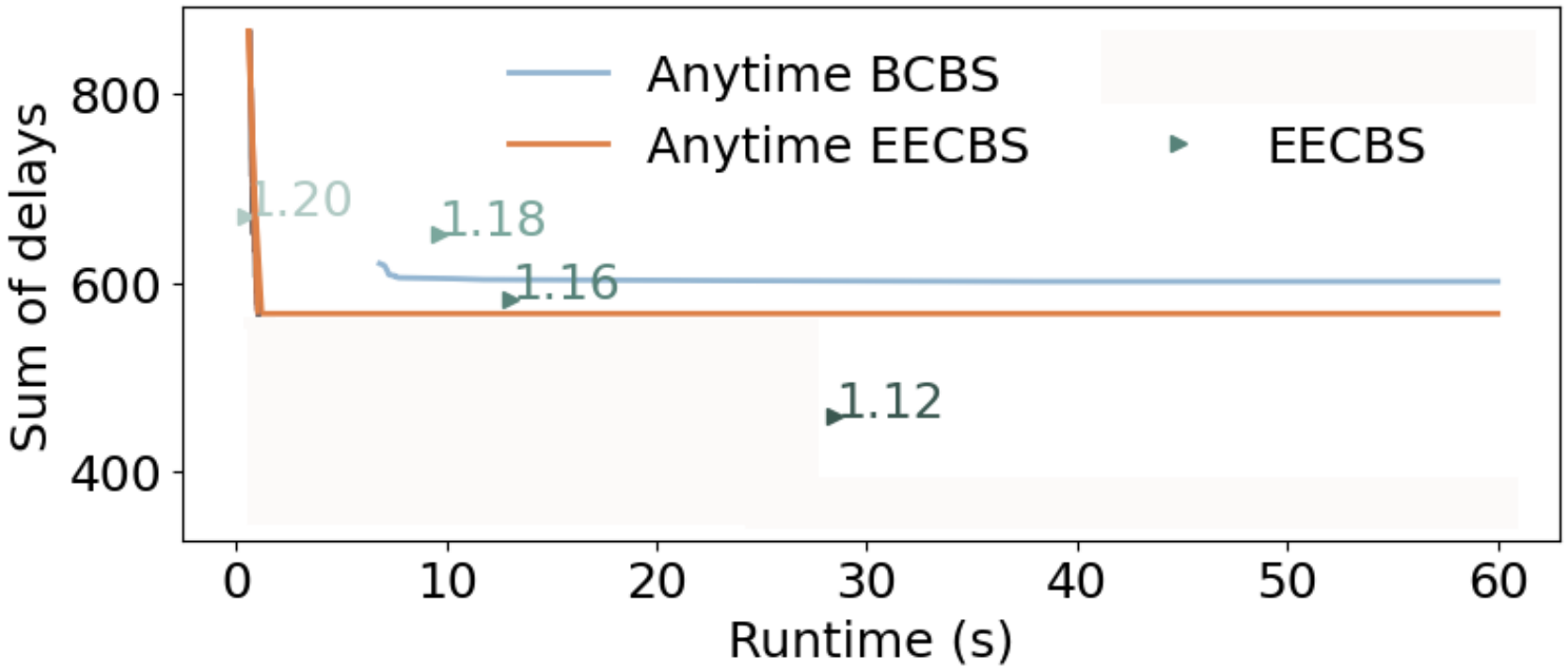
We want MAPF solvers that are fast, scalable and maximise throughput.
The approaches we have seen so far:

- Compute feasible solutions fast (e.g., PIBT)
- Compute close-to-optimal solutions more slowly (e.g., EECBS)
- Scale to many hundreds (even thousands) of agents.

But they are all strongly-coupled, all-or-nothing solvers:

- They tackle the whole problem, in one shot.
- They return a single solution, or failure.

The story so far

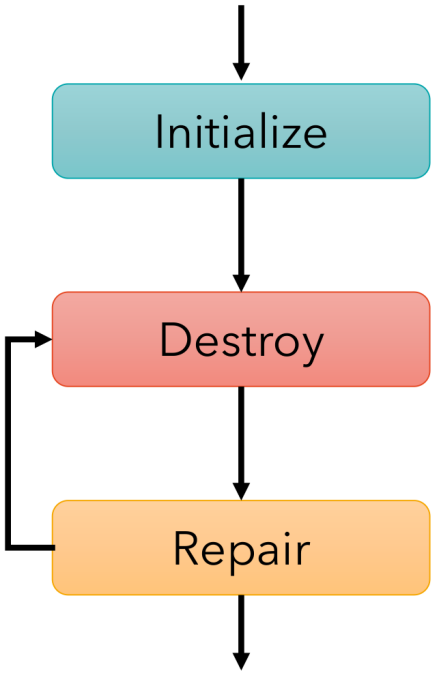


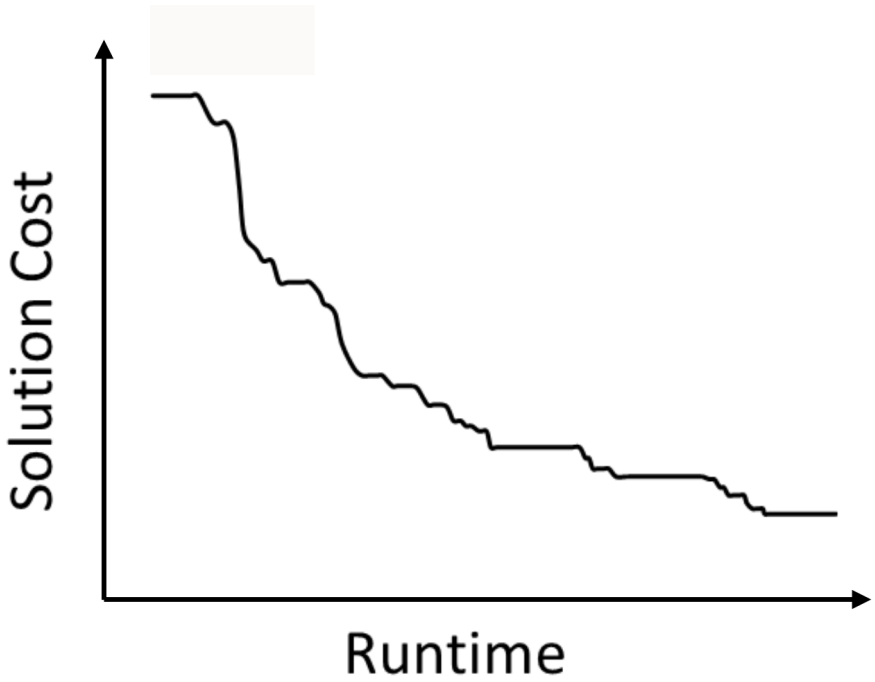
Behaviour of anytime optimal and near-optimal CBS variants
(this experiment: 150 agents on 32x32 map with 20% random obstacles)

Large Neighbourhood Search for MAPF (MAPF-LNS2)

Idea:

Solve MAPF problems incrementally; improve the solution over time.





Initialize

Assign collision-minimising paths to each agents.

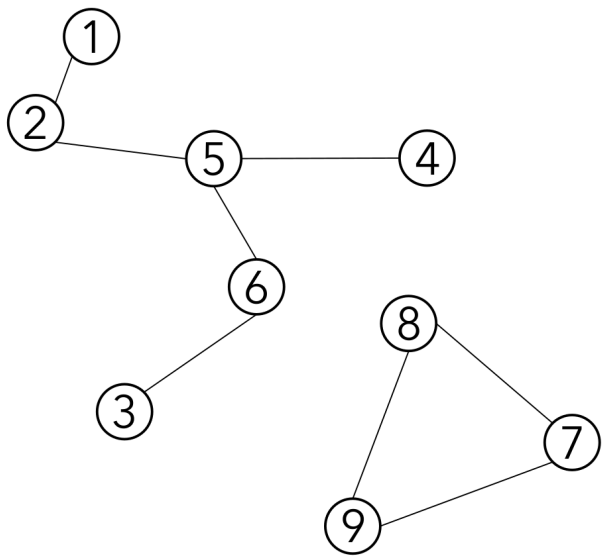
Here, we adapt a version of prioritised planning:

- Plan a path for the first agent (random order)
- Plan a path that **minimises** the number of collisions with the planned path.
- Repeat steps 1 and 2 until every agent has a path

NB: The first solution could also be computed with any other solver [Li *et al.*, 2021a].

Destroy

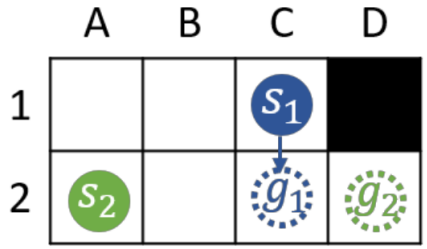
Select a subset of agents that are in collision. We switch between several different strategies that try to identify highly coupled sets of agents.

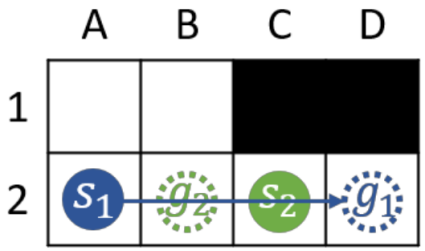


Collision neighbourhood is based on a conflict graph. We choose k agents from a random selected connected component.

Destroy

Select a subset of agents that are in collision. We switch between several different strategies that try to identify highly coupled sets of agents.

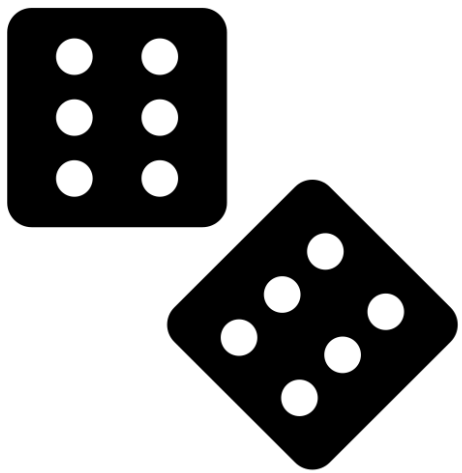




Failure neighbourhood. We choose an in-collsion agent a_i and then select other agents that prevent a_i from completing its plan, collision-free.

Destroy

Select a subset of agents that are in collision. We switch between several different strategies that try to identify highly coupled sets of agents.



Random neighbourhood. We choose k agents at random.

Repair

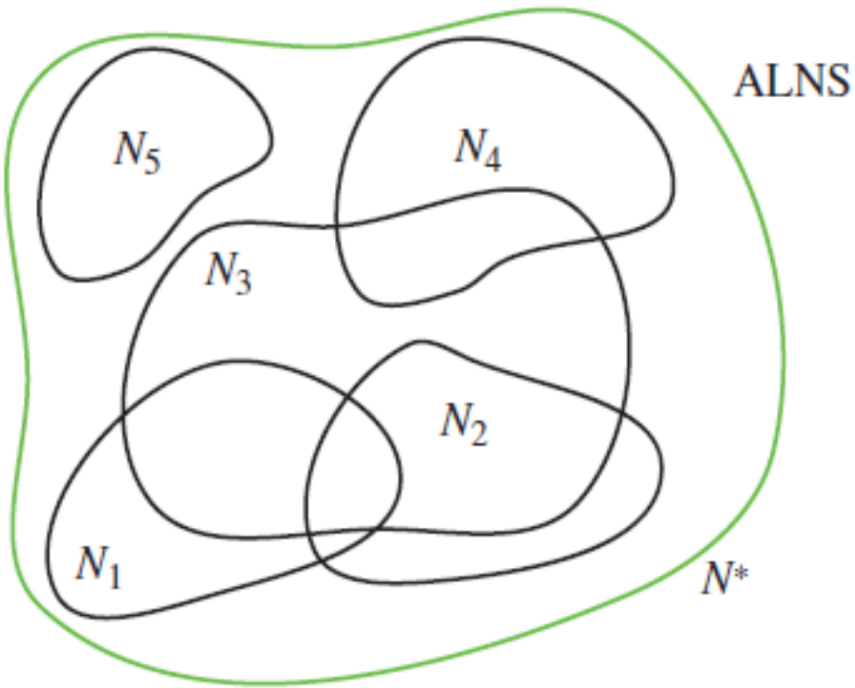
Assign new collision-minimising paths to the agents in the destroy neighbourhood.

Here, we use the same prioritised planning method as before:

- Plan a path for the first agent (random order)
- Plan a path that **minimises** the number of collisions with the planned path.
- Repeat steps 1 and 2 until every agent has a path

Adaptive Neighbourhood Selection

LNS tracks how effectively a neighbourhood improves the objective (# collisions) and switches between them dynamically.



Each neighbourhood N_i is assigned a weight w_i that tracks its recent success: in reducing the number of collisions. We select N_i with probability $\frac{w_i}{\sum_j w_j}$.

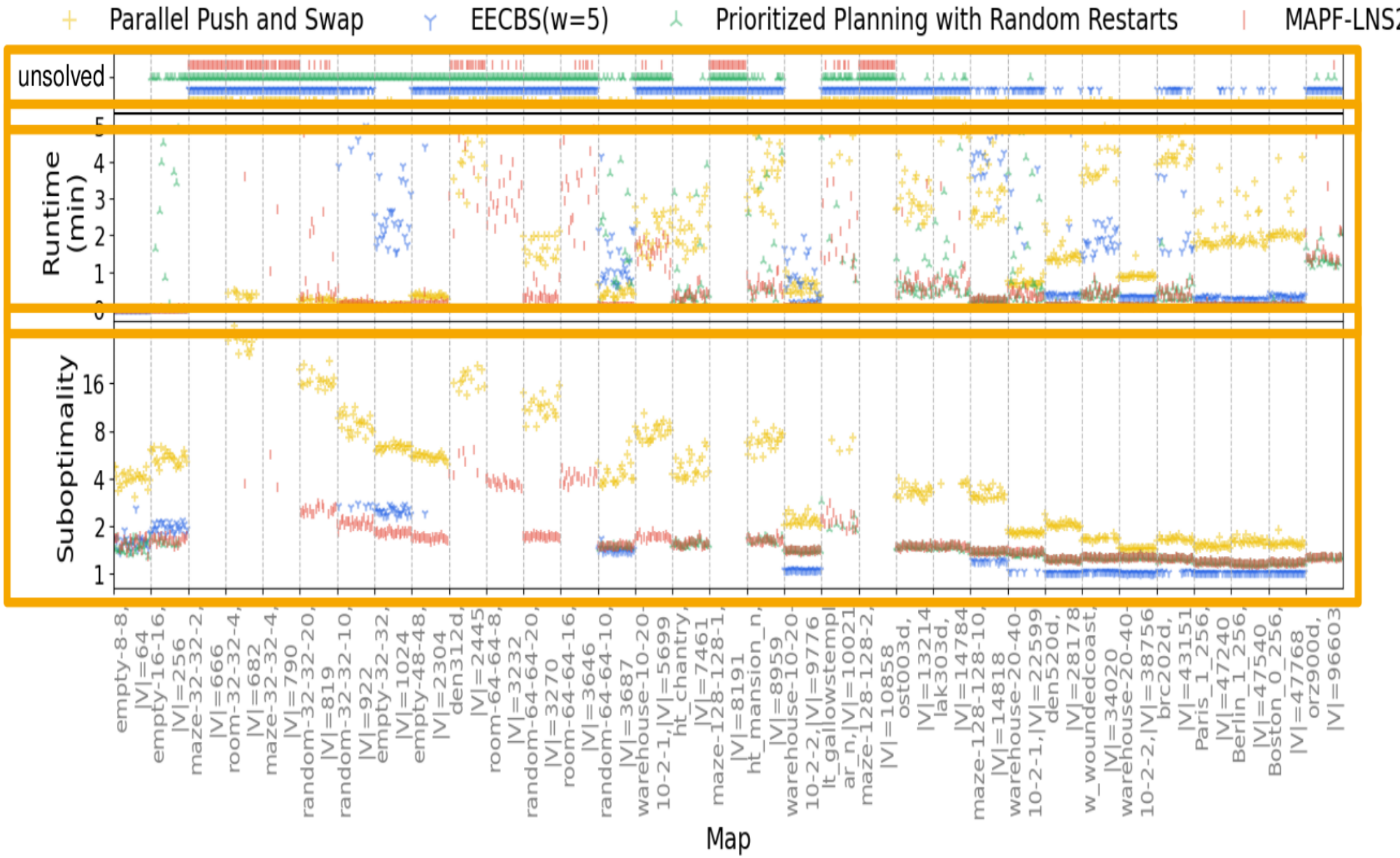
Experiments

We compare MAPF-LNS2 with a variety of suboptimal solvers:

- Prioritized Planning (random restarts)
- Parallel Push and Swap (rule-based solver)
- EECBS (boundes suboptimal)

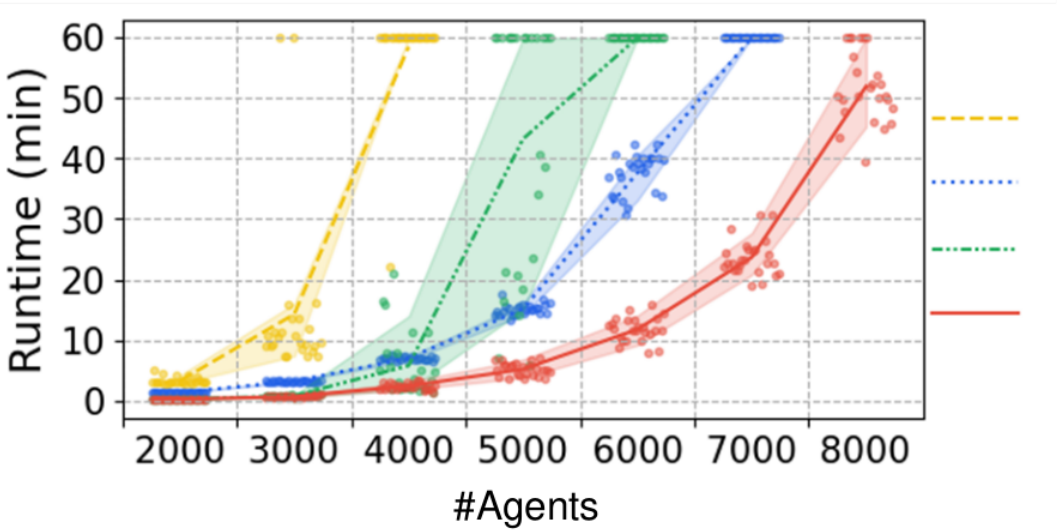
- 33 maps from a standard (MovingAI) benchmarks [Stern *et al.*, 2019].
- Small maps (up to 50% agent density)
- Large maps (up to 1000 agents)

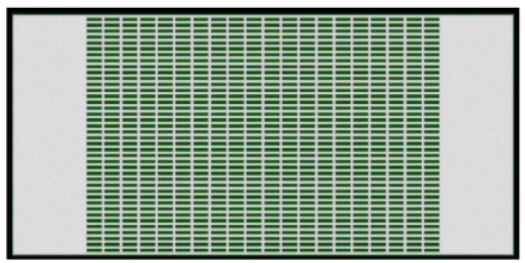
Results


Parallel Push and Swap
EECBS(w=5)
Prioritized Planning with Random Restarts
MAPF-LNS
unsolved
Runtime (min)
Suboptimality
Map

Results

In this experiment we test scalability on a **single warehouse map**

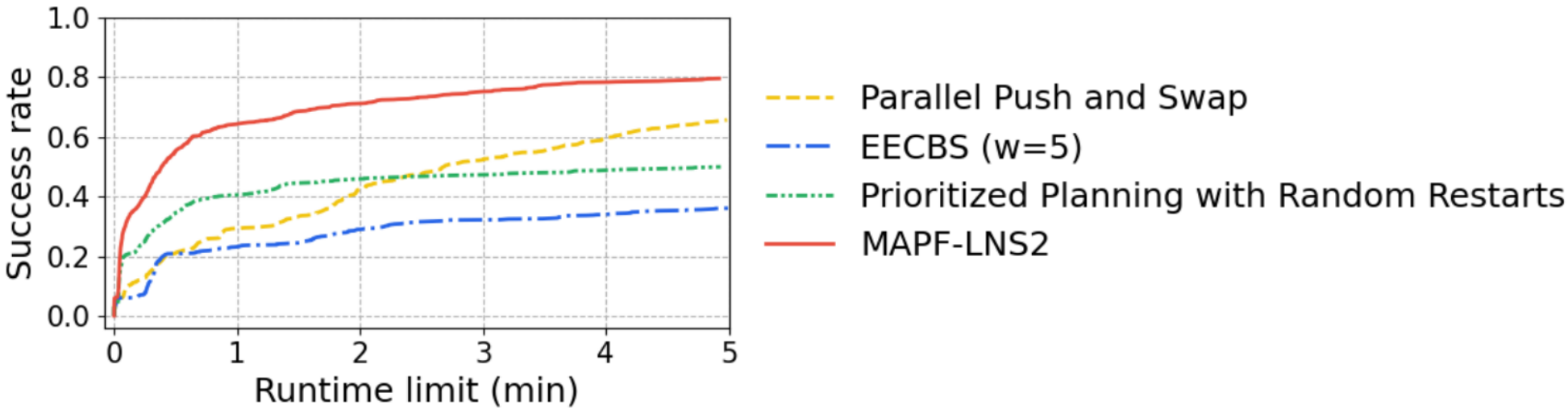




warehouse-20-40-10-2-2.map
340x164 grid with 38,756 empty cells

Results

In this experiment we look at success rate **across all 33 maps**



Recent developments in this area

LNS is a workhorse meta-heuristic in Operations Research [Shaw, 1998]. Effectiveness depends strongly on good strategies for **destroy** and **repair**

- MAPF-LNS first appears in [Li *et al.*, 2021a]. This version starts from an initially feasible plan and iteratively improves.
- Later work (MAPF-LNS2) [Li *et al.*, 2022] improves performance and adds support for infeasible first solutions.
- Other recent work considers how ML can be used to derive new types of destroy heuristics [Huang *et al.*, 2022].

Variants of MAPF-LNS obtained **first place** at the 2020 and 2021 NeurIPS Flatland Challenge [Li *et al.*, 2021b].