Практика 2

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Содержание

1 Гамма фунция

$$\int_0^{+\infty} t^{x-1}e^{-t}dt = \Gamma(x) \quad ,x > 0$$

$$\Gamma(n+1) = n!$$

$$\int_0^1 t^{x-1}(1-t)^{y-1}dt = B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \approx \frac{1}{\binom{x+y}{x}}$$

Формула понижения:

$$\Gamma(x+1) = x\cdot\Gamma(x)$$

Формула дополнения:

$$\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin \pi x}$$

$$\Gamma(\frac{1}{2}) = \sqrt{\pi}$$

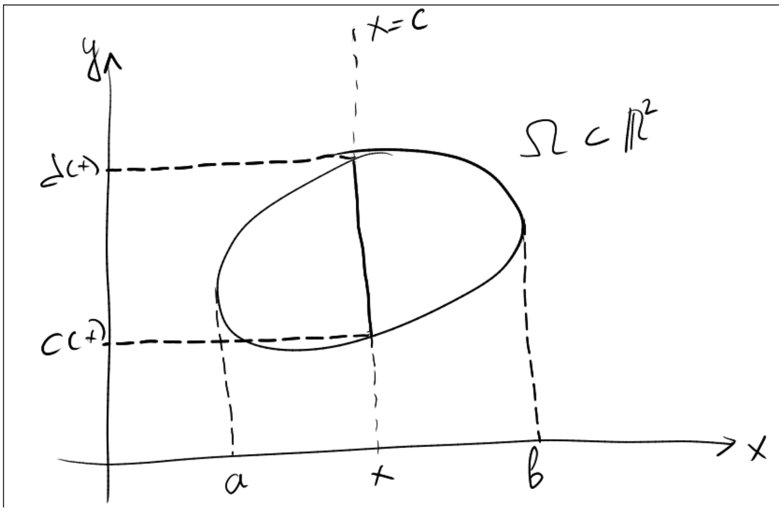
Пример.

$$\int_0^{\frac{\pi}{2}} \sin^{2021} x dx = \begin{bmatrix} t = \sin^2 x \\ dt = 2\sin x \cos x dx \\ dx = \frac{dt}{2t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}} \end{bmatrix} = \int_0^1 t^{1010,5}\cdot\frac{1}{2}t^{-\frac{1}{2}}(1-t)^{-\frac{1}{2}}dt =$$

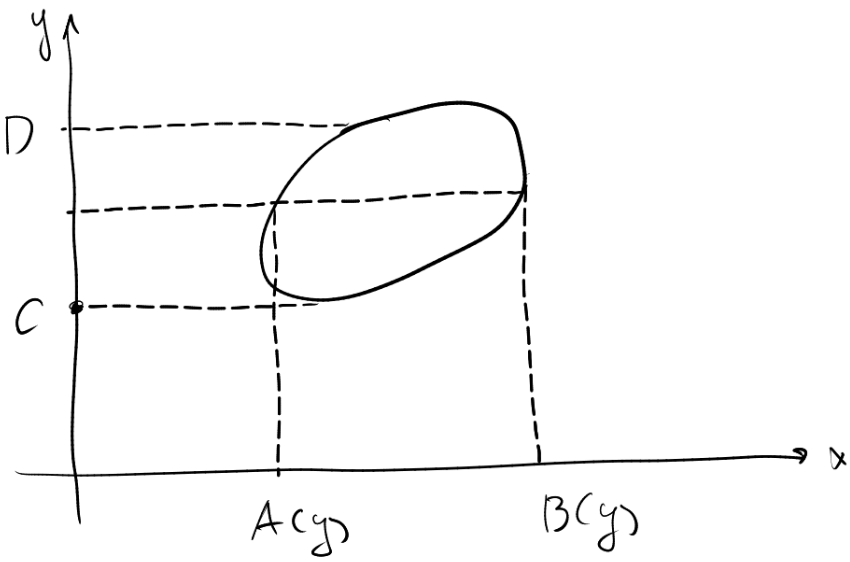
$$= \frac{1}{2}\int_0^1 t^{1010}(1-t)^{-\frac{1}{2}}dt = \frac{1}{2}B(1011,\frac{1}{2}) = \frac{1}{2}\frac{\Gamma(1011)\Gamma(\frac{1}{2})}{\Gamma(1011+\frac{1}{2})} =$$

$$= \frac{1}{2}\frac{1010!\sqrt{\pi}}{\frac{2021}{2}\cdot\frac{2019}{2}\cdot\ldots\cdot\frac{1}{2}\cdot\sqrt{\pi}} = \frac{1010!\cdot 2^{1009?}}{2021!!}$$

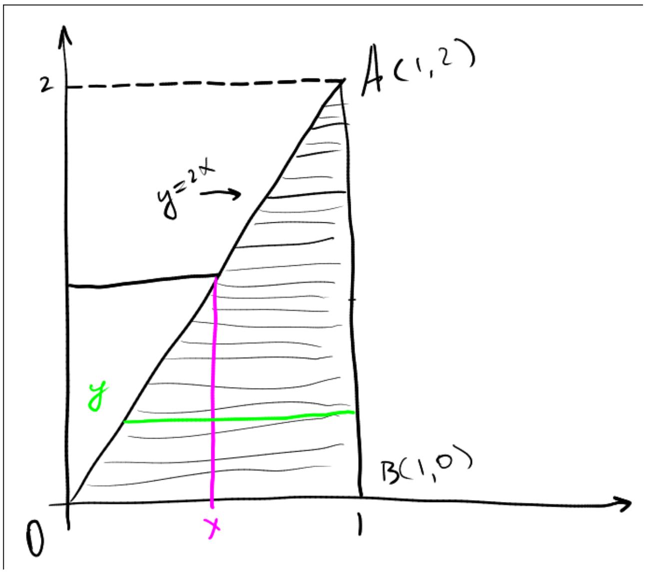
2 Кратные интегралы



$$\iint f dxdy = \int_a^b \left(\int_{c(x)}^{d(x)} f(x,y)dy \right) dx =$$
$$= \int_C^D \left(\int_{A(y)}^{B(y)} f(x,y)dx \right) dy$$



Пример.



$$\iint_{\triangle AOB} f dxdy = \int_0^1 dx \int_0^{2x} f dy =$$

$$= \int_0^2 dy \int_{\frac{y}{2}}^1 f dx$$

2.1 Изменение порядка интегрирования

$$\int_a^b dx \int_{c(x)}^{d(x)} f dy$$

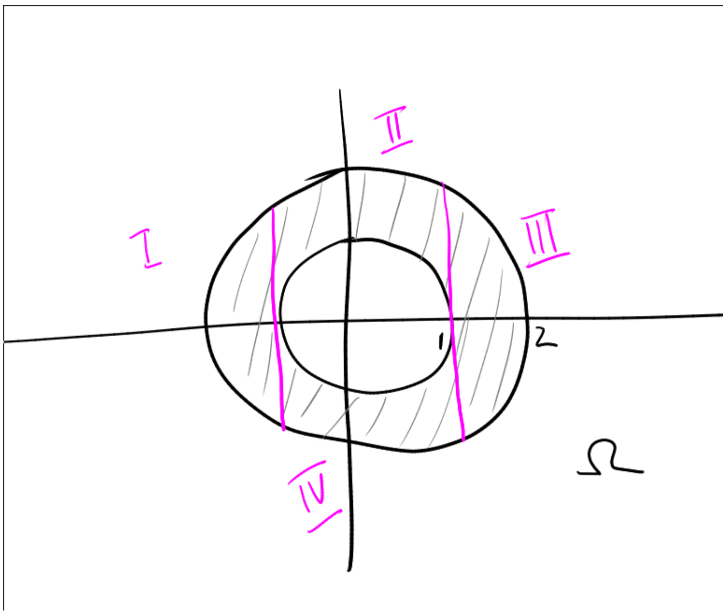
Пример.

$$\iint_{x^2+y^2\le y} f dxdy$$

- $x^2 + (y - \frac{1}{2})^2 \le \frac{1}{4}$ — окружность радиуса $\frac{1}{2}$ с центром в $(0, \frac{1}{2})$

Пример.

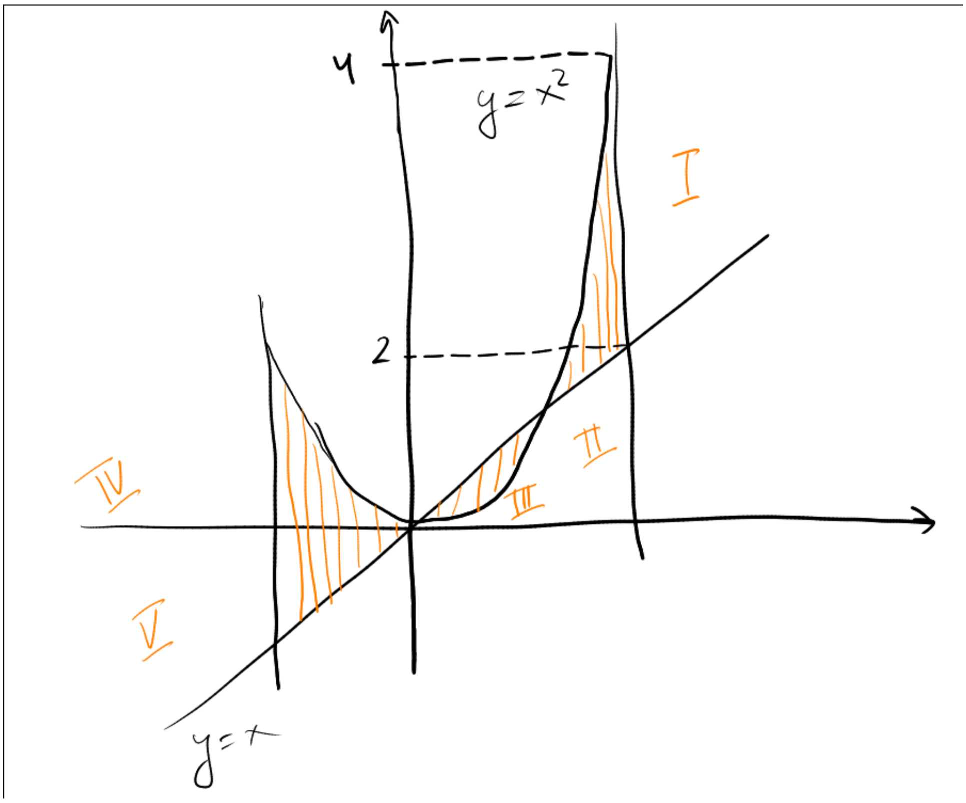
$$\iint_\Omega f dxdy$$



$$I = \int_{-2}^{-1} dx \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dy$$

$$II = \int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{4-x^2}} f dy$$

Пример.



$$\int_{-1}^{2} dx \int_{x}^{x^2} f dy = \int_{2}^{4} dy \int_{\sqrt{y}}^{2} f dx + \int_{1}^{2} dy \int_{\sqrt{y}}^{y} f dx - \int_{0}^{1} dy \int_{y}^{\sqrt{y}} f dx + \int_{0}^{1} dy \int_{-1}^{-\sqrt{y}} f dx + \int_{-1}^{0} dy \int_{-1}^{y} f dx$$