Практика 3

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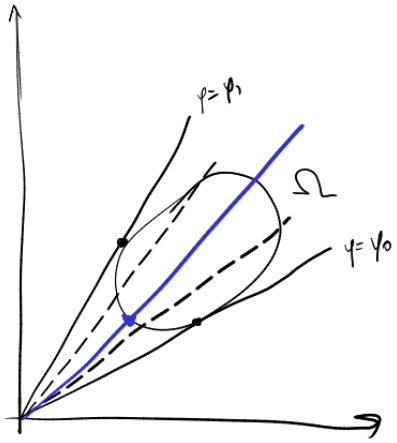
Содержание

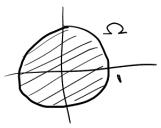
1 Полярные координаты

- $x = r\cos\varphi$
- $y = r\sin\varphi$

$$\iint_\Omega f dxdy = \int_{\varphi_0}^{\varphi_1} d\varphi \int_{r_0(\varphi)}^{r_1(\varphi)} f(r\cos\varphi, r\sin\varphi)\cdot J\ dr = \int_{r_0}^{r_1} dr \int_{\varphi_0 r}^{\varphi_1(r)} f\cdot J\ d\varphi$$

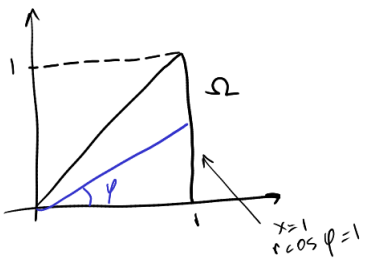


Задача 1.



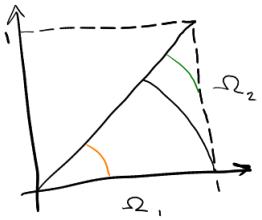
$$\iint_\Omega f dxdy = \int_0^{2\pi} d\varphi \int_0^1 f\cdot J\ dr$$

Задача 2.



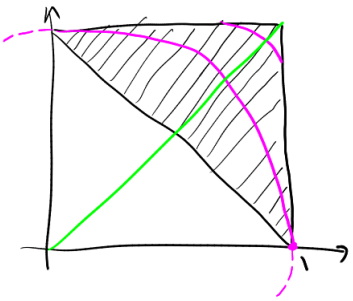
$$\iint_\Omega f dxdy = \int_0^{\frac{\pi}{4}} d\varphi \int_0^{\frac{1}{\cos\varphi}} dr\ f \cdot J = \int dr \int d\varphi\ f \cdot J$$

Задача 3.



$$\iint_\Omega f dxdy = \int_0^1 dr \int_0^{\frac{\pi}{4}} d\varphi\ f \cdot J + \int_1^{\sqrt{2}} dr \int_{\arccos\frac{1}{2}}^{\frac{\pi}{4}}$$

Задача 4.



$$\iint_\Omega = \int_0^{\frac{\pi}{4}} d\varphi \int_{\frac{1}{\cos\varphi+\sin\varphi}}^{\frac{1}{\cos\varphi}} dr \cdots + \int_{\frac{\pi}{4}}^{\frac{pi}{2}} d\varphi \int_{\frac{1}{\cos\varphi+\sin\varphi}}^{\frac{1}{\sin\varphi}} dr \cdots =$$

$$= \int_{\frac{\sqrt{2}}{2}}^1 dr \int$$

Задача 5.

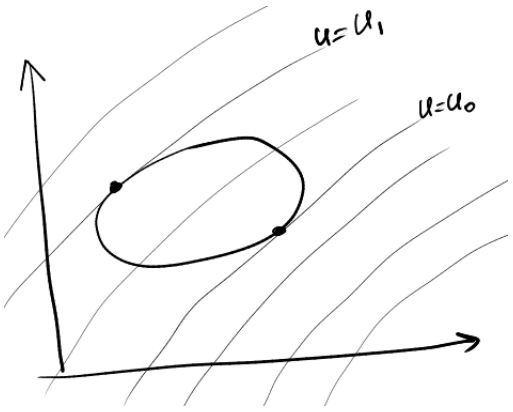
$$x^2 + y^2 \le \alpha x$$

$$r = \alpha \cos\varphi$$

$$\iint_\Omega = \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} d\varphi \int_0^{\alpha\cos\varphi} f \cdot J\ dr = \int_0^\alpha dr \int_{-\arccos\frac{\alpha}{2}}^{\arccos\frac{\alpha}{2}} f \cdot J d\varphi$$

2 Замена переменных

- $x = x(u, v)$
- $y = y(u, v)$



Фиксируем $u = u_0$:

$$\begin{cases} x = x(u_0, v) \\ y = y(u_0, v) \end{cases}$$

$$\iint_\Omega f dx dy = \int_{u_0}^{u_1} du \int_{v_0(u)}^{v_1(u)} f(x(u,v), y(u,v)) \left(\det \begin{pmatrix} x'_u & x'_v \\ y'_u & y'_v \end{pmatrix} \right) dv$$

$$\int f(x) dx = \int f(x(t)) x' dt$$