Практика 4

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17 марта 2021 г.

Содержание

1 Практика 4

- $x = ar\cos^\alpha\varphi$
- $y = br\sin^\alpha\varphi$

$$J = \alpha abr\cos^{\alpha-1}\varphi\sin^{\alpha-1}\varphi \tag{1}$$
$$\frac{y}{x} = \text{const}\cdot\operatorname{tg}^\alpha\varphi$$

Задача 1.

$$\frac{x^3}{a^3}+\frac{y^3}{b^3}=\frac{x^2}{h^2}+\frac{y^2}{k^2}\quad x=0,y=0$$

Решение.

$$\alpha=\frac{2}{3}\ r=\frac{a^2}{h^2}\cos^{\frac{4}{3}}\varphi+\frac{b^2}{k^2}\sin^{\frac{4}{3}}\varphi$$

Из 1 $\alpha=\frac{2}{3}$

$$\lambda\Omega=\iint_\Omega 1dxdy=\int_0^{\frac{\pi}{2}}du\int_0^{r(u)}\frac{2}{3}abr\cos^{\alpha-1}\varphi\sin^{\alpha-1}\varphi dr=$$
$$=\frac{1}{3}ab\int_0^{\frac{\pi}{2}}\cos^{\alpha-1}\varphi\sin^{\alpha-1}\varphi\left(\frac{a^2}{h^2}\cos^{\frac{4}{3}}\varphi+\frac{b^2}{k^2}\sin^{\frac{4}{3}}\varphi\right)^2d\varphi=\mathrm{I}+\mathrm{II}+\mathrm{III}$$
$$\mathrm{I}=\int_0^{\frac{\pi}{2}}\frac{a^4}{h^4}\cos^{\frac{7}{3}}\varphi\sin^{-\frac{1}{3}}\varphi d\varphi=\left|\begin{matrix}t=\sin^2\varphi\\ d\varphi=\frac{1}{2t^{\frac{1}{2}}(1-t)^{\frac{1}{2}}}\end{matrix}\right.=\frac{a^4}{b^4}\int_0^1(1-t)^{\frac{7}{6}-\frac{1}{2}}t^{-\frac{1}{6}-\frac{1}{2}}=\frac{a^4}{2b^4}B(\frac{5}{3},\frac{1}{3})$$

Задача 2.

$$(x^3+y^3)^2=x^2+y^2$$

, $x\ge 0, y\ge 0\to$ полярные координаты

Решение.

$$r^4(\cos^3\varphi+\sin^3\varphi)=1$$
$$\lambda(\Omega)=\int_0^{\frac{\pi}{2}}d\varphi\int_0^{r(\varphi)}\underset{\text{Якобиан}}{\underset{\uparrow}{r}}dr=\frac{1}{2}\int_0^{\frac{\pi}{2}}\frac{d\varphi}{\cos^3\varphi+\sin^3\varphi}=\frac{1}{2}\int_0^{+\infty}\frac{\sqrt{t^2+1}}{t^3+1}dt$$
$$\int\frac{d\varphi}{(\cos\varphi+\sin\varphi)(1-\cos\varphi\sin\varphi)}$$
$$\int\frac{dt}{\left(\frac{2t^2}{1+t^2}\right)^3+\left(\frac{2t}{1+t^2}\right)^3}\cdot\frac{2}{1+t^2}$$

Задача 3.

$$\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$$
$$\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=4$$

, $\frac{x}{a}=\frac{y}{b}$, $8\frac{x}{a}=\frac{y}{b}$

Решение.

$$x = ar\cos\varphi^3$$
$$y = br\sin^3\varphi$$

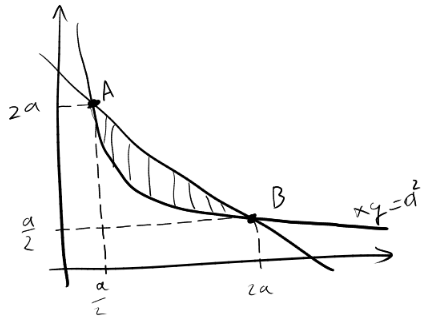
- $r = 1, r = 8$
- $\varphi = 1, \varphi = \operatorname{arctg} 2$

$$\lambda(\Omega) = \int_1^8 dr \int_1^{\operatorname{arctg} 2} 3abr\cos^2\varphi\sin^2\varphi d\varphi = \frac{3}{2}abr^2 8_1 \cdot \underbrace{\int_1^{\operatorname{arctg} 2} \sin^2 2\varphi d\varphi}_{\frac{1-\cos 4\varphi}{2}}$$

Задача 4.

$$xy = a^2$$
$$x + y = \frac{5}{2}$$

Решение.



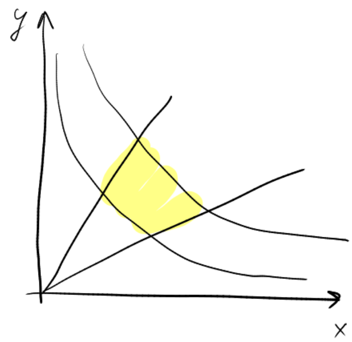
$$\int_{\frac{a}{2}}^{2a} \int_{\frac{a^2}{x}}^{\frac{5}{2}a - x} dy = q^2\frac{15}{8} - 2q^2\ln 2$$
$$= \int_{\frac{a}{2}}^{2a} \frac{5}{2}a - x - \frac{a^2}{x}dx = \frac{5}{2}a \cdot \frac{3}{2}a - \frac{x^2}{2}\bigg|_{\frac{a}{2}}^{2a} - a^2\ln x\bigg|_{\frac{a}{2}}^{2a}$$

Задача 5.

$$xy = a^2$$
$$xy = 2a^2$$
$$y = x$$
$$y = 2x$$

, $x, y > 0$

Решение.



$$u = xy,\ v = \frac{y}{x}$$

$$x = \sqrt{\frac{u}{v}}$$

$$y = \sqrt{uv}$$

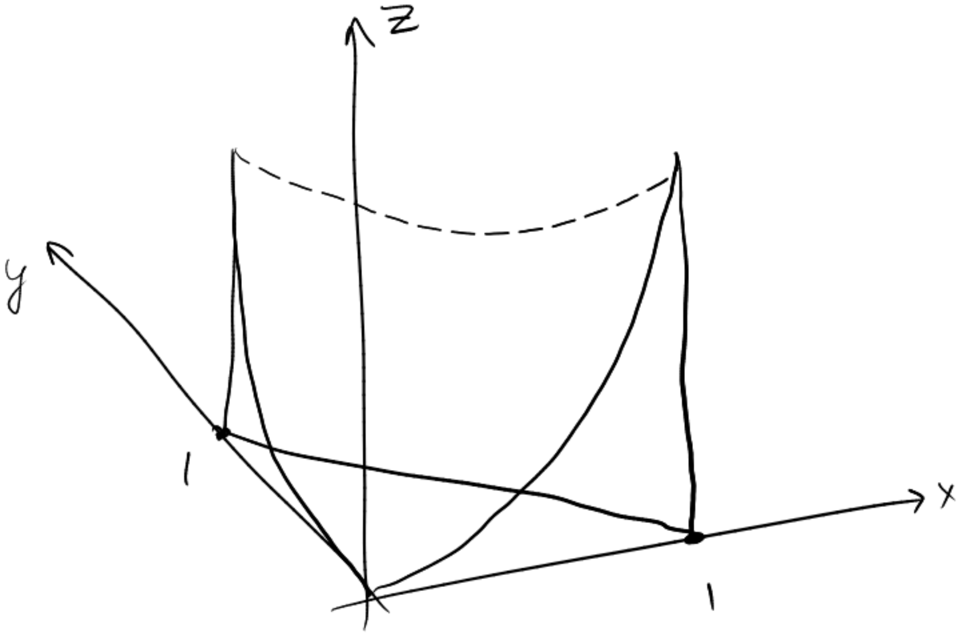
$$J = \begin{vmatrix} \frac{1}{2\sqrt{uv}} & -\frac{1}{2}\frac{\sqrt{u}}{\sqrt{v}} \\ \frac{1}{2}\frac{\sqrt{v}}{\sqrt{u}} & \frac{1}{2}\frac{\sqrt{u}}{\sqrt{v}} \end{vmatrix} = \frac{1}{2v}$$

$$\lambda(\Omega) = \int_{a^2}^{2a^2} du \int_1^2 \frac{1}{2v} dv = a^2 \int_1^2 \frac{1}{2v} dv = \frac{a^2}{2} \ln 2$$

Задача 6.

$$V = \int_0^1 dx \int_0^{1-x} x^2 + y^2 dy$$

Определим что эта за фигура в плоскости xOy

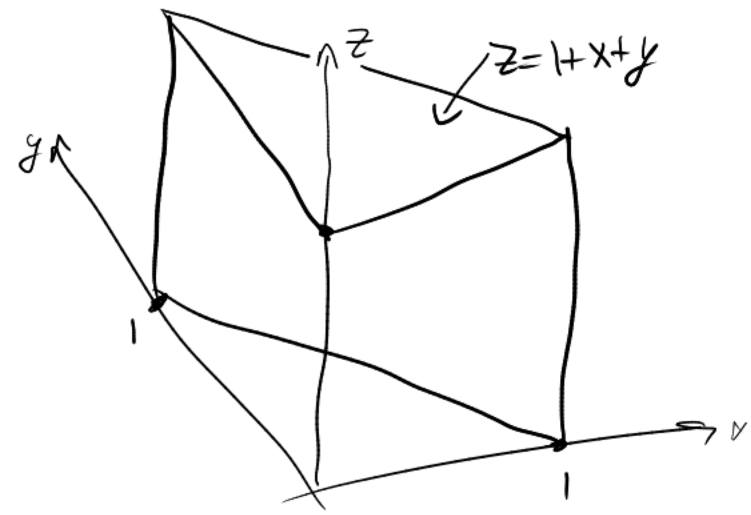


Задача 7.

$$\left[\begin{array}{l} z = 1 + x + y \\ z = 0 \end{array}\right.$$

$$\left[\begin{array}{l} x + y = 1 \\ x = 0 \\ y = 0 \end{array}\right.$$

Цилинрическая форма

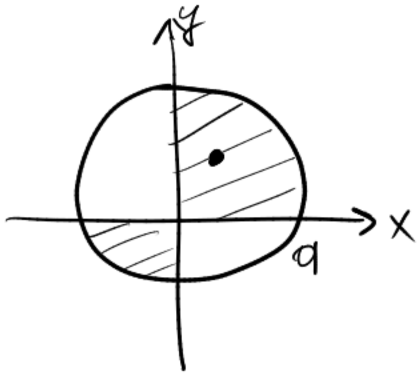


Решение.

$$\lambda(\Omega) = \int_0^1 dx \int_0^{1-x} 1 + x + ydy = \int_0^1 (1+x)(1-x) + \frac{y^2}{2} 1 - x_0 dx$$

Задача 8.

$$\left[\begin{array}{l} z^2 = xy \\ x^2 + y^2 = a^2 \end{array} \right.$$



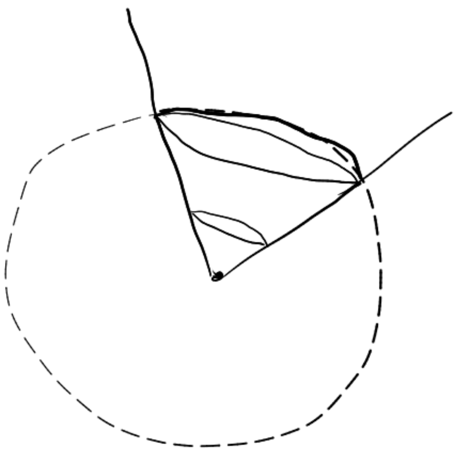
Решение.

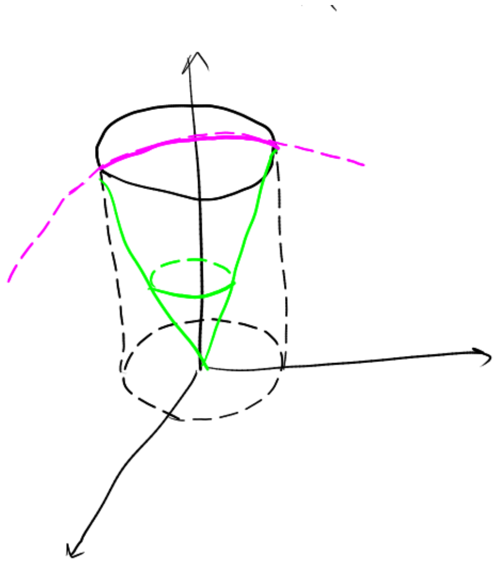
$$\lambda(\Omega) = 4 \iint \sqrt{xy} dxdy = \int_0^{\frac{\pi}{2}} d\varphi \int_0^a r^2 \sqrt{\cos\varphi \sin\varphi} dr = \frac{a^3}{3} \int_0^{\frac{\pi}{2}} \cos^{\frac{1}{2}} \varphi \sin^{\frac{1}{2}} \varphi d\varphi$$

Задача 9.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$





Решение.

$$V = \iint z_2(x,y) - z_1(x,y) \tag{2}$$

$$\frac{2z^2}{c^2} = 1$$

$$z = \frac{c}{\sqrt{2}}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$$

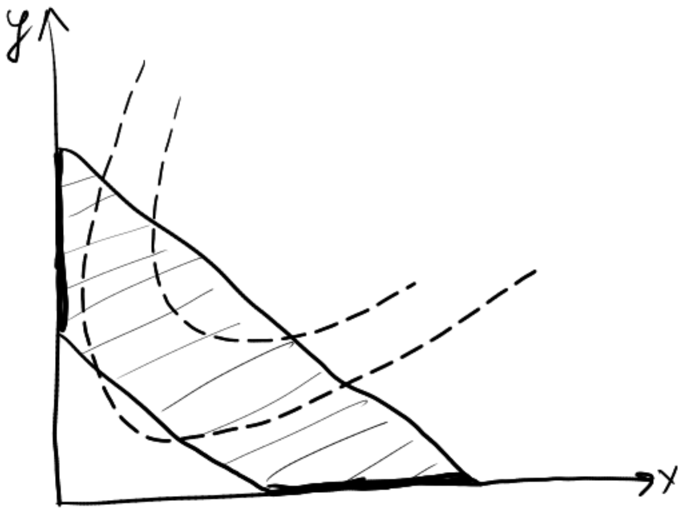
$$2 = \left| \begin{array}{l} x = ar\cos\varphi \\ y = br\sin \end{array} \right. = \int_0^{2\pi} d\varphi \int_0^{\frac{1}{\sqrt{2}}} abr(-c^2 \cdot r^2 + c^2(1-r^2)) = 2\pi \int_0^{\frac{1}{\sqrt{2}}}$$

Задача 10.

$$z^2 = xy$$

$$x + y = a$$

$$x + y = b$$



Решение.

$$2\int_a^b dx \int_0^{a-x} \sqrt{xy}dy + \int_0^a dx \int_{a-x}^{b-x} \sqrt{xy}dy$$