

# Практика 8

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## Содержание

Задача 1 (4089).

$$z = x^2 + y^2$$

Решение.

$$r \sin \psi = r^2 \cos^2 \psi \Leftrightarrow r = \frac{\sin \psi}{\cos^2 \psi}$$

$$x = y \Leftrightarrow \varphi = \frac{\pi}{4}$$

$$\cos \varphi = \sin \phi$$

- $x = 1, r \cos \varphi \cos \psi = 1$
- $r = 0, \psi = 0$
- $y = 0, r \sin \varphi \cos \psi = 0, \phi = 0$

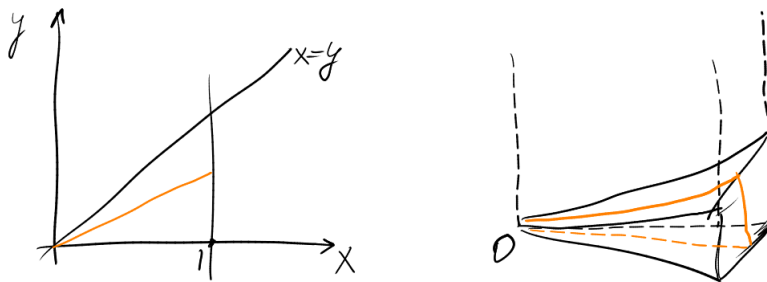


Рис. 1: Фиксируем  $\varphi$

$$\int_0^{\frac{\pi}{4}} d\varphi \int_0^{\arctg \frac{1}{\cos \phi}} d\psi \int_{\frac{\sin \psi}{\cos^2 \psi}}^{\frac{1}{\cos \phi \cos \psi}} f(r) r^2 \cos \psi dr$$

Переходим в полярные координаты в плоскости  $xOy$ :

- $x = r \cos \varphi$
- $y = r \sin \varphi$
- $z = z$

$$J = r$$

Задача 2 (4118.2).

$$\sqrt[3]{\frac{x}{a}} + \sqrt[3]{\frac{y}{b}} + \sqrt[3]{\frac{z}{c}} = 1 \quad x, y, z \geq 0$$

Решение.

- $x = ar \cos^6 \varphi \cos^6 \psi$

- $y = br \sin^6 \varphi \cos^6 \psi$
- $z = cr \cos^6 \psi$

$$r^{\frac{1}{3}} = 1$$

$$V = \iiint 1 \, dx \, dy \, dz = \int_0^1 dr \int_0^{\frac{\pi}{2}} d\varphi \int_0^{\frac{\pi}{2}} ab \cdot 36 \cdot \cos^5 \phi \sin^5 \phi \cos^{11} \psi \sin^5 \psi$$

**Задача 3** (4105).

$$az = a^2 - x^2 - y^2$$

$$z = a - x - y$$

$$x = 0 \quad y = 0 \quad z = 0 \quad a > 0$$

*Решение.*



$$V = \iiint 1 \, dx \, dy \, dz = \iint dx \, dy \int_0^{(a^2 - x^2 - y^2)} 1 \, dz - \iint dy \int_0^{a-x} 1 \, dz =$$

$$= \int_0^a dr \int_0^{\frac{\pi}{2}} d\varphi \frac{a^2 - r^2}{a} \cdot r - \int_0^a dx \int_0^{a-x} (a - x - y) \, dy$$

**Задача 4** (4108).

$$(x^2 + y^2 + z^2)^2 = a^2(x^2 + y^2 - z^2)$$

*Решение.*

$$r^4 = a^2(\cos^2 \psi - \sin^2 \psi)r^2$$

$$r = a\sqrt{\cos(2\psi)}$$

$$V = \iiint_{\Omega} 1 \, dx \, dy \, dz = 2 \int_0^{\frac{\pi}{2}} d\psi \int_0^{\sqrt{\cos(2\psi)}} dr \int_0^{2\pi} a^3 \cos \psi \cdot r^2 d\varphi =$$

$$= \frac{4\pi}{3} \int_0^{\frac{\pi}{4}} d\psi \cos \psi \cos^{\frac{3}{2}}(2\psi)$$

**Задача 5** (4121).

$$(x^2 + y^2 + z^2)^3 = \frac{a^6 z^2}{x^2 + y^2}$$

*Решение.*

- $x = ar \cos \varphi \cos \psi$
- $y = ar \sin \varphi \cos \psi$
- $z = ar \sin \psi$

$$r^6 = \frac{\sin^2 \psi}{\cos^2 \psi}$$

$$r = \sqrt[3]{\operatorname{tg} \psi}$$

$$V = \iiint 1 \, dx \, dy \, dz = 2 \int_0^{\frac{\pi}{2}} d\psi \int_0^{\sqrt[3]{\operatorname{tg} \psi}} dr \int_0^{2\pi} a^3 \cos \psi r^2 d\varphi =$$

$$= \int_0^{\frac{\pi}{2}} \frac{2a^3}{3} \cdot 2\pi \operatorname{tg} \psi \cos \psi d\psi = \frac{4a^3\pi}{3}$$

**Задача 6** (4124).

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = \ln \frac{\frac{x}{a} + \frac{y}{b} + \frac{z}{c}}{\frac{x}{a} + \frac{y}{b}}$$

$$x = 0, \, z = 0, \, \frac{y}{b} + \frac{z}{c} = 0, \, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

*Решение.*

- $u = \frac{x}{a}$
- $v = \frac{x}{a} + \frac{y}{b}$
- $w = \frac{x}{a} + \frac{y}{b} + \frac{z}{c}$

$$J = abc$$

$$w = \ln \frac{w}{v}, \, u = 0, \, w - u = 0, \, w = 1, \, z = 0 \rightsquigarrow w - v = 0$$

$$w = \ln w - \ln v \Rightarrow v = e^{\ln w - w} = \frac{w}{e^w}$$

$$V = abc \int_0^1 dw \int_0^w du \int_{we^{-w}}^w 1 dv$$

**Задача 7** (4012).

*Решение.* Доделать

**Задача 8** (4115).

*Решение.* Доделать

**Задача 9** (4120).

*Решение.* Доделать

**Задача 10** (4123).

*Решение.* Доделать

**Задача 11** (4129).

*Решение.* Доделать