RIGA TECHNICAL UNIVERSITY

TELECOMMUNICATIONS THEORY

Course Project

Tasks and guidelines

Course project individual parameters for course:
«Telecommunications Theory»

No	Name, Surname	Source No	Symbol duration, ns	Signal to Noise Ratio, dB	Source code	Error correction code	Error correction code parameters	Carrier Frequency, GHz	Modulation	BFSK mod. index	BPSK phase shift
1.	Elyorbek Akhmedov	1	120	10.1	Huffman	Group, systematic	$m = 22$	0.5	BFSK	3.0	–
2.	Ruveyda Ceren Gokce	2	110	11.5	Shannon-Fano	Group, systematic	$m = 26$	0.5	BPSK	–	π
3.	Endri Goxhaj	3	110	8.2	Huffman	Cyclic	$m = 15$ $z^4 \oplus z^2 \oplus 1$	2.5	BPSK	–	π
4.	Samyukdha Madhavan	4	10	8.9	Shannon-Fano	Group, non-systematic	$m = 27$	1.0	BASK	–	–
5.	Salvador Moreno Carrillo	5	80	10.8	Shannon-Fano	Group, systematic	$m = 25$	2.0	BPSK	–	π
6.	Hamza Ahmad Abdelmuhsen Tawayha	6	150	11.3	Shannon-Fano	Group, non-systematic	$m = 24$	2.5	BFSK	4.0	–
7.	Alessandro Trigolo	7	60	8.1	Shannon-Fano	Cyclic	$m = 31$ $z^5 \oplus z^2 \oplus 1$	2.5	BPSK	–	π
8.		8	130	11.4	Shannon-Fano	Cyclic	$m = 31$ $z^5 \oplus z^3 \oplus 1$	0.5	BFSK	2.0	–
9.		9	140	11.9	Huffman	Group, systematic	$m = 29$	1.0	BPSK	–	π
10.	Chen Tianhua	10	150	10.2	Shannon-Fano	Group, non-systematic	$m = 30$	2.0	BPSK	–	π
11.	*Elans Grabs (Examples)*	*11*	*60*	*12.0*	*Huffman Shannon-Fano*	*Cyclic, Group (syst. and non-syst)*	$m = 7$ $z^3 \oplus z \oplus 1$	*2.0*	*BASK BFSK BPSK*	*4.0*	π

The tables of source probability distribution for Huffman or Shannon-Fano source coding and calculations in the course project.

Source No. 1		**Source No. 2**		**Source No. 3**		**Source No. 4**		**Source No. 5**		**Source No. 6**	
Sym.	Prob.	Sym.	Prob.	Sym.	Prob.	Sym.	Prob.	Sym.	Prob.	Sym.	Prob.
a_1	0.16	a_1	0.03	a_1	0.05	a_1	0.18	a_1	0.15	a_1	0.22
a_2	0.11	a_2	0.03	a_2	0.04	a_2	0.02	a_2	0.19	a_2	0.07
a_3	0.04	a_3	0.08	a_3	0.12	a_3	0.02	a_3	0.16	a_3	0.03
a_4	0.04	a_4	0.17	a_4	0.08	a_4	0.13	a_4	0.03	a_4	0.18
a_5	0.17	a_5	0.06	a_5	0.06	a_5	0.07	a_5	0.06	a_5	0.05
a_6	0.04	a_6	0.07	a_6	0.15	a_6	0.17	a_6	0.09	a_6	0.09
a_7	0.17	a_7	0.14	a_7	0.14	a_7	0.01	a_7	0.02	a_7	0.06
a_8	0.04	a_8	0.11	a_8	0.02	a_8	0.2	a_8	0.1	a_8	0.02
a_9	0.04	a_9	0.16	a_9	0.07	a_9	0.02	a_9	0.12	a_9	0.11
a_{10}	0.18	a_{10}	0.15	a_{10}	0.13	a_{10}	0.18	a_{10}	0.08	a_{10}	0.17
a_{11}	0.01	a_{11}		a_{11}	0.01	a_{11}		a_{11}		a_{11}	
a_{12}		a_{12}		a_{12}	0.13	a_{12}		a_{12}		a_{12}	

Source No. 7		**Source No. 8**		**Source No. 9**		**Source No. 10**		**Source No. 11**		**Source No. 12**	
Sym.	Prob.	Sym.	Prob.	Sym.	Prob.	Sym.	Prob.	Sym.	Prob.	Sym.	Prob.
a_1	0.11	a_1	0.15	a_1	0.17	a_1	0.11	a_1	0.12	a_1	
a_2	0.07	a_2	0.14	a_2	0.07	a_2	0.01	a_2	0.14	a_2	
a_3	0.09	a_3	0.08	a_3	0.15	a_3	0.07	a_3	0.07	a_3	
a_4	0.01	a_4	0.07	a_4	0.13	a_4	0.07	a_4	0.11	a_4	
a_5	0.06	a_5	0.13	a_5	0.02	a_5	0.14	a_5	0.12	a_5	
a_6	0.06	a_6	0.05	a_6	0.15	a_6	0.08	a_6	0.06	a_6	
a_7	0.13	a_7	0.07	a_7	0.07	a_7	0.17	a_7	0.08	a_7	
a_8	0.14	a_8	0.18	a_8	0.02	a_8	0.11	a_8	0.15	a_8	
a_9	0.13	a_9	0.13	a_9	0.02	a_9	0.05	a_9	0.15	a_9	
a_{10}	0.05	a_{10}		a_{10}	0.02	a_{10}	0.19	a_{10}		a_{10}	
a_{11}	0.11	a_{11}		a_{11}	0.12	a_{11}		a_{11}		a_{11}	
a_{12}	0.04	a_{12}		a_{12}	0.06	a_{12}		a_{12}		a_{12}	

Course Project Tasks

The course project must be formatted as a single report, which contains solutions, program code and plots for the following tasks:

1. Draw a diagram of digital information transmission system and describe with 1-2 sentences the purpose of each component.

2. Prepare a calculation program (Matlab, Python, C, etc.) for analysis of data source. Based on specified probabilities of each symbol, calculate the following:
   a. The source entropy and maximum possible value;
   b. The source redundancy.

3. Perform source encoding (Shannon-Fano or Huffman method) and improve the program from task 2, by calculating the following values:
   a. Average codeword length;
   b. Probability of “1” and “0” symbol and binary entropy;
   c. Source data generation rate (after coding);
   d. Compression ratio.

4. Analyze Shannon theorem’s condition, by adding following calculations into program developed for Task 2 and Task 3:
   a. Channel capacity without noise;
   b. Bit Error Rate (BER) for specified SNR and modulation type;
   c. Channel capacity with noise.

   Make a conclusion, whether Shannon’s theorem condition is met or not, based on Your calculations.

5. Analyze error correction code with specified parameters by performing coding and decoding operations for Your data block:
   a. Encoding and writing systematic/non-systematic codeword;
   b. Decoding codeword without introduced errors;
   c. Decoding codeword after introducing a single error;
   d. Decoding codeword after introducing a double error.

   Design and add into report the schematic diagrams for encoding and decoding devices.

6. Develop a program (Matlab, Python, C, etc.) for AWGN channel simulation with Your specified modulation type and codeword length *m*. Simulate two cases:
   a. without Hamming error correction code;
   b. with Hamming error correction code.

   Obtain BER curves for each case in SNR range from 0 dB to 15 dB with a step of 0.5 dB. For uncoded simulation mark theoretical value calculated in Task 4.b. Make conclusion on used code efficiency.

7. Calculate and plot modulated signal spectrums for Your specified parameters (symbol duration, modulation type, carrier frequency. Include in Your report following plots:
   a. Magnitude (amplitude) spectrum for infinite periodic sequence “1 0 1 0 1 0 1 0 …”;
   b. Spectral Power Density (energy spectrum) for random sequence of “1” and “0”.

8. Evaluate the probability of more than 2 errors per codeword for Your specified Hamming code length and calculated BER value.

Course Project Guidelines

Task 1. Diagram of communications system.
Please, see the 1-st lecture slides/record. Only brief description for each component of system is needed: what this component does in the entire system.

Task 2. Calculation of source parameters.
Please, use the following formulas to calculate required values for Your specified source table. Here P_i is the probability of a source selecting the symbol a_i:

Source entropy	$H(A) = -\sum_{i=1}^{N} P_i \cdot \log_2 P_i$
Maximum entropy	$H_{\max}(A) = \log_2 N$, where N is the number of symbols in alphabet
Source redundancy	$\rho = 1 - \frac{H(A)}{H_{\max}(A)}$

Task 3. Calculation of source parameters.
Please, see lecture slides/record for examples on Shannon-Fano and Huffman code design procedure. The result of the code design should be presented as a table of codewords along with specified number of symbols, “1” symbols and “0” symbols:

Codeword	Length, m_i	Number of “1”, m_{1i}	Number of “0”, m_{0i}

Please, use the following formulas to calculate required values:

Average codeword length	$\bar{m} = \sum_{i=1}^{N} P_i \cdot m_i$
Average number of “1” symbols	$\bar{m}_1 = \sum_{i=1}^{N} P_i \cdot m_{1i}$
Average number of “0” symbols	$\bar{m}_0 = \sum_{i=1}^{N} P_i \cdot m_{0i}$
Probability of “1” symbol	$P_{"1"} = \frac{\bar{m}_1}{\bar{m}}$
Probability of “0” symbol	$P_{"0"} = \frac{\bar{m}_0}{\bar{m}}$
Binary entropy	$H_{bin}(A) = -P_{"0"} \cdot \log_2 P_{"0"} - P_{"1"} \cdot \log_2 P_{"1"}$

Data rate	$R = \frac{H(A)}{\bar{m}\tau}$, where τ is binary symbol duration.
Compression ratio	$K = \frac{\bar{m}}{H(A)}$

Compare $P_{"1"}$ and $P_{"0"}$, to see how close we are to 50% probability for each binary symbol.

Task 4. Channel capacity and Shannon Limit

Please, use the following formulas to calculate required values. Note, that τ is binary symbol duration specified in Your individual parameters, and You must calculate error probability for Your specified modulation type only.

Also, please note, that for P_{err} formulas $\frac{E_b}{N_0} = 10^{SNR/10}$, where *SNR* is specified in Your individual parameters in dB units.

Noiseless channel capacity	$C_{bin} = \frac{1}{\tau}$
Error probability for BASK modulation	$P_{err} = 1 - \Phi\left(\sqrt{\frac{1}{2} \cdot \frac{E_b}{N_0}}\right)$
Error probability for BFSK modulation	$P_{err} = 1 - \Phi\left(\sqrt{\frac{E_b}{N_0}}\right)$
Error probability for BPSK modulation	$P_{err} = 1 - \Phi\left(\sqrt{2 \cdot \frac{E_b}{N_0}}\right)$
Channel capacity with noise	$C_{chan} = \frac{1 + P_{err} \cdot \log_2 P_{err} + (1 - P_{err}) \cdot \log_2(1 - P_{err})}{\tau}$

Please note, that in order to calculate $\Phi(x)$ function in Octave, ERF() function can be used:

$$\Phi(x) = \frac{1}{2} \cdot \left(1 + ERF\left(\frac{x}{\sqrt{2}}\right)\right)$$

<u>Task 7. Calculation of the spectrums.</u>

Please, see corresponding lecture presentation on full derivation of the spectrum formulas. Use the following formulas to calculate spectrum for Your specified parameters.

BASK	Amplitude spectrum: $$\dot{C}_{BASK}[k] = j\frac{U}{4} \cdot \frac{\sin\left((k\Omega - \omega_0)\frac{\tau}{2}\right)}{(k\Omega - \omega_0)\frac{\tau}{2}}$$ Power spectral density: $$G_{BASK}(\omega) = 2\tau \cdot \left|\dot{C}_{BASK}(\omega)\right|^2$$

where $\dot{C}_{BASK}(\omega)$ is the continuous equivalent of $\dot{C}_{BASK}[k]$, obtained by substituting $\omega = k\Omega$. For easier calculation (0/0 type uncertainty calculation), it is recommended to use *sinc()* function:

$$\mathrm{sinc}(x) = \frac{\sin(x \cdot \pi)}{x \cdot \pi},$$

Be sure to compensate for π, by dividing argument!

Also, it is advisable to define k range centered around carrier frequency. The index for carrier frequency can be calculated as:

$$k_0 = \frac{\omega_0}{\Omega},$$

where $\Omega = \pi/\tau$.
Then k is integer variable in range $k_0 - 10$ to $k_0 + 10$.

BFSK	Amplitude spectrum: $$\dot{C}_{BFSK}[k] = \dot{C}_{BASK1}[k] \cdot e^{+jk\Omega\frac{\tau}{2}} + \dot{C}_{BASK2}[k] \cdot e^{-jk\Omega\frac{\tau}{2}}$$ Power spectral density: $$G_{BASK}(\omega) = G_{BASK1}(\omega) + G_{BASK2}(\omega)$$

where $\dot{C}_{BASK1}[k]$ is calculated for sub-carrier frequency ω_1 and $\dot{C}_{BASK2}[k]$ calculated for ω_2. Similar is true for $G_{BASK1}(\omega)$ and $G_{BASK2}(\omega)$.

These two sub-carrier frequencies can be calculated based on specified carrier frequency ω_0 and modulation index m_F:

$$\omega_1 = \omega_0 - \frac{\Delta\omega}{2} \qquad \text{and} \qquad \omega_2 = \omega_0 + \frac{\Delta\omega}{2},$$

where $\Delta\omega = m_F \cdot \Omega$.

BPSK	Amplitude spectrum: $$\dot{C}_{PFSK}[k] = \dot{C}_{BASK}[k] \cdot e^{+jk\Omega\frac{\tau}{2}} - \dot{C}_{BASK}[k] \cdot e^{-jk\Omega\frac{\tau}{2}}$$ Power spectral density: $$G_{BASK}(\omega) = 2G_{BASK}(\omega)$$

For BPSK modulation, the calculations are like BFSK case with two exceptions:

- "–" sign instead of "+" for amplitude spectrum;
- Both BASK modulations are calculated for the same carrier frequency ω_0.

Note:

- For periodic signals, the spectrum is discrete, so use "*stem*()" function to plot it.
- For non-periodic (random) signals, the continuous density is calculated instead. Use "*plot*()" function for this case.

Task 8. The probability of 2+ errors per m bits long codeword

Please, use the following generalized formula. Here P_{err} is error probability value calculated in Task 4. Also, please note, that for Hamming code $g = 1$.

$$P_{uncor} = 1 - (1 - P_{err})^m - \sum_{i=1}^{g} C_m^i P_{err}^i (1 - P_{err})^{m-i}$$

This is a probability of uncorrectable error for Your specified course project parameters. You can compare this value with P_{err} and conclude, whether selected code is a good choice.