Morphing

A Guide to Mathematical Transformations for Architects and Designers

Joseph Choma

To Ting-Ting

Morphing

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Contents

‘The way in which a problem is decomposed imposes fundamental constraints on the way in which people attempt to solve that problem.’

(Rodney Brooks, 1999)

A shape can be defined as anything with a geometric boundary. Yet, when describing a shape with mathematics, precision is crucial.

Dictionaries define words, but these words do not necessarily define our understanding of the world in which we perceive and create. The word 'cube' is defined as a shape whose boundary is composed of six congruent square faces. Imagine cutting six square pieces of paper and gluing the edges together. The cube, in this case, is created by six square planes. In mathematics, these planes are considered discrete elements. Because each plane in the paper cube meets the others at a sharp edge, technically they are not connected, but are separate parts, each defined by a unique parametric equation.

Now, imagine a ball of clay. Roll it around on the tabletop to make it into a sphere. To flatten it into a cube, the ball can be simply compressed in multiple directions (rather than forming six planar sides that are joined together, as above). Gradually, the sphere could transform into a six-sided shape. Most physical cubes in the world have a certain degree of rounded corners. If we accept this definition for a cube, then a cube could be defined with one parametric equation. The framework that is used becomes critical to deciding how to go about defining a shape mathematically.

If a shape maintains its topological continuity, it can be defined by a single equation. A break in continuity, such as a sharp edge, requires another equation to define the other 'part'.

Breaking continuity and having sharp edges can create aesthetic effects that allow curves to be expressed in a more objectified manner. Breaking the continuity of a shape can also sometimes facilitate the fabrication of particular geometries out of flat sheet material. However, the scope of this guide has been constrained to the definition of shapes with a single parametric equation.

A **parametric equation** is one way of defining values of coordinates (x, y, z) for shapes with parameters (u, v, w). All of the mathematical equations in this guide are presented as parametric equations. Think of x, y and z as dimensions in the Cartesian coordinate system – like a three-dimensional grid. Think of u, v and w as a range of values or parameters rather than a single integer. A single integer would be like a single point along a line, while the range (u, v or w) would define the end points of that line and then draw all the points between them.

Why trigonometry?

Tools inherently constrain the way individuals design; however, designers are often unaware of their tools' influences and biases. Digital tools in particular are becoming increasingly complex and filled with hierarchical symbolic heuristics, creating a black box in which designers do not understand what is 'under the hood' of the tools they 'drive'. Many contemporary digital tools use a fixed symbolic interface, like a visual dictionary. When a designer wants to create a sphere, he or she clicks on the sphere icon and draws a radius. The resulting sphere, defined by a single symbol, can only be manipulated as a whole. Like a ball of clay, the sphere can be stretched, twisted and pulled.

If, however, the shape had been defined by a parametric equation, it would be defined by a rule-based logic that encompasses both the whole sphere and its parts. When the designer manipulates the shape's trigonometry (or 'DNA'), it becomes clear that he or she has a new range of geometric freedom that could not have been imagined in the other framework. Since the designer can manipulate the smallest building blocks of the shapes, understanding how each function influences a particular transformation becomes straightforward. The designer is no longer designing within a black box, but rather within a transparent box.

Because mathematical models can be based on a global Cartesian coordinate system, the designer can constantly redefine a shape by redefining its parametric equations, avoiding the chore of telling the computer how to redraw it every time. For example, a smooth continuous surface can be confined to the boundary of a cube with a single transformation. Designers can begin to think and manipulate in a less linear fashion and constantly redefine the 'world' that they create and perceive. The Cartesian coordinate system becomes a blank canvas at which any type of paint can be thrown!

This pedagogical guide embraces the thought that all shapes could potentially be described by the trigonometric functions of sine and cosine. But the utility of this guide does not depend on whether that idea is true or not: this guide does not invent a new field of mathematics, but develops a cognitive narrative within the existing discipline, emphasizing the interconnected and plastic nature of shapes. Within this guide, sine and cosine are the only functions that are examined.

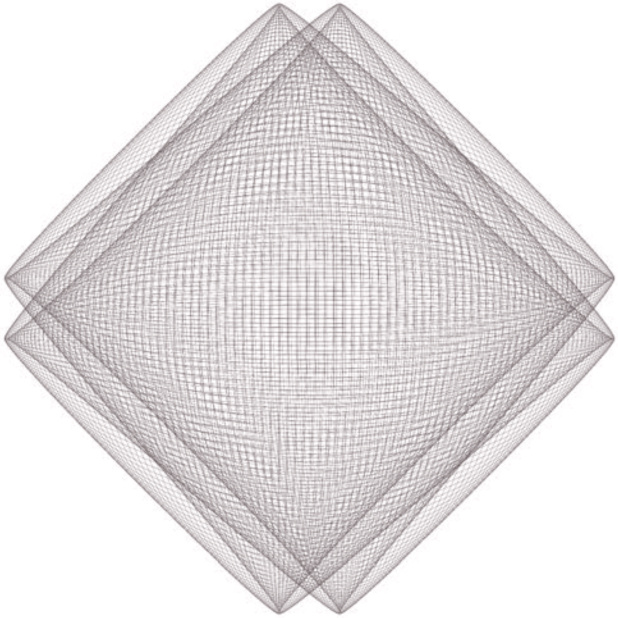
A word of warning: trigonometry may seem like the tool of a designer's dreams, but just because every shape could be potentially described by trigonometric functions does not mean that it is necessarily easy to make every shape. It is important to remember that all tools have biases, even mathematics. For instance, in order to make a cube with a single parametric equation one must first make a sphere; therefore, shapes that are initially round become easier to produce.

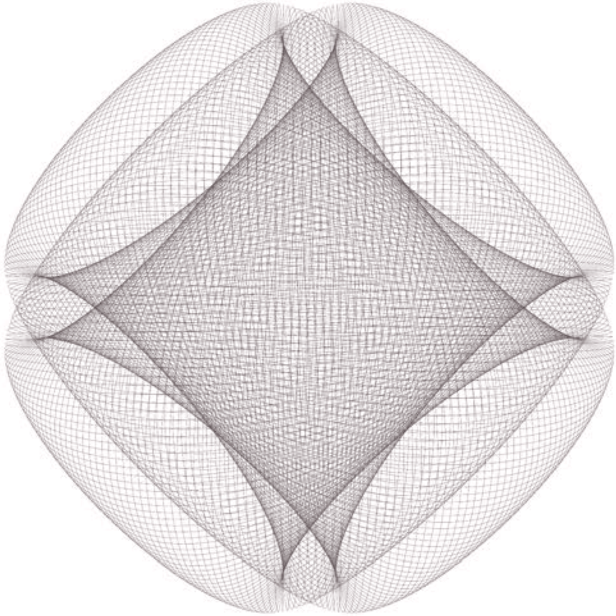
A **tool** is a device that augments an individual's ability to perform a particular task.

In this context, **heuristics** refers to the strategies used to solve problems within software. Think of heuristics as the rules that govern how a machine 'thinks' and calculates solutions.

Functions (in mathematics) associate an input (independent variable) with an output (dependent variable). For example if $x \rightarrow y$, x would be considered the independent variable while y would be considered the dependent variable. The function would be what makes x map to y. In trigonometry, sine and cosine are functions. Note that trigonometric functions are a type of periodic functions, whose values repeat in regular periods or intervals.

A **pedagogical guide** is neither purely a technical reference nor a theoretical text, but rather the teaching of an inquiry through a series of instructional frameworks.





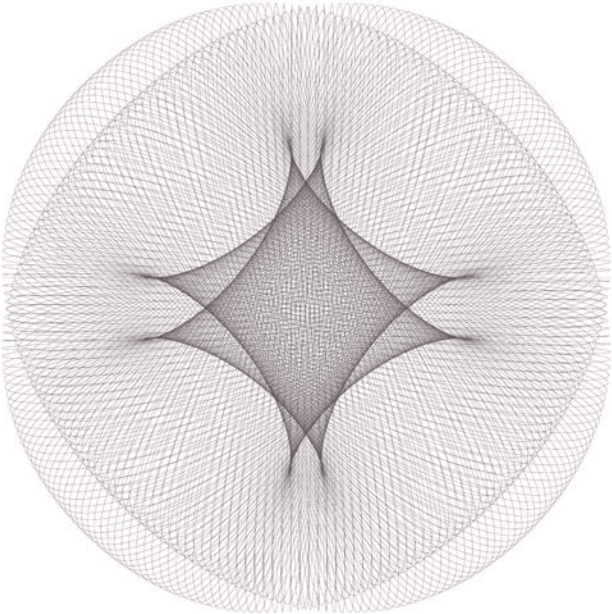
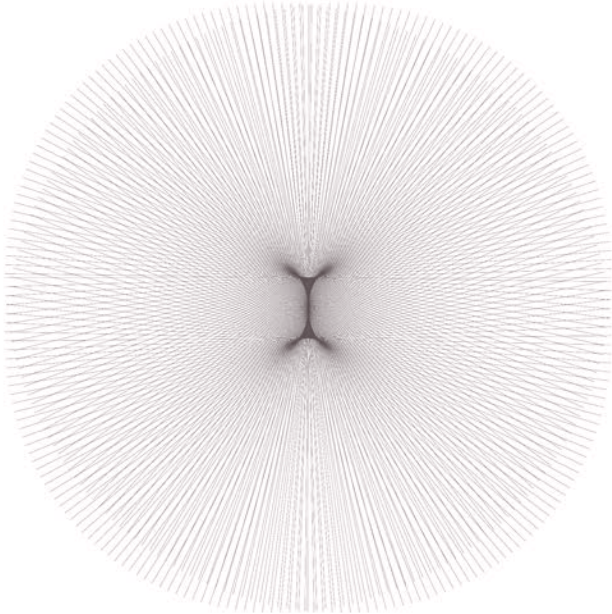
A point of view

{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ 2π }

x = sin(u)10(cos(v))
y = sin(v)+cos(u)10(cos(v))
z = cos(v)+10(sin(v))sin(u)

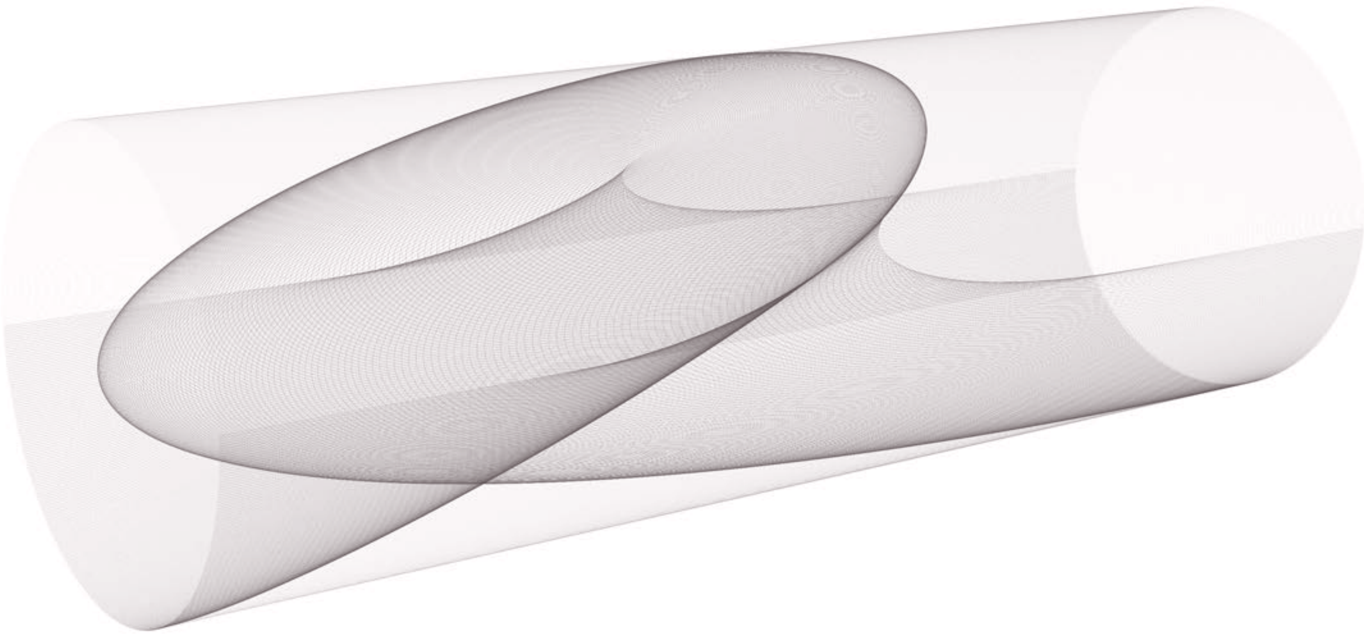
As we circle around an object, our perceptual understanding of it is transformed. We move in dialogue with the observed object, as in a dance performance. Similarly, the pages within this book document shapes according to particular, chosen perspectives. Most of the shapes within this guide are three dimensional, but their representation is constrained to the two-dimensional plane of the page. Some shapes may be difficult to comprehend in their entirety because of this singular view.

In the series above, there are four distinct shapes. On paper, they look different; however, they are defined by the same mathematical formula. As an individual moves around the object anti-clockwise, the shape begins to reveal an illusion. Initially, the shape appears to be rectangles overlapping

one another. Then, diamond-like shapes appear. Eventually, the diamond-like shapes shrink and disappear, as the perimeter of the shape transforms into a circle.

The selection of a particular view may seem trivial, but it is not. As with any scientific recording, the point of view from which data is presented biases the understanding of its contents. Within this guide, points of view remain constant within each morphing series. However, the selected views influence the visual documentation of each mathematical description.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)$
$y = u+u(\sin(v)\sin(u))$
$z = \sin(v)\cos(u)$

Boundaries

A boundary is anything that defines a limit. Numerically, determining a boundary may be straightforward, but perceptually it is often more ambiguous.

Spheres, cylinders, helicoids and cones are all shapes that, when viewed from one direction, share a circular profile. Embedded within each of their mathematical DNA are the same functions, which define a circle. Throughout this guide, simple, known shapes are transformed into many shapes which have never been visualized before. But by starting with a basic shape, such as a cylinder or sphere, the boundary condition of the resultant shape could maintain the initial signature of the ancestor.

For example, the shape on the left was a sphere which morphed under a *spiralling* and then *ascending* mathematical transformation. The result is a self-intersecting surface, which is bounded inside a cylinder.

The shape is not recognizable or defined within the dictionary, yet it shares the cylinder's front- and top-view boundary profiles: the top view of the shape is a rectangle, while its front view is a circle. The seemingly foreign shape appears familiar because of its profile projections.

As shapes transform throughout this guide, visualize the shape's two-dimensional projections. Like an architect's dissection of a building with plan, section and elevation, complex shapes can be simplified by analyzing parts of their parametric equation. As the coordinates (x, y, z) within the equation are altered, the projections will change; which coordinate is altered will determine which projection will change.

Mathematical transformations refer to the categorized rules for manipulating shapes with mathematical operations. Throughout this book, the names of transformations and other operations that are discussed in detail are set in italics for ease of reference.

Transformations

‘Evidently, there is scarcely anything that one can say about a “single sensation” by itself, but we can often say much more when we can make comparisons.’

(Marvin Minsky, 1985)

We all know what a ball is. A ball is a sphere. But when a ball is compressed, it is no longer a sphere, but a squished sphere. The known shape has transformed into an undefined shape. Rather than describing arbitrary shapes with mathematics, known shapes are utilized as consistent starting-points within this chapter. Think of this chapter and the others that follow as a collection of design experiments. Within this framework, the sine curve, circle, cylinder or sphere is the initial starting shape for each morphing series. They are the constants, while the mathematical transformations are the variables.

Thirteen mathematical transformations are introduced: *translating*, *cutting*, *rotating*, *reflecting*, *scaling*, *modulating*, *ascending*, *spiralling*, *texturing*, *bending*, *pinching*, *flattening* and *thickening*.

Although this guide challenges words and definitions, each mathematical transformation is labelled with a verb. The words are not always perfect, but they are used as placeholders. These placeholders allow me to reference and write about different transformations throughout this guide without explicitly restating the mathematics. The words labelling each transformation are more like nicknames for the mathematics, rather than the mathematics defining the word.

Each of these transformations is revealed through a morphing series; like motion-capture photography, each shape is recorded as it is transformed iteratively by the same operation. Under each shape in the series is a parametric equation that defines a particular instance. As the shape transforms, look at the equation below it. In the rule or logic behind the change in the equation, you can find the transformation.

Transformations

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

Shaping

Within *shaping*, the four starting shapes are introduced: a sine curve, circle, cylinder and sphere. A sine curve and circle are defined with two coordinates (x, y), while three-dimensional shapes (cylinder and sphere) are defined by x, y and z. A sine curve is simply defined by a function of sine and the u-parameter. A circle is defined by a function of sine and cosine. The v-parameter allows the circle to transform into a surface: a cylinder is an extruded circle, or a circle with a z-coordinate value of v. A cylinder can easily morph into a helix, helicoid, cone or sphere.

Shaping

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = 0$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \cos(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi/3 \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq 2\pi/3 \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

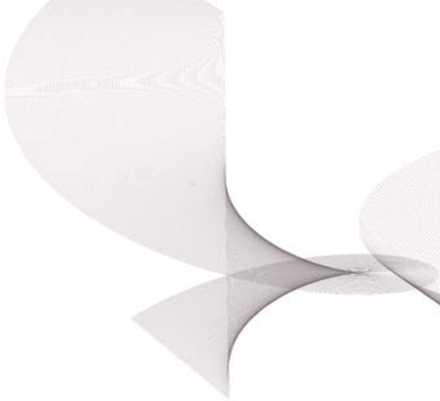
$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

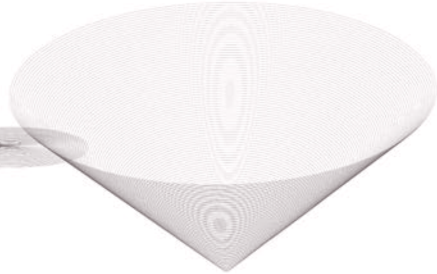
$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = u$



$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = v(\cos(u))$
$y = v(\sin(u))$
$z = u$



$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = v(\cos(u))$
$y = v(\sin(u))$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

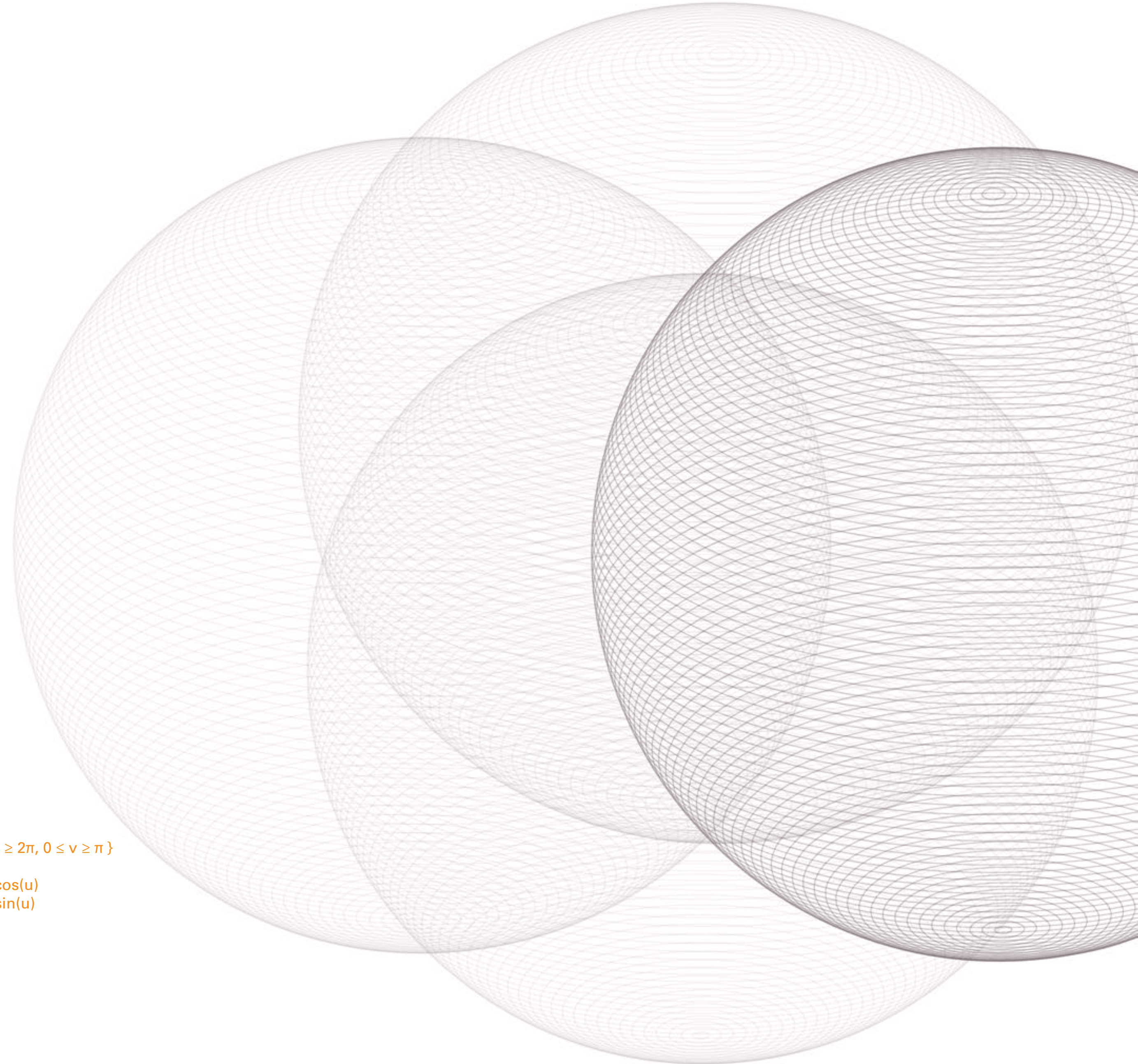
$x = \sin(v)\cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

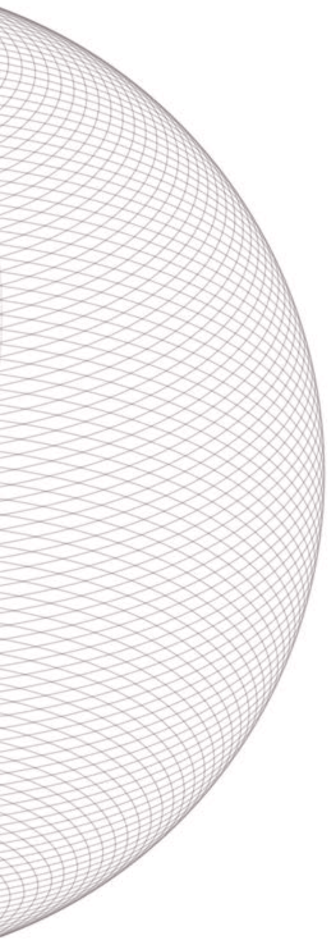
$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = 1+\sin(v)\cos(u)$
$y = 1+\sin(v)\sin(u)$
$z = \cos(v)$

Translating



A coffee mug is lifted off a table. The mug still appears to be the same shape. Physically, the material of the mug has not transformed; however, its location is different. According to mathematics, the location of a shape is part of the shape's inherent DNA. Throughout this guide, parametric equations include: coordinates (x, y, z), parameters (u, v, w) and functions (sine, cosine). Parameters and functions define the value of the coordinates, and the coordinates define the locations of the shape's points in space. If an integer is added to a particular coordinate, the shape moves in that direction. Remember, the parametric equation is the DNA of any given shape. Forget about words and definitions! If the equation changes, so does the shape. Not all spheres have the same mathematical description.

Transformations

Translating

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

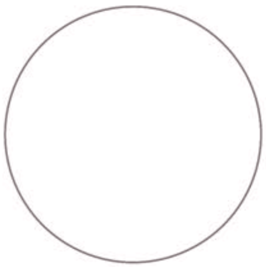
$x = u$
$y = 1+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = 1+u$
$y = \sin(u)$

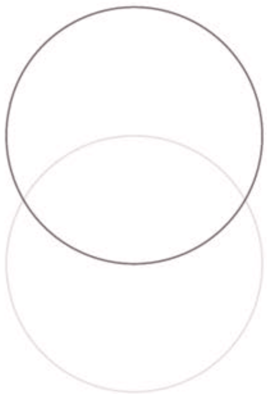
$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = 1+u$
$y = 1+\sin(u)$



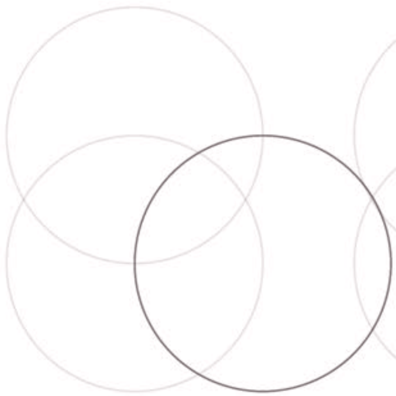
$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$



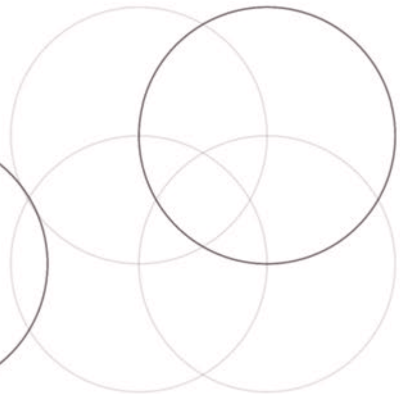
$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = 1+\sin(u)$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = 1+\cos(u)$
$y = \sin(u)$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = 1+\cos(u)$
$y = 1+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = 1+\sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = 1+\cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

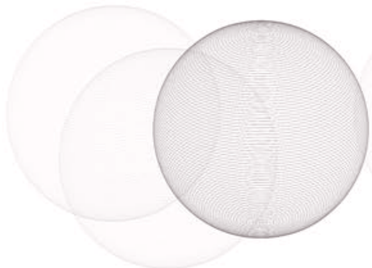
$x = 1+\cos(u)$
$y = 1+\sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

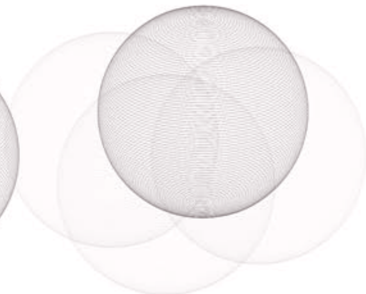
$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = 1+\sin(v)\sin(u)$
$z = \cos(v)$



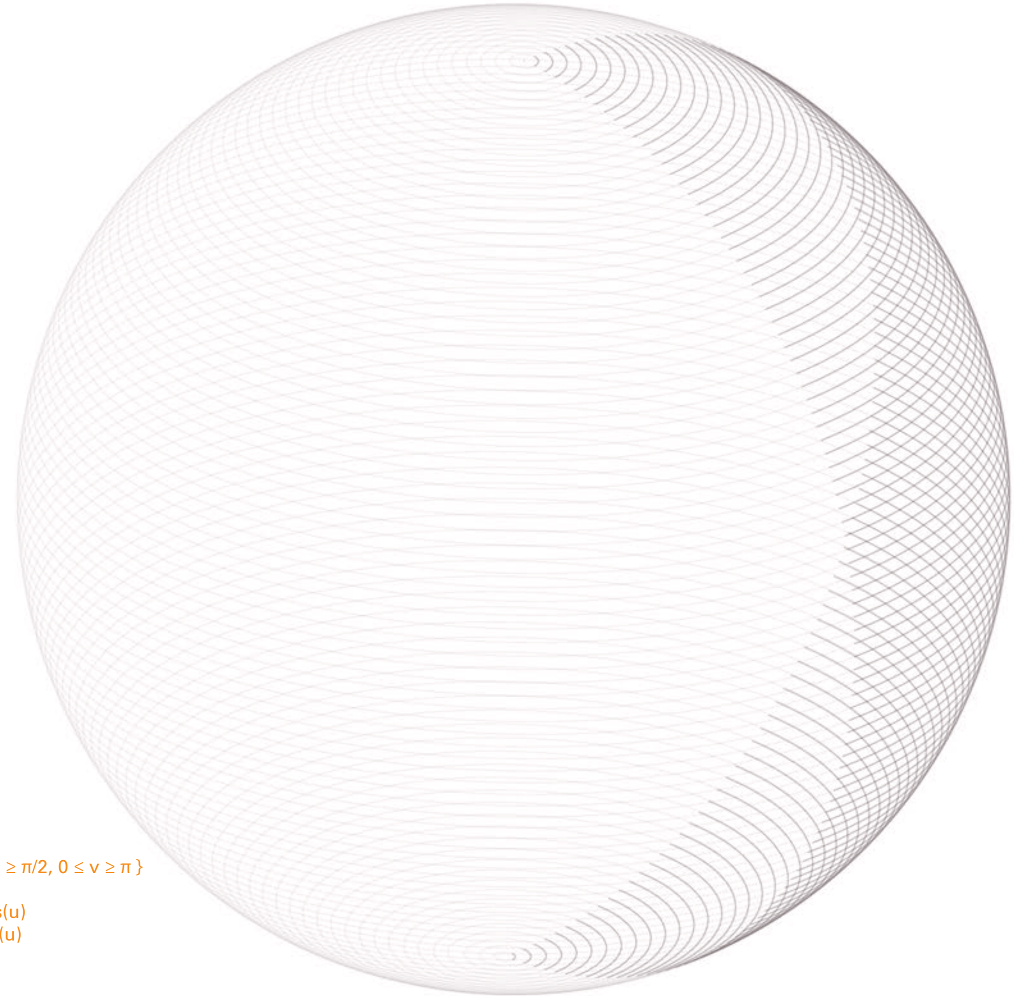
$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = 1+\sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = 1+\sin(v)\cos(u)$
$y = 1+\sin(v)\sin(u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq \pi/2, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

Cutting

A curve is defined by a series of points. The points on the curve are described as a range of values. The limits of this range are defined by the two end points on the curve. In the parametric equations that follow, there are two parameters (u, v), which control these end points. The u-parameter 'sees' only the local curves, which define a particular shape, while the v-parameter 'sees' the elevations of the entire shape. For example, in the case of the cylinder, the v-parameter would see a rectangle, while the u-parameter would see a circle. Limiting the numerical range of the parameters (u and v) will effectively cut the shape.

Cutting

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 3\pi/2 \}$

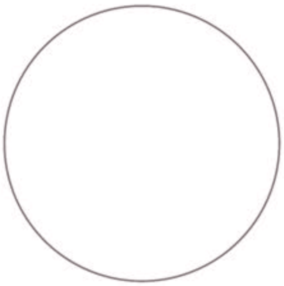
$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq \pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq \pi/2 \}$

$x = u$
$y = \sin(u)$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 3\pi/2 \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq \pi/2 \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 3\pi/2, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge \pi/2, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 3\pi/2, 0 \le v \ge \pi \}$

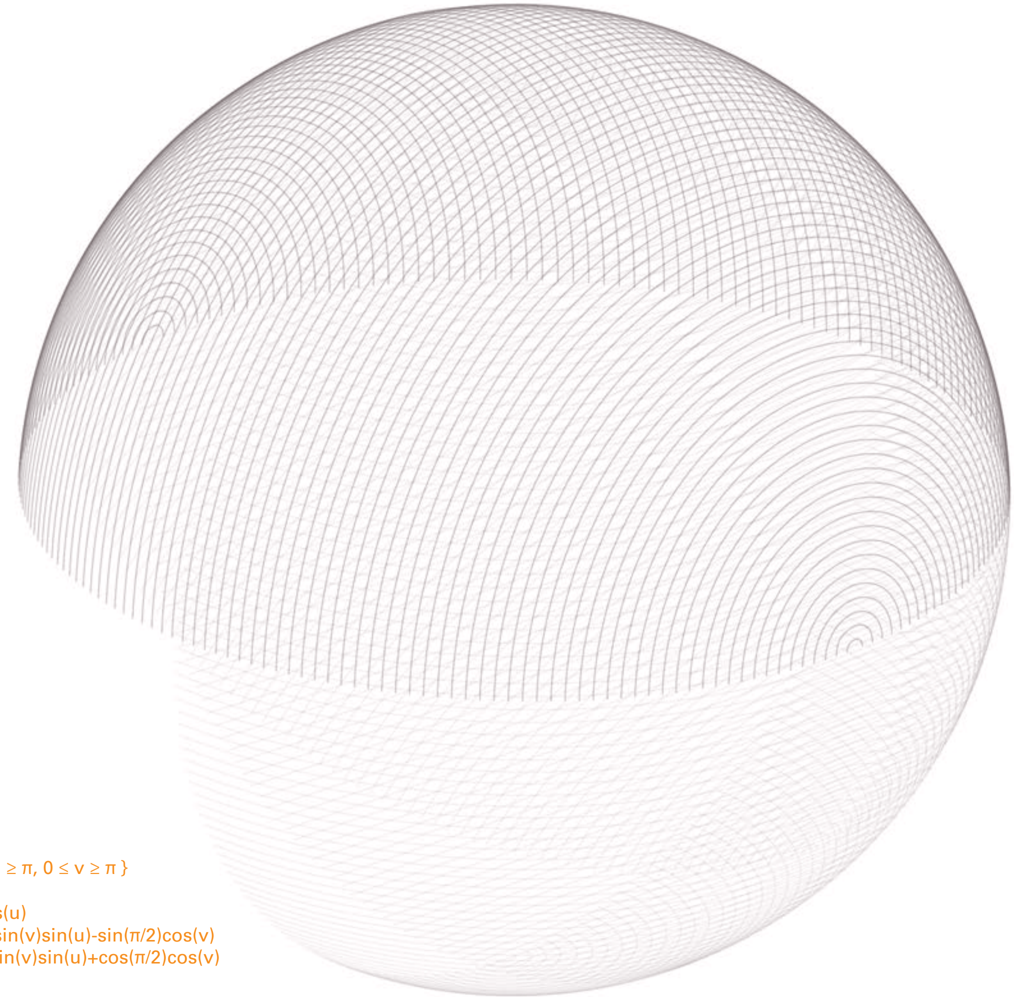
$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi/2, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq \pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \cos(\pi/2)\sin(v)\sin(u)-\sin(\pi/2)\cos(v)$
$z = \sin(\pi/2)\sin(v)\sin(u)+\cos(\pi/2)\cos(v)$

Rotating

In order to rotate a shape within most software, a rotational axis must first be defined. Similarly, within mathematics, an initial geometry is 'plugged into' a rotation matrix. Below are a series of templates to rotate a shape around each axis (x, y, z). The x_a,y_a, z_a within each of the templates below are the initial descriptions of the original shape, prior to its rotation. For example, a cylinder is defined by a parametric equation where $x_a = \cos(u)$, $y_a = \sin(u)$, and $z_a = v$. To rotate the cylinder, insert each of these definitions into the templates below.

Typically, we speak and think about rotation in terms of degrees, not radians. This guide has been written in radians, but it is easy to move between the two units. Some common conversions between degrees and radians include: 30 degrees = π/6 radians, 45 degrees = π/4 radians, 60 degrees = π/3 radians, 90 degrees = π/2 radians, 120 degrees = 2π/3 radians, 135 degrees = 3π/4 radians, 150 degrees = 5π/6 radians, 180 degrees = π radians. Each of the templates below rotates a shape by π/2 radians (90 degrees).

Rotating around x-axis

$x = x_a$
$y = \cos(\pi/2)y_a - \sin(\pi/2)z_a$
$z = \sin(\pi/2)y_a + \cos(\pi/2)z_a$

Rotating around y-axis

$x = \cos(\pi/2)x_a + \sin(\pi/2)z_a$
$y = y_a$
$z = -\sin(\pi/2)x_a + \cos(\pi/2)z_a$

Rotating around z-axis

$x = \cos(\pi/2)x_a - \sin(\pi/2)y_a$
$y = \sin(\pi/2)x_a + \cos(\pi/2)y_a$
$z = z_a$

Transformations

Rotating

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(\pi/6)u - \sin(\pi/6)\sin(u)$
$y = \sin(\pi/6)u + \cos(\pi/6)\sin(u)$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(\pi/3)u - \sin(\pi/3)\sin(u)$
$y = \sin(\pi/3)u + \cos(\pi/3)\sin(u)$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(\pi/2)u - \sin(\pi/2)\sin(u)$
$y = \sin(\pi/2)u + \cos(\pi/2)\sin(u)$

$\{ u \mid 0 \le u \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge \pi \}$

$x = \cos(\pi/6)\cos(u) - \sin(\pi/6)\sin(u)$
$y = \sin(\pi/6)\cos(u) + \cos(\pi/6)\sin(u)$

$\{ u \mid 0 \le u \ge \pi \}$

$x = \cos(\pi/3)\cos(u) - \sin(\pi/3)\sin(u)$
$y = \sin(\pi/3)\cos(u) + \cos(\pi/3)\sin(u)$

$\{ u \mid 0 \le u \ge \pi \}$

$x = \cos(\pi/2)\cos(u) - \sin(\pi/2)\sin(u)$
$y = \sin(\pi/2)\cos(u) + \cos(\pi/2)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \cos(\pi/6)\sin(u)-\sin(\pi/6)v$
$z = \sin(\pi/6)\sin(u)+\cos(\pi/6)v$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \cos(\pi/3)\sin(u)-\sin(\pi/3)v$
$z = \sin(\pi/3)\sin(u)+\cos(\pi/3)v$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

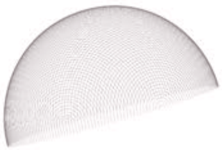
$x = \cos(u)$
$y = \cos(\pi/2)\sin(u)-\sin(\pi/2)v$
$z = \sin(\pi/2)\sin(u)+\cos(\pi/2)v$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \cos(\pi/6)\sin(v)\sin(u)-\sin(\pi/6)\cos(v)$
$z = \sin(\pi/6)\sin(v)\sin(u)+\cos(\pi/6)\cos(v)$

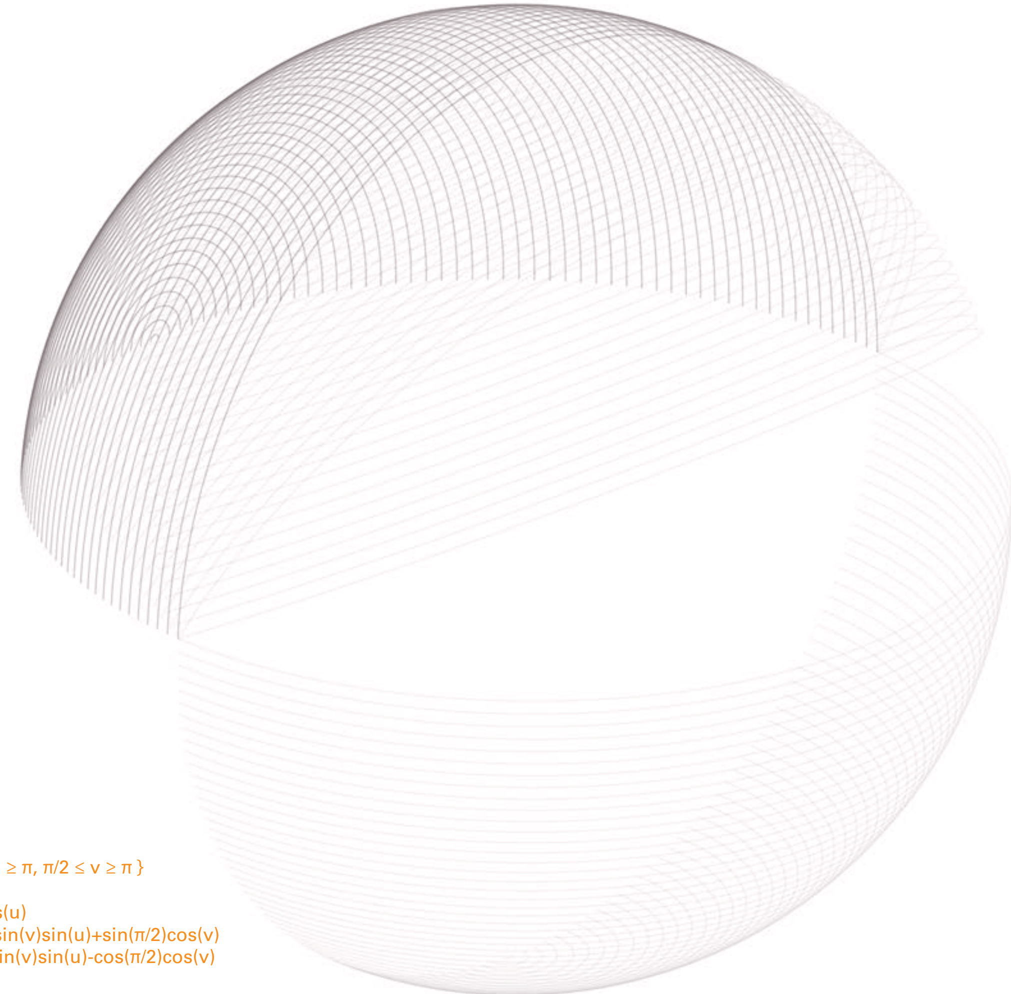


$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \cos(\pi/3)\sin(v)\sin(u)-\sin(\pi/3)\cos(v)$
$z = \sin(\pi/3)\sin(v)\sin(u)+\cos(\pi/3)\cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \cos(\pi/2)\sin(v)\sin(u)-\sin(\pi/2)\cos(v)$
$z = \sin(\pi/2)\sin(v)\sin(u)+\cos(\pi/2)\cos(v)$



{ (u,v) | 0 ≤ u ≥ π, π/2 ≤ v ≥ π }

x = sin(v)cos(u)
y = cos(π/2)sin(v)sin(u)+sin(π/2)cos(v)
z = sin(π/2)sin(v)sin(u)-cos(π/2)cos(v)

Reflecting

In *rotating*, an initial shape was inserted into a rotation matrix. *Reflecting* utilizes a similar template. When referring back to the *rotating* around x-axis template, you will notice subtraction in the y-coordinate and addition in the z-coordinate definition. If subtraction is replaced with addition, and addition is replaced with subtraction, an initial shape will be reflected about a plane that is rotated (at a defined angle) about the x-axis. However, the input angle defined is not the angle through which the shape is reflected, but half the angle reflected.

Imagine a vertical mirror on a wall. This book is closed and brought perpendicular to the mirror. Within the mirror, the book reflects and forms a flat plane, π radians (180 degrees). The angle between the book and mirror on each side is π/2 radians (90 degrees), or one half the input angle. Each of the templates below reflects a shape by π/4 radians (45 degrees), but contains an input angle of π/2 radians (90 degrees).

Reflecting about a plane (x-axis)

$x = x_a$
$y = \cos(\pi/2)y_a+\sin(\pi/2)z_a$
$z = \sin(\pi/2)y_a-\cos(\pi/2)z_a$

Reflecting about a plane (y-axis)

$x = -\cos(\pi/2)x_a+\sin(\pi/2)z_a$
$y = y_a$
$z = \sin(\pi/2)x_a+\cos(\pi/2)z_a$

Reflecting about a plane (z-axis)

$x = \cos(\pi/2)x_a+\sin(\pi/2)y_a$
$y = \sin(\pi/2)x_a-\cos(\pi/2)y_a$
$z = z_a$

Transformations

Reflecting

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(\pi/6)u+\sin(\pi/6)\sin(u)$
$y = \sin(\pi/6)u-\cos(\pi/6)\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(\pi/3)u+\sin(\pi/3)\sin(u)$
$y = \sin(\pi/3)u-\cos(\pi/3)\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(\pi/2)u+\sin(\pi/2)\sin(u)$
$y = \sin(\pi/2)u-\cos(\pi/2)\sin(u)$

$\{ u \mid 0 \leq u \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq \pi \}$

$x = \cos(\pi/6)\cos(u)+\sin(\pi/6)\sin(u)$
$y = \sin(\pi/6)\cos(u)-\cos(\pi/6)\sin(u)$

$\{ u \mid 0 \leq u \geq \pi \}$

$x = \cos(\pi/3)\cos(u)+\sin(\pi/3)\sin(u)$
$y = \sin(\pi/3)\cos(u)-\cos(\pi/3)\sin(u)$

$\{ u \mid 0 \leq u \geq \pi \}$

$x = \cos(\pi/2)\cos(u)+\sin(\pi/2)\sin(u)$
$y = \sin(\pi/2)\cos(u)-\cos(\pi/2)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \cos(\pi/6)\sin(u)+\sin(\pi/6)v$
$z = \sin(\pi/6)\sin(u)-\cos(\pi/6)v$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

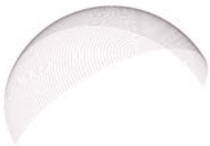
$x = \cos(u)$
$y = \cos(\pi/3)\sin(u)+\sin(\pi/3)v$
$z = \sin(\pi/3)\sin(u)-\cos(\pi/3)v$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \cos(\pi/2)\sin(u)+\sin(\pi/2)v$
$z = \sin(\pi/2)\sin(u)-\cos(\pi/2)v$

$\{ (u,v) \mid 0 \le u \ge \pi, \pi/2 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \le u \ge \pi, \pi/2 \le v \ge \pi \}$

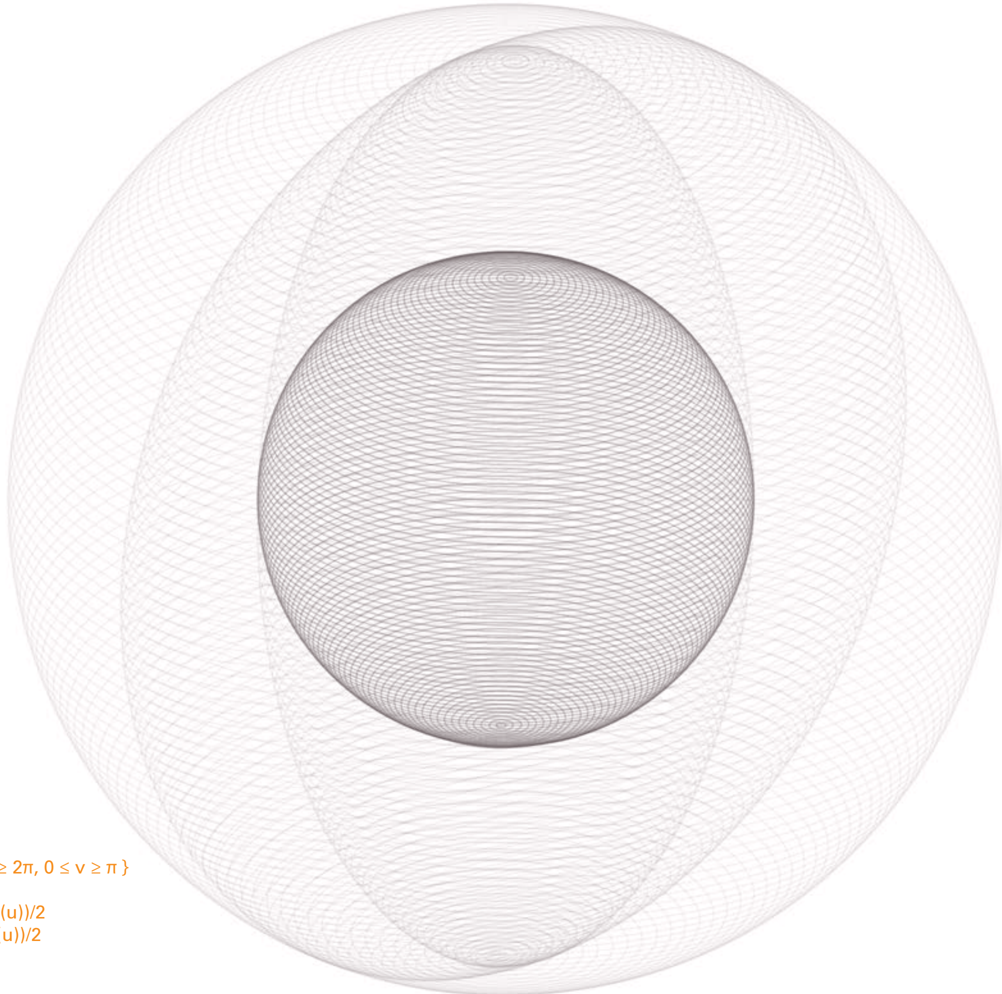
$x = \sin(v)\cos(u)$
$y = \cos(\pi/6)\sin(v)\sin(u)+\sin(\pi/6)\cos(v)$
$z = \sin(\pi/6)\sin(v)\sin(u)-\cos(\pi/6)\cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi, \pi/2 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \cos(\pi/3)\sin(v)\sin(u)+\sin(\pi/3)\cos(v)$
$z = \sin(\pi/3)\sin(v)\sin(u)-\cos(\pi/3)\cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi, \pi/2 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \cos(\pi/2)\sin(v)\sin(u)+\sin(\pi/2)\cos(v)$
$z = \sin(\pi/2)\sin(v)\sin(u)-\cos(\pi/2)\cos(v)$



{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = (sin(v)cos(u))/2
y = (sin(v)sin(u))/2
z = cos(v)/2

Scaling

According to basic multiplication, if an integer is multiplied by another integer, its value increases according to the multiplier. If an integer is divided by another integer, its value decreases according to the divider. These same rules apply to *scaling* shapes. Multiplying a shape by an integer will scale that shape accordingly. If only the x-coordinate is multiplied or divided, the shape will scale only in that axis.

Transformations

Scaling

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

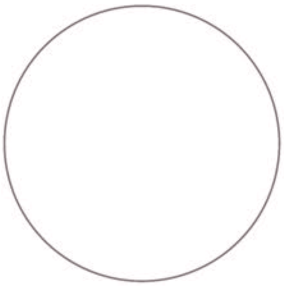
$x = u$
$y = \sin(u)/2$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u/2$
$y = \sin(u)$

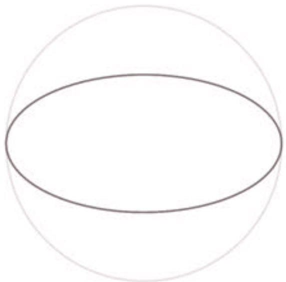
$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u/2$
$y = \sin(u)/2$



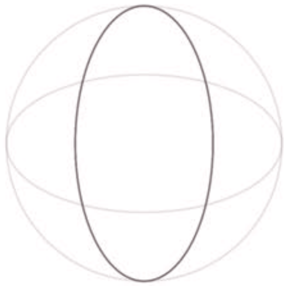
$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$



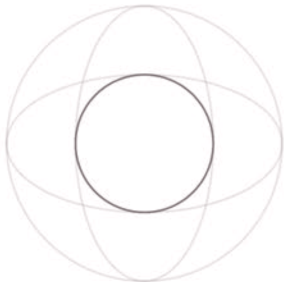
$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)/2$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)/2$
$y = \sin(u)$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)/2$
$y = \sin(u)/2$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)/2$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)/2$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)/2$
$y = \sin(u)/2$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

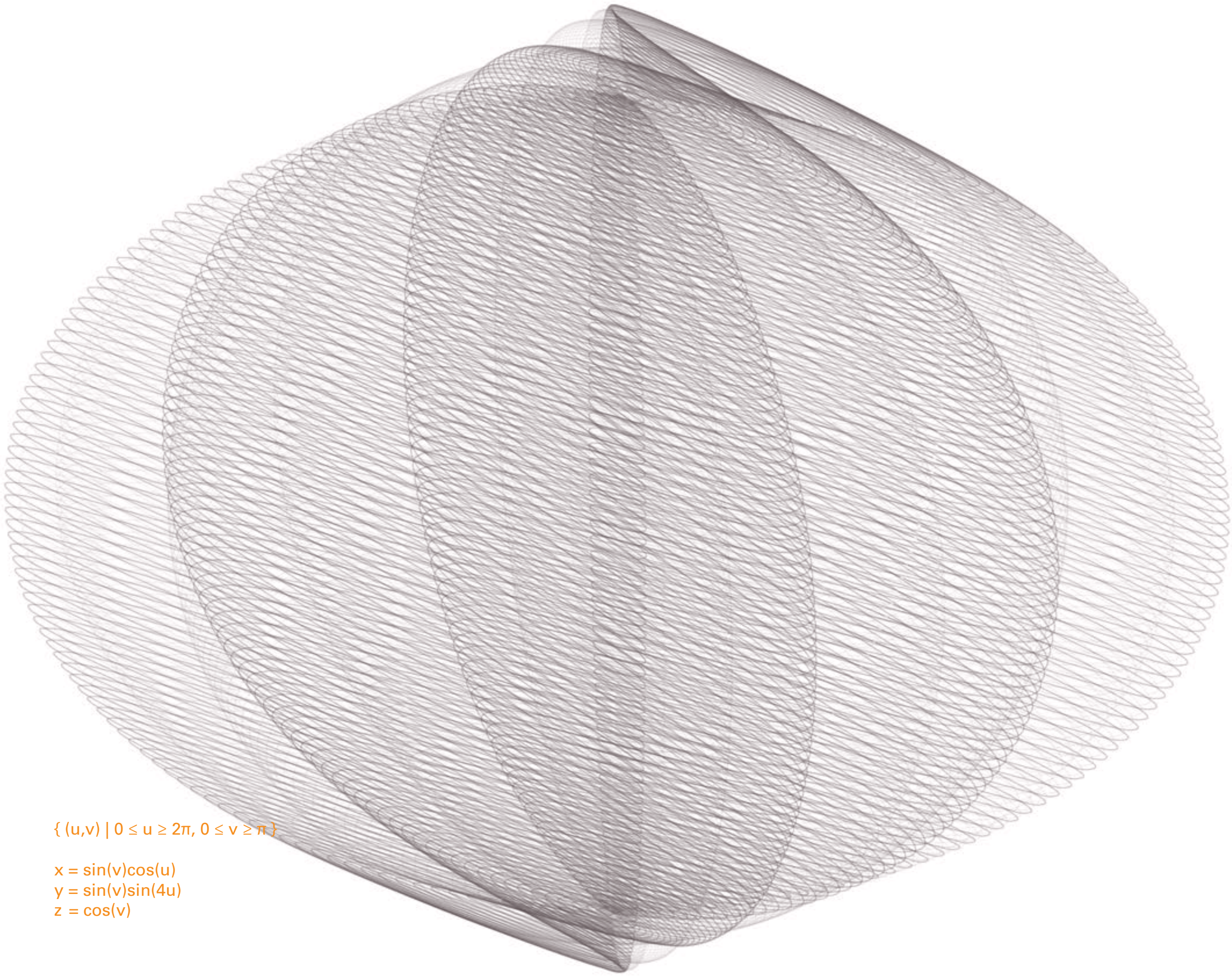
$x = \sin(v)\cos(u)$
$y = (\sin(v)\sin(u))/2$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (\sin(v)\cos(u))/2$
$y = (\sin(v)\sin(u))/2$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (\sin(v)\cos(u))/2$
$y = (\sin(v)\sin(u))/2$
$z = \cos(v)/2$



$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(4u)$
$z = \cos(v)$

Modulating

Within the trigonometric functions (sine, cosine) there are parameters (u, v). Within *cutting*, these parameters control the end points of a curve. However, when these parameters are scaled while embedded inside a trigonometric function [for example, sin(A(u)), where A is a scaling factor], they control the frequency of the curve defined. For example, if the parameter is doubled, the output would yield double the number of periods over the same length. Multiplying the parameter also multiplies the frequency of the curve.

Modulating

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge 2\pi \}$

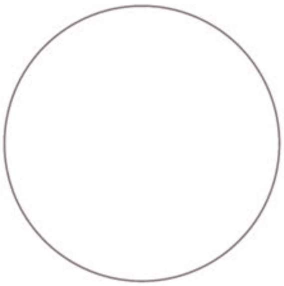
$x = u$
$y = \sin(2u)$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = u$
$y = \sin(3u)$

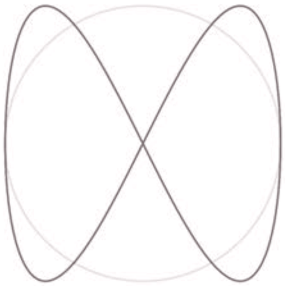
$\{ u \mid 0 \le u \ge 2\pi \}$

$x = u$
$y = \sin(4u)$



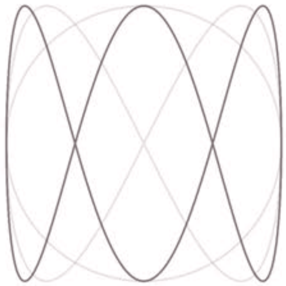
$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$



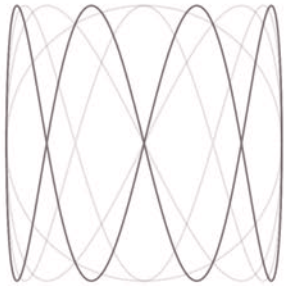
$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(u)$
$y = \sin(2u)$



$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(u)$
$y = \sin(3u)$



$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(u)$
$y = \sin(4u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(2u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(3u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

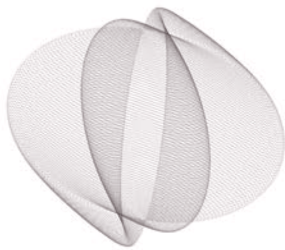
$x = \cos(u)$
$y = \sin(4u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

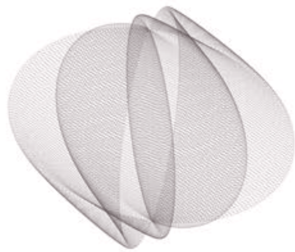
$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(2u)$
$z = \cos(v)$



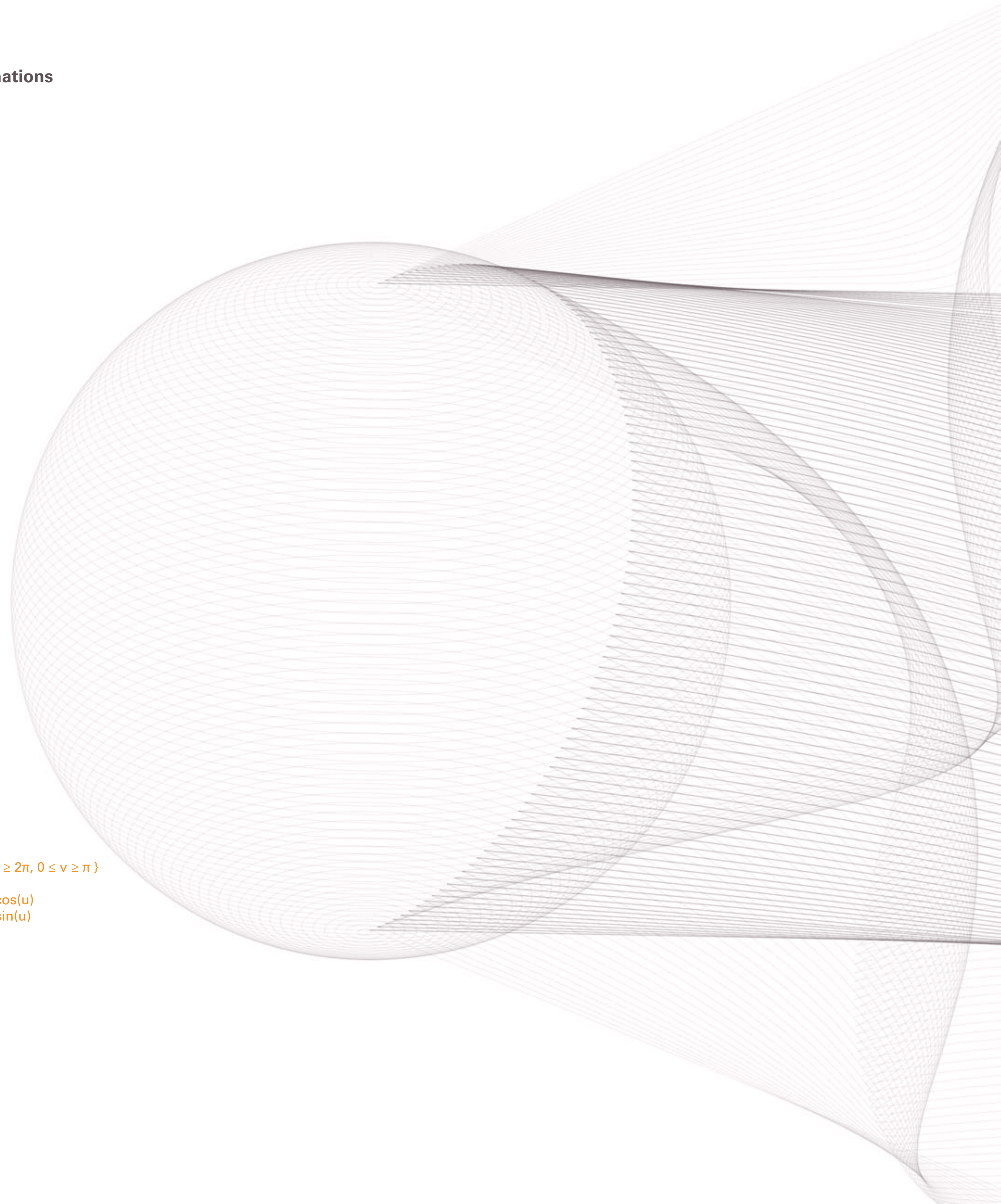
$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(3u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(4u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+\sin(v)\cos(u)$
$y = u+\sin(v)\sin(u)$
$z = \cos(v)$

Ascending

Imagine a piece of graph paper with an x-axis and a y-axis. If a sine curve was drawn on the x-axis, its period would start and end on the x-axis. Plotting a sine curve is much like plotting a horizontal line, with undulations of a specific amplitude and frequency. Adding a u-parameter outside the trigonometric function [for example, u+sin(u)] is much like plotting a diagonal line at an angle to the horizontal. For every cycle the function completes, it incrementally moves vertically. The centre of the sine curve would no longer follow the x-axis; instead, it would follow the newly plotted diagonal line. Adding an integer to a function would give a linear translation, while adding a u-parameter would shift the shape and produce a diagonal trajectory.

Transformations

Ascending

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = u+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u+u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u+u$
$y = u+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = u+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u+\cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u+\cos(u)$
$y = u+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = u+\sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+\cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+\cos(u)$
$y = u+\sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

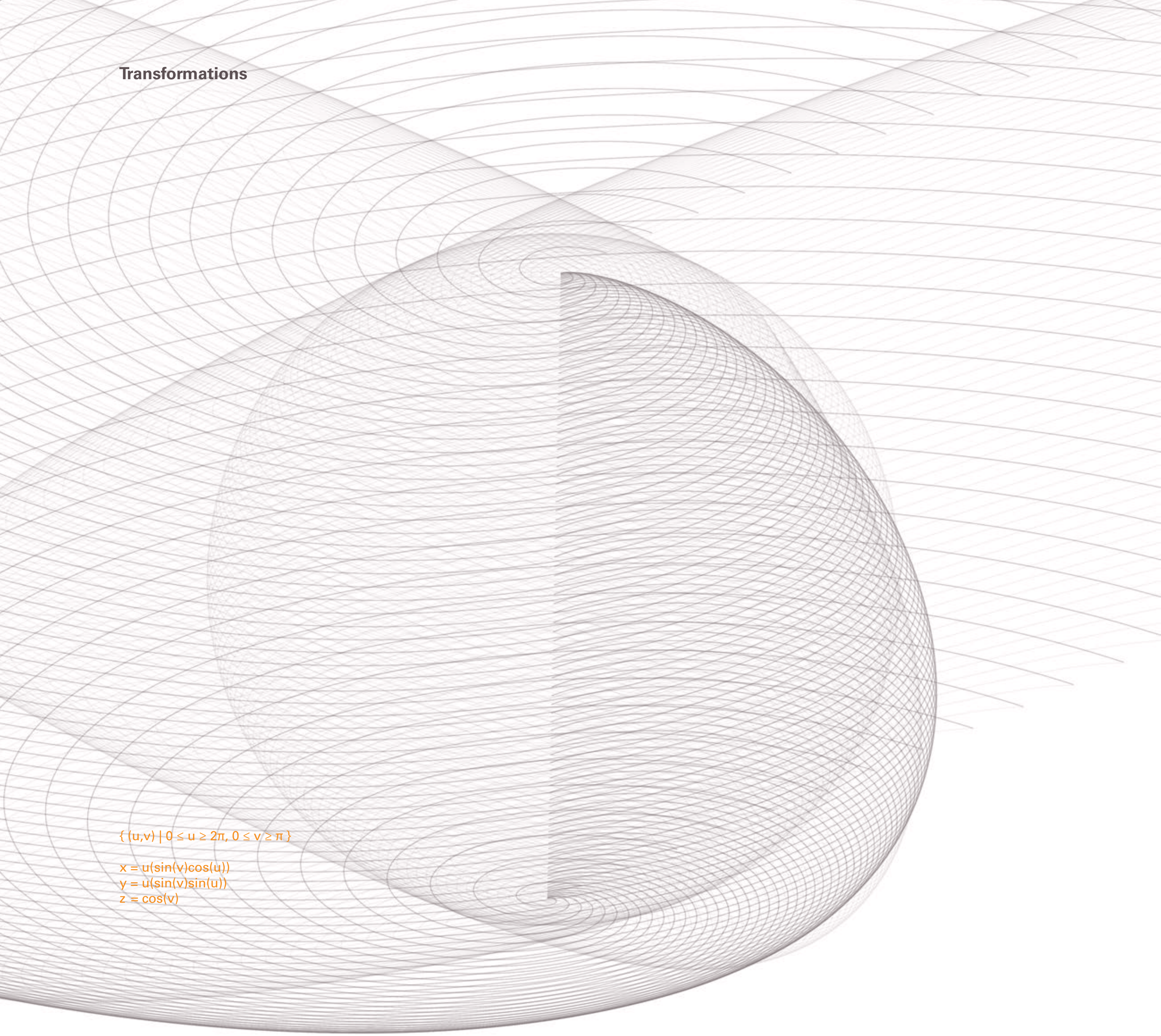
$x = \sin(v)\cos(u)$
$y = u+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+\sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+\sin(v)\cos(u)$
$y = u+\sin(v)\sin(u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

Spiralling

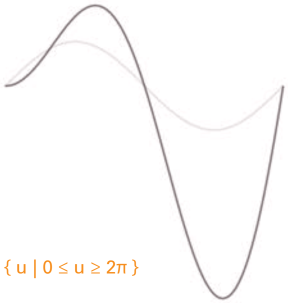
As stated in *ascending*, adding the u-parameter is much like plotting a diagonal line. The operation of adding is a means of displacing a shape, like shifting a sine curve from the x-axis to a diagonal trajectory. Multiplication is quite different. If the u-parameter is multiplied outside the trigonometric function [for example, $u(\sin(v))$], the radius of the curve drawn from the centre origin increases incrementally. The curve is not being displaced; it is being stretched iteratively outward according to the u-parameter.

Transformations

$\{ u \mid 0 \leq u \geq 2\pi \}$

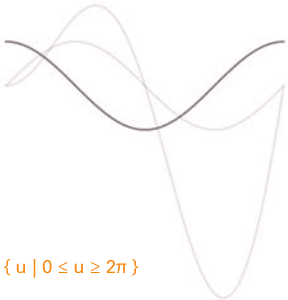
$x = u$
$y = \sin(u)$

Spiralling



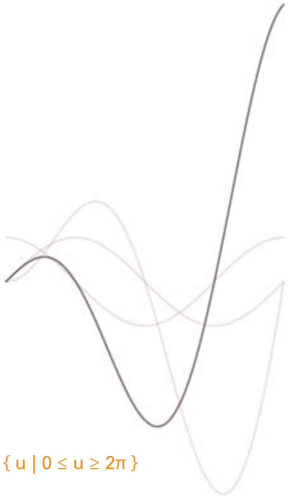
$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = u(\sin(u))$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \cos(u)$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = u(\cos(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = u(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

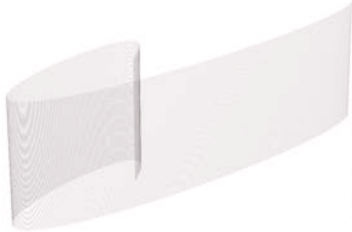
$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = u(\sin(u))$
$z = v$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(u))$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(u))$
$y = u(\sin(u))$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

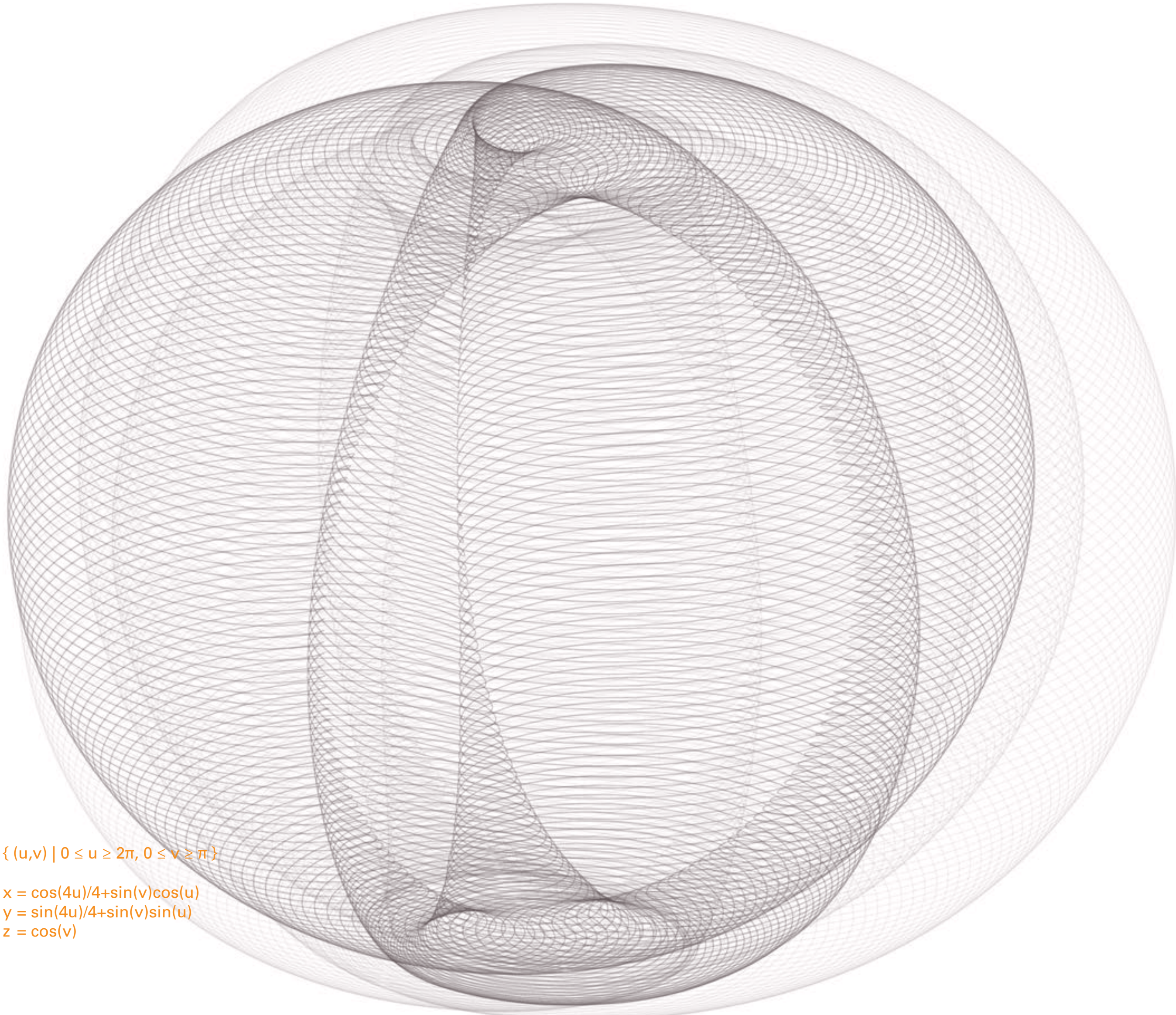
$x = \sin(v)\cos(u)$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(4u)/4+\sin(v)\cos(u)$
$y = \sin(4u)/4+\sin(v)\sin(u)$
$z = \cos(v)$

Texturing

When adding two trigonometric functions of different frequencies, one frequency is drawn inside the other. Like a curve undulating within the envelope formed by another curve, the two functions independently control different parameters of the resulting curve. Although this seems straightforward, a curve inside a curve can get out of control easily. It is highly recommended that if the frequency of a curve is increased by a certain value, its amplitude should be decreased by that same amount. This will maintain the amplitude of the overall curve (the envelope) while independently increasing the frequency of the texture (the inner curve).

Transformations

Texturing

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

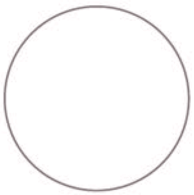
$x = u$
$y = \sin(2u)/2+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(3u)/3+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(4u)/4+\sin(u)$

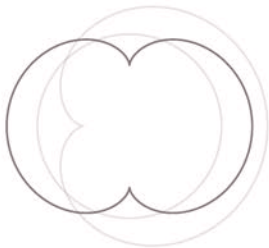


$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(2u)/2+\cos(u)$
$y = \sin(2u)/2+\sin(u)$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(3u)/3+\cos(u)$
$y = \sin(3u)/3+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(4u)/4+\cos(u)$
$y = \sin(4u)/4+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \cos(2u)/2+\cos(u)$
$y = \sin(2u)/2+\sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

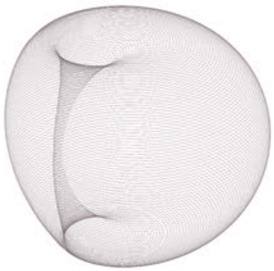
$x = \cos(3u)/3+\cos(u)$
$y = \sin(3u)/3+\sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \cos(4u)/4+\cos(u)$
$y = \sin(4u)/4+\sin(u)$
$z = v$

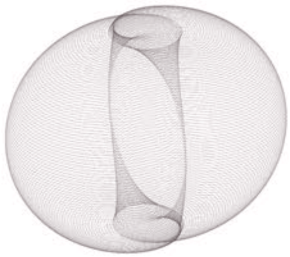
$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$



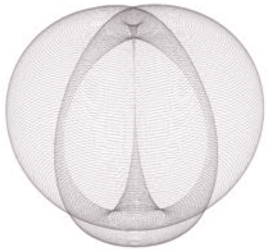
$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \cos(2u)/2+\sin(v)\cos(u)$
$y = \sin(2u)/2+\sin(v)\sin(u)$
$z = \cos(v)$



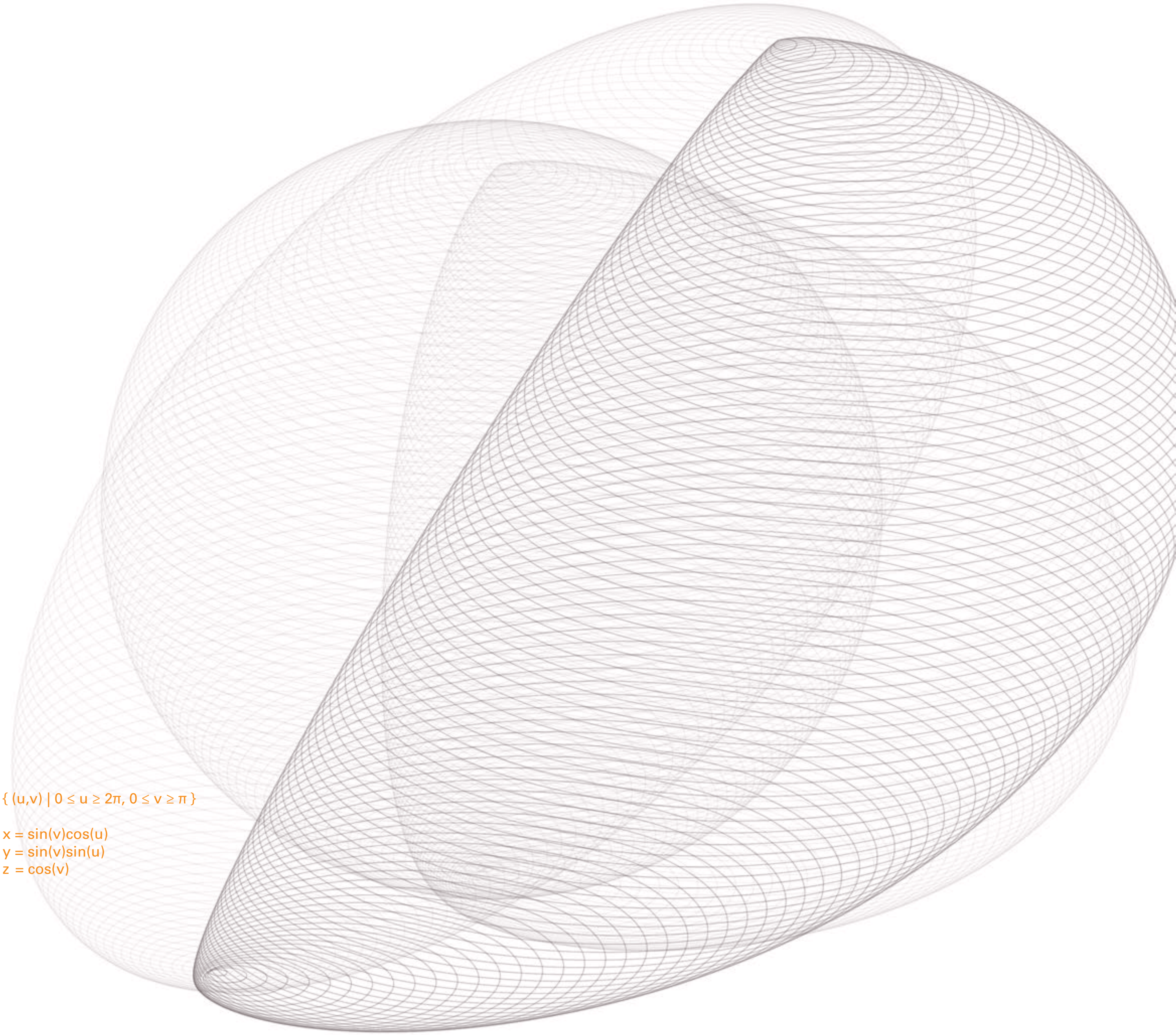
$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \cos(3u)/3+\sin(v)\cos(u)$
$y = \sin(3u)/3+\sin(v)\sin(u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \cos(4u)/4+\sin(v)\cos(u)$
$y = \sin(4u)/4+\sin(v)\sin(u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

Bending

Within *shaping*, a cylinder was squished into a sphere by multiplying trigonometric functions. In *ascending*, a curve was plotted according to a diagonal trajectory. In the case of *bending*, an entire shape is swept along a curve. This is more like *ascending* than the squishing transformation portrayed in *shaping*. The geometry of the initial shape is not squished and deformed, but instead is plotted to follow a path other than a straight line. Adding a trigonometric function to a cylinder will not distort the circle that composes it. Rather, the circle, instead of extruding vertically in a straight line, will follow a curve and bend.

Transformations

Bending

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u$
$y = \sin(v)+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+u$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+u$
$y = \sin(v)+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(v)+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+\cos(u)$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+\cos(u)$
$y = \sin(v)+\sin(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(v)+\sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(v)+\cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(v)+\cos(u)$
$y = \sin(v)+\sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

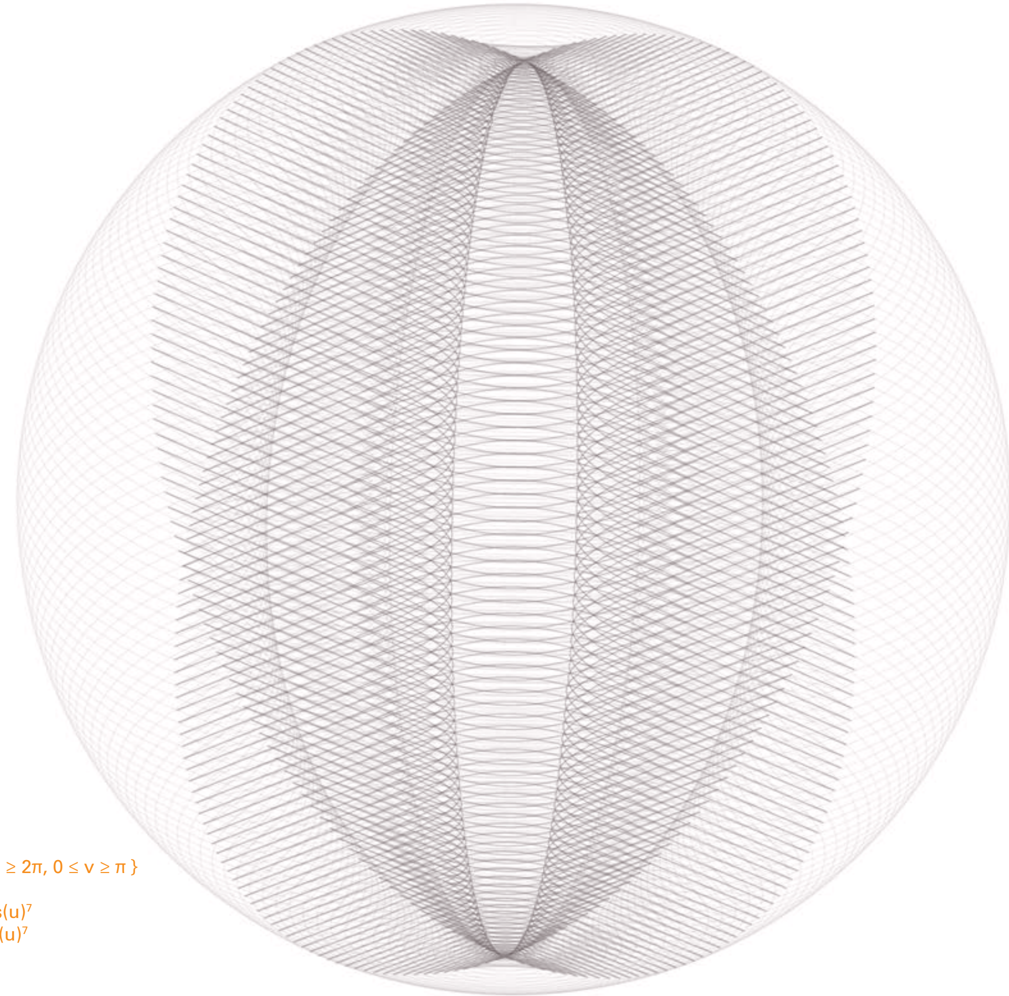
$x = \sin(v)\cos(u)$
$y = \sin(v)+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(v)+\sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(v)+\sin(v)\cos(u)$
$y = \sin(v)+\sin(v)\sin(u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)^7$
$y = \sin(v)\sin(u)^7$
$z = \cos(v)$

Pinching

By raising a trigonometric function to an exponent, the degree of curvature will increase, emphasizing the outermost boundaries of that particular shape. The higher the power, the steeper the apex of the shape will be. If a shape is raised to an even power, its orientation will be mirrored. For example, if all odd powers point upwards, all even powers will point downward. Although *pinching* will give the illusion of an edge, the edge will have a slight rounding to maintain the property of continuity.

Pinching

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

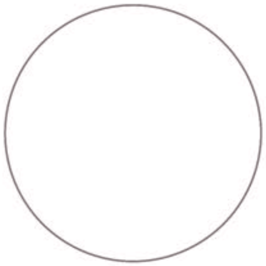
$x = u$
$y = \sin(u)^3$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)^5$

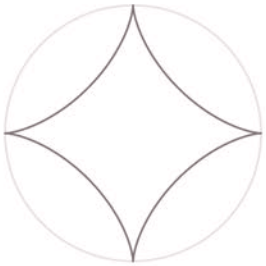
$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)^7$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)^3$
$y = \sin(u)^3$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)^5$
$y = \sin(u)^5$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)^7$
$y = \sin(u)^7$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)^3$
$y = \sin(u)^3$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)^5$
$y = \sin(u)^5$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)^7$
$y = \sin(u)^7$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)^3$
$y = \sin(v)\sin(u)^3$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)^5$
$y = \sin(v)\sin(u)^5$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)^7$
$y = \sin(v)\sin(u)^7$
$z = \cos(v)$

{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = sin(sin(sin(sin(v)cos(u))))
y = sin(sin(sin(sin(v)sin(u))))
z = sin(sin(sin(cos(v))))

Flattening

As an equation is placed inside a function of sine, its degree of curvature decreases. The more recursions of sine functions that are embedded inside each other, the more flattened the shape will become. Like smashing a ball of clay into a cube, as the sphere is smashed material is compressed. *Flattening* a shape also gradually scales the shape. The more cube-like the sphere becomes, the smaller it grows. A sphere can transform into a cube with one parametric equation, if slightly rounded corners are acceptable. Discrete edges would break the continuity and would require additional equations.

Transformations **Flattening**

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

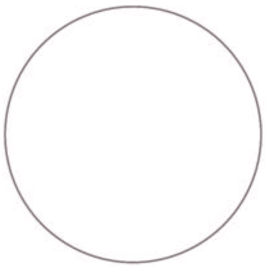
$x = u$
$y = \sin(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(\sin(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(\sin(\sin(\sin(u))))$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\cos(u))$
$y = \sin(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\sin(\cos(u)))$
$y = \sin(\sin(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\sin(\sin(\cos(u))))$
$y = \sin(\sin(\sin(\sin(u))))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\cos(u))$
$y = \sin(\sin(u))$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\sin(\cos(u)))$
$y = \sin(\sin(\sin(u)))$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\sin(\sin(\cos(u))))$
$y = \sin(\sin(\sin(\sin(u))))$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

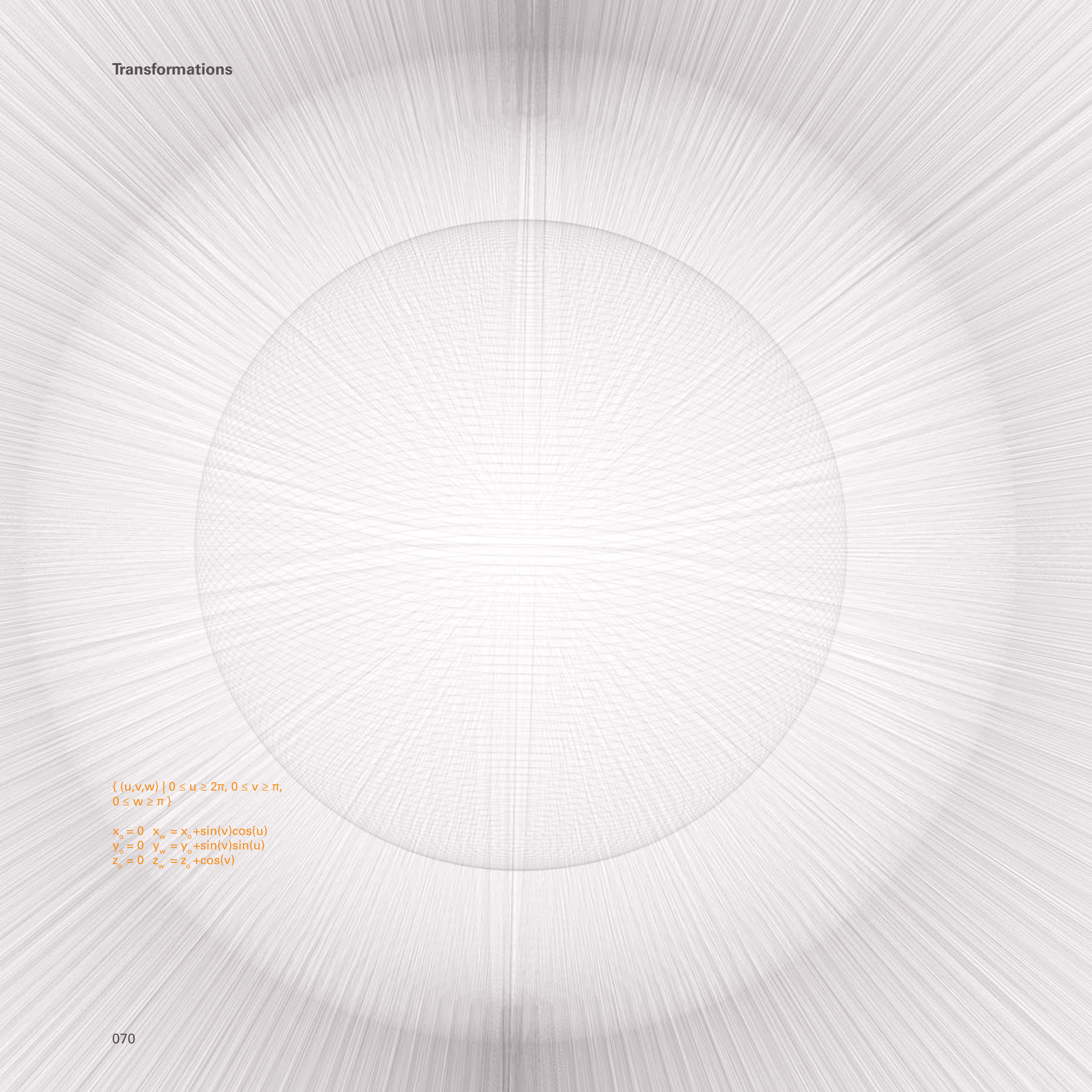
$x = \sin(\sin(v)\cos(u))$
$y = \sin(\sin(v)\sin(u))$
$z = \sin(\cos(v))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\sin(\sin(v)\cos(u)))$
$y = \sin(\sin(\sin(v)\sin(u)))$
$z = \sin(\sin(\cos(v)))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\sin(\sin(\sin(v)\cos(u))))$
$y = \sin(\sin(\sin(\sin(v)\sin(u))))$
$z = \sin(\sin(\sin(\cos(v))))$



$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,$
$0 \leq w \geq \pi \}$

$x_o = 0 \quad x_w = x_o + \sin(v)\cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(v)\sin(u)$
$z_o = 0 \quad z_w = z_o + \cos(v)$

Thickening

A three-dimensional grid in space comprises the x-, y- and z-coordinates in a parametric equation. Imagine a sphere within this three-dimensional grid. On the sphere itself is another grid, formed by the u- and v-parameters. In order to give a sphere thickness, an additional parameter must be introduced. The w-parameter is like an arrow coming off the surface of a sphere. It relates to w as z relates to x and y (or, to put it a different way, w is like the z-coordinate of u and v).

Because a sphere with thickness has two boundary conditions, an inner and an outer, two different sets of x, y and z must be established. One set describes the inner boundary of the sphere and the second defines its thickness (and therefore implicitly also describes its outer boundary).

The w-parameter controls the thickness of the shape. Note that thickness is used to describe the two boundary conditions. The shape is still a single-thickness geometry but the geometry itself bounces back and forth between two boundary conditions, giving the illusion of thickness.

Transformations

Thickening

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u$
$y = \sin(u)$

$\{ (u,w) \mid 0 \leq u \geq 2\pi, 0 \leq w \geq \pi/3 \}$

$x_o = 0 \quad x_w = x_o + u$
$y_o = 0 \quad y_w = y_o + \sin(u)$

$\{ (u,w) \mid 0 \leq u \geq 2\pi, 0 \leq w \geq 2\pi/3 \}$

$x_o = 0 \quad x_w = x_o + u$
$y_o = 0 \quad y_w = y_o + \sin(u)$

$\{ (u,w) \mid 0 \leq u \geq 2\pi, 0 \leq w \geq \pi \}$

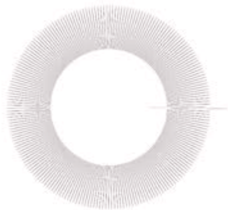
$x_o = 0 \quad x_w = x_o + u$
$y_o = 0 \quad y_w = y_o + \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

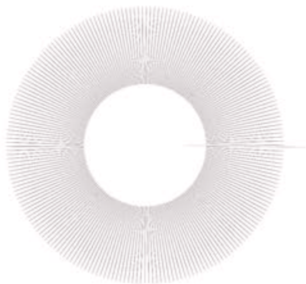
$\{ (u,w) \mid 0 \leq u \geq 2\pi, 0 \leq w \geq \pi/3 \}$

$x_o = 0 \quad x_w = x_o + \cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(u)$



$\{ (u,w) \mid 0 \leq u \geq 2\pi, 0 \leq w \geq 2\pi/3 \}$

$x_o = 0 \quad x_w = x_o + \cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(u)$



$\{ (u,w) \mid 0 \leq u \geq 2\pi, 0 \leq w \geq \pi \}$

$x_o = 0 \quad x_w = x_o + \cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v,w) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi,$
$0 \leq w \geq \pi/3 \}$

$x_o = 0 \quad x_w = x_o + \cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(u)$
$z_o = 0 \quad z_w = z_o + v$

$\{ (u,v,w) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi,$
$0 \leq w \geq 2\pi/3 \}$

$x_o = 0 \quad x_w = x_o + \cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(u)$
$z_o = 0 \quad z_w = z_o + v$

$\{ (u,v,w) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi,$
$0 \leq w \geq \pi \}$

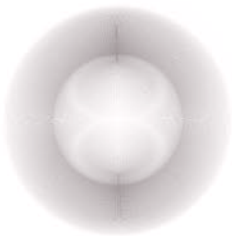
$x_o = 0 \quad x_w = x_o + \cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(u)$
$z_o = 0 \quad z_w = z_o + v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

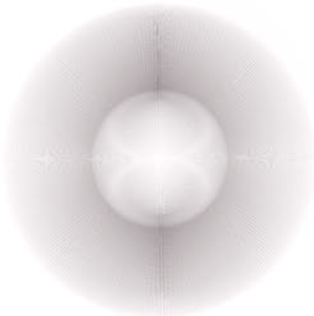
$\{ (u,v,w) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi,$
$0 \leq w \geq \pi/3 \}$

$x_o = 0 \quad x_w = x_o + \sin(v)\cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(v)\sin(u)$
$z_o = 0 \quad z_w = z_o + \cos(v)$



$\{ (u,v,w) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi,$
$0 \leq w \geq 2\pi/3 \}$

$x_o = 0 \quad x_w = x_o + \sin(v)\cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(v)\sin(u)$
$z_o = 0 \quad z_w = z_o + \cos(v)$



$\{ (u,v,w) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi,$
$0 \leq w \geq \pi \}$

$x_o = 0 \quad x_w = x_o + \sin(v)\cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(v)\sin(u)$
$z_o = 0 \quad z_w = z_o + \cos(v)$

Combining Transformations

‘The forces coming from without which transform the point into a line, can be very diverse. The variation in lines depends upon the number of these forces and upon their combinations.’

(Wassily Kandinsky, 1947)

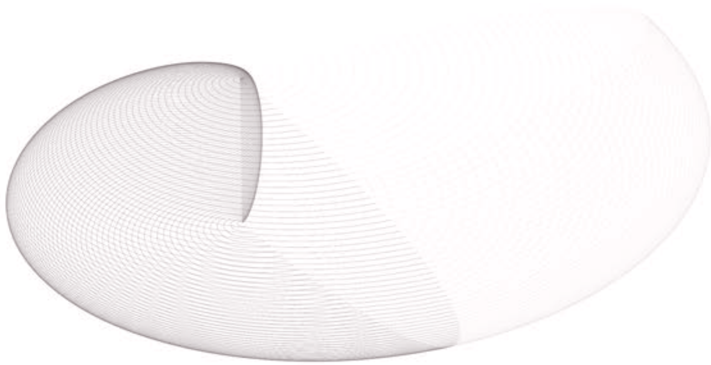
Red, yellow and blue are considered primary colours. If the colour desired is a shade or tint of those primary colours, black or white can be added. But, if green was a desired output, yellow and blue would have to combine. The mathematical transformations in the previous chapter establish a palette of tools to manipulate shapes. When shapes are transformed with only one rule, they can only transform iteratively. After a single transformation, it becomes quite predictable how a shape will continue to change with repetitions of the same transformation. Mathematical transformations may seem trivial or limited in possibilities, but when they combine, shapes can morph in much less predictable and more complex manners. It seems possible to generate almost any shape imaginable.

As in the previous chapter – or, indeed, any experiment – there must be both constants and variables. The starting shapes throughout this chapter will be a spiral and spiralled sphere. In addition to morphing series, which show a linear progression, this chapter also introduces taxonomies. Taxonomies are like tables or graphs of transformations, where the left-most column uses only one type of transformation and the bottom-most row uses another, while the shapes along the table's diagonal transform under both.

In order to combine transformations with control, it is imperative to remember that the order of operations in an equation will have a determining effect on the shape created. For example, a sphere could be spiralled and then flattened or could be flattened and then spiralled; the change in order will produce a change in the resulting shape.

Cutting and spiralling

The period of a shape is controlled by *cutting*. Since *cutting* controls how much a particular shape is expressed (by limiting the numerical ranges of u and v), all transformations occur in parallel with *cutting*. Here, the parallel transformation is the incremental increase in the radius of a sphere through *spiralling*.



$\{ (u,v) \mid 0 \leq u \geq 3\pi/2, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

{ (u,v) | 0 ≤ u ≥ π/2, 0 ≤ v ≥ π }

x = u(sin(v)cos(u))
y = u(sin(v)sin(u))
z = cos(v)

Cutting and spiralling

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ u \mid 0 \le u \ge 3\pi/2 \}$

$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ u \mid 0 \le u \ge \pi \}$

$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ u \mid 0 \le u \ge \pi/2 \}$

$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge 3\pi/2 \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge \pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge \pi/2 \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge 3\pi/2 \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge \pi/2 \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 3\pi/2, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi/2, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 3\pi/2, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi/2, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 3\pi/2, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge \pi/2, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

Scaling and spiralling

After a sphere's size is altered through *scaling,* its radius is incrementally increased (*spiralling*). The order of operations does not matter for *spiralling* and *scaling*; either can transform the shape first.

{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = u(sin(v)cos(u))
y = (u(sin(v)sin(u)))/2
z = cos(v)

{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = (u(sin(v)cos(u)))/2
y = (u(sin(v)sin(u)))/2
z = cos(v)/2

Scaling and spiralling

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = u(\cos(u))$
$y = (u(\sin(u)))/2$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = (u(\cos(u)))/2$
$y = u(\sin(u))$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = (u(\cos(u)))/2$
$y = (u(\sin(u)))/2$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)/2$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = (u(\cos(u)))/2$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = (u(\cos(u)))/2$
$y = \sin(u)/2$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(u)$
$y = \sin(u)/2$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(u)/2$
$y = \sin(u)$

$\{ u \mid 0 \le u \ge 2\pi \}$

$x = \cos(u)/2$
$y = \sin(u)/2$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = (u(\sin(v)\sin(u)))/2$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (u(\sin(v)\cos(u)))/2$
$y = (u(\sin(v)\sin(u)))/2$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (u(\sin(v)\cos(u)))/2$
$y = (u(\sin(v)\sin(u)))/2$
$z = \cos(v)/2$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = (\sin(v)\sin(u))/2$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (u(\sin(v)\cos(u)))/2$
$y = (\sin(v)\sin(u))/2$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (u(\sin(v)\cos(u)))/2$
$y = (\sin(v)\sin(u))/2$
$z = \cos(v)/2$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = (\sin(v)\sin(u))/2$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

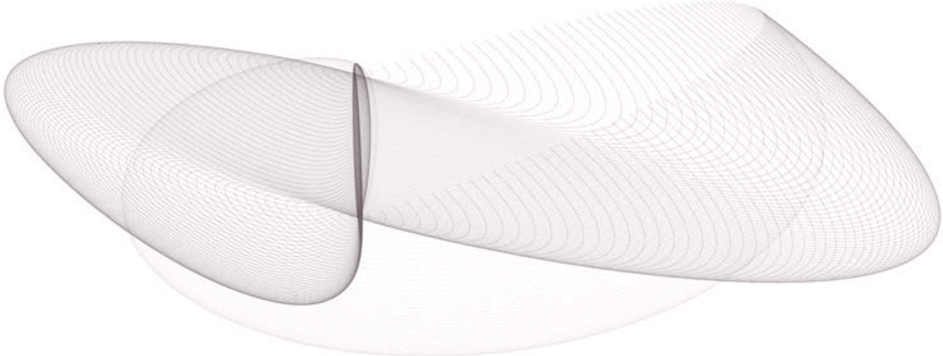
$x = (\sin(v)\cos(u))/2$
$y = (\sin(v)\sin(u))/2$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (\sin(v)\cos(u))/2$
$y = (\sin(v)\sin(u))/2$
$z = \cos(v)/2$

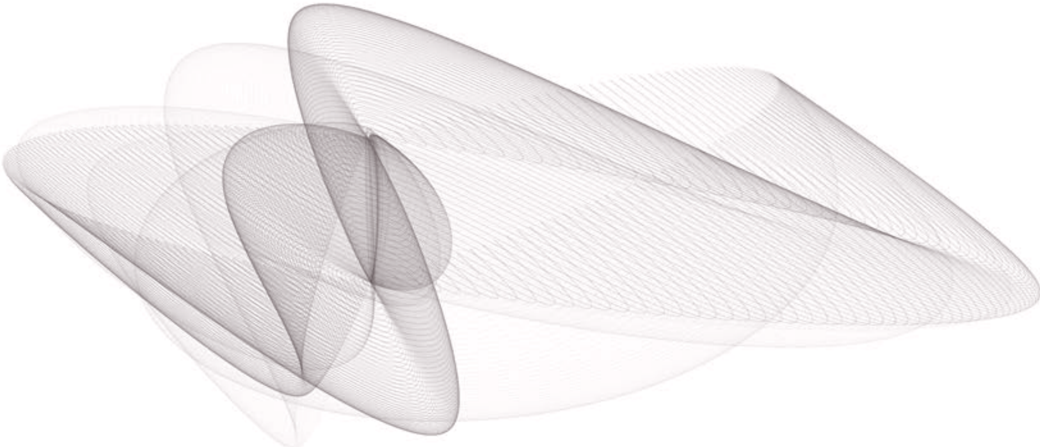
Modulating and spiralling

In addition to the *spiralling* transformation, the frequency of the shape is altered through *modulating*. First, the shape's frequency increases and then the shape's radius incrementally increases.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(2u))$
$z = \cos(v)$



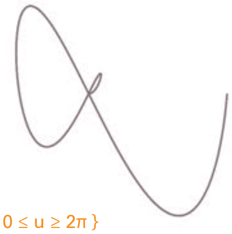
$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(4u))$
$z = \cos(v)$

Modulating and spiralling

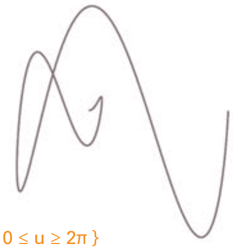
$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(u))$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(2u))$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(3u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(4u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(2u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(3u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(4u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(2u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(3u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(4u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

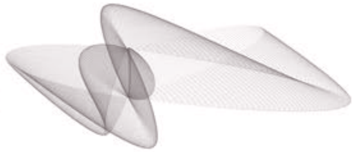
$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(2u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(3u))$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(4u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(2u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(3u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(4u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(2u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

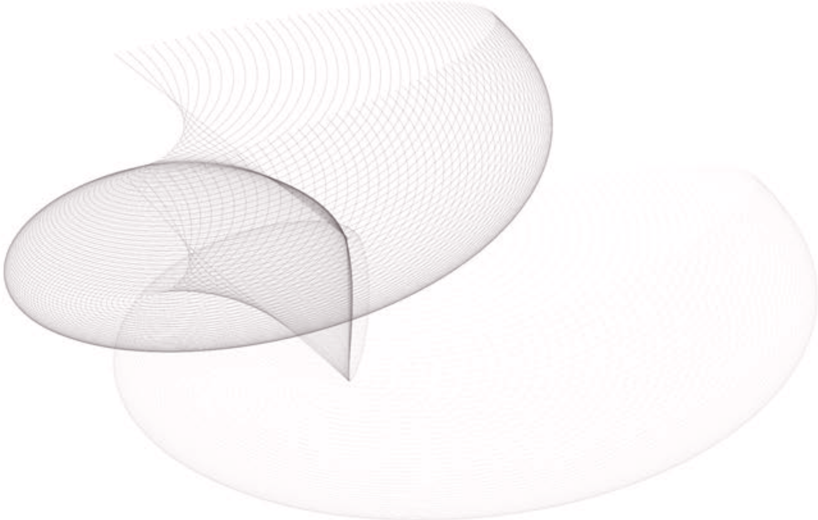
$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(3u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(4u)$
$z = \cos(v)$

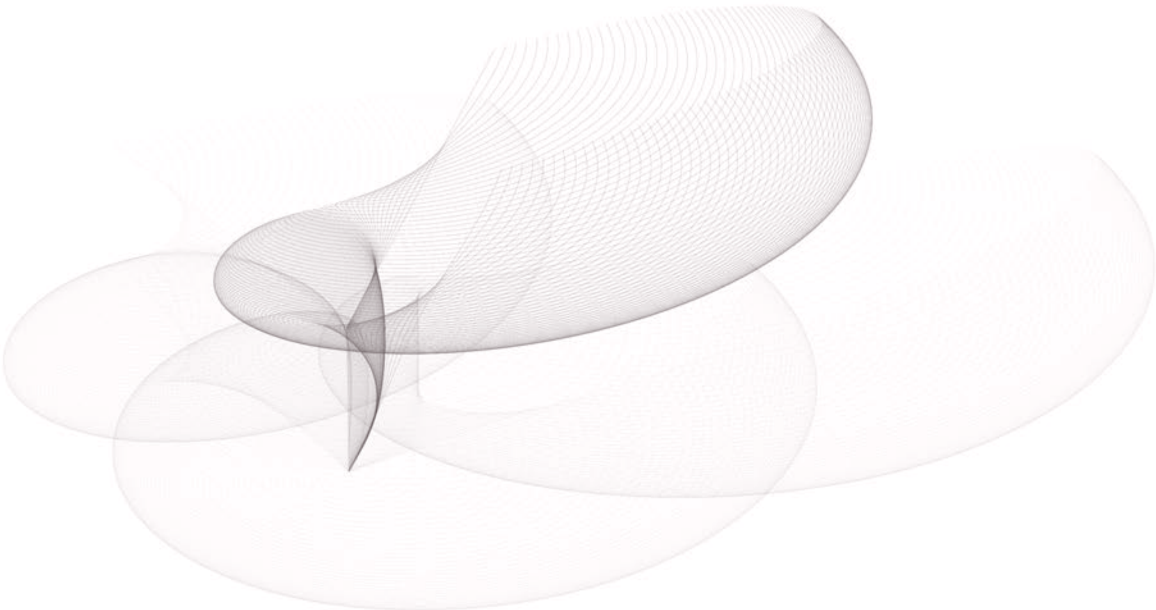
Spiralling and ascending

After the shape's radius incrementally increases, the shape is iteratively plotted according to a diagonal trajectory. Since *ascending* is occurring after *spiralling*, some of the resulting shapes are self-intersecting.



{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = u(sin(v)cos(u))
y = u+(u(sin(v)sin(u)))
z = cos(v)



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+(u(\sin(v)\cos(u)))$
$y = u+(u(\sin(v)\sin(u)))$
$z = \cos(v)$

Combining Transformations

Spiralling and ascending

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u+(u(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u+(u(\cos(u)))$
$y = u(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u+(u(\cos(u)))$
$y = u+(u(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u+(u(\cos(u)))$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u+(u(\cos(u)))$
$y = u+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = u+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u+\cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u+\cos(u)$
$y = u+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u+(u(\sin(v)\sin(u)))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+(u(\sin(v)\cos(u)))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+(u(\sin(v)\cos(u)))$
$y = u+(u(\sin(v)\sin(u)))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

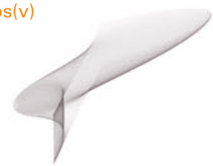
$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+(u(\sin(v)\cos(u)))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+(u(\sin(v)\cos(u)))$
$y = u+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = u+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

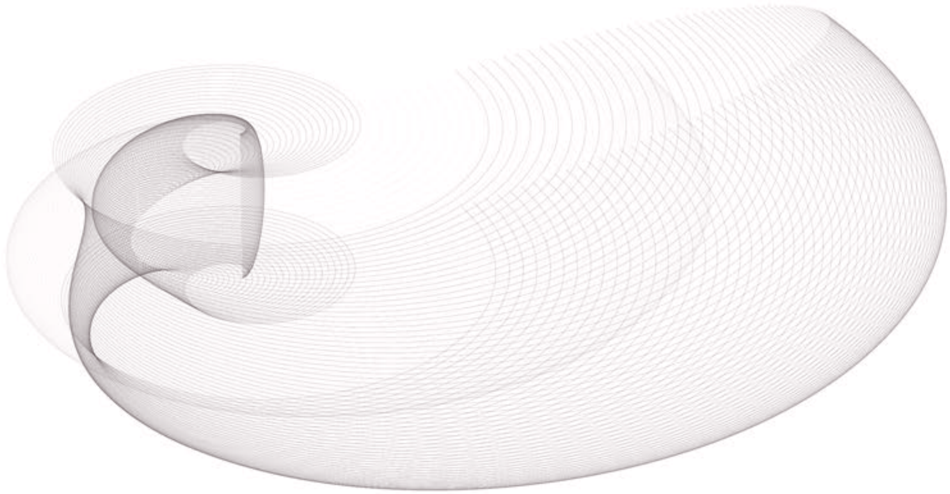
$x = u+\sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u+\sin(v)\cos(u)$
$y = u+\sin(v)\sin(u)$
$z = \cos(v)$

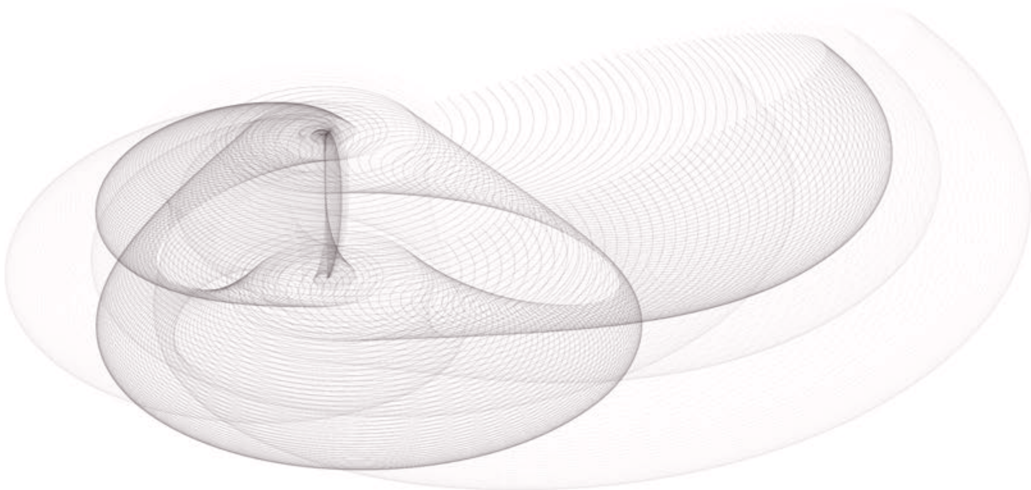
Texturing and spiralling

Before the shape is spiralled, it is transformed through *texturing*. When a textured sphere's radius incrementally increases under *spiralling*, its texture also incrementally becomes larger.



{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = u(cos(2u)/2+sin(v)cos(u))
y = u(sin(2u)/2+sin(v)sin(u))
z = cos(v)



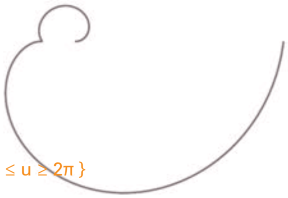
{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = u(cos(4u)/4+sin(v)cos(u))
y = u(sin(4u)/4+sin(v)sin(u))
z = cos(v)

Texturing and spiralling

{ u | 0 ≤ u ≥ 2π }

x = u(cos(u))
y = u(sin(u))



{ u | 0 ≤ u ≥ 2π }

x = u(cos(2u)/2+cos(u))
y = u(sin(2u)/2+sin(u))

{ u | 0 ≤ u ≥ 2π }

x = u(cos(3u)/3+cos(u))
y = u(sin(3u)/3+sin(u))

{ u | 0 ≤ u ≥ 2π }

x = u(cos(4u)/4+cos(u))
y = u(sin(4u)/4+sin(u))

{ u | 0 ≤ u ≥ 2π }

x = u(cos(u))
y = sin(u)

{ u | 0 ≤ u ≥ 2π }

x = u(cos(2u)/2+cos(u))
y = sin(2u)/2+sin(u)

{ u | 0 ≤ u ≥ 2π }

x = u(cos(3u)/3+cos(u))
y = sin(3u)/3+sin(u)

{ u | 0 ≤ u ≥ 2π }

x = u(cos(4u)/4+cos(u))
y = sin(4u)/4+sin(u)

{ u | 0 ≤ u ≥ 2π }

x = cos(u)
y = sin(u)

{ u | 0 ≤ u ≥ 2π }

x = cos(2u)/2+cos(u)
y = sin(2u)/2+sin(u)

{ u | 0 ≤ u ≥ 2π }

x = cos(3u)/3+cos(u)
y = sin(3u)/3+sin(u)

{ u | 0 ≤ u ≥ 2π }

x = cos(4u)/4+cos(u)
y = sin(4u)/4+sin(u)

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

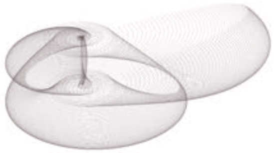
$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(2u)/2+\sin(v)\cos(u))$
$y = u(\sin(2u)/2+\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(3u)/3+\sin(v)\cos(u))$
$y = u(\sin(3u)/3+\sin(v)\sin(u))$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(4u)/4+\sin(v)\cos(u))$
$y = u(\sin(4u)/4+\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(2u)/2+\sin(v)\cos(u))$
$y = \sin(2u)/2+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(3u)/3+\sin(v)\cos(u))$
$y = \sin(3u)/3+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(4u)/4+\sin(v)\cos(u))$
$y = \sin(4u)/4+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(2u)/2+\sin(v)\cos(u)$
$y = \sin(2u)/2+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

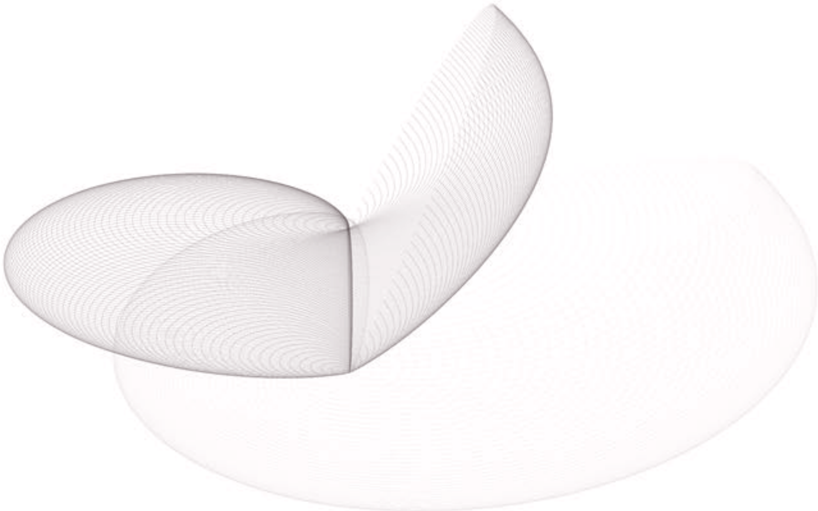
$x = \cos(3u)/3+\sin(v)\cos(u)$
$y = \sin(3u)/3+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(4u)/4+\sin(v)\cos(u)$
$y = \sin(4u)/4+\sin(v)\sin(u)$
$z = \cos(v)$

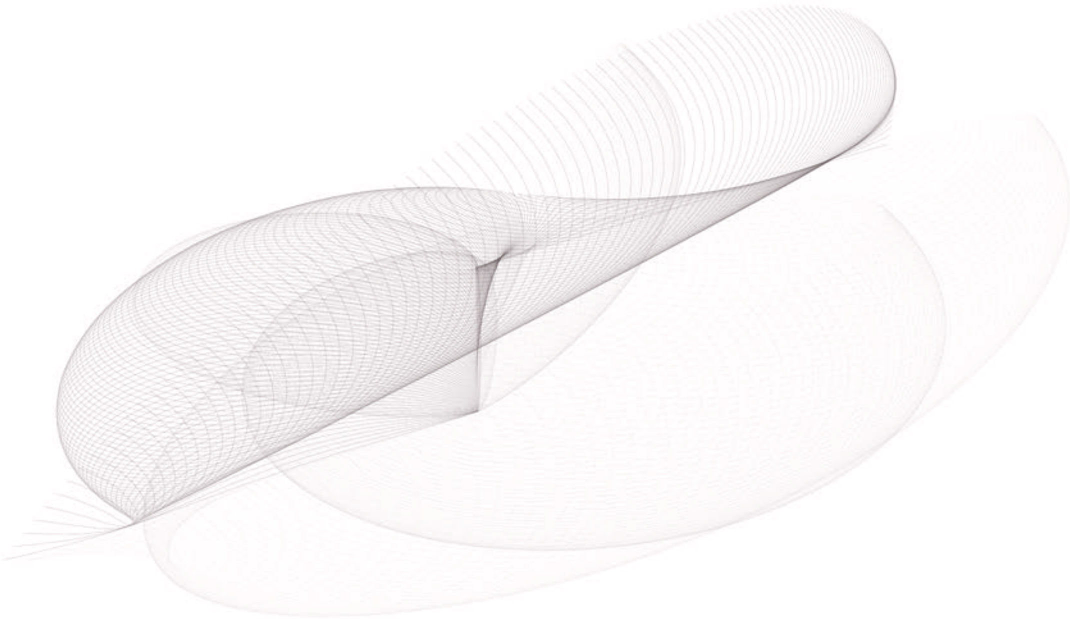
Bending and spiralling

In this instance, the shape is first transformed through *bending* and then the bent shape is transformed through *spiralling*. The order of operations is very important with these two transformations.



{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = u(sin(v)cos(u))
y = u(sin(v)+sin(v)sin(u))
z = cos(v)



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(v)+\sin(v)\cos(u))$
$y = u(\sin(v)+\sin(v)\sin(u))$
$z = \cos(v)$

Bending and spiralling

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

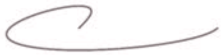
$x = u(\cos(u))$
$y = u(\sin(v)+\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(v)+\cos(u))$
$y = u(\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(v)+\cos(u))$
$y = u(\sin(v)+\sin(u))$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(u))$
$y = \sin(v)+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(v)+\cos(u))$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(v)+\cos(u))$
$y = \sin(v)+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(v)+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+\cos(u)$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+\cos(u)$
$y = \sin(v)+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)+\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(v)+\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(v)+\sin(v)\cos(u))$
$y = u(\sin(v)+\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(v)+\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(v)+\sin(v)\cos(u))$
$y = \sin(v)+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

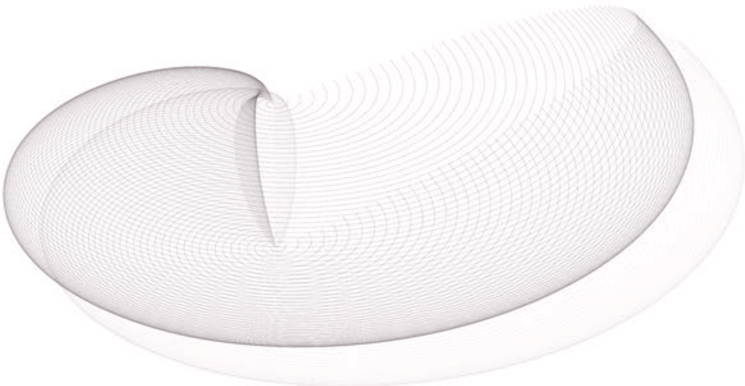
$x = \cos(v)+\sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+\sin(v)\cos(u)$
$y = \sin(v)+\sin(v)\sin(u)$
$z = \cos(v)$

Spiralling and bending

In this iteration, the shape is first spiralled and then bent. By *spiralling* the shape first, the *bending* operation becomes more subtle than it is in the previous combination.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)+(u(\sin(v)\sin(u)))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+(u(\sin(v)\cos(u)))$
$y = \sin(v)+(u(\sin(v)\sin(u)))$
$z = \cos(v)$

Spiralling and bending

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(u))$
$y = \sin(v)+(u(\sin(u)))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+(u(\cos(u)))$
$y = u(\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+(u(\cos(u)))$
$y = \sin(v)+(u(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\cos(u))$
$y = \sin(v)+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+(u(\cos(u)))$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+(u(\cos(u)))$
$y = \sin(v)+\sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(v)+\sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+\cos(u)$
$y = \sin(u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(v)+\cos(u)$
$y = \sin(v)+\sin(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)+(u(\sin(v)\sin(u)))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \cos(v)+(u(\sin(v)\cos(u)))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \cos(v)+(u(\sin(v)\cos(u)))$
$y = \sin(v)+(u(\sin(v)\sin(u)))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \cos(v)+(u(\sin(v)\cos(u)))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \cos(v)+(u(\sin(v)\cos(u)))$
$y = \sin(v)+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)+\sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

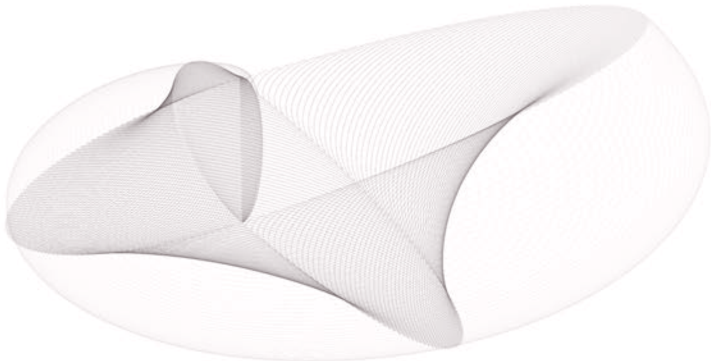
$x = \cos(v)+\sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \cos(v)+\sin(v)\cos(u)$
$y = \sin(v)+\sin(v)\sin(u)$
$z = \cos(v)$

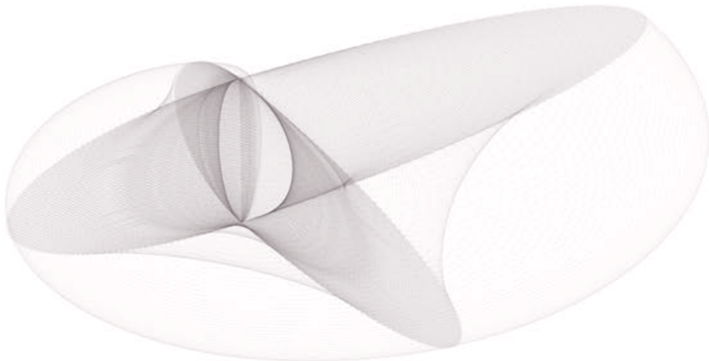
Pinching and spiralling

When a shape is first transformed under *pinching* and then with *spiralling*, its pinched edges exponentially increase. As with *texturing and spiralling*, the underlying spiral order is maintained.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u)^3)$
$y = u(\sin(v)\sin(u)^3)$
$z = \cos(v)$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u)^7)$
$y = u(\sin(v)\sin(u)^7)$
$z = \cos(v)$

Pinching and spiralling

$\{ u \mid 0 \leq u \geq 2\pi \}$

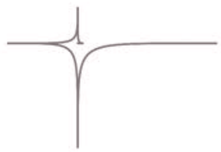
$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u)^3)$
$y = u(\sin(u)^3)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u)^5)$
$y = u(\sin(u)^5)$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u)^7)$
$y = u(\sin(u)^7)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u)^3)$
$y = \sin(u)^3$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u)^5)$
$y = \sin(u)^5$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u)^7)$
$y = \sin(u)^7$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)^3$
$y = \sin(u)^3$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)^5$
$y = \sin(u)^5$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)^7$
$y = \sin(u)^7$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u)^3)$
$y = u(\sin(v)\sin(u)^3)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u)^5)$
$y = u(\sin(v)\sin(u)^5)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u)^7)$
$y = u(\sin(v)\sin(u)^7)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u)^3)$
$y = \sin(v)\sin(u)^3$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u)^5)$
$y = \sin(v)\sin(u)^5$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u)^7)$
$y = \sin(v)\sin(u)^7$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)^3$
$y = \sin(v)\sin(u)^3$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)^5$
$y = \sin(v)\sin(u)^5$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)^7$
$y = \sin(v)\sin(u)^7$
$z = \cos(v)$

Flattening and spiralling

Before the shape spirals, it is transformed through *flattening*. Because *spiralling* is the last operation, the global order is a spiral with locally flattened edges. The order of operations is critical.

{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = u(sin(sin(v)cos(u)))
y = u(sin(sin(v)sin(u)))
z = sin(cos(v))

{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = u(sin(sin(sin(sin(v)cos(u)))))
y = u(sin(sin(sin(sin(v)sin(u)))))
z = sin(sin(sin(cos(v))))

Flattening and spiralling

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\sin(\cos(u)))$
$y = u(\sin(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\sin(\sin(\cos(u))))$
$y = u(\sin(\sin(\sin(u))))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\sin(\sin(\sin(\cos(u)))))$
$y = u(\sin(\sin(\sin(\sin(u)))))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\sin(\cos(u)))$
$y = \sin(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\sin(\sin(\cos(u))))$
$y = \sin(\sin(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\sin(\sin(\sin(\cos(u)))))$
$y = \sin(\sin(\sin(\sin(u))))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\cos(u))$
$y = \sin(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\sin(\cos(u)))$
$y = \sin(\sin(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\sin(\sin(\cos(u))))$
$y = \sin(\sin(\sin(\sin(u))))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(\sin(v)\cos(u)))$
$y = u(\sin(\sin(v)\sin(u)))$
$z = \sin(\cos(v))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(\sin(\sin(v)\cos(u))))$
$y = u(\sin(\sin(\sin(v)\sin(u))))$
$z = \sin(\sin(\cos(v)))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(\sin(\sin(\sin(v)\cos(u)))))$
$y = u(\sin(\sin(\sin(\sin(v)\sin(u)))))$
$z = \sin(\sin(\sin(\cos(v))))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(\sin(v)\cos(u)))$
$y = \sin(\sin(v)\sin(u))$
$z = \sin(\cos(v))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(\sin(\sin(v)\cos(u))))$
$y = \sin(\sin(\sin(v)\sin(u)))$
$z = \sin(\sin(\cos(v)))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u(\sin(\sin(\sin(\sin(v)\cos(u)))))$
$y = \sin(\sin(\sin(\sin(v)\sin(u))))$
$z = \sin(\sin(\sin(\cos(v))))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(\sin(v)\cos(u))$
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$z = \sin(\cos(v))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

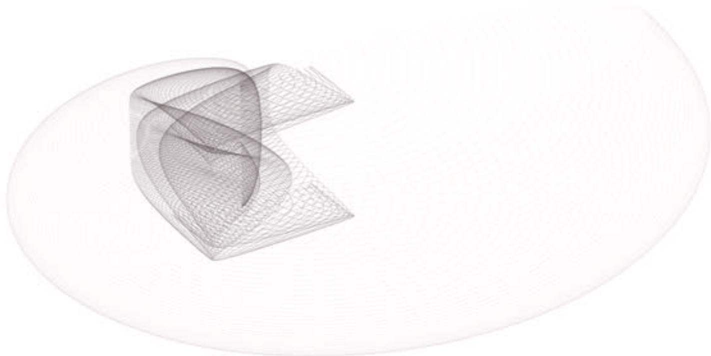
$x = \sin(\sin(\sin(v)\cos(u)))$
$y = \sin(\sin(\sin(v)\sin(u)))$
$z = \sin(\sin(\cos(v)))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(\sin(\sin(\sin(v)\cos(u))))$
$y = \sin(\sin(\sin(\sin(v)\sin(u))))$
$z = \sin(\sin(\sin(\cos(v))))$

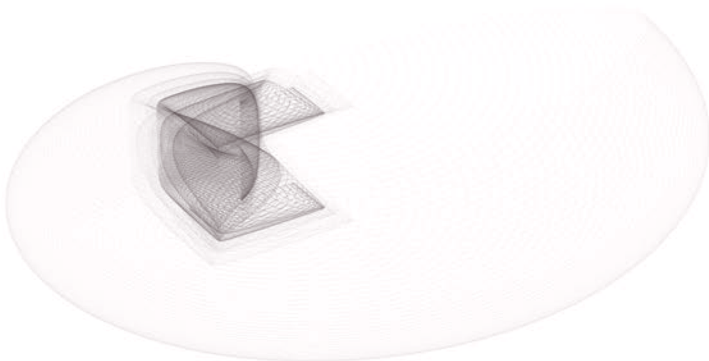
Spiralling and flattening

Since the last operation is a *flattening* transformation, the spiralled sphere is contained to the boundary of a cube. The overall boundary of the shape is determined by the last operation.



{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = sin(u(sin(v)cos(u)))
y = sin(u(sin(v)sin(u)))
z = sin(cos(v))



{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = sin(sin(sin(u(sin(v)cos(u)))))
y = sin(sin(sin(u(sin(v)sin(u)))))
z = sin(sin(sin(cos(v))))

Spiralling and flattening

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = u(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(u(\cos(u)))$
$y = \sin(u(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\sin(u(\cos(u))))$
$y = \sin(\sin(u(\sin(u))))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\sin(\sin(u(\cos(u)))))$
$y = \sin(\sin(\sin(u(\sin(u)))))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = u(\cos(u))$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(u(\cos(u)))$
$y = \sin(\sin(u))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\sin(u(\cos(u))))$
$y = \sin(\sin(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\sin(\sin(u(\cos(u)))))$
$y = \sin(\sin(\sin(\sin(u))))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \cos(u)$
$y = \sin(u)$

$\{ u \mid 0 \leq u \geq 2\pi \}$

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$y = \sin(\sin(u))$

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$x = \sin(\sin(\cos(u)))$
$y = \sin(\sin(\sin(u)))$

$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = \sin(\sin(\sin(\cos(u))))$
$y = \sin(\sin(\sin(\sin(u))))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(u(\sin(v)\cos(u)))$
$y = \sin(u(\sin(v)\sin(u)))$
$z = \sin(\cos(v))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\sin(u(\sin(v)\cos(u))))$
$y = \sin(\sin(u(\sin(v)\sin(u))))$
$z = \sin(\sin(\cos(v)))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\sin(\sin(u(\sin(v)\cos(u)))))$
$y = \sin(\sin(\sin(u(\sin(v)\sin(u)))))$
$z = \sin(\sin(\sin(\cos(v))))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(u(\sin(v)\cos(u)))$
$y = \sin(\sin(v)\sin(u))$
$z = \sin(\cos(v))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\sin(u(\sin(v)\cos(u))))$
$y = \sin(\sin(\sin(v)\sin(u)))$
$z = \sin(\sin(\cos(v)))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\sin(\sin(u(\sin(v)\cos(u)))))$
$y = \sin(\sin(\sin(\sin(v)\sin(u))))$
$z = \sin(\sin(\sin(\cos(v))))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\sin(v)\cos(u))$
$y = \sin(\sin(v)\sin(u))$
$z = \sin(\cos(v))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(\sin(\sin(v)\cos(u)))$
$y = \sin(\sin(\sin(v)\sin(u)))$
$z = \sin(\sin(\cos(v)))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

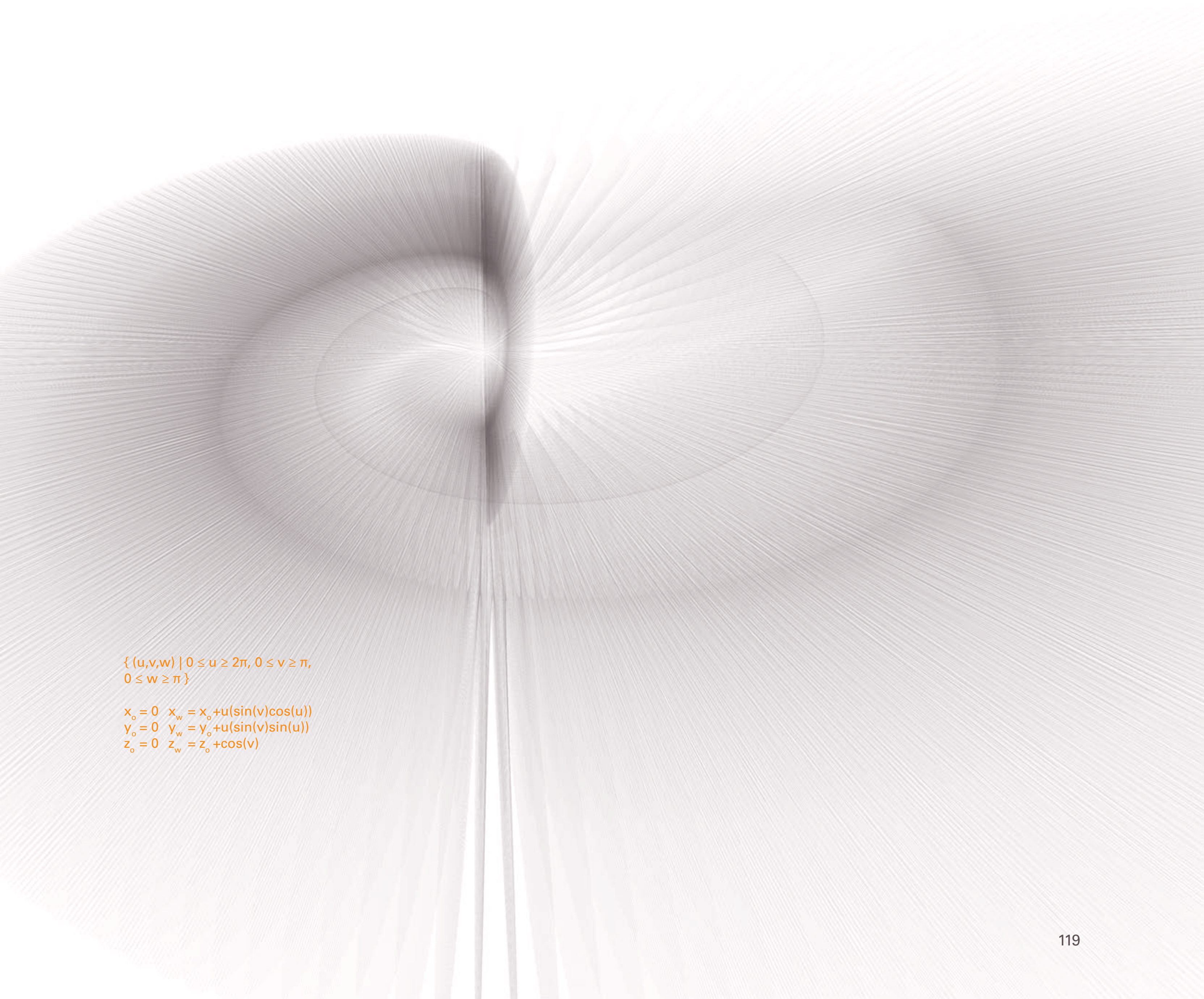
$x = \sin(\sin(\sin(\sin(v)\cos(u))))$
$y = \sin(\sin(\sin(\sin(v)\sin(u))))$
$z = \sin(\sin(\sin(\cos(v))))$

Spiralling and thickening

***Thickening* introduces a new parameter, w, and a second subset of x, y and z. *Thickening* does not transform the *spiralling* order; instead, it allows a different range of its shape to be expressed.**

$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,$
$0 \leq w \geq \pi/3 \}$

$x_o = 0 \quad x_w = x_o + u(\sin(v)\cos(u))$
$y_o = 0 \quad y_w = y_o + u(\sin(v)\sin(u))$
$z_o = 0 \quad z_w = z_o + \cos(v)$



$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,$
$0 \leq w \geq \pi \}$

$x_o = 0 \quad x_w = x_o + u(\sin(v)\cos(u))$
$y_o = 0 \quad y_w = y_o + u(\sin(v)\sin(u))$
$z_o = 0 \quad z_w = z_o + \cos(v)$

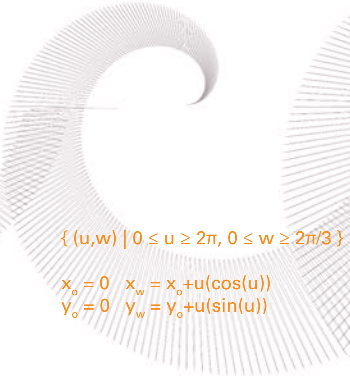
Spiralling and thickening

{ u | 0 ≤ u ≥ 2π }

$x = u(\cos(u))$
$y = u(\sin(u))$

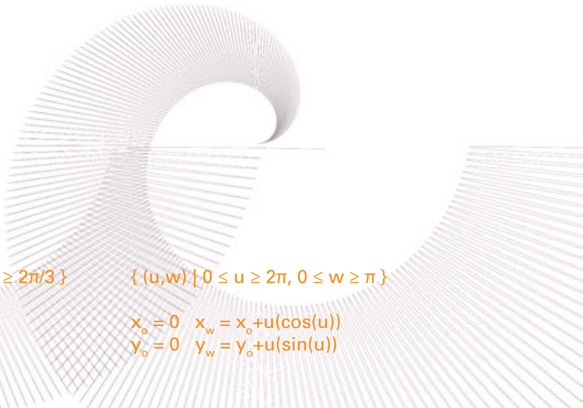
{ (u,w) | 0 ≤ u ≥ 2π, 0 ≤ w ≥ π/3 }

$x_o = 0 \quad x_w = x_o + u(\cos(u))$
$y_o = 0 \quad y_w = y_o + u(\sin(u))$



{ (u,w) | 0 ≤ u ≥ 2π, 0 ≤ w ≥ 2π/3 }

$x_o = 0 \quad x_w = x_o + u(\cos(u))$
$y_o = 0 \quad y_w = y_o + u(\sin(u))$



{ (u,w) | 0 ≤ u ≥ 2π, 0 ≤ w ≥ π }

$x_o = 0 \quad x_w = x_o + u(\cos(u))$
$y_o = 0 \quad y_w = y_o + u(\sin(u))$

{ u | 0 ≤ u ≥ 2π }

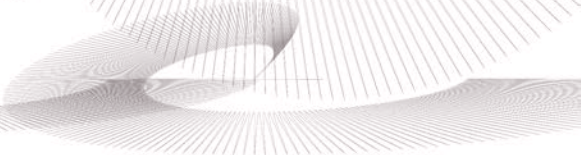
$x = u(\cos(u))$
$y = \sin(u)$

{ (u,w) | 0 ≤ u ≥ 2π, 0 ≤ w ≥ π/3 }

$x_o = 0 \quad x_w = x_o + u(\cos(u))$
$y_o = 0 \quad y_w = y_o + \sin(u)$

{ (u,w) | 0 ≤ u ≥ 2π, 0 ≤ w ≥ 2π/3 }

$x_o = 0 \quad x_w = x_o + u(\cos(u))$
$y_o = 0 \quad y_w = y_o + \sin(u)$



{ (u,w) | 0 ≤ u ≥ 2π, 0 ≤ w ≥ π }

$x_o = 0 \quad x_w = x_o + u(\cos(u))$
$y_o = 0 \quad y_w = y_o + \sin(u)$

{ u | 0 ≤ u ≥ 2π }

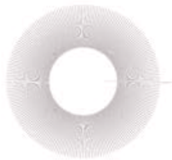
$x = \cos(u)$
$y = \sin(u)$

{ (u,w) | 0 ≤ u ≥ 2π, 0 ≤ w ≥ π/3 }

$x_o = 0 \quad x_w = x_o + \cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(u)$

{ (u,w) | 0 ≤ u ≥ 2π, 0 ≤ w ≥ 2π/3 }

$x_o = 0 \quad x_w = x_o + \cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(u)$



{ (u,w) | 0 ≤ u ≥ 2π, 0 ≤ w ≥ π }

$x_o = 0 \quad x_w = x_o + \cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = u(\sin(v)\cos(u))$
$y = u(\sin(v)\sin(u))$
$z = \cos(v)$

$\{ (u,v,w) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi,$
$0 \le w \ge \pi/3 \}$

$x_o = 0 \quad x_w = x_o + u(\sin(v)\cos(u))$
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$\{ (u,v,w) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi,$
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$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

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$\{ (u,v,w) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi,$
$0 \le w \ge \pi \}$

$x_o = 0 \quad x_w = x_o + u(\sin(v)\cos(u))$
$y_o = 0 \quad y_w = y_o + \sin(v)\sin(u)$
$z_o = 0 \quad z_w = z_o + \cos(v)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v,w) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi,$
$0 \le w \ge \pi/3 \}$

$x_o = 0 \quad x_w = x_o + \sin(v)\cos(u)$
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$\{ (u,v,w) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi,$
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$z_o = 0 \quad z_w = z_o + \cos(v)$

$\{ (u,v,w) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi,$
$0 \le w \ge \pi \}$

$x_o = 0 \quad x_w = x_o + \sin(v)\cos(u)$
$y_o = 0 \quad y_w = y_o + \sin(v)\sin(u)$
$z_o = 0 \quad z_w = z_o + \cos(v)$

Combining Shapes

‘For the artist communication with nature remains the most essential condition. The artist is human, himself nature; part of nature within natural space.’

(Paul Klee, 1944)

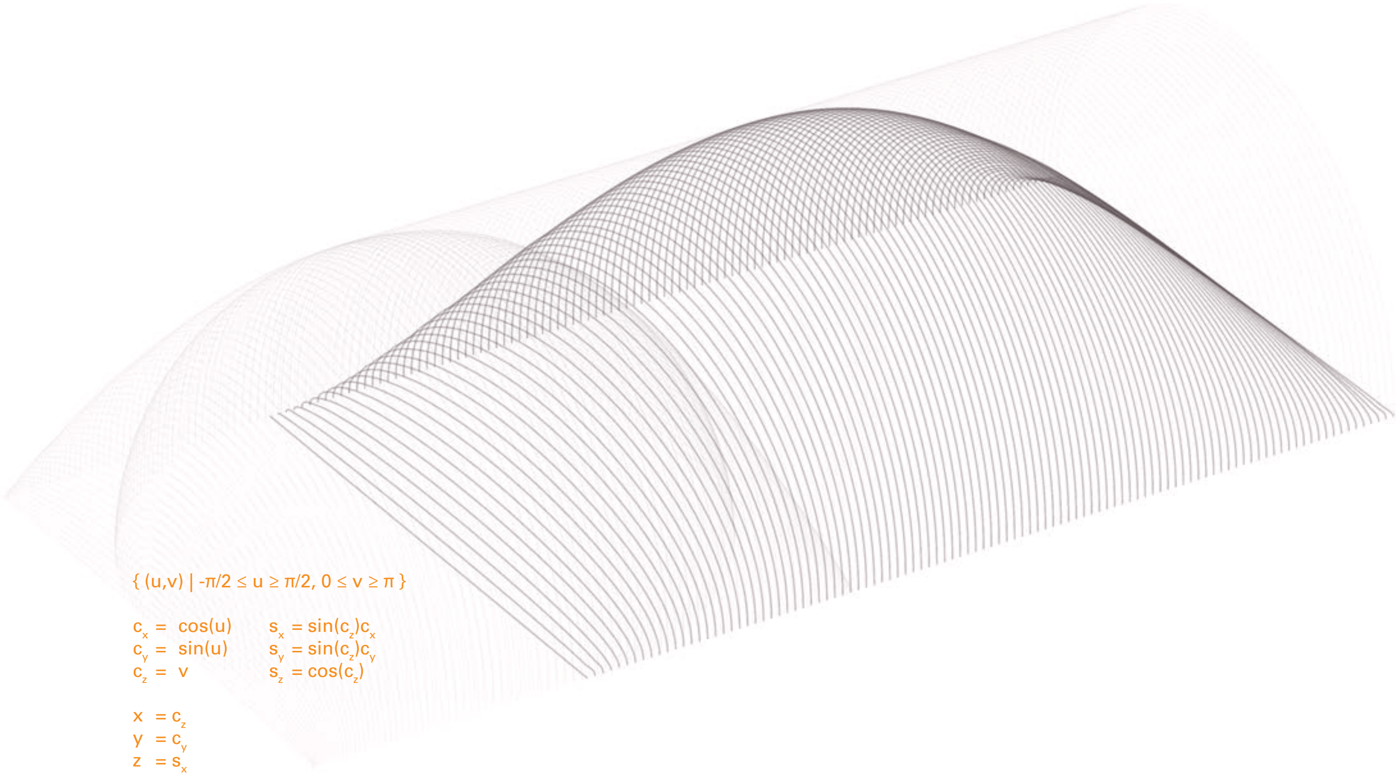
If you imagine a particular shape as a species with a specific DNA, think of the previous two chapters as manipulating or mutating a shape's DNA. This chapter is akin to geometric breeding. Here, it is the shapes that combine with one another. Instead of simply using sine and cosine as inputs, the shapes that are defined by sine and cosine become the inputs. Calculating with pieces larger than sine and cosine gives a different type of geometric deformation to calculating with sine and cosine themselves. The manner in which a shape distorts is more like moulding with a ball of clay. The periodic nature of trigonometry is slightly less obvious within these types of combinations.

Within this chapter, a cylinder's and a sphere's x-, y- and z-coordinates are defined as the parts to calculate with. Initially, the parts of the cylinder and sphere are utilized to generate a barrel vault and a dome. Then elements defining these two shapes are used to form a mound-like shape similar to a hill or a slope. This landscape-like mound becomes the constant starting shape throughout the rest of the chapter. As the shape transforms, the resultants suggest other natural land formations.

Throughout this chapter, a cylinder's x, y and z are defined as c_x, c_y and c_z and a sphere's x, y and z are defined as s_x, s_y and s_z.

A mound

Reversing the x and z of a shape alters that shape's orientation, making it perpendicular to its previous orientation. Here, a cylinder and a sphere are reorientated along the horizontal axis, and cut into a barrel vault and dome. By combining parts of the cylinder and the sphere, a *mound* is generated. The *mound* is confined to either a square or a rectangular plan, depending on which shape's x-coordinate is expressed.



$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u) \qquad s_x = \sin(c_z)c_x$
$c_y = \sin(u) \qquad s_y = \sin(c_z)c_y$
$c_z = v \qquad s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = 0$
$y = c_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = 0$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = 0$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = c_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = s_z$
$y = s_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = s_z$
$y = c_y$
$z = s_x$

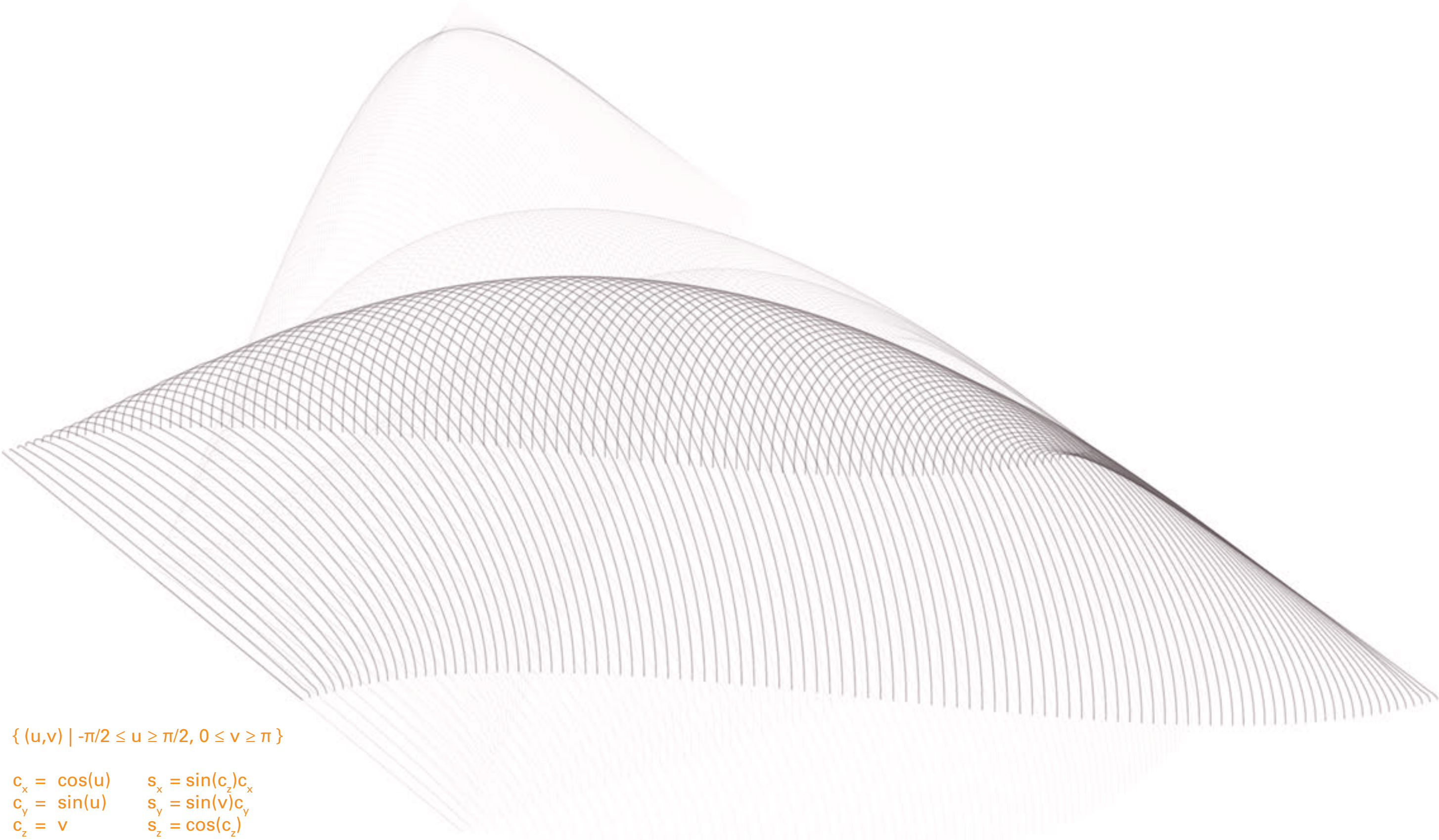
$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x$

A meandering mound

Adding parts of shapes together in the y-coordinate transforms a shape's plan-view figure. For instance, adding the y-coordinate of a sphere would bulge the plan of the *mound*. Adding the z-coordinate of a sphere forms a *meandering mound*.



$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $\quad s_y = \sin(v)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = c_z$
$y = c_y + s_z$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = 0$
$y = c_y+s_z$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+s_z$
$z = 0$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = 0$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+s_z$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+c_z$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+s_y$
$z = s_x$

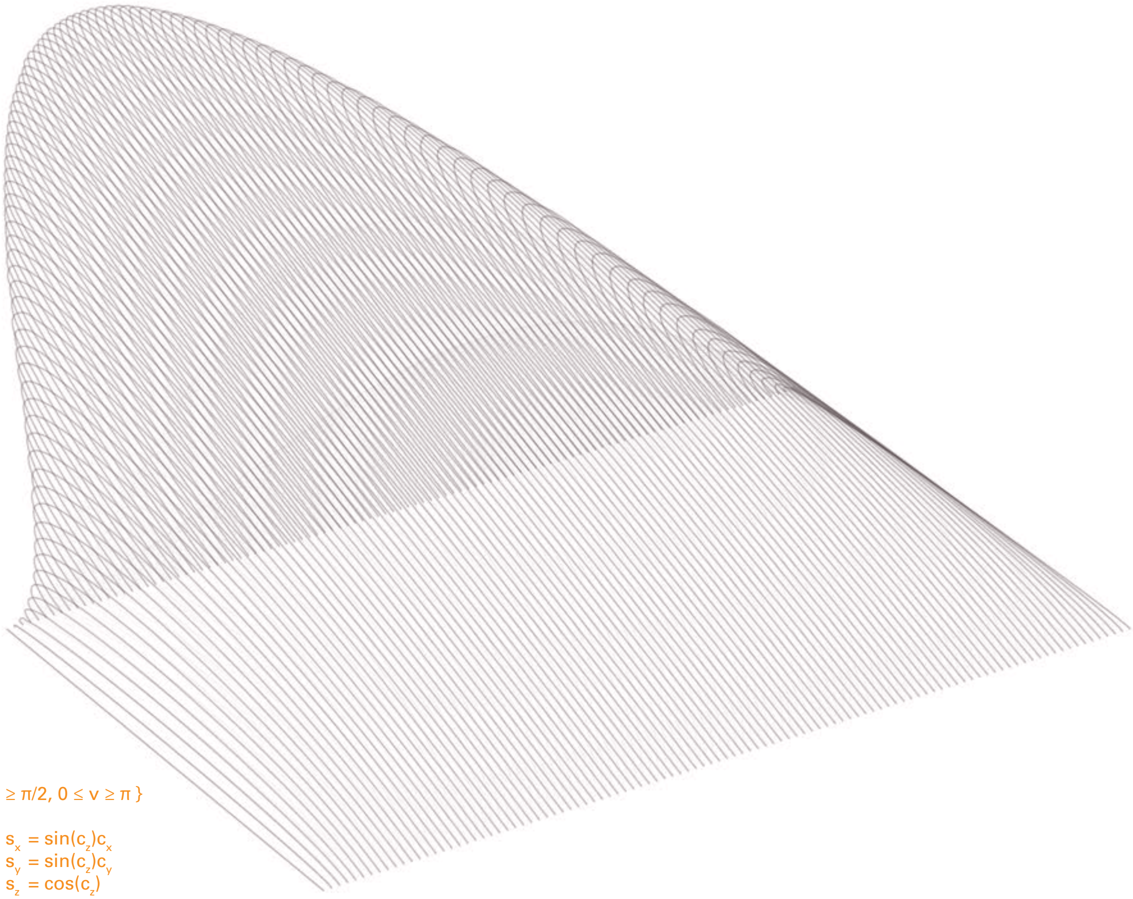
$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+s_z$
$z = s_x$

A leaning mound

Adding a sphere's x-coordinate to the y-coordinate of a *mound* causes it to lean. The lean increases as duplicates of the sphere's x-coordinate are added, but its rectilinear base is never altered.



$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+s_x+s_x+s_x$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = 0$
$y = c_y+s_x+s_x+s_x$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+s_x+s_x+s_x$
$z = 0$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = 0$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+s_x+s_x+s_x$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+s_x$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+s_x+s_x$
$z = s_x$

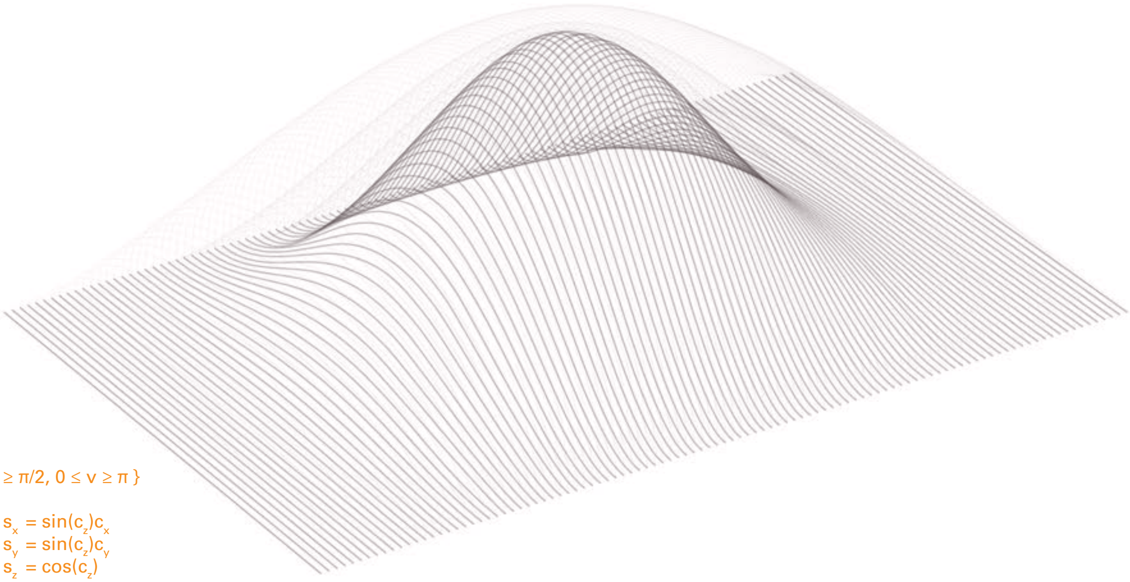
$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y+s_x+s_x+s_x$
$z = s_x$

A steeper mound

Multiplying the x-coordinate of a sphere in the z-coordinate of a *mound* increases its steepness. As more x-coordinates of a sphere are multiplied, the *mound* becomes even steeper.



$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u) \qquad s_x = \sin(c_z)c_x$
$c_y = \sin(u) \qquad s_y = \sin(c_z)c_y$
$c_z = v \qquad s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x s_x s_x s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2, 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = 0$
$y = c_y$
$z = s_x s_x s_x s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2, 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = 0$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2, 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = 0$
$z = s_x s_x s_x s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2, 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x s_x s_x s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2, 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2, 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2, 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x s_x s_x$

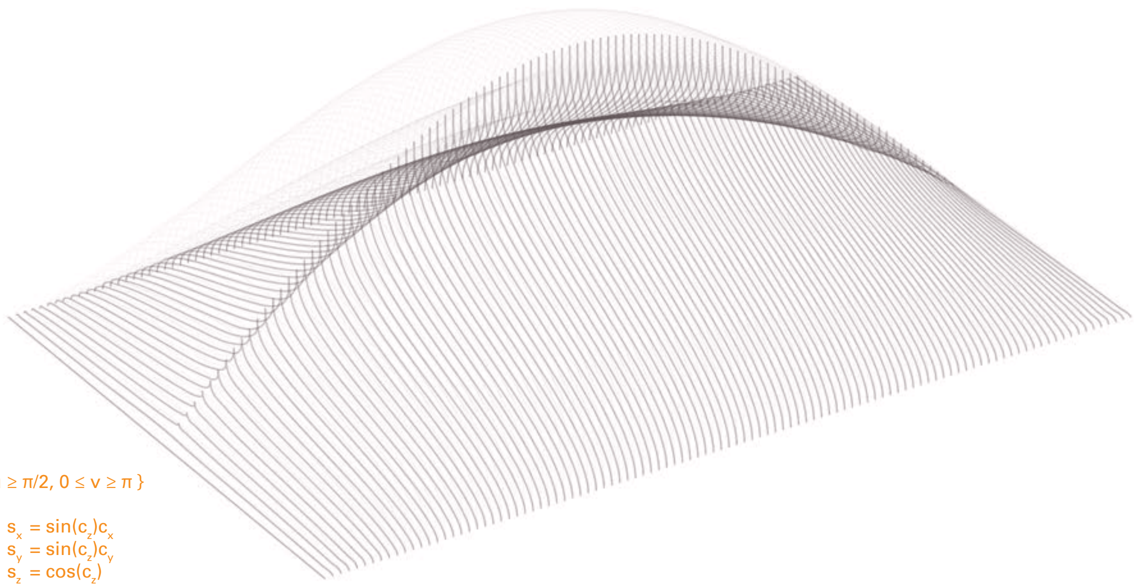
$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2, 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x s_x s_x s_x$

A creased mound

A *mound* is pinched or combed into a mohawk-like crease. As the y-coordinate of a cylinder is multiplied iteratively in the y-coordinate of the *mound*, the crease increases in depth.



$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u) \qquad s_x = \sin(c_z)c_x$
$c_y = \sin(u) \qquad s_y = \sin(c_z)c_y$
$c_z = v \qquad s_z = \cos(c_z)$

$x = c_z$
$y = c_y c_y c_y c_y c_y c_y c_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = 0$
$y = c_yc_yc_yc_yc_yc_yc_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_yc_yc_yc_yc_yc_yc_y$
$z = 0$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = 0$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_yc_yc_yc_yc_yc_yc_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_yc_yc_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_yc_yc_yc_yc_y$
$z = s_x$

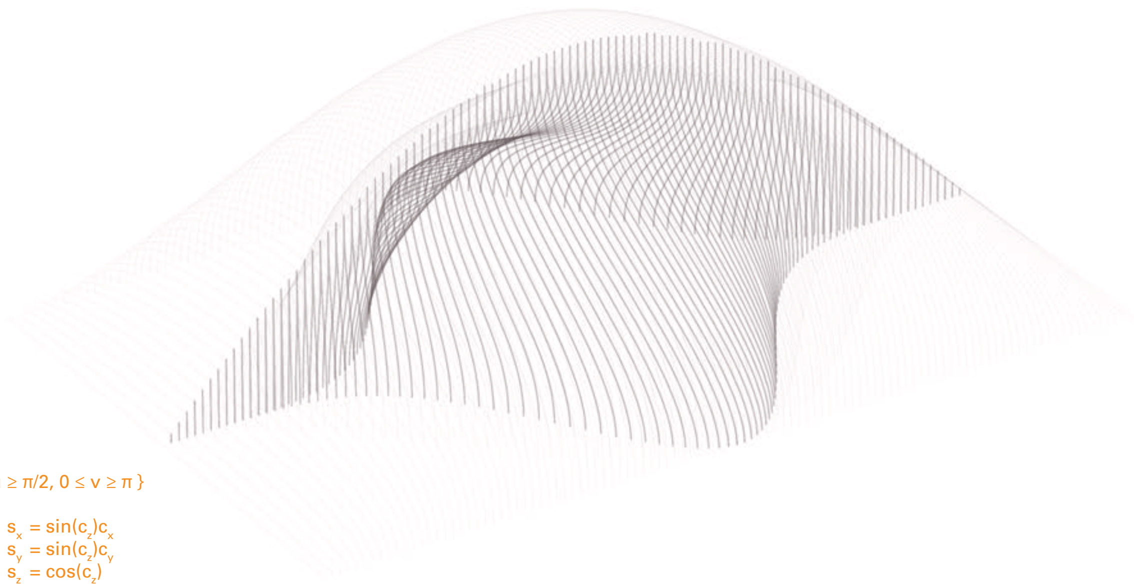
$\{ (u,v) \mid -\pi/2 \le u \ge \pi/2,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_yc_yc_yc_yc_yc_yc_y$
$z = s_x$

A creased and pinched mound

A *mound* is creased and pinched in elevation and plan. The pinching increases as the y-coordinate of a sphere is multiplied more times in the y-coordinate of the *mound*.



{ (u,v) | -π/2 ≤ u ≥ π/2, 0 ≤ v ≥ π }

$c_x = \cos(u)$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $\quad s_y = \sin(c_z)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_y s_y s_y s_y s_y s_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = 0$
$y = c_y s_y s_y s_y s_y s_y s_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_y s_y s_y s_y s_y s_y$
$z = 0$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = 0$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_y s_y s_y s_y s_y s_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_y s_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_y s_y s_y s_y$
$z = s_x$

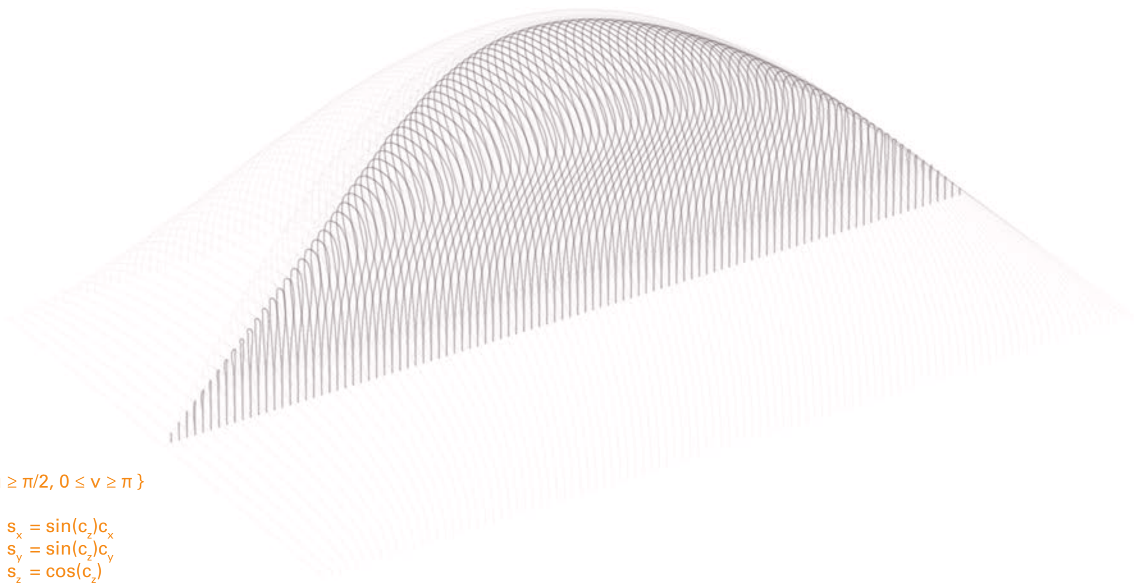
$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_y s_y s_y s_y s_y s_y$
$z = s_x$

A wedge

By multiplying the x-coordinate of a sphere in the y-coordinate of a *mound*, its underside is pinched. The rectilinear base, previously defined by two lines, is transformed into a single line.



$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $\quad s_y = \sin(c_z)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_x s_x s_x$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = 0$
$y = c_y s_x s_x s_x$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_x s_x s_x$
$z = 0$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = 0$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_x s_x s_x$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_x$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = c_z$
$y = c_y s_x s_x$
$z = s_x$

$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

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$x = c_z$
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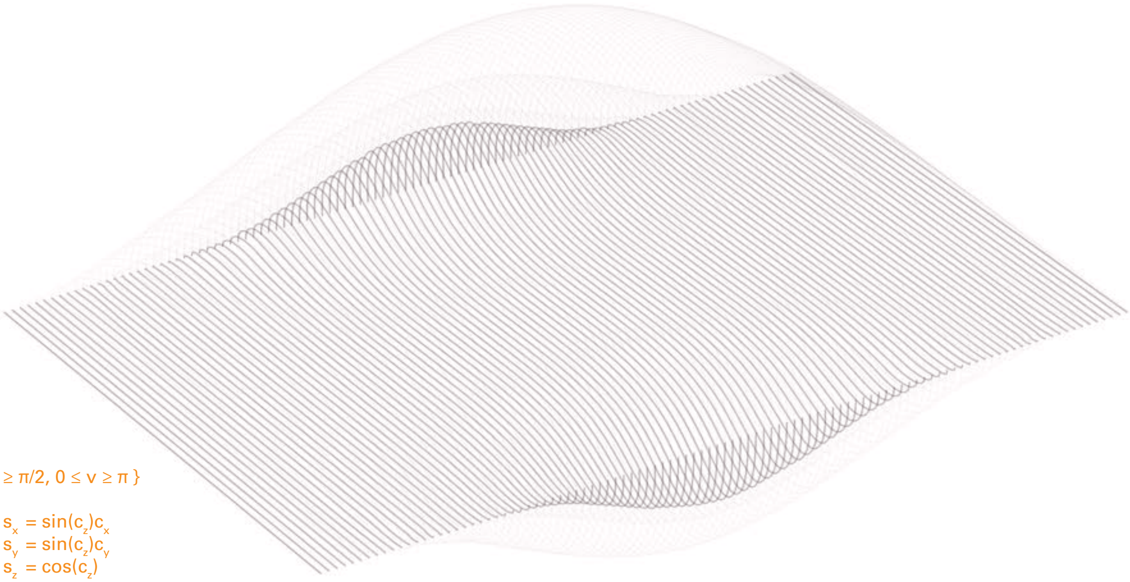
$z = s_x$

A ridge and trench

When cutting through the apex of a *mound*, an arched section is revealed. After the y-coordinate of a sphere is multiplied in the z-coordinate of the *mound*, the section transforms to a sine-curve-like profile. The *mound* is transformed into a *ridge and trench*.



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$x = c_z$
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$x = 0$
$y = c_y$
$z = s_x s_y s_y s_y s_y s_y$

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$x = c_z$
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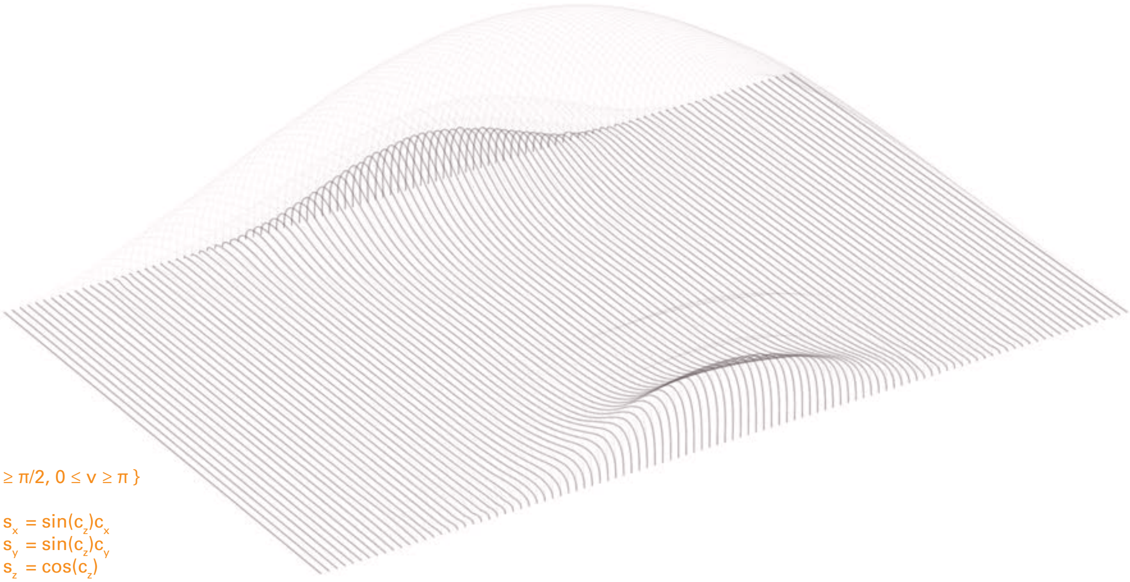
$z = s_x s_y s_y s_y s_y s_y$

Two ridges

Multiplying the y-coordinate of a sphere an even number of times in the z-coordinate of a *mound* transforms it into *two ridges*. If an odd number of the y-coordinate are multiplied, as on the previous page, the result is a *ridge and trench*.



{ (u,v) | -π/2 ≤ u ≥ π/2, 0 ≤ v ≥ π }

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
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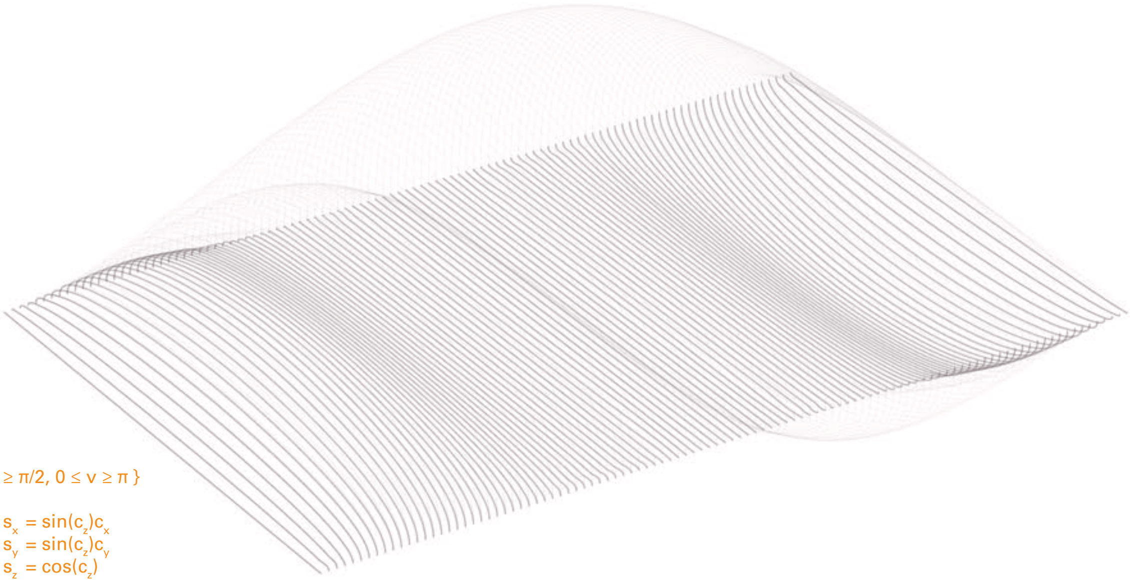
Another ridge and trench

By multiplying the z-coordinate of a sphere in the z-coordinate of a *mound*, another *ridge and trench* is formed. This *ridge and trench* is orientated perpendicular to the previous one.



$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

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$x = c_z$
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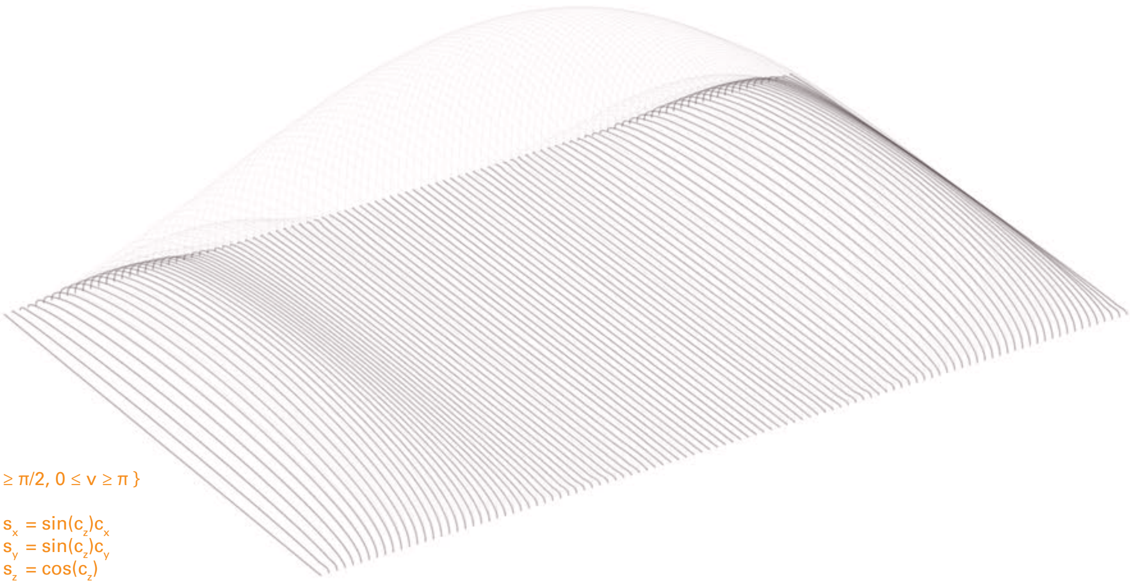
$z = s_x s_z s_z s_z s_z s_z$

A valley

When multiplying the z-coordinate of a sphere in the z-coordinate of a *mound* an even number of times, it is transformed into another set of *two ridges*, which together define a *valley*. If an odd number of these z-coordinates are multiplied, the result is a *ridge and trench*.



$\{ (u,v) \mid -\pi/2 \leq u \geq \pi/2,\ 0 \leq v \geq \pi \}$

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$x = c_z$
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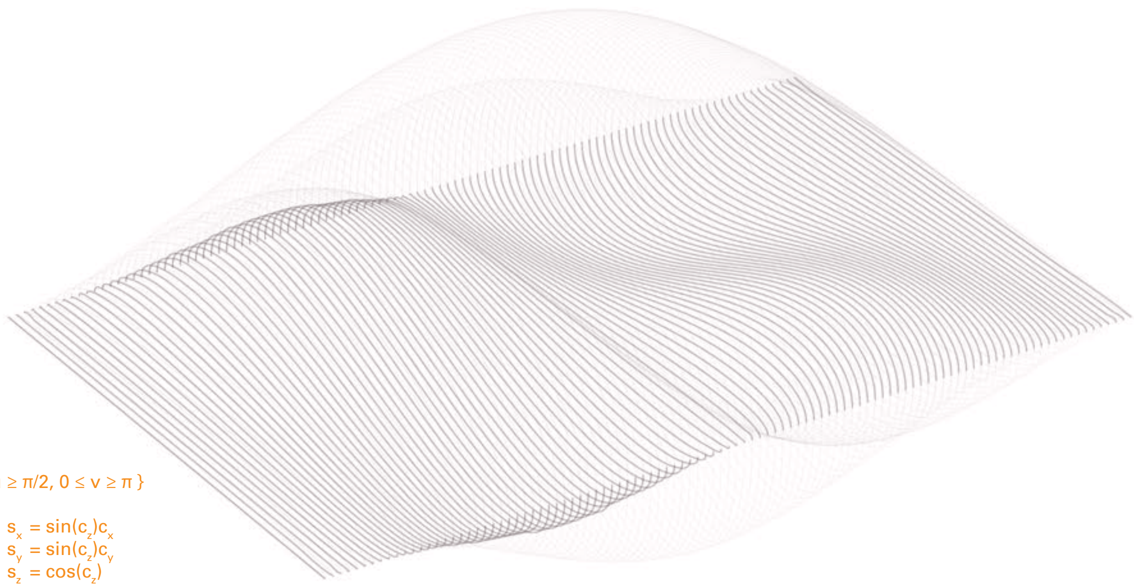
Moguls

If the y- and z-coordinates of a sphere are multiplied in the z-coordinate of a *mound*, it is transformed into *moguls*. The y- and z-coordinates independently produce ridges and trenches in opposite orientations. When they are combined, a saddle-like curvature arises.



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Analyzing

‘If the essentials of architecture lie in spheres, cones and cylinders, the generating and accusing lines of these forms are on a basis of pure geometry.’

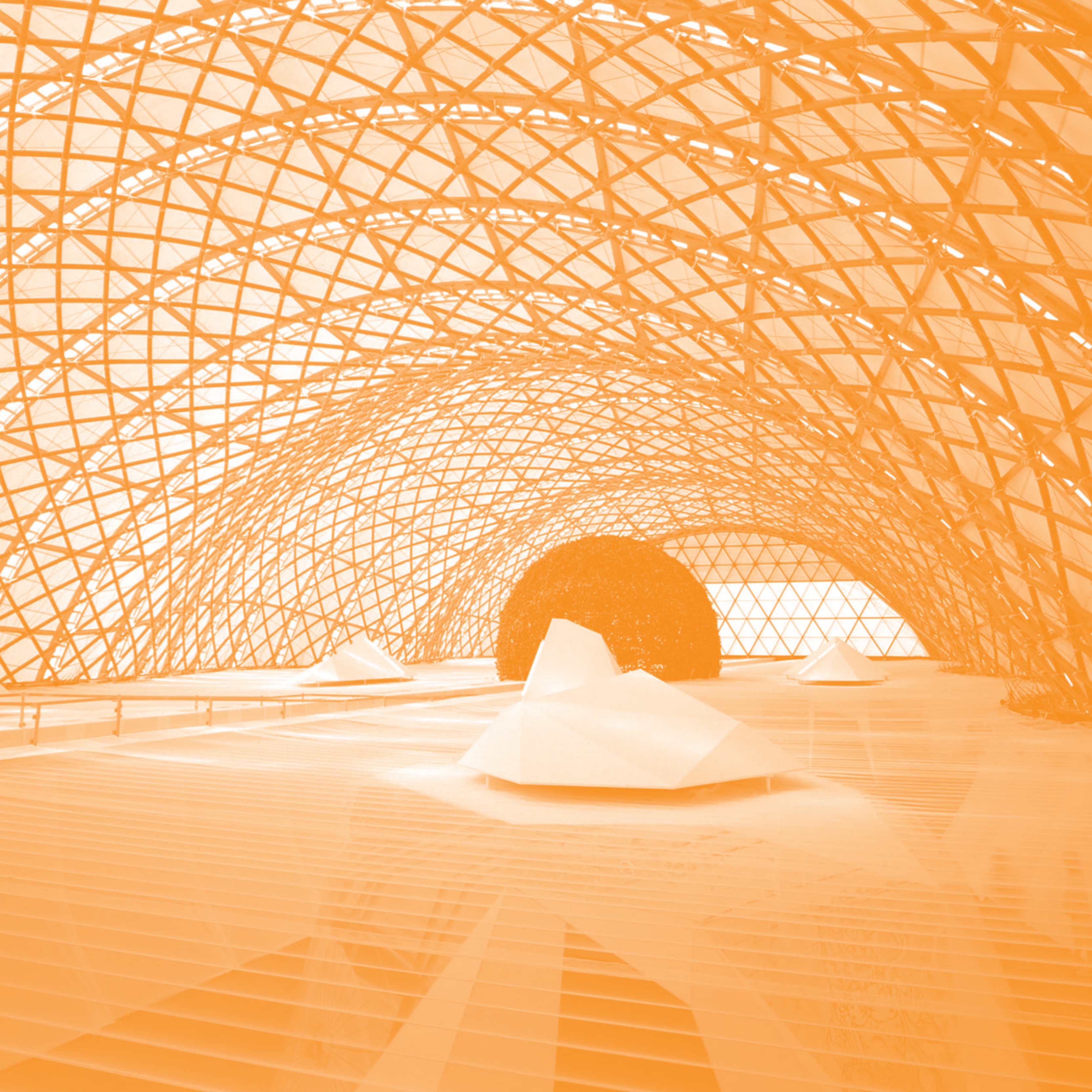
(Le Corbusier, 1931)

Precedent studies have become a tradition in architectural education; however, these analytical studies rarely translate directly into useful design devices. This chapter is a unique catalogue of architectural designs analyzed using mathematical equations. The chapter not only presents the built shape and its equation, but transforms each building's most basic topological ancestor (cylinder or sphere) into the building's final shape. After the final built shape is generated, the morphing process projects a series of mathematical variations beyond the building's form. This emphasizes that the act of studying precedents can be a generative mechanism.

Tools can constrain the way an individual designs. Similarly, the tool that is used to analyze a precedent study will influence how an individual constructs an understanding of that building. It may be possible to understand the spatial hierarchies, symmetries, proportions or generative rules behind a plan or section. However, a curved surface is significantly more difficult to analyze with conventional architectural tools.

Mathematics is a very specific analytical tool. Typically, buildings are categorized according to their type, such as a library or a museum. When analyzing buildings through mathematics, they become categorized according to their plastic shape. As catalogued in this chapter, buildings may relate to one another according to their mathematical DNA, but not necessarily according to their materiality or typology.

Each of the buildings within this chapter expresses a particular periodic function. After each building was analyzed, design variations were generated. The consistent design objective was to break the symmetrical relationships in the transverse and longitudinal axis of each project, without removing the elegant rhythmic nature. These design variations could not have been imagined without trigonometric functions. The choice of tool or medium will always help determine the set of possible design solutions. This is not necessarily the best way to analyze precedents, but one particular way – a method that should perhaps be considered especially when there is an interest in the formal qualities of a project.



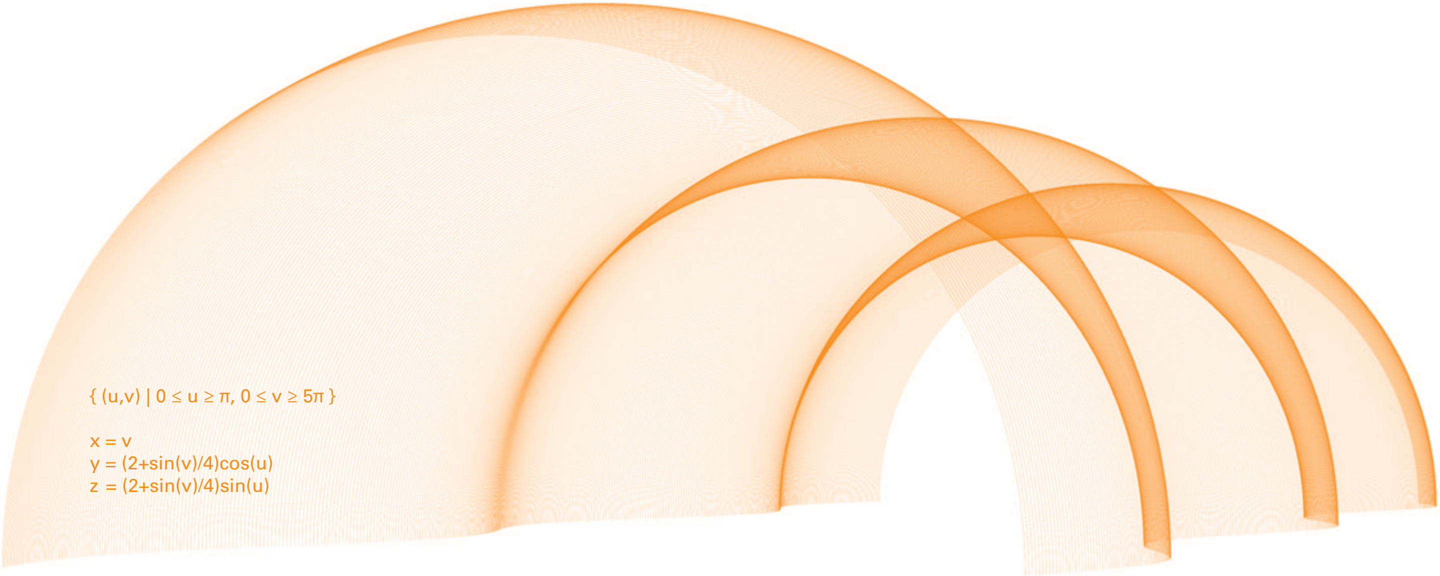
Japan Pavilion in Hanover, Germany, 2000
Shigeru Ban Architects, Tokyo, Japan

Shigeru Ban collaborated with architect Frei Otto to design a gridshell structure 72 m (235 ft) in length. The tunnel-arch geometry was primarily made out of paper tubing. After Expo 2000 concluded, the structure was recycled to paper pulp.

Interior and exterior photographs by Hiroyuki Hirai. Courtesy of Shigeru Ban Architects.

Japan Pavilion a mathematical recipe

First, transform a cylinder into a barrel vault by *cutting* the period of the u-parameter. Second, multiply both the y- and z-coordinates by a function of sine. A sine curve can be read in both plan and longitudinal elevations. Third, add an integer in front of the newly added sine functions. Adding this integer will increase the radius of the tube. Finally, divide the v-parameter within the sine functions, to control the curvature of each sine-curve profile.



$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge 5\pi \}$

$x = v$
$y = \cos(u)$
$z = \sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge 5\pi \}$

$x = v$
$y = \sin(v)\cos(u)$
$z = \sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge 5\pi \}$

$x = v$
$y = \sin(v)\cos(u)$
$z = \sin(v)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge 5\pi \}$

$x = v$
$y = (1+\sin(v))\cos(u)$
$z = (1+\sin(v))\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge 5\pi \}$

$x = v$
$y = (2+\sin(v))\cos(u)$
$z = (2+\sin(v))\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge 5\pi \}$

$x = v$
$y = (2+\sin(v)/2)\cos(u)$
$z = (2+\sin(v)/2)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge 5\pi \}$

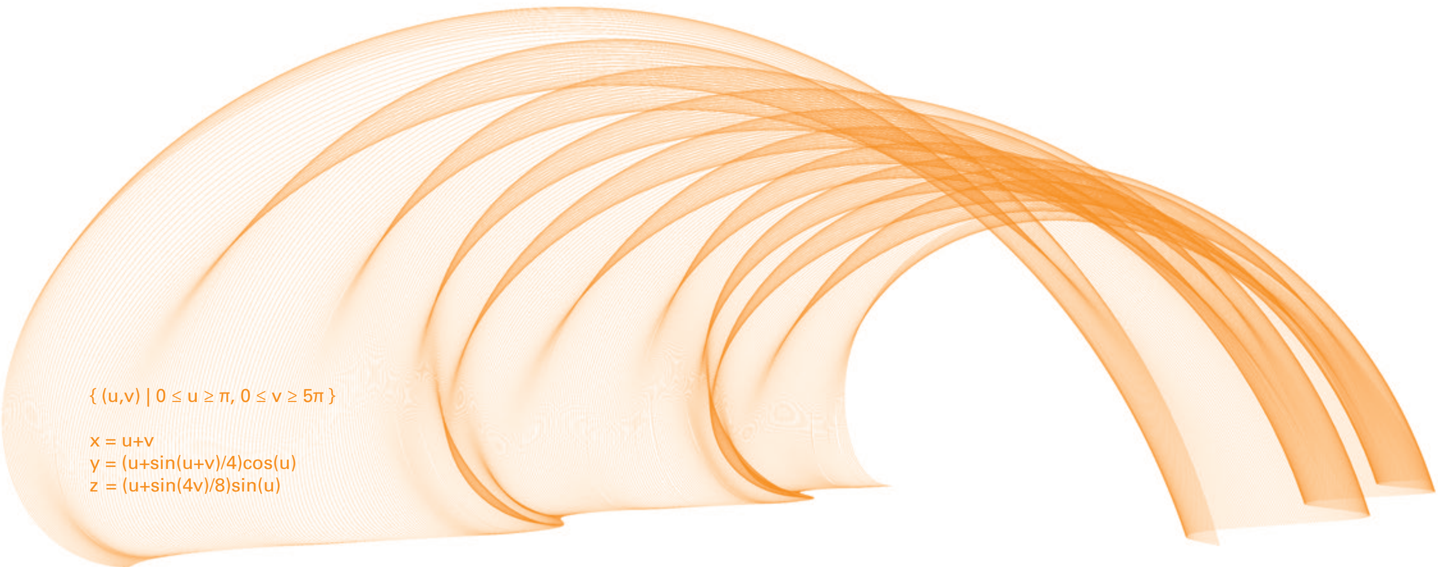
$x = v$
$y = (2+\sin(v)/3)\cos(u)$
$z = (2+\sin(v)/3)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi, 0 \le v \ge 5\pi \}$

$x = v$
$y = (2+\sin(v)/4)\cos(u)$
$z = (2+\sin(v)/4)\sin(u)$

Japan Pavilion design variations

First, add u to v within the sine function in the y-coordinate, transforming the symmetrical plan into a meandering tube. Second, add u to the x-coordinate, shifting the grain of the tube according to a diagonal (*ascending*). Third, replace the integer in front of the sine function in the y- and z-coordinates with u. As the radius of the arch gradually increases, a spiral-like transverse section appears. Finally, increase the frequency of the sine function in the z-coordinate (*modulating*), bifurcating the rhythm between the elevation and plan.



$\{ (u,v) \mid 0 \leq u \geq \pi,\ 0 \leq v \geq 5\pi \}$

$x = u+v$
$y = (u+\sin(u+v)/4)\cos(u)$
$z = (u+\sin(4v)/8)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi,\ 0 \le v \ge 5\pi \}$

$x = v$
$y = (2+\sin(v)/4)\cos(u)$
$z = (2+\sin(v)/4)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi,\ 0 \le v \ge 5\pi \}$

$x = v$
$y = (2+\sin(u+v)/4)\cos(u)$
$z = (2+\sin(v)/4)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi,\ 0 \le v \ge 5\pi \}$

$x = u+v$
$y = (2+\sin(u+v)/4)\cos(u)$
$z = (2+\sin(v)/4)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi,\ 0 \le v \ge 5\pi \}$

$x = u+v$
$y = (u+\sin(u+v)/4)\cos(u)$
$z = (u+\sin(v)/4)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi,\ 0 \le v \ge 5\pi \}$

$x = u+v$
$y = (u+\sin(u+4v)/4)\cos(u)$
$z = (u+\sin(4v)/4)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi,\ 0 \le v \ge 5\pi \}$

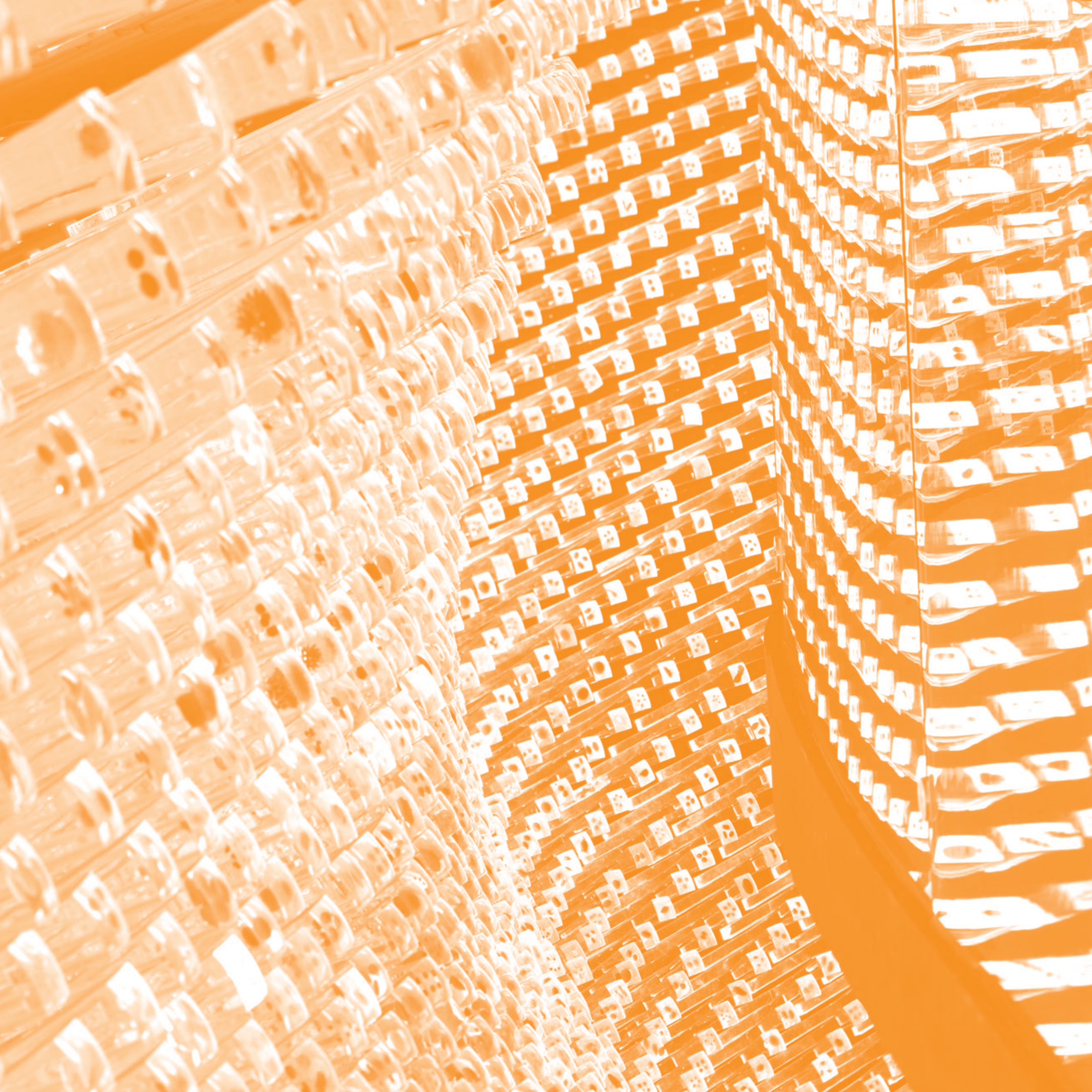
$x = u+v$
$y = (u+\sin(u+4v)/8)\cos(u)$
$z = (u+\sin(4v)/8)\sin(u)$

$\{ (u,v) \mid 0 \le u \ge \pi,\ 0 \le v \ge 5\pi \}$

$x = u+v$
$y = (u+\sin(u+4v)/8)\cos(u)$
$z = (u+\sin(v)/4)\sin(u)$

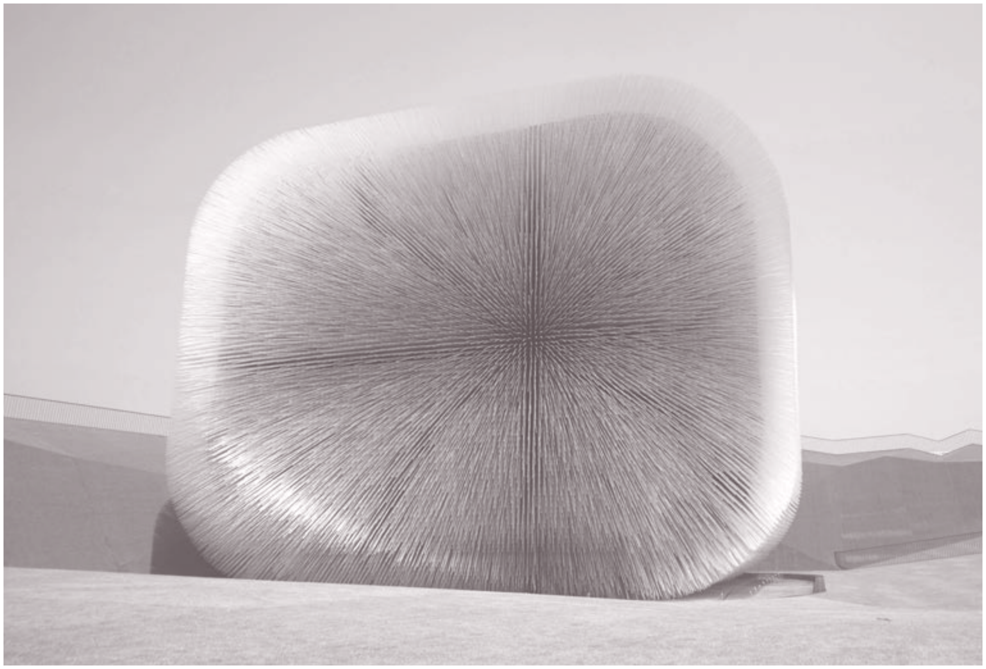
$\{ (u,v) \mid 0 \le u \ge \pi,\ 0 \le v \ge 5\pi \}$

$x = u+v$
$y = (u+\sin(u+v)/4)\cos(u)$
$z = (u+\sin(4v)/8)\sin(u)$



UK Pavilion in Shanghai, China, 2010
Heatherwick Studio, London, UK

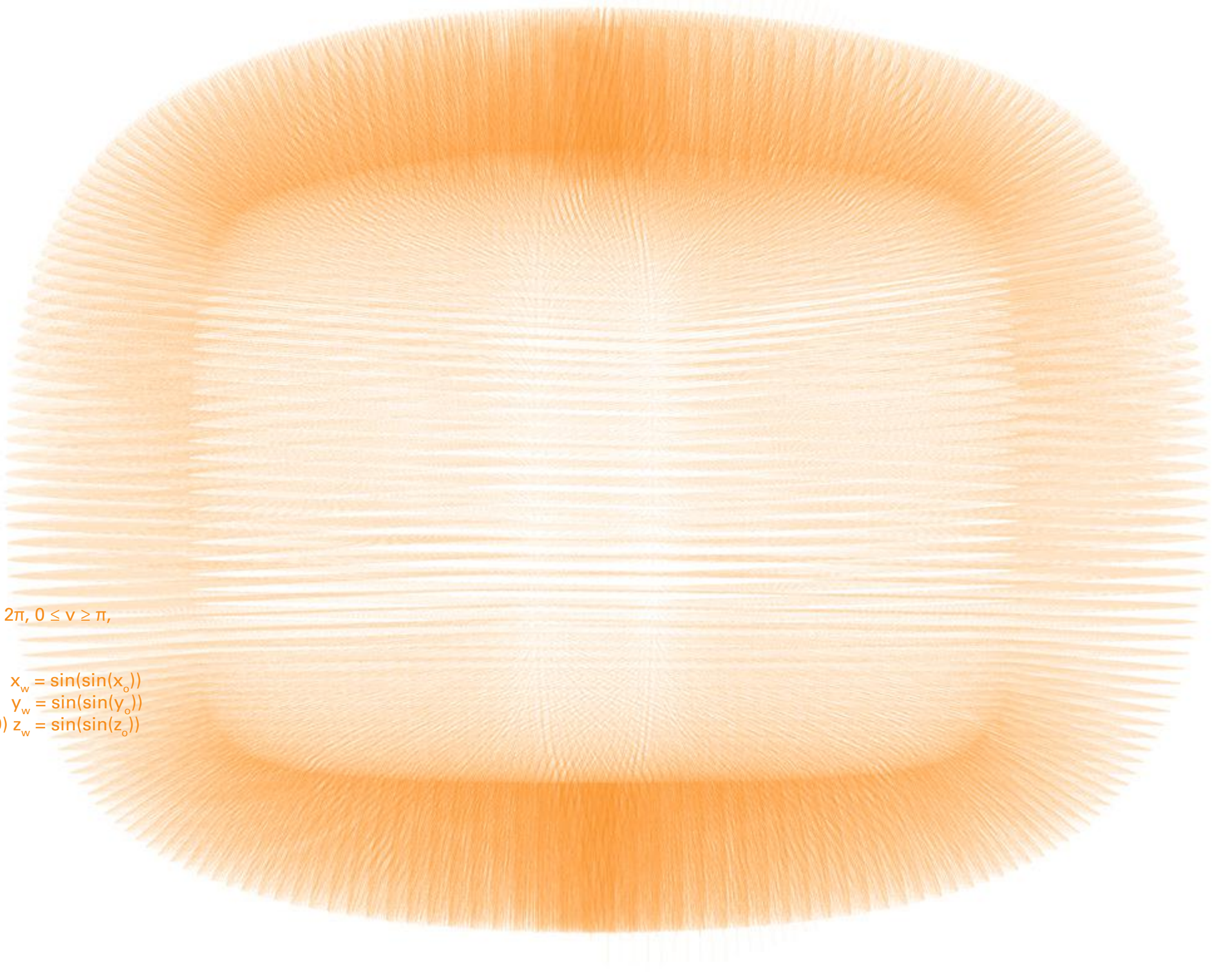
Heatherwick Studio designed a 'Seed Cathedral', 15 m (50 ft) high and 10 m (33 ft) tall. The pavilion was composed of 60,000 clear acrylic rods, each measuring 7.5 m (25 ft) in length. More than 8 million people visited it during Expo 2010.



Interior photograph by Charlie Xia. Exterior photograph by Katarina Stübe.

UK Pavilion a mathematical recipe

First, transform a sphere into a rounded cube by *flattening* its curvature. Second, divide the z-coordinate by 13/10, *scaling* the height of the shape. Third, introduce *thickening*. The first set of x_o, y_o, z_o is embedded with the initial sphere, while the second set of x_w, y_w, z_w contains the *flattening* transformation. As the shape flattens inward, it simultaneously thickens inward.



$\{ (u,v,w) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi, 0 \leq w \geq 2\pi \}$

$x_o = \sin(v)\cos(u)$ $x_w = \sin(\sin(x_o))$
$y_o = \sin(v)\sin(u)$ $y_w = \sin(\sin(y_o))$
$z_o = \cos(v)/(13/10)$ $z_w = \sin(\sin(z_o))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(v)\cos(u)$
$y = \sin(v)\sin(u)$
$z = \cos(v)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(\sin(v)\cos(u))$
$y = \sin(\sin(v)\sin(u))$
$z = \sin(\cos(v))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(\sin(\sin(v)\cos(u)))$
$y = \sin(\sin(\sin(v)\sin(u)))$
$z = \sin(\sin(\cos(v)))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(\sin(\sin(v)\cos(u)))$
$y = \sin(\sin(\sin(v)\sin(u)))$
$z = \sin(\sin(\cos(v)))/(13/10)$

$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,$
$0 \leq w \geq \pi/2 \}$

$x_o = \sin(v)\cos(u) \quad x_w = \sin(\sin(x_o))$
$y_o = \sin(v)\sin(u) \quad y_w = \sin(\sin(y_o))$
$z_o = \cos(v)/(13/10) \quad z_w = \sin(\sin(z_o))$

$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,$
$0 \leq w \geq \pi \}$

$x_o = \sin(v)\cos(u) \quad x_w = \sin(\sin(x_o))$
$y_o = \sin(v)\sin(u) \quad y_w = \sin(\sin(y_o))$
$z_o = \cos(v)/(13/10) \quad z_w = \sin(\sin(z_o))$

$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,$
$0 \leq w \geq 3\pi/2 \}$

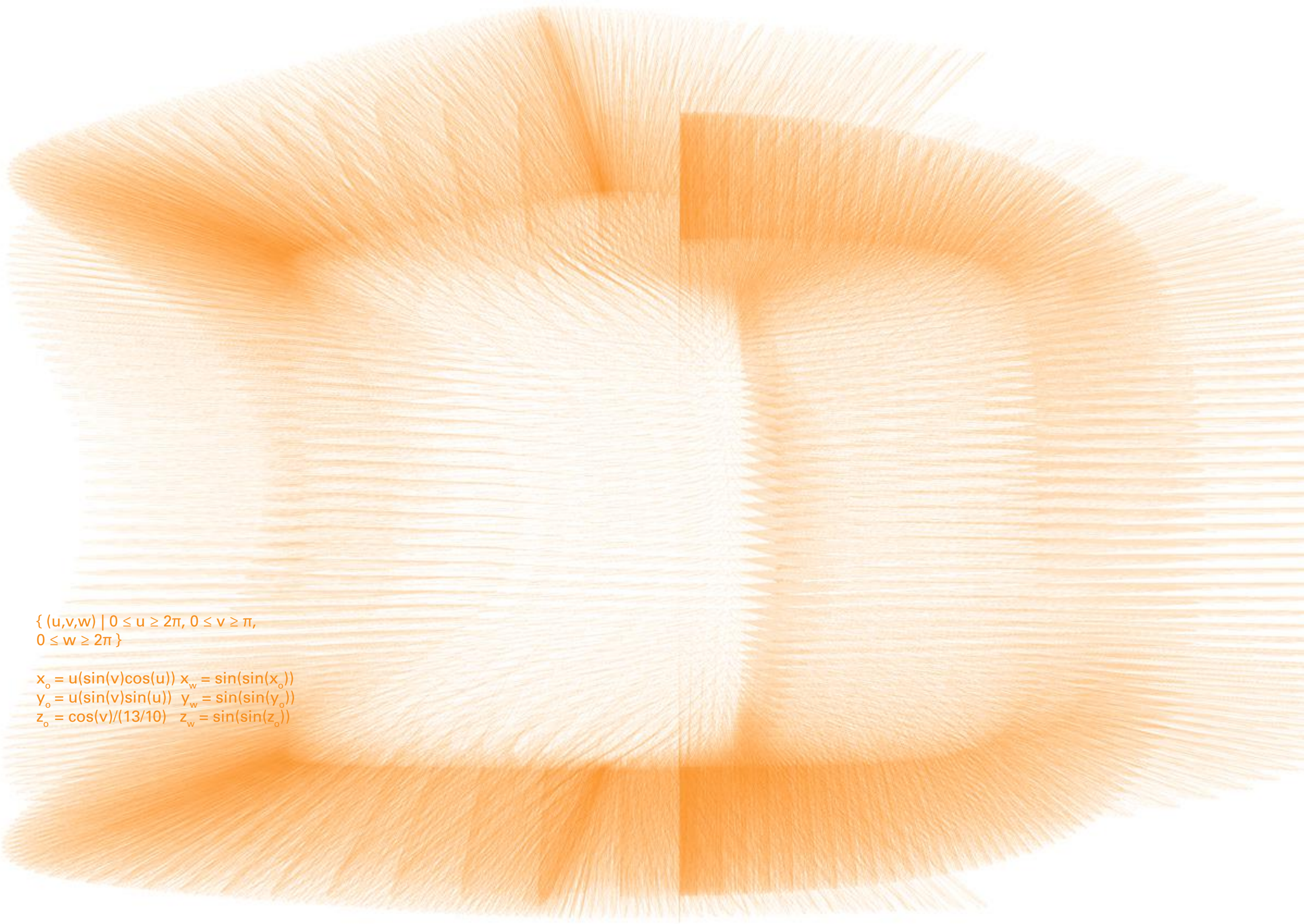
$x_o = \sin(v)\cos(u) \quad x_w = \sin(\sin(x_o))$
$y_o = \sin(v)\sin(u) \quad y_w = \sin(\sin(y_o))$
$z_o = \cos(v)/(13/10) \quad z_w = \sin(\sin(z_o))$

$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,$
$0 \leq w \geq 2\pi \}$

$x_o = \sin(v)\cos(u) \quad x_w = \sin(\sin(x_o))$
$y_o = \sin(v)\sin(u) \quad y_w = \sin(\sin(y_o))$
$z_o = \cos(v)/(13/10) \quad z_w = \sin(\sin(z_o))$

UK Pavilion design variations

First, introduce *spiralling* to the x-coordinate, opening the closed shape. Second, also apply *spiralling* to the y-coordinate, causing the thickened box to gradually twist at its core.

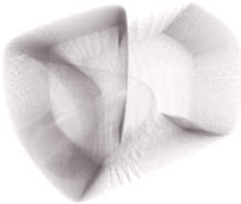


$\{ (u,v,w) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi,\ 0 \le w \ge 2\pi \}$

$x_o = u(\sin(v)\cos(u))$ $x_w = \sin(\sin(x_o))$
$y_o = u(\sin(v)\sin(u))$ $y_w = \sin(\sin(y_o))$
$z_o = \cos(v)/(13/10)$ $z_w = \sin(\sin(z_o))$

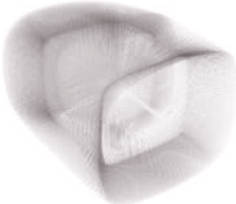
$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,\ 0 \leq w \geq 2\pi \}$

$x_o = \sin(v)\cos(u) \quad x_w = \sin(\sin(x_o))$
$y_o = \sin(v)\sin(u) \quad y_w = \sin(\sin(y_o))$
$z_o = \cos(v)/(13/10) \quad z_w = \sin(\sin(z_o))$



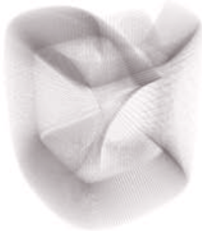
$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,\ 0 \leq w \geq 2\pi \}$

$x_o = u(\sin(v)\cos(u)) \quad x_w = \sin(\sin(x_o))$
$y_o = \sin(v)\sin(u) \quad y_w = \sin(\sin(y_o))$
$z_o = \cos(v)/(13/10) \quad z_w = \sin(\sin(z_o))$



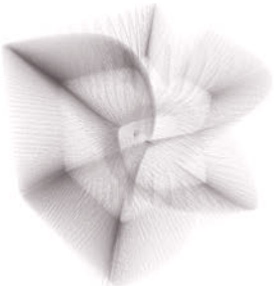
$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,\ 0 \leq w \geq 2\pi \}$

$x_o = \sin(v)\cos(u) \quad x_w = \sin(\sin(x_o))$
$y_o = u(\sin(v)\sin(u)) \quad y_w = \sin(\sin(y_o))$
$z_o = \cos(v)/(13/10) \quad z_w = \sin(\sin(z_o))$



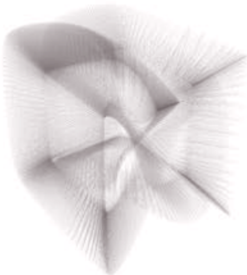
$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,\ 0 \leq w \geq 2\pi \}$

$x_o = \sin(v)\cos(u) \quad x_w = \sin(\sin(x_o))$
$y_o = \sin(v)\sin(u) \quad y_w = \sin(\sin(y_o))$
$z_o = u(\cos(v)/(13/10)) \quad z_w = \sin(\sin(z_o))$



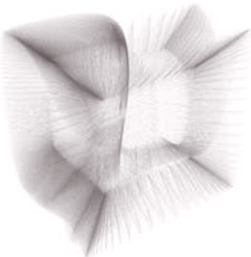
$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,\ 0 \leq w \geq 2\pi \}$

$x_o = u(\sin(v)\cos(u)) \quad x_w = \sin(\sin(x_o))$
$y_o = u(\sin(v)\sin(u)) \quad y_w = \sin(\sin(y_o))$
$z_o = u(\cos(v)/(13/10)) \quad z_w = \sin(\sin(z_o))$



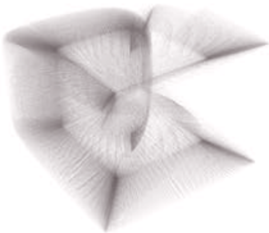
$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,\ 0 \leq w \geq 2\pi \}$

$x_o = \sin(v)\cos(u) \quad x_w = \sin(\sin(x_o))$
$y_o = u(\sin(v)\sin(u)) \quad y_w = \sin(\sin(y_o))$
$z_o = u(\cos(v)/(13/10)) \quad z_w = \sin(\sin(z_o))$



$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,\ 0 \leq w \geq 2\pi \}$

$x_o = u(\sin(v)\cos(u)) \quad x_w = \sin(\sin(x_o))$
$y_o = \sin(v)\sin(u) \quad y_w = \sin(\sin(y_o))$
$z_o = u(\cos(v)/(13/10)) \quad z_w = \sin(\sin(z_o))$

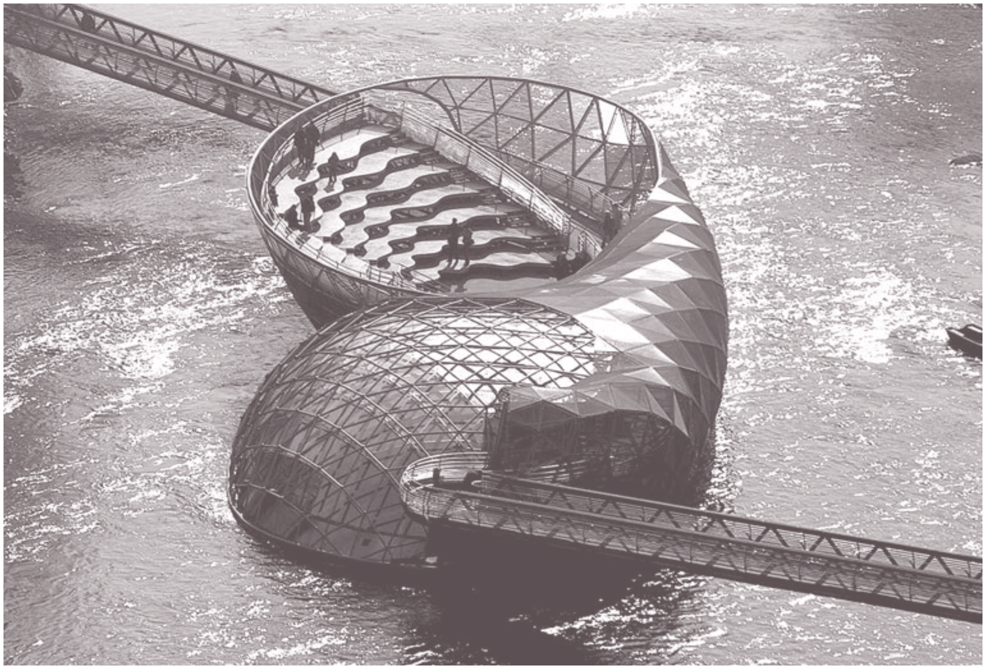


$\{ (u,v,w) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi,\ 0 \leq w \geq 2\pi \}$

$x_o = u(\sin(v)\cos(u)) \quad x_w = \sin(\sin(x_o))$
$y_o = u(\sin(v)\sin(u)) \quad y_w = \sin(\sin(y_o))$
$z_o = \cos(v)/(13/10) \quad z_w = \sin(\sin(z_o))$

Mur Island in Graz, Austria, 2003
Acconci Studio, New York, USA

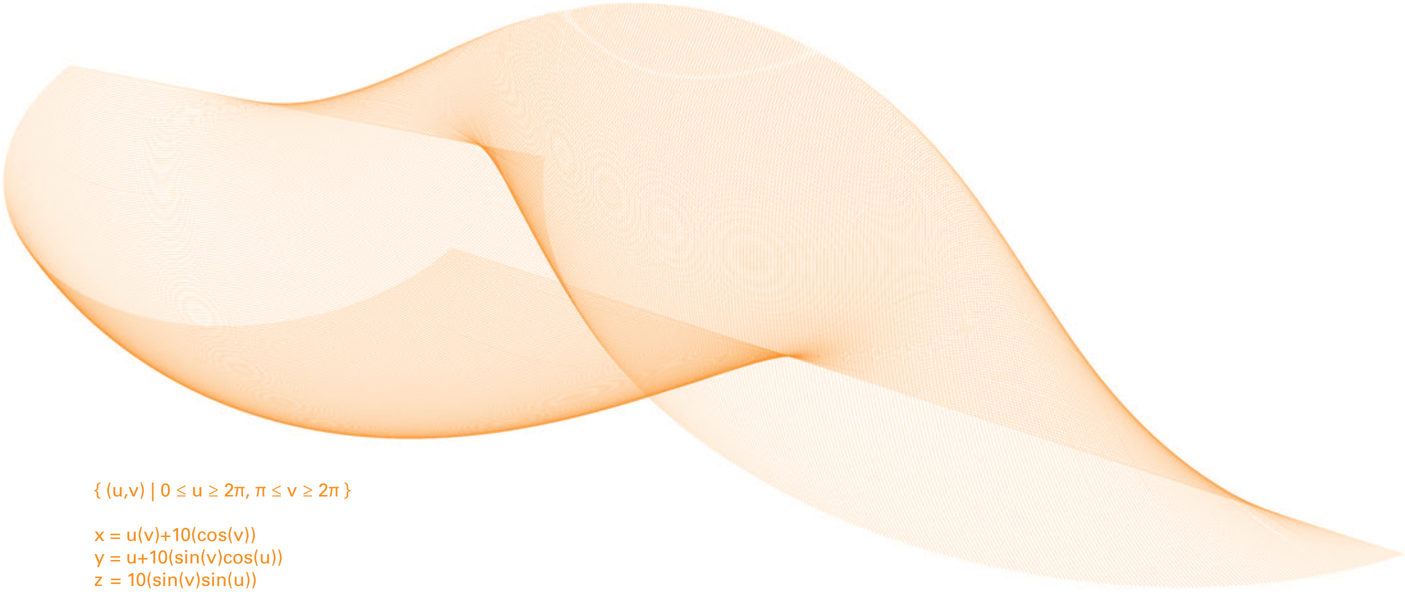
Acconci Studio designed an artificial floating island on the Mur River. The building measures 47 m (150 ft) in length and is reached by two pedestrian bridges. The twisting of the steel and glass structure forms an outdoor ampitheatre.



Interior photograph by Acconci Studio. Exterior photograph by Harry Schiffer.

Mur Island a mathematical recipe

First, orientate a sphere's poles along the x-axis and scale the sphere by ten in each coordinate. *Scaling* the sphere will make for subtler *ascending* transformations. Second, transform a closed sphere into a sphere that peels open by adding a u-parameter to the y-coordinate (*ascending*). Next, multiply the u- and v-parameters; when u(v) is added to the x-coordinate, the shape twists. Finally, as the v-parameter shifts half a period, π to 2π (*cutting*), the shape grows to an appropriate proportion.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi \leq v \geq 2\pi \}$

$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = 10(\cos(v))$
$y = 10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = 10(\cos(v))$
$y = (u/3)+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = 10(\cos(v))$
$y = (u/2)+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = 10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = (u/2)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = u+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \le u \ge 2\pi, \pi \le v \ge 2\pi \}$

$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

Mur Island design variations

Continue to transform the shape through *cutting*, by changing the period of the v-parameter from (π to 2π) to (2π to 4π). As the shape grows, it continues to loop and twist around itself, without any self-intersections.

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 2\pi \leq v \geq 4\pi \}$

$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi \leq v \geq 2\pi \}$

$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi \leq v \geq 5\pi/2 \}$

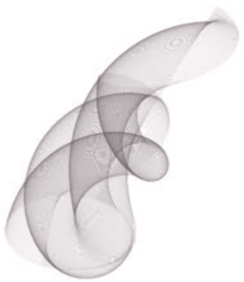
$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi \leq v \geq 3\pi \}$

$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi \leq v \geq 7\pi/2 \}$

$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi \leq v \geq 4\pi \}$

$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi \leq v \geq 9\pi/2 \}$

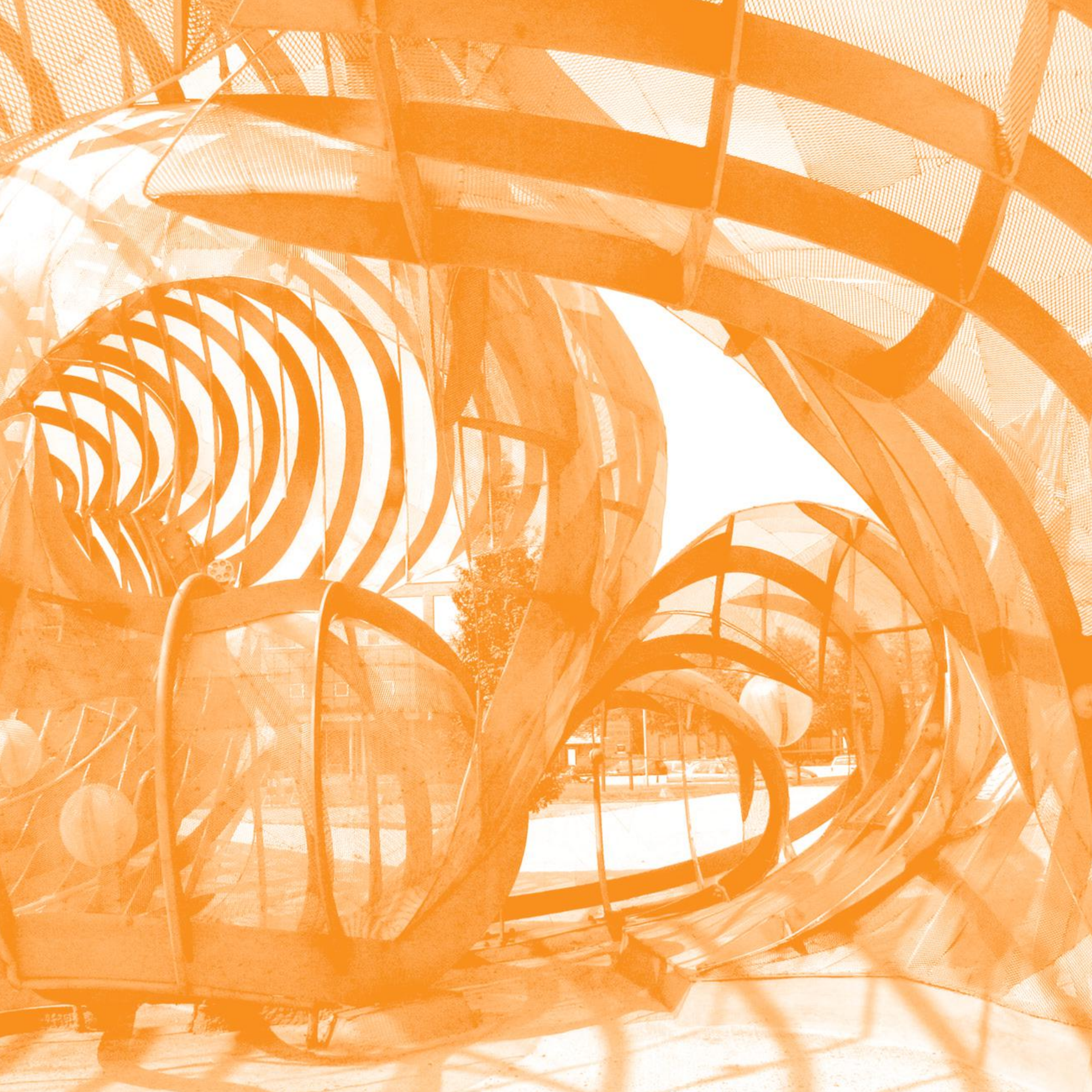
$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 3\pi/2 \leq v \geq 4\pi \}$

$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 2\pi \leq v \geq 4\pi \}$

$x = u(v)+10(\cos(v))$
$y = u+10(\sin(v)\cos(u))$
$z = 10(\sin(v)\sin(u))$



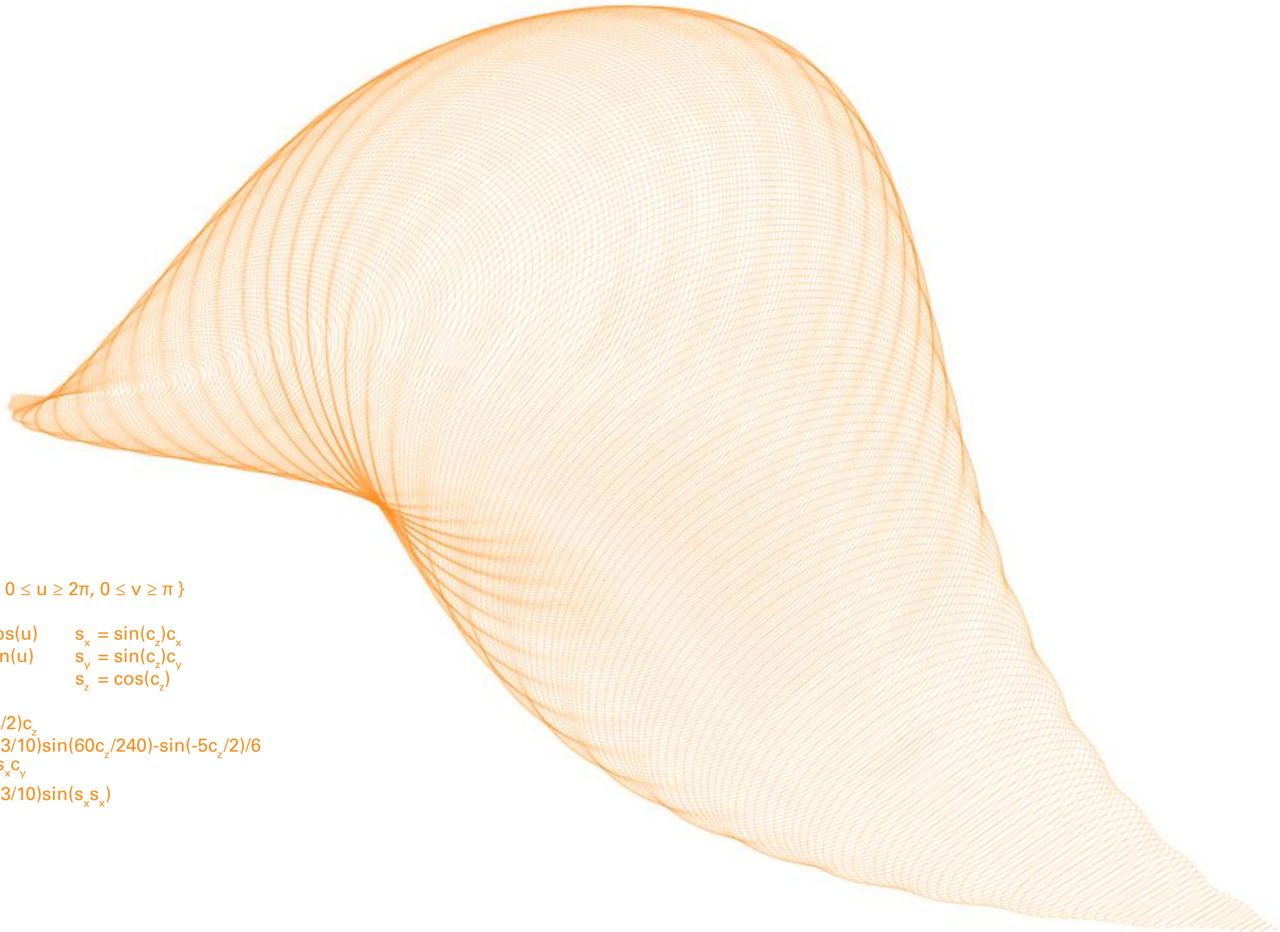
Son-O-House in Son en Breugel, Netherlands, 2004
NOX, Rotterdam, Netherlands

Lars Spuybroek designed a 'house where sound lives'. The public pavilion contains 23 motion sensors that indirectly influence the music. Sound artist Edwin van der Heide composed and programmed the system of sounds.

Interior and exterior photographs courtesy of NOX/Lars Spuybroek.

Son-O-House a mathematical recipe

First, combine parts of shapes to transform a cylinder into a *mound*. Second, multiply the x-coordinate of a sphere in the z-coordinate of a *mound*, creating a *steeper mound*. Third, multiply the x-coordinate of a sphere in the y-coordinate, creating a *wedge*. Fourth, add a sine function to the y-coordinate, creating a *meandering mound* that follows the curve. Finally, apply a subtle *texturing* to ripple the surface.



$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$c_x = \cos(u) \qquad s_x = \sin(c_z)c_x$
$c_y = \sin(u) \qquad s_y = \sin(c_z)c_y$
$c_z = v \qquad s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6 + s_x c_y$
$z = (13/10)\sin(s_x s_x)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)c_y$
$z = (13/10)c_x$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)c_y$
$z = (13/10)s_x$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)s_x c_y$
$z = (13/10)s_x$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)c_y$
$z = (13/10)s_x s_x$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)s_x c_y$
$z = (13/10)s_x s_x$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)s_x c_y$
$z = (13/10)\sin(s_x s_x)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)(-\sin(-5c_z/2)/6+s_x c_y)$
$z = (13/10)\sin(s_x s_x)$

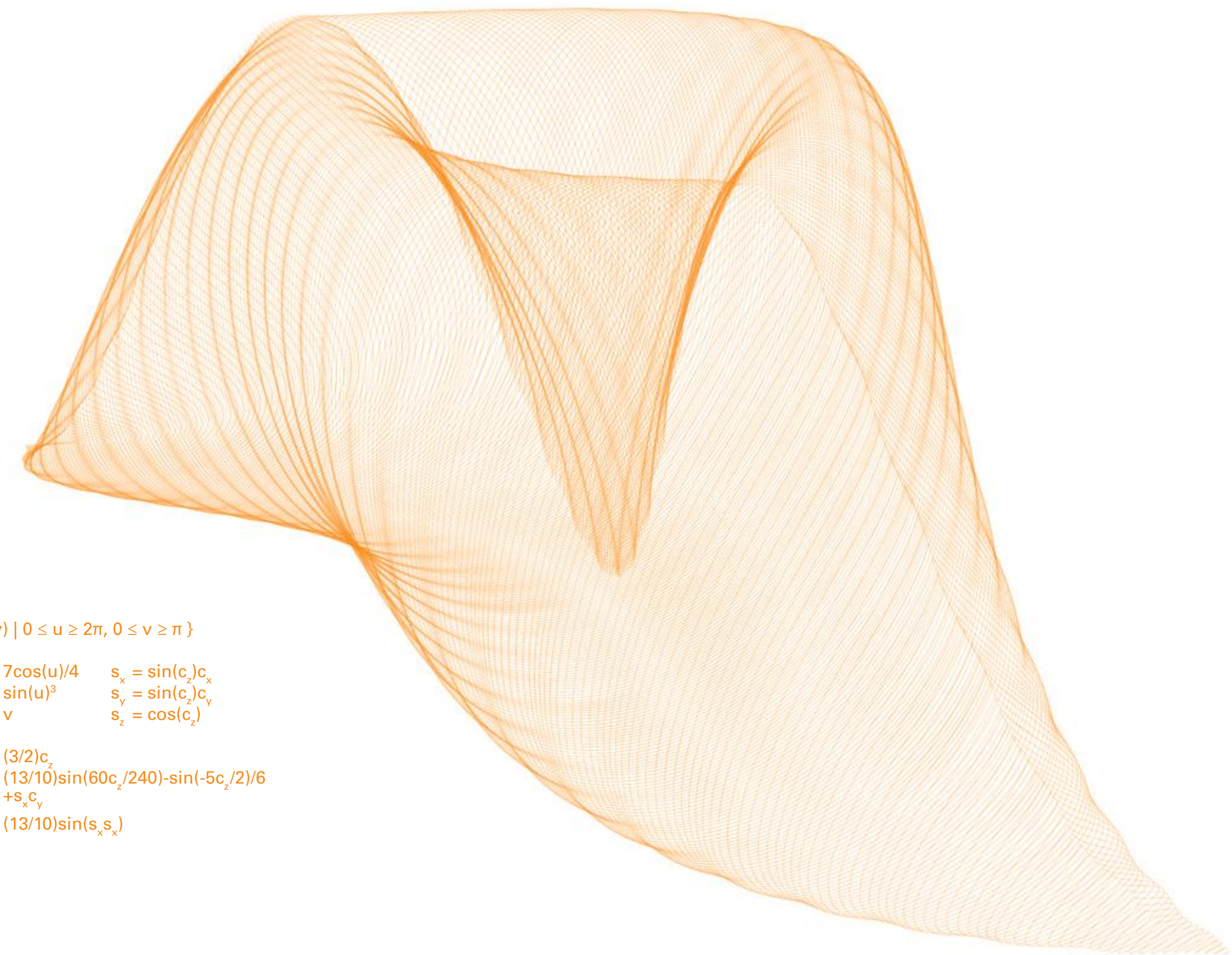
$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $s_y = \sin(c_z)c_y$
$c_z = v$ $s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6$
$+s_x c_y$
$z = (13/10)\sin(s_x s_x)$

Son-O-House design variations

First, apply *pinching* to the y-coordinate of the cylinder definition. A crease appears along the centre spine of the shape. Second, apply *scaling* to the x-coordinate of the cylinder definition. As it increases, the centre spine of the shape inflects up and down, following a cosine curve.



$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$c_x = 7\cos(u)/4 \qquad s_x = \sin(c_z)c_x$
$c_y = \sin(u)^3 \qquad s_y = \sin(c_z)c_y$
$c_z = v \qquad s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6 + s_x c_y$
$z = (13/10)\sin(s_x s_x)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)$ $\quad s_y = \sin(c_z)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6 + s_x c_y$
$z = (13/10)\sin(s_x s_x)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$c_x = \cos(u)$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)^3$ $\quad s_y = \sin(c_z)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6 + s_x c_y$
$z = (13/10)\sin(s_x s_x)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$c_x = 9\cos(u)/8$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)^3$ $\quad s_y = \sin(c_z)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6 + s_x c_y$
$z = (13/10)\sin(s_x s_x)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$c_x = 5\cos(u)/4$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)^3$ $\quad s_y = \sin(c_z)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6 + s_x c_y$
$z = (13/10)\sin(s_x s_x)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$c_x = 11\cos(u)/8$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)^3$ $\quad s_y = \sin(c_z)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6 + s_x c_y$
$z = (13/10)\sin(s_x s_x)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$c_x = 6\cos(u)/4$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)^3$ $\quad s_y = \sin(c_z)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6 + s_x c_y$
$z = (13/10)\sin(s_x s_x)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$c_x = 13\cos(u)/8$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)^3$ $\quad s_y = \sin(c_z)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6 + s_x c_y$
$z = (13/10)\sin(s_x s_x)$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$c_x = 7\cos(u)/4$ $\quad s_x = \sin(c_z)c_x$
$c_y = \sin(u)^3$ $\quad s_y = \sin(c_z)c_y$
$c_z = v$ $\quad s_z = \cos(c_z)$

$x = (3/2)c_z$
$y = (13/10)\sin(60c_z/240)-\sin(-5c_z/2)/6 + s_x c_y$
$z = (13/10)\sin(s_x s_x)$

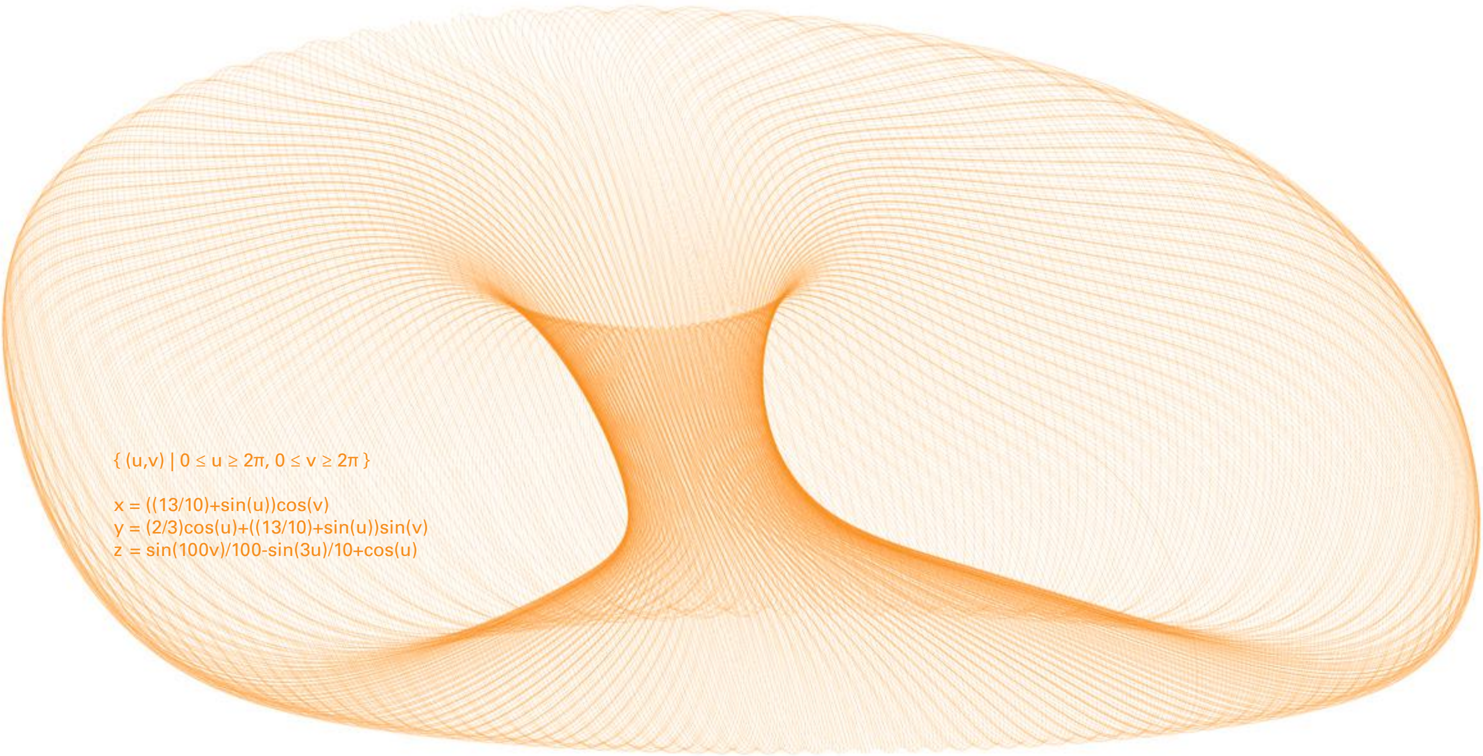
Ark Nova in Matsushima, Japan, 2013
Arata Isozaki, Tokyo, Japan and Anish Kapoor, London, UK

Arata Isozaki collaborated with sculptor Anish Kapoor to design the world's first inflatable concert hall. The pneumatic structure toured areas of Japan affected by the 2011 earthquake and tsunami, hosting performances as part of the Lucerne Festival.

Interior and exterior photographs courtesy of Lucerne Festival Ark Nova.

Ark Nova a mathematical recipe

First, reverse the u- and v-parameters within each function, rotating the orientation of the grain on the geometry. Second, transform a sphere into a torus. Third, shear the torus's funnel along a diagonal by adding a cosine function to the y-coordinate. Next, undulate the z-coordinate to create an apex on the top of the shape. Finally, apply a gentle *texturing*.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq 2\pi \}$

$x = ((13/10)+\sin(u))\cos(v)$
$y = (2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \sin(100v)/100-\sin(3u)/10+\cos(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = \sin(u)\cos(v)$
$y = \sin(u)\sin(v)$
$z = \cos(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = (1+\sin(u))\cos(v)$
$y = (1+\sin(u))\sin(v)$
$z = \cos(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge 2\pi \}$

$x = (1+\sin(u))\cos(v)$
$y = (1+\sin(u))\sin(v)$
$z = \cos(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge 2\pi \}$

$x = ((13/10)+\sin(u))\cos(v)$
$y = ((13/10)+\sin(u))\sin(v)$
$z = \cos(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge 2\pi \}$

$x = ((13/10)+\sin(u))\cos(v)$
$y = (2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \cos(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge 2\pi \}$

$x = ((13/10)+\sin(u))\cos(v)$
$y = (2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = -\sin(3u)/5+\cos(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge 2\pi \}$

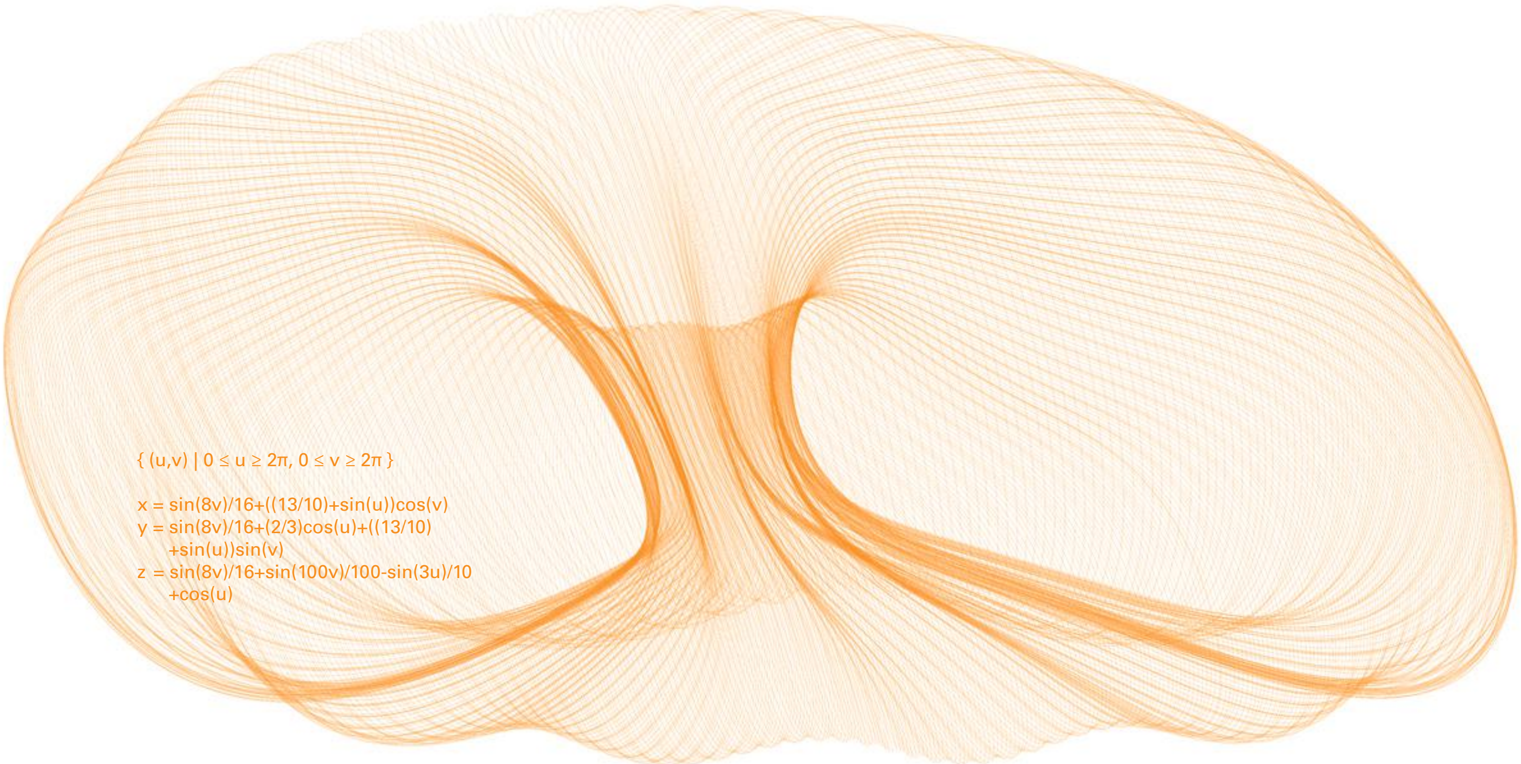
$x = ((13/10)+\sin(u))\cos(v)$
$y = (2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = -\sin(3u)/10+\cos(u)$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge 2\pi \}$

$x = ((13/10)+\sin(u))\cos(v)$
$y = (2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \sin(100v)/100-\sin(3u)/10+\cos(u)$

Ark Nova design variations

Continue to transform the shape through *texturing*, by adding an additional sine function to the x-, y- and z-coordinates. The frequency and amplitude of the sine function increases incrementally with each instance in the morphing series until, eventually, a wrinkled torus appears.

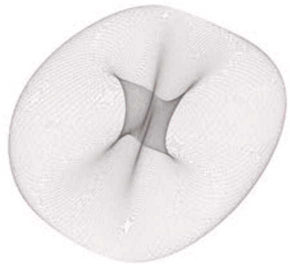


$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge 2\pi \}$

$x = ((13/10)+\sin(u))\cos(v)$
$y = (2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \sin(100v)/100-\sin(3u)/10+\cos(u)$

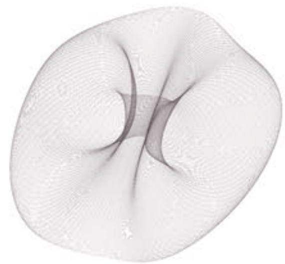
$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge 2\pi \}$

$x = \sin(2v)/10+((13/10)+\sin(u))\cos(v)$
$y = \sin(2v)/10+(2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \sin(2v)/10+\sin(100v)/100-\sin(3u)/10+\cos(u)$



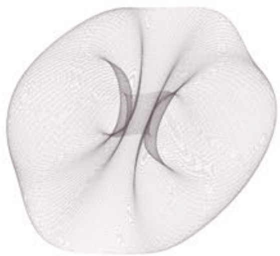
$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge 2\pi \}$

$x = \sin(3v)/11+((13/10)+\sin(u))\cos(v)$
$y = \sin(3v)/11+(2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \sin(3v)/11+\sin(100v)/100-\sin(3u)/10+\cos(u)$



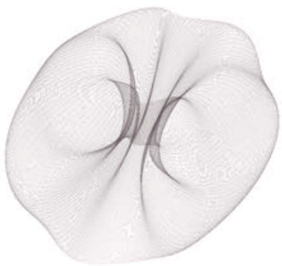
$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge 2\pi \}$

$x = \sin(4v)/12+((13/10)+\sin(u))\cos(v)$
$y = \sin(4v)/12+(2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \sin(4v)/12+\sin(100v)/100-\sin(3u)/10+\cos(u)$



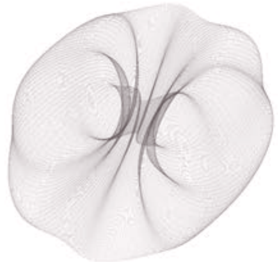
$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge 2\pi \}$

$x = \sin(5v)/13+((13/10)+\sin(u))\cos(v)$
$y = \sin(5v)/13+(2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \sin(5v)/13+\sin(100v)/100-\sin(3u)/10+\cos(u)$



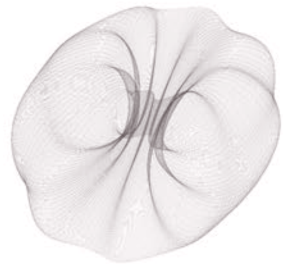
$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge 2\pi \}$

$x = \sin(6v)/14+((13/10)+\sin(u))\cos(v)$
$y = \sin(6v)/14+(2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \sin(6v)/14+\sin(100v)/100-\sin(3u)/10+\cos(u)$



$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge 2\pi \}$

$x = \sin(7v)/15+((13/10)+\sin(u))\cos(v)$
$y = \sin(7v)/15+(2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \sin(7v)/15+\sin(100v)/100-\sin(3u)/10+\cos(u)$



$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge 2\pi \}$

$x = \sin(8v)/16+((13/10)+\sin(u))\cos(v)$
$y = \sin(8v)/16+(2/3)\cos(u)+((13/10)+\sin(u))\sin(v)$
$z = \sin(8v)/16+\sin(100v)/100-\sin(3u)/10+\cos(u)$

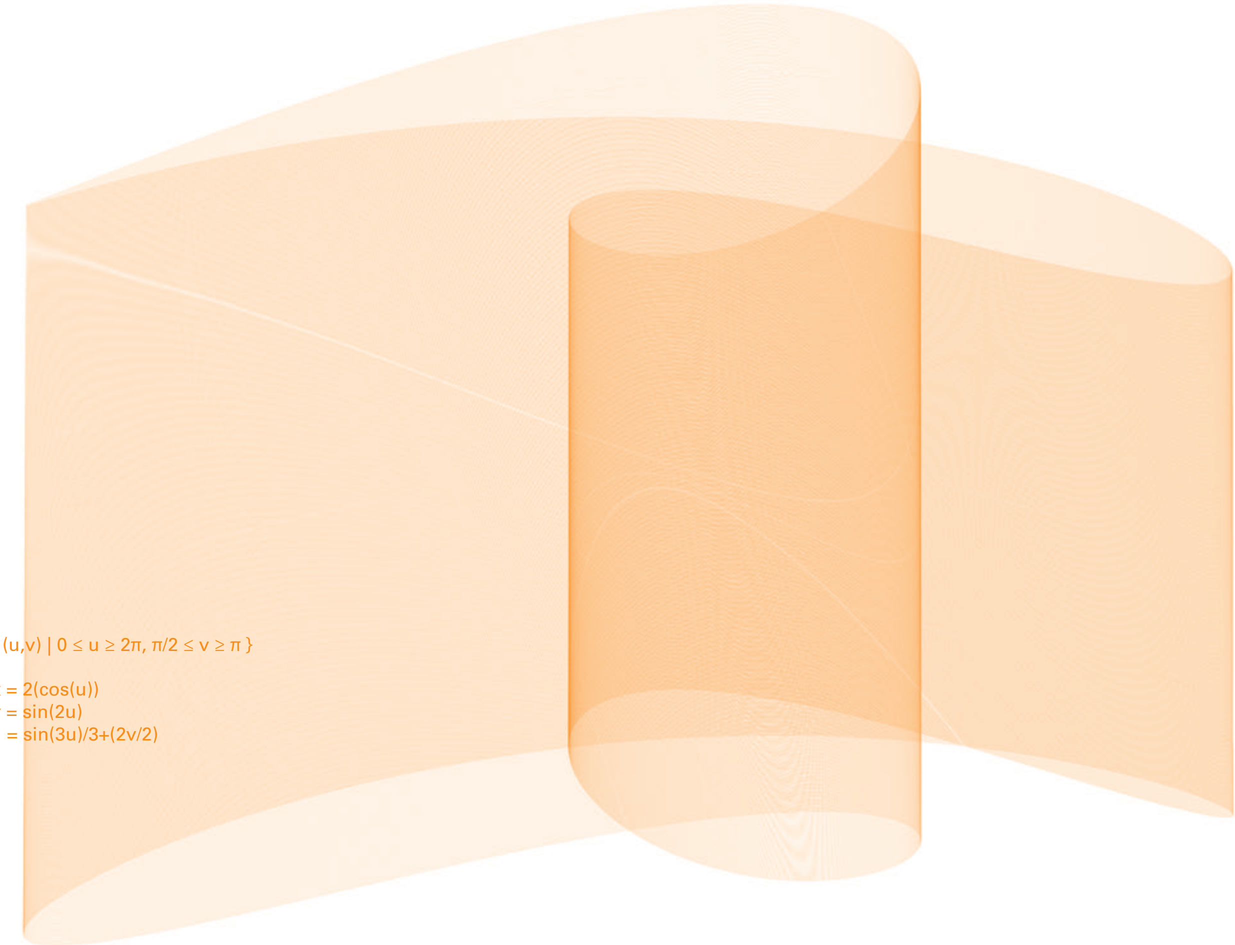
Looptecture F in Minamiawaji, Japan, 2010
Endo Shuhei Architect Institute, Osaka, Japan

Endo Shuhei designed Looptecture F, a two-storey tsunami-disaster preventive control centre at the Port of Fukura. The form is constructed out of a continuous 7.3 m (24 ft) wide steel 'belt', which intersects itself several times as it curves.

Interior and exterior photographs by Yoshiharu Matsumura. Courtesy of Endo Shuhei Architect Institute.

Looptecture F a mathematical recipe

First, a cylinder is sheared by adding a sine function in the z-coordinate. Second, that sine function's frequency is increased (*modulating*), resulting in an undulating loop. Third, the y-coordinate is transformed through *modulating* as well, leading to a self-intersection. Finally, the shape is scaled in the x- and z-coordinates.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = \sin(2u)$
$z = \sin(3u)/3+(2v/2)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = \sin(u)+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = \sin(2u)+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = \sin(2u)/2+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = \sin(3u)/3+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(2u)$
$z = \sin(3u)/3+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

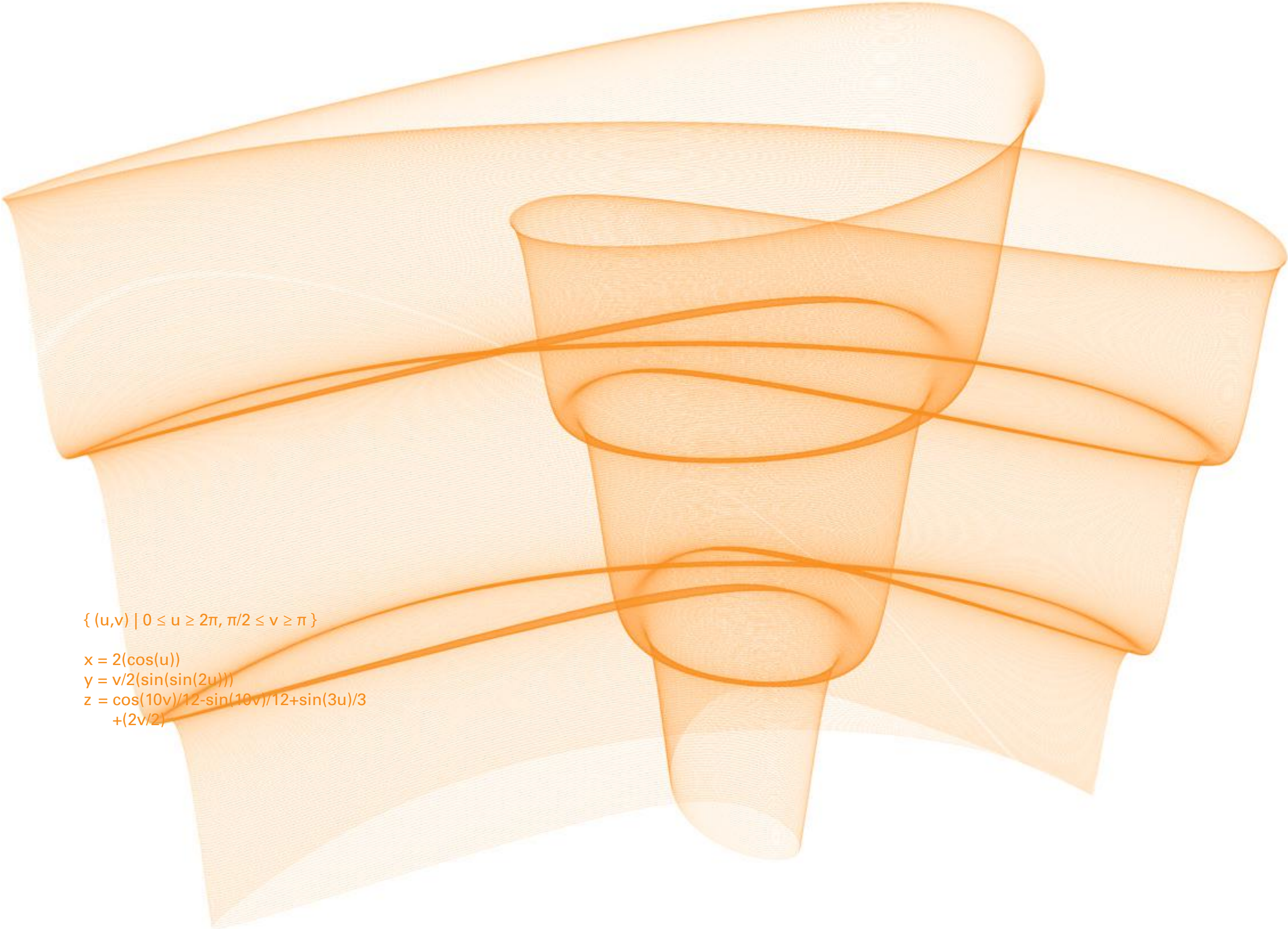
$x = 2(\cos(u))$
$y = \sin(2u)$
$z = \sin(3u)/3+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = \sin(2u)$
$z = \sin(3u)/3+(2v/2)$

Looptecture F design variations

First, transform the parallel, vertically extruded sides of the loop into a conical projection by multiplying a v-parameter in the y-coordinate. Second, apply *flattening* to the y-coordinate. Finally, apply *texturing* to the z-coordinate.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = v/2(\sin(\sin(2u)))$
$z = \cos(10v)/12-\sin(10v)/12+\sin(3u)/3$
$\quad +(2v/2)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = \sin(2u)$
$z = \sin(3u)/3+(2v/2)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = (v/2)(\sin(2u))$
$z = \sin(3u)/3+(2v/2)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = (v/2)(\sin(\sin(2u)))$
$z = \sin(3u)/3+(2v/2)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = (v/2)(\sin(\sin(2u)))$
$z = \cos(5v)/6+\sin(3u)/3+(2v/2)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = (v/2)(\sin(\sin(2u)))$
$z = \cos(5v)/6-\sin(5v)/6+\sin(3u)/3$
$+(2v/2)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = (v/2)(\sin(\sin(2u)))$
$z = \cos(10v)/12-\sin(5v)/6+\sin(3u)/3$
$+(2v/2)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = (v/2)(\sin(\sin(2u)))$
$z = \cos(5v)/6-\sin(10v)/12+\sin(3u)/3$
$+(2v/2)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, \pi/2 \leq v \geq \pi \}$

$x = 2(\cos(u))$
$y = (v/2)(\sin(\sin(2u)))$
$z = \cos(10v)/12-\sin(10v)/12+\sin(3u)/3$
$+(2v/2)$

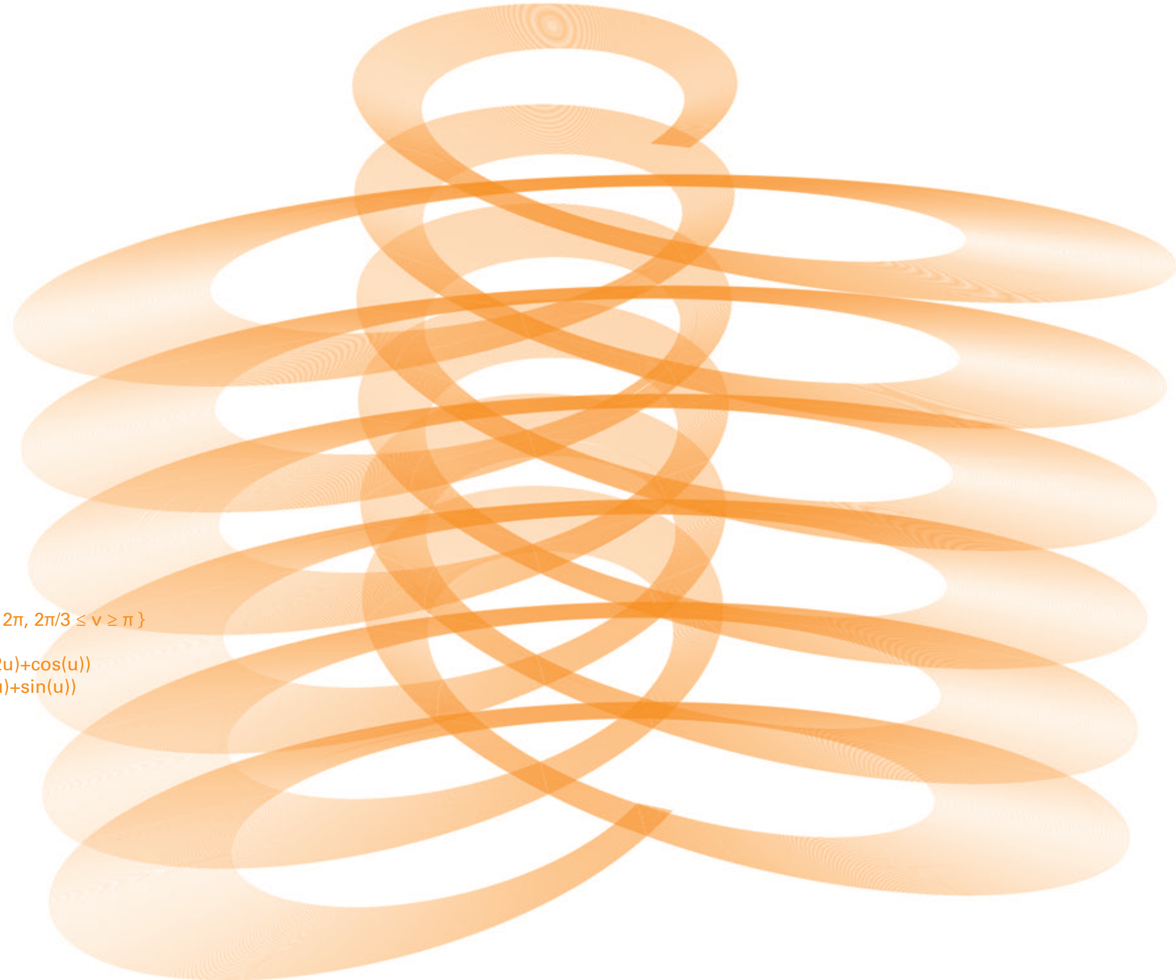
Mercedes-Benz Museum in Stuttgart, Germany, 2006
UNStudio, Amsterdam, Netherlands

Next to the Daimler factory, UNStudio designed a 32,000 m^2 (350,000 sq ft) museum, which also includes stores, offices, a restaurant and an auditorium. The inner helical ramps spiral up and around a central void, a design inspired by a trefoil knot.

Interior and exterior photographs by Brigida Gonzalez. Courtesy of UNStudio.

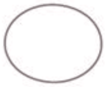
Mercedes-Benz Museum a mathematical recipe

First, transform a circle into a trefoil knot by adding a cosine function to the x-coordinate (*texturing*) and a sine function to the y- and z-coordinates. Second, multiply the x- and y-coordinates by a v-parameter, transforming the line into a surface. Third, replace the sine function in the z-coordinate with a u-parameter, breaking the closed loop and allowing it to spiral upwards. Finally, increase the period of the ramp (*cutting*).



$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(2u)+\cos(u))$
$y = v((3/2)\sin(2u)+\sin(u))$
$z = u/4$

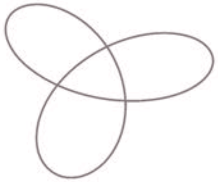


$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = 3(\cos(u))$
$y = 3(\sin(u))$

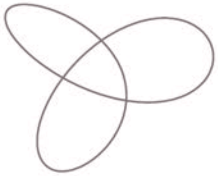
$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = 3((-3/2)\cos(2u)+\cos(u))$
$y = 3(\sin(u))$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = 3((-3/2)\cos(2u)+\cos(u))$
$y = 3((3/2)\sin(2u)+\sin(u))$



$\{ u \mid 0 \leq u \geq 2\pi \}$

$x = 3((-3/2)\cos(2u)+\cos(u))$
$y = 3((3/2)\sin(2u)+\sin(u))$
$z = \sin(3u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 2\pi/3 \leq v \geq \pi \}$

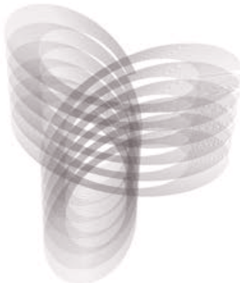
$x = v((-3/2)\cos(2u)+\cos(u))$
$y = v((3/2)\sin(2u)+\sin(u))$
$z = \sin(3u)$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(2u)+\cos(u))$
$y = v((3/2)\sin(2u)+\sin(u))$
$z = u/4$

$\{ (u,v) \mid 0 \leq u \geq 7\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(2u)+\cos(u))$
$y = v((3/2)\sin(2u)+\sin(u))$
$z = u/4$

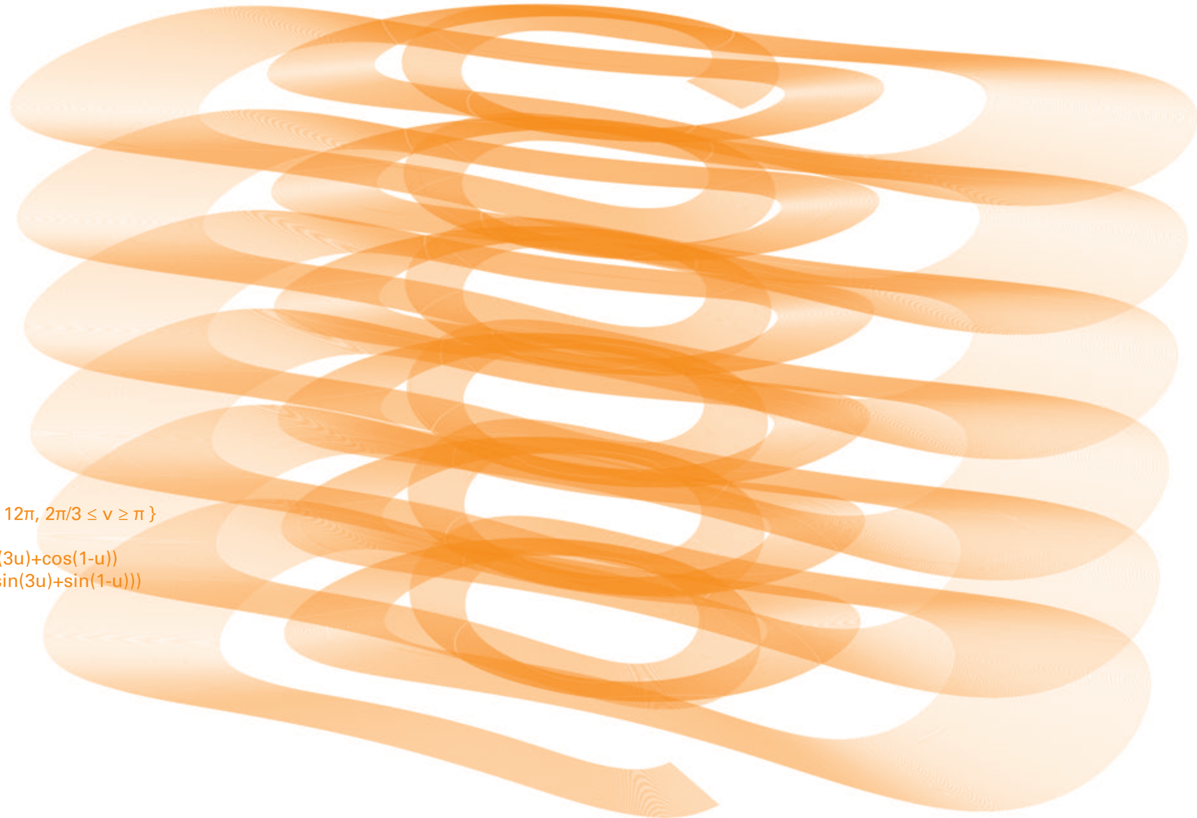


$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(2u)+\cos(u))$
$y = v((3/2)\sin(2u)+\sin(u))$
$z = u/4$

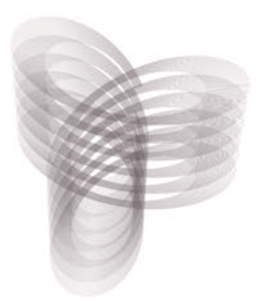
Mercedes-Benz Museum design variations

First, change the configuration of the ramp by altering the frequency of the cosine and sine functions in the x- and y-coordinates respectively. When both curves are of frequency three (*modulating*), the three leaves of the ramp become four. Next, subtract u from 1 within the sine and cosine functions in the x- and y-coordinates, breaking the rotational symmetry. Finally, use *flattening* to contain the ramp within a rectilinear boundary.



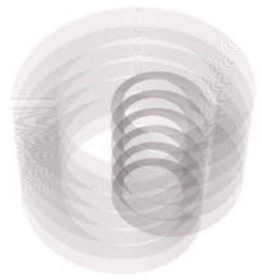
$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(3u)+\cos(1-u))$
$y = v(\sin((3/2)\sin(3u)+\sin(1-u)))$
$z = u/4$



$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(2u)+\cos(u))$
$y = v((3/2)\sin(2u)+\sin(u))$
$z = u/4$

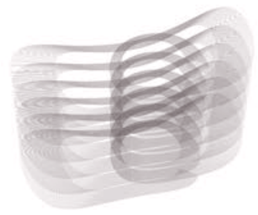


$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(2u)+\cos(1-u))$
$y = v((3/2)\sin(2u)+\sin(1-u))$
$z = u/4$

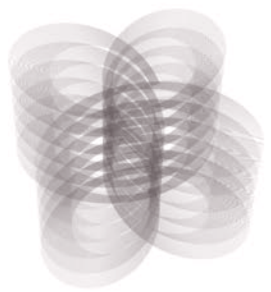
$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v(\sin((-3/2)\cos(2u)+\cos(1-u)))$
$y = v((3/2)\sin(2u)+\sin(1-u))$
$z = u/4$



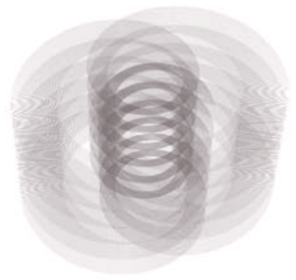
$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(2u)+\cos(1-u))$
$y = v(\sin((3/2)\sin(2u)+\sin(1-u)))$
$z = u/4$



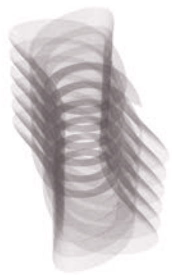
$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(3u)+\cos(u))$
$y = v((3/2)\sin(3u)+\sin(u))$
$z = u/4$



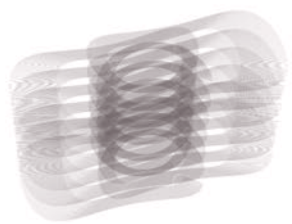
$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(3u)+\cos(1-u))$
$y = v((3/2)\sin(3u)+\sin(1-u))$
$z = u/4$



$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

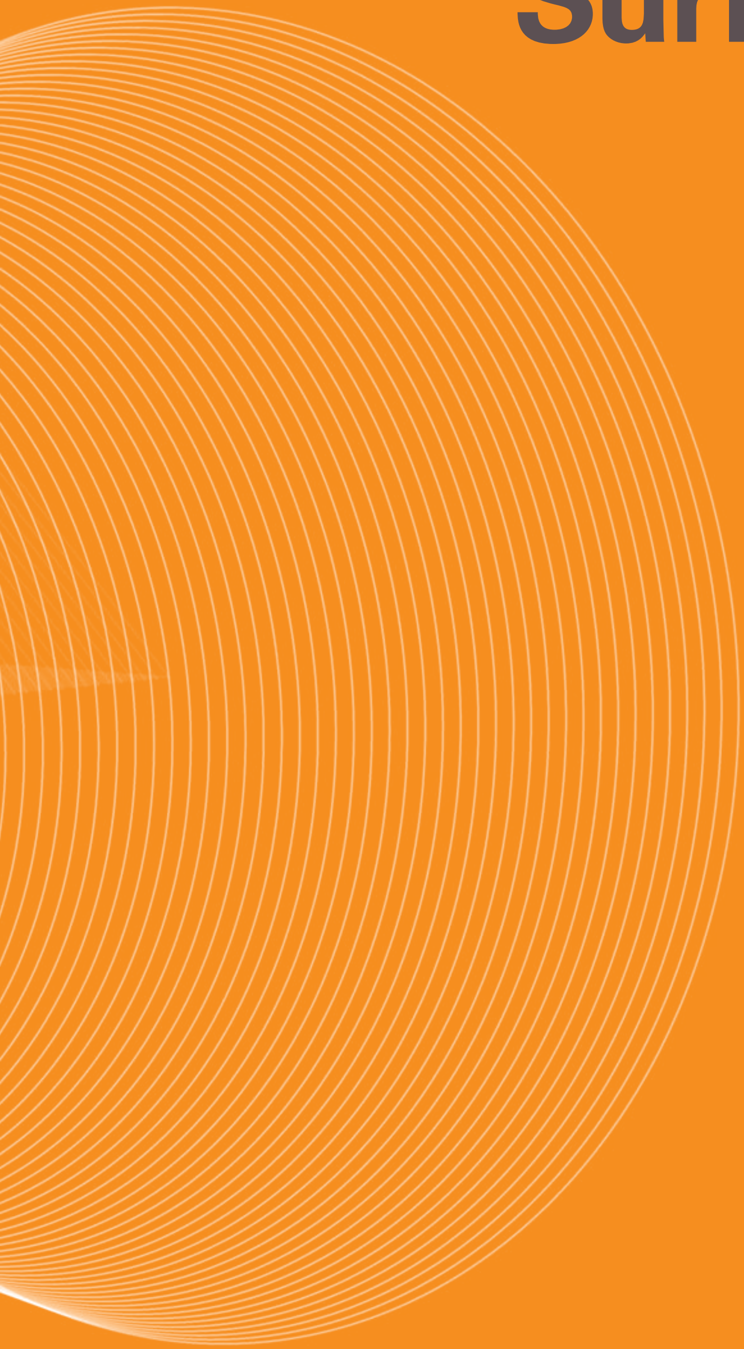
$x = v(\sin((-3/2)\cos(3u)+\cos(1-u)))$
$y = v((3/2)\sin(3u)+\sin(1-u))$
$z = u/4$



$\{ (u,v) \mid 0 \leq u \geq 12\pi,\ 2\pi/3 \leq v \geq \pi \}$

$x = v((-3/2)\cos(3u)+\cos(1-u))$
$y = v(\sin((3/2)\sin(3u)+\sin(1-u)))$
$z = u/4$

Developable Surfaces



‘That’s an important word, “understanding”. When I use that word, it means, “what the mind can do to find relationships”. I have been giving you relationships and relationships and relationships.’

(Buckminster Fuller, 1980)

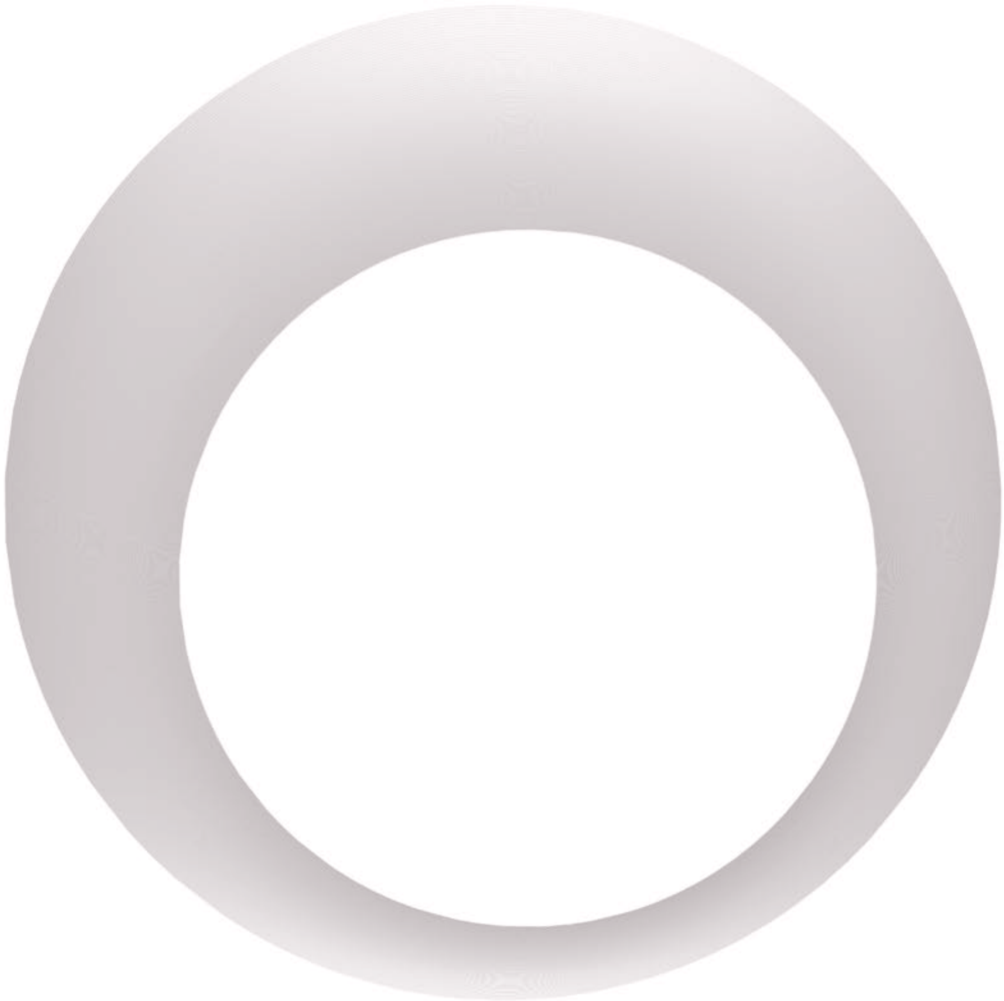
A sheet of paper on a desk is like a rectangle or plane in a Cartesian coordinate system. Lift the paper off from the desk, arch it, and connect the two short ends together. A plane has transformed into a cylinder. If the sheet of paper was initially cut into a fan-like shape, connecting the two edges could create a cone. Cylinders and cones are examples of developable surfaces. A developable surface is a surface with zero Gaussian curvature; in other words, a three-dimensional surface that can be unrolled to a flat two-dimensional surface without stretching or compressing.

Throughout this guide, shapes have morphed under mathematical transformations. It is possible to transform developable surfaces without compromising their zero Gaussian curvature.

In this chapter, the cylinder and cone are manipulated under a series of mathematical transformations while maintaining their developable surface logic. Above each three-dimensional developable surface is a flat two-dimensional unrolled surface; in each case, the two-dimensional unrolled surface could be used to construct the three-dimensional shape. Like a template or a cut file, each shape could be constructed out of a flat sheet of material. As the shapes transform, look at how their unrolled surfaces change. The object of this experiment is to design an understanding of developable surfaces by drawing relationships between shapes and their two-dimensional unrolled constructs.

Plane to cylinder

A plane can be transformed into a cylinder by defining a circle in the x- and y-coordinates. The plane and cylinder, both defined by the same periods, can be constructed out of the same two-dimensional plane. The extruded cosine and sine curves also share a common two-dimensional unrolled surface.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u$
$y = \cos(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

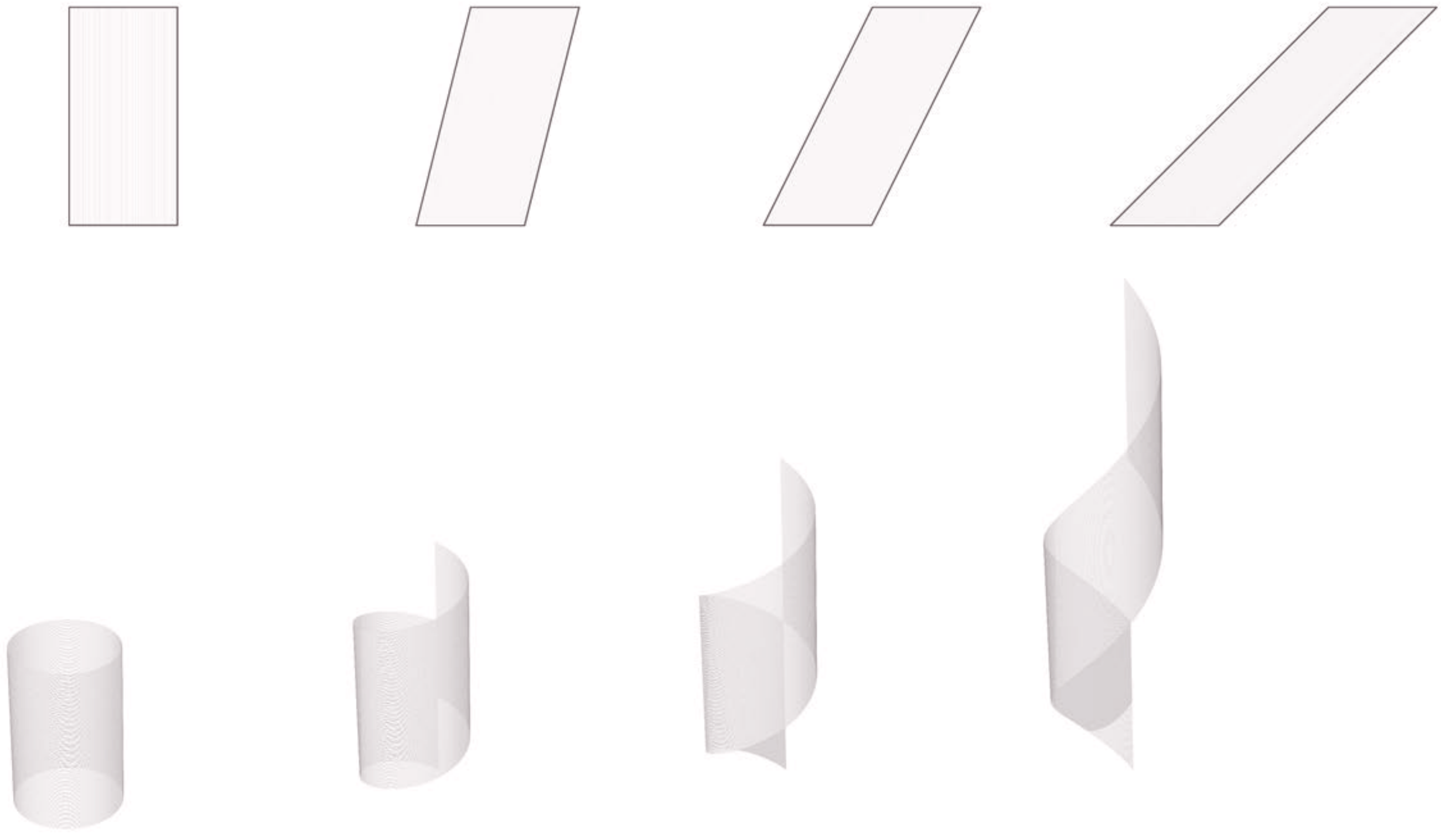
$x = \cos(u)$
$y = \sin(u)$
$z = v$

Cylinder with ascending

As a cylinder transforms by *ascending* in the z-coordinate, the closed circular section peels open to follow a helix. As the shape ascends, its unrolled two-dimensional surface shears to the right; the rectangle becomes a parallelogram.

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = u+v$



$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = u/4+v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = u/2+v$

$\{ (u,v) \mid 0 \le u \ge 2\pi,\ 0 \le v \ge \pi \}$

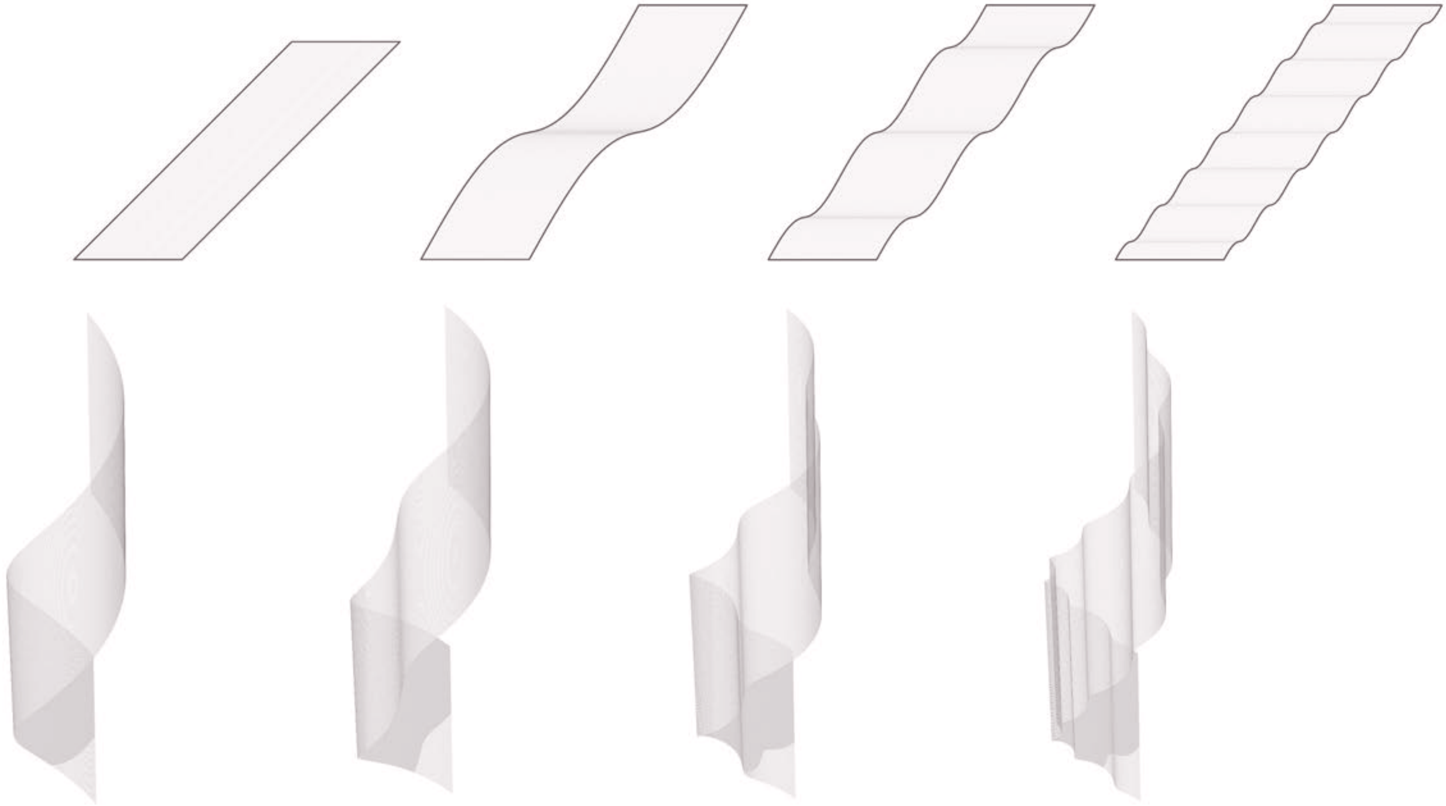
$x = \cos(u)$
$y = \sin(u)$
$z = u+v$

Cylinder with texturing and ascending

Texturing **is applied to the previous shape. As the frequency of the texture increases, the ripple in the flat two-dimensional unrolled surface also increases.**

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(8u)/10+\cos(u)$
$y = \sin(8u)/10+\sin(u)$
$z = u+v$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(u)$
$y = \sin(u)$
$z = u+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(2u)/(5/2)+\cos(u)$
$y = \sin(2u)/(5/2)+\sin(u)$
$z = u+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \cos(4u)/5+\cos(u)$
$y = \sin(4u)/5+\sin(u)$
$z = u+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

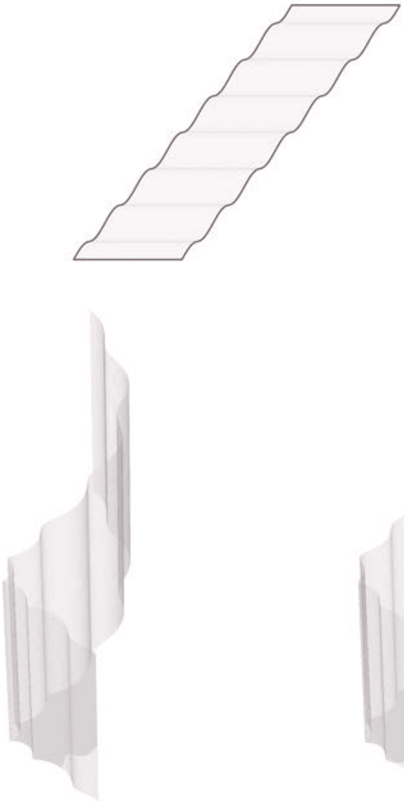
$x = \cos(8u)/10+\cos(u)$
$y = \sin(8u)/10+\sin(u)$
$z = u+v$

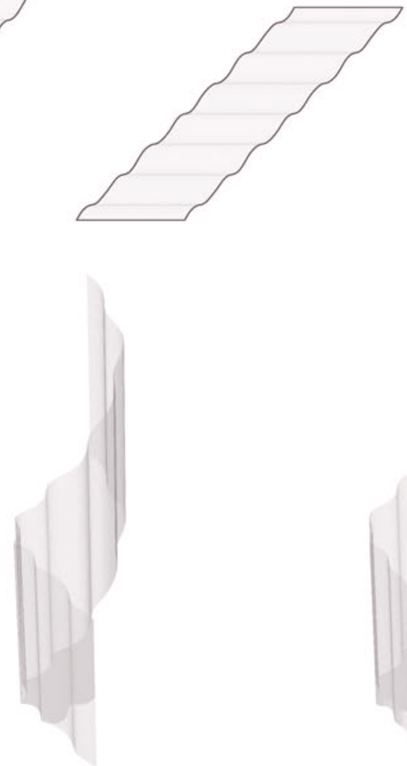
Cylinder with flattening, texturing and ascending

On top of *texturing*, *flattening* is applied to the initial shape. As the developable surface flattens over a series of recursions, the two-dimensional construct scales in one direction.

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(\sin(\sin(\sin(\sin(\cos(8u)/10 + \cos(u))))))$
$y = \sin(\sin(\sin(\sin(\sin(\sin(8u)/10 + \sin(u))))))$
$z = u+v$





$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \cos(8u)/10+\cos(u)$
$y = \sin(8u)/10+\sin(u)$
$z = u+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(\cos(8u)/10+\cos(u))$
$y = \sin(\sin(8u)/10+\sin(u))$
$z = u+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(\sin(\sin(\cos(8u)/10+\cos(u))))$
$y = \sin(\sin(\sin(\sin(8u)/10+\sin(u))))$
$z = u+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

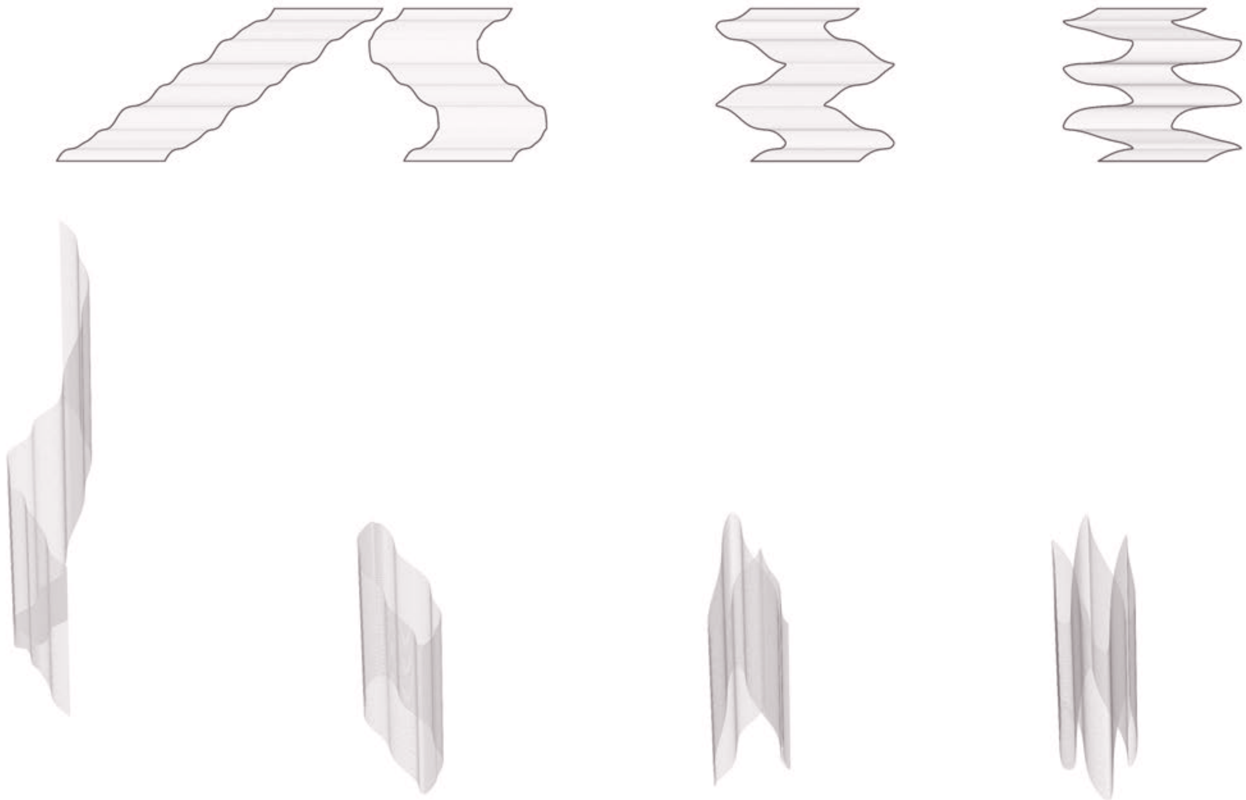
$x = \sin(\sin(\sin(\sin(\sin(\cos(8u)/10+\cos(u))))))$
$y = \sin(\sin(\sin(\sin(\sin(\sin(8u)/10+\sin(u))))))$
$z = u+v$

Cylinder with modulating, flattening and texturing

When the *ascending* transformation (adding a u-parameter in the z-coordinate) is replaced with a function of sine, the helical surface reconnects into a closed loop. After the shape is closed, *modulating* can be applied to vary the frequency at which the shape undulates up and down. As the three-dimensional shape modulates, so does the two-dimensional unrolled surface.

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = \sin(\sin(\sin(\sin(\sin(\cos(8u)/10 + \cos(u))))))$
$y = \sin(\sin(\sin(\sin(\sin(\sin(8u)/10 + \sin(u))))))$
$z = \sin(3u)+v$



$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(\sin(\sin(\sin(\sin(\cos(8u)/10$
$+\cos(u))))))$
$y = \sin(\sin(\sin(\sin(\sin(\sin(8u)/10$
$+\sin(u))))))$
$z = u+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(\sin(\sin(\sin(\sin(\cos(8u)/10$
$+\cos(u))))))$
$y = \sin(\sin(\sin(\sin(\sin(\sin(8u)/10$
$+\sin(u))))))$
$z = \sin(u)+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

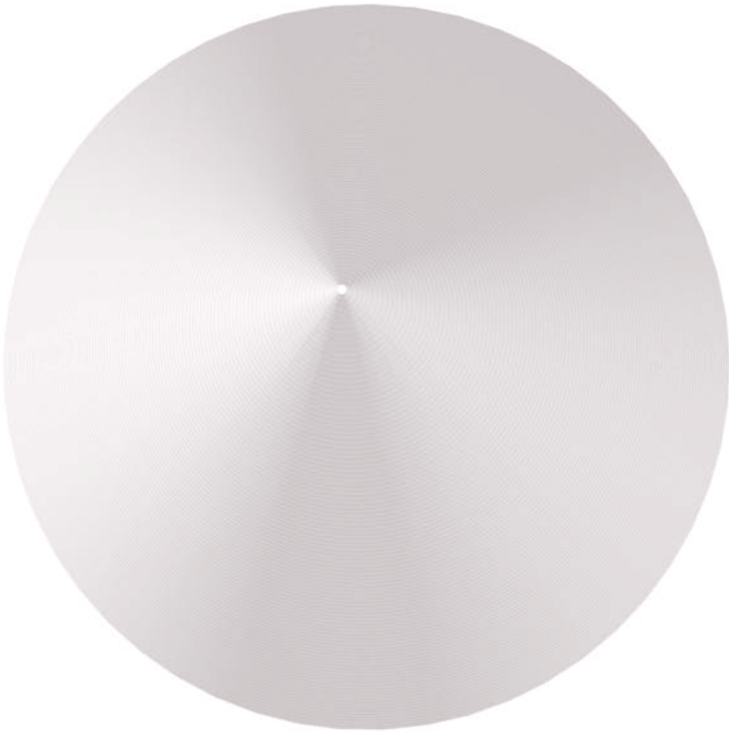
$x = \sin(\sin(\sin(\sin(\sin(\cos(8u)/10$
$+\cos(u))))))$
$y = \sin(\sin(\sin(\sin(\sin(\sin(8u)/10$
$+\sin(u))))))$
$z = \sin(2u)+v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = \sin(\sin(\sin(\sin(\sin(\cos(8u)/10$
$+\cos(u))))))$
$y = \sin(\sin(\sin(\sin(\sin(\sin(8u)/10$
$+\sin(u))))))$
$z = \sin(3u)+v$

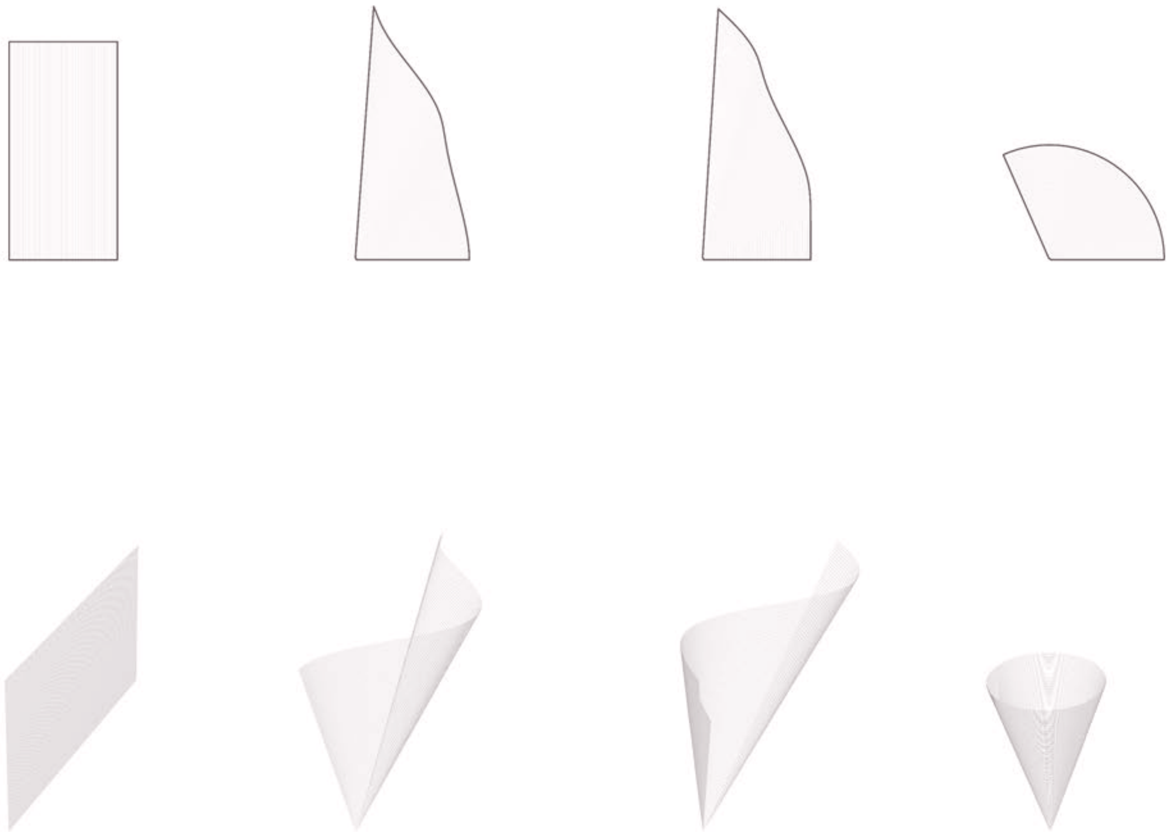
Plane to cone

A plane can be transformed into a cone by defining a circle (using cosine and sine) and multiplying with a v-parameter in the x- and y-coordinates. The shape no longer extrudes in one direction; instead, it tapers to a point. The unrolled developable surface transforms from a rectangle to a fan-like shape.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)\cos(u)$
$y = (v/3)\sin(u)$
$z = v$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = u$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)u$
$y = (v/3)\cos(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

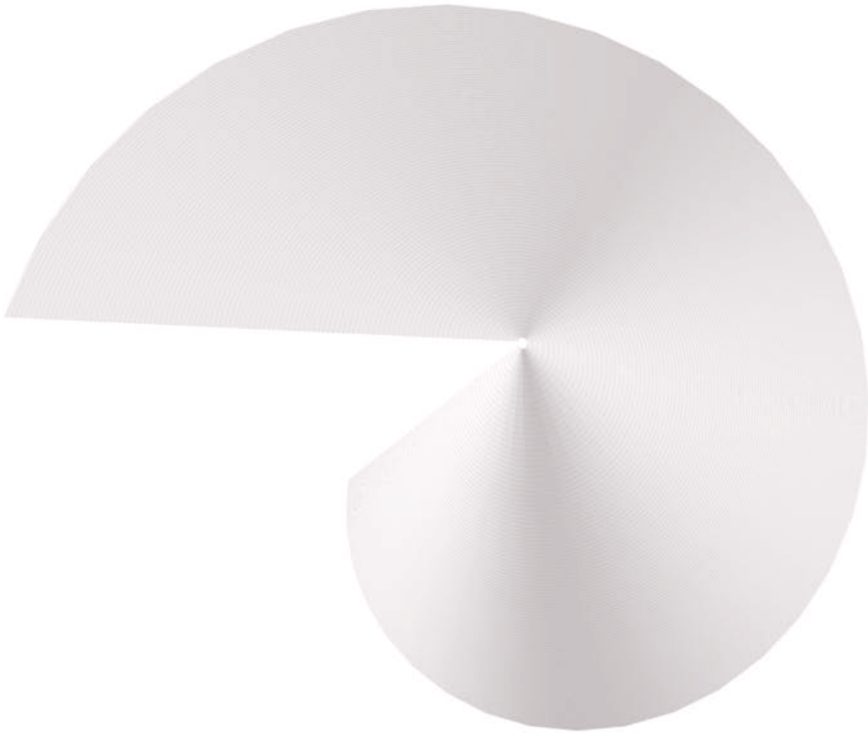
$x = (v/3)u$
$y = (v/3)\sin(u)$
$z = v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)\cos(u)$
$y = (v/3)\sin(u)$
$z = v$

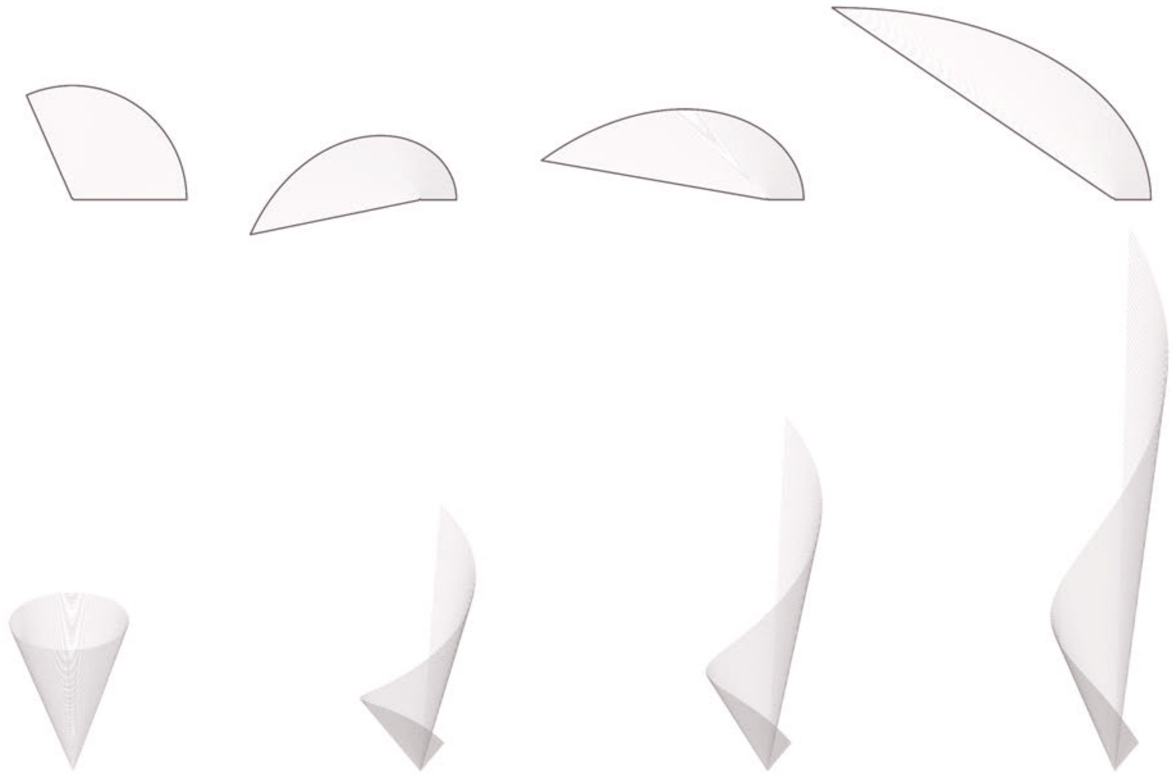
Cone with spiralling

As *spiralling* is applied to the z-coordinate, the circle profile transforms into a helical spiral, while maintaining its taper to a point. As the three-dimensional shape morphs, the radius of the two-dimensional unrolled surface increases incrementally.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)\cos(u)$
$y = (v/3)\sin(u)$
$z = (u/2)v$



{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = (v/3)cos(u)
y = (v/3)sin(u)
z = v

{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = (v/3)cos(u)
y = (v/3)sin(u)
z = (u/4)v

{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

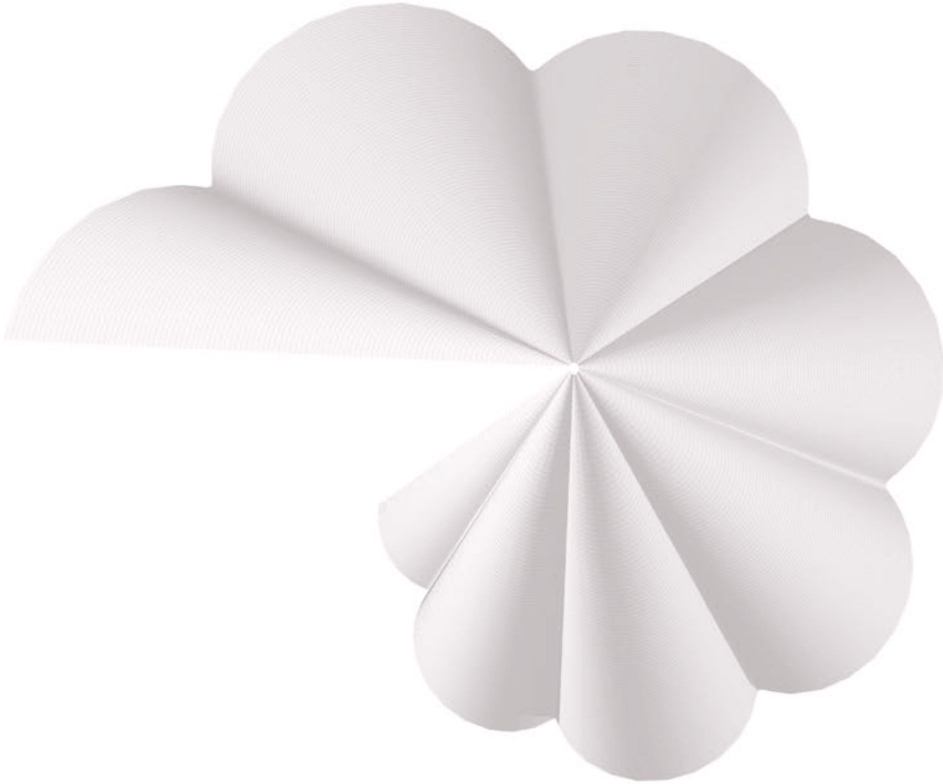
x = (v/3)cos(u)
y = (v/3)sin(u)
z = (u/3)v

{ (u,v) | 0 ≤ u ≥ 2π, 0 ≤ v ≥ π }

x = (v/3)cos(u)
y = (v/3)sin(u)
z = (u/2)v

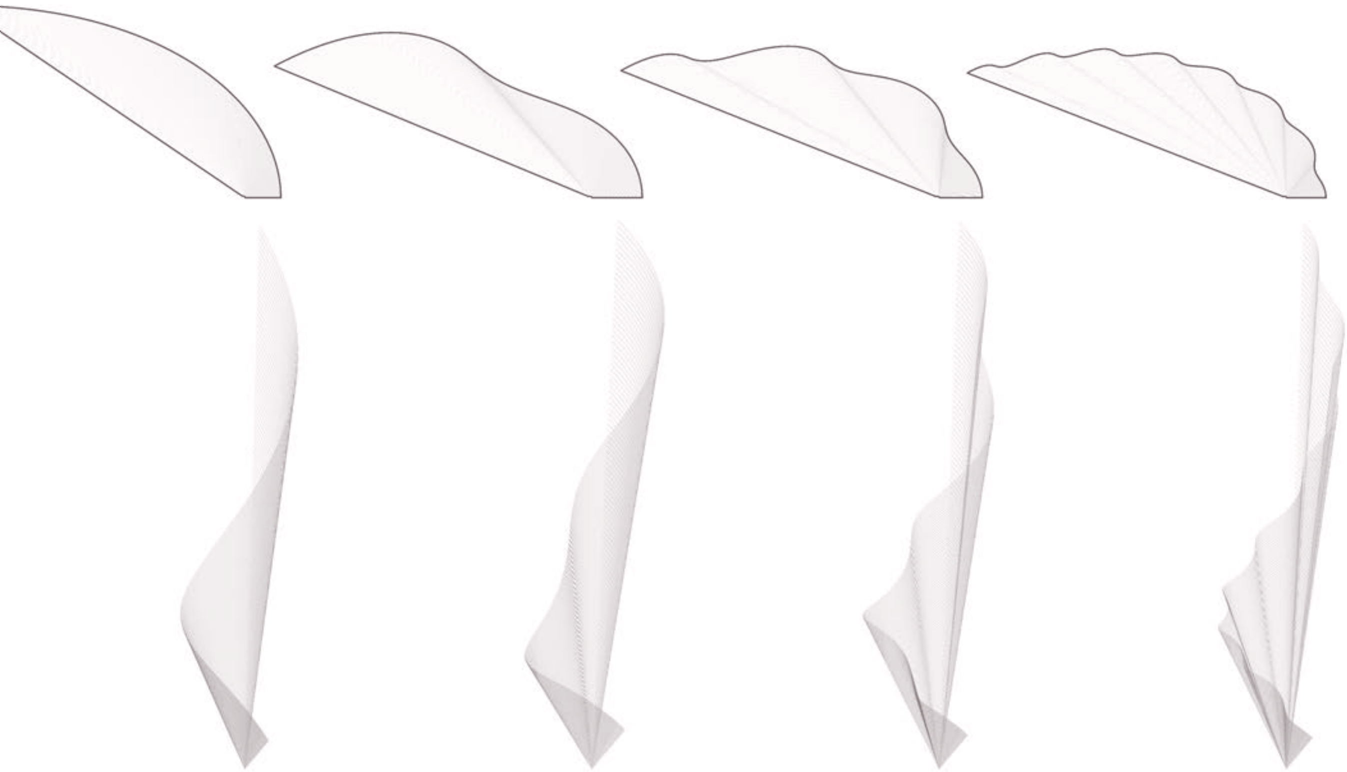
Cone with spiralling and texturing

Texturing **is added to the helical cone from the previous page. The size of the texture applied to the surface is directly related to the variation in the shape's radius. As the frequency of the texture increases, the two-dimensional surface's frequency also increases.**



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)(\cos(8u)/10+\cos(u))$
$y = (v/3)(\sin(8u)/10+\sin(u))$
$z = (u/2)v$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)\cos(u)$
$y = (v/3)\sin(u)$
$z = (u/2)v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)(\cos(2u)/(5/2)+\cos(u))$
$y = (v/3)(\sin(2u)/(5/2)+\sin(u))$
$z = (u/2)v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)(\cos(4u)/5+\cos(u))$
$y = (v/3)(\sin(4u)/5+\sin(u))$
$z = (u/2)v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)(\cos(8u)/10+\cos(u))$
$y = (v/3)(\sin(8u)/10+\sin(u))$
$z = (u/2)v$

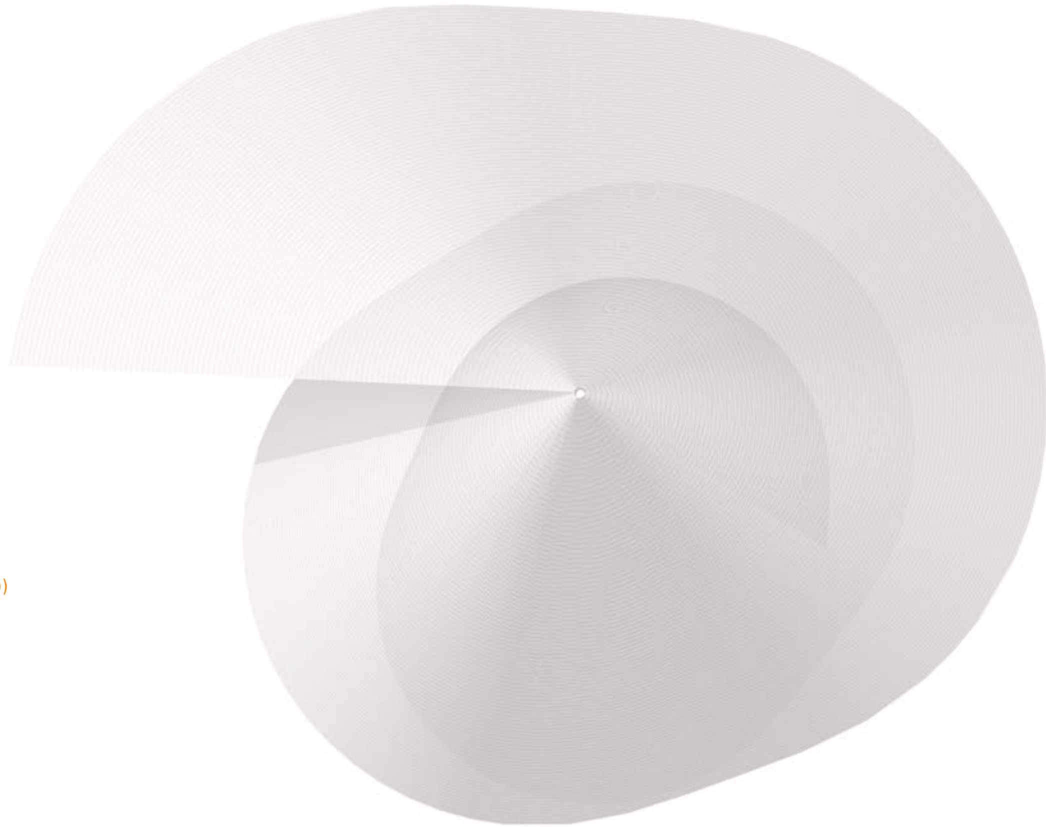
Developable Surfaces:
Plane to Cone

Cone with spiralling, texturing and modulating

Modulating **alters the frequency of a curve. When *modulating* is applied to a textured, helical, spiralled cone, the shape incrementally spirals inward. As the shape spirals in on itself, its surface area also increases, resulting in a two-dimensional unrolled surface that grows and spirals anti-clockwise.**

$\{ (u,v) \mid 0 \leq u \geq 2\pi,\ 0 \leq v \geq \pi \}$

$x = (v/3)(\cos(8u)/10+\cos((5/2)u))$
$y = (v/3)(\sin(8u)/10+\sin((5/2)u))$
$z = (u/2)v$



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)(\cos(8u)/10+\cos(u))$
$y = (v/3)(\sin(8u)/10+\sin(u))$
$z = (u/2)v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)(\cos(8u)/10+\cos((3/2)u))$
$y = (v/3)(\sin(8u)/10+\sin((3/2)u))$
$z = (u/2)v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

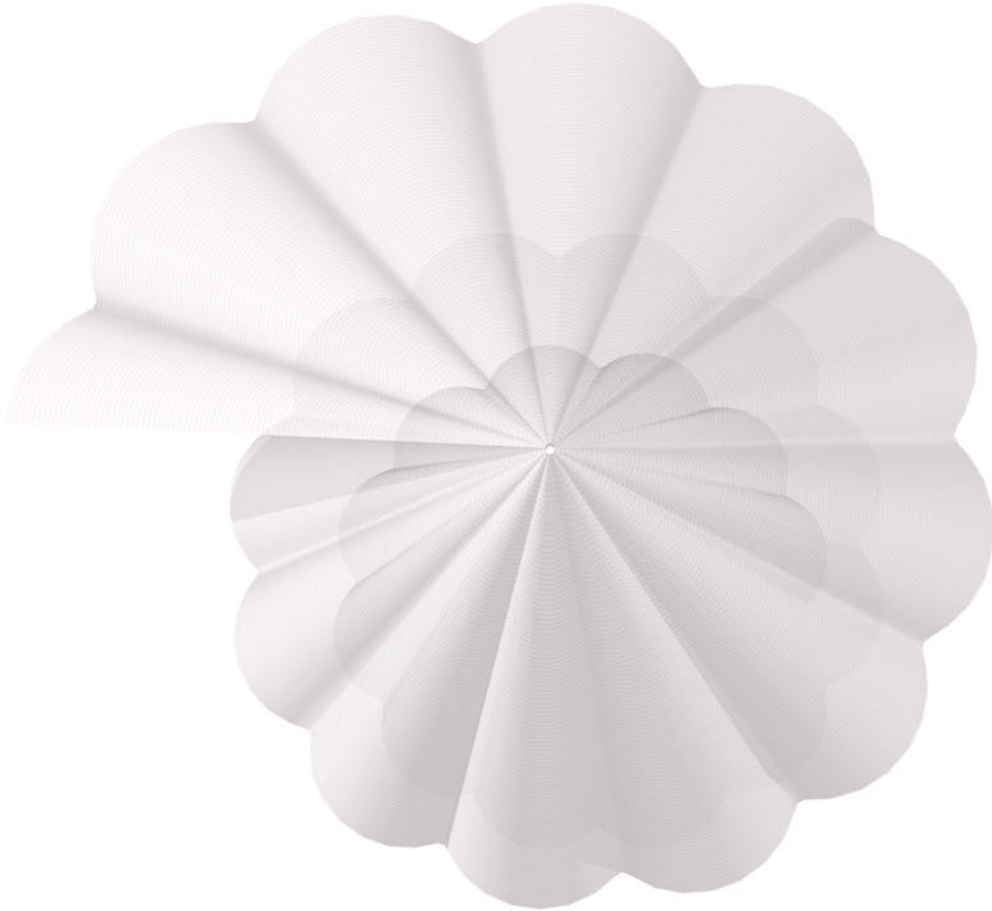
$x = (v/3)(\cos(8u)/10+\cos(2u))$
$y = (v/3)(\sin(8u)/10+\sin(2u))$
$z = (u/2)v$

$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)(\cos(8u)/10+\cos((5/2)u))$
$y = (v/3)(\sin(8u)/10+\sin((5/2)u))$
$z = (u/2)v$

Cone with spiralling, texturing and more modulating

Further *modulating* is applied to the previous shape. In this morphing series, the frequency of the texture is altered. As the frequency increases, the previously disintegrated texture reappears. A fine ripple emerges in the two-dimensional unrolled surface.



$\{ (u,v) \mid 0 \leq u \geq 2\pi, 0 \leq v \geq \pi \}$

$x = (v/3)(\cos(32u)/19+\cos((5/2)u))$
$y = (v/3)(\sin(32u)/19+\sin((5/2)u))$
$z = (u/2)v$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

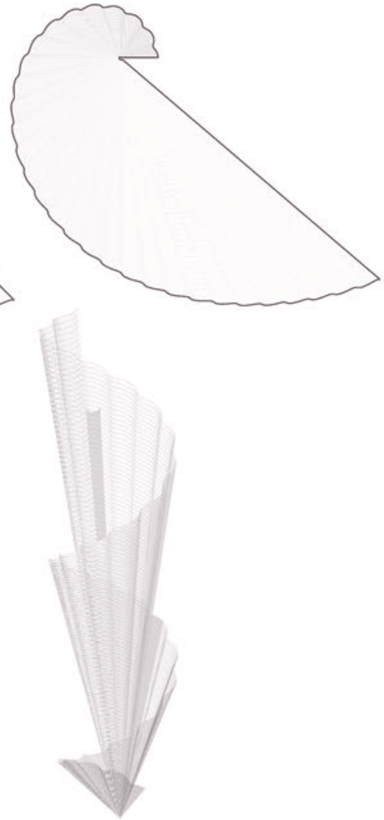
$x = (v/3)(\cos(8u)/10+\cos((5/2)u))$
$y = (v/3)(\sin(8u)/10+\sin((5/2)u))$
$z = (u/2)v$

$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = (v/3)(\cos(16u)/13+\cos((5/2)u))$
$y = (v/3)(\sin(16u)/13+\sin((5/2)u))$
$z = (u/2)v$

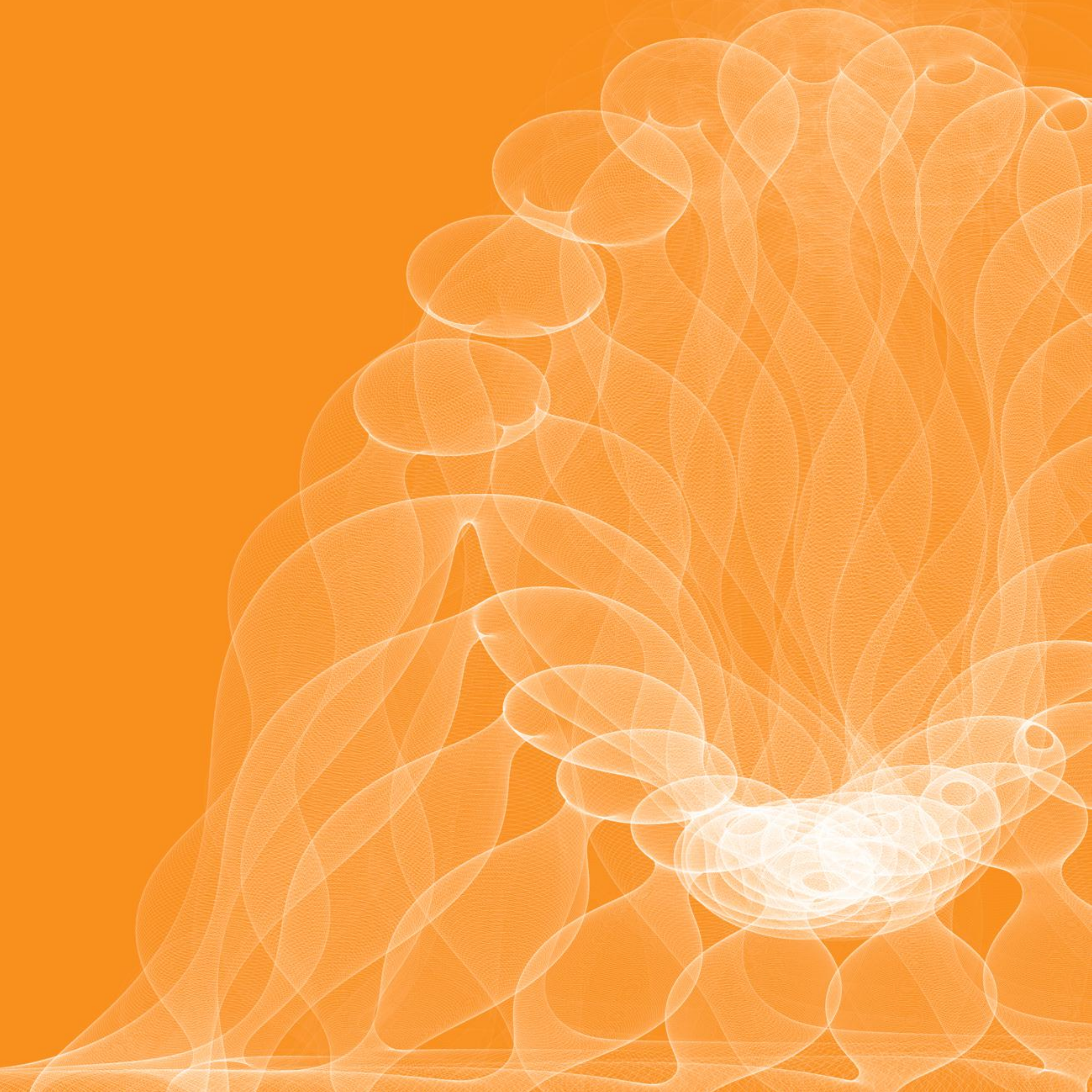
$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = (v/3)(\cos(24u)/16+\cos((5/2)u))$
$y = (v/3)(\sin(24u)/16+\sin((5/2)u))$
$z = (u/2)v$



$\{ (u,v) \mid 0 \le u \ge 2\pi, 0 \le v \ge \pi \}$

$x = (v/3)(\cos(32u)/19+\cos((5/2)u))$
$y = (v/3)(\sin(32u)/19+\sin((5/2)u))$
$z = (u/2)v$



Assumptions

‘His numerous legs, which were pitifully thin compared to the rest of his bulk, waved helplessly before his eyes.’

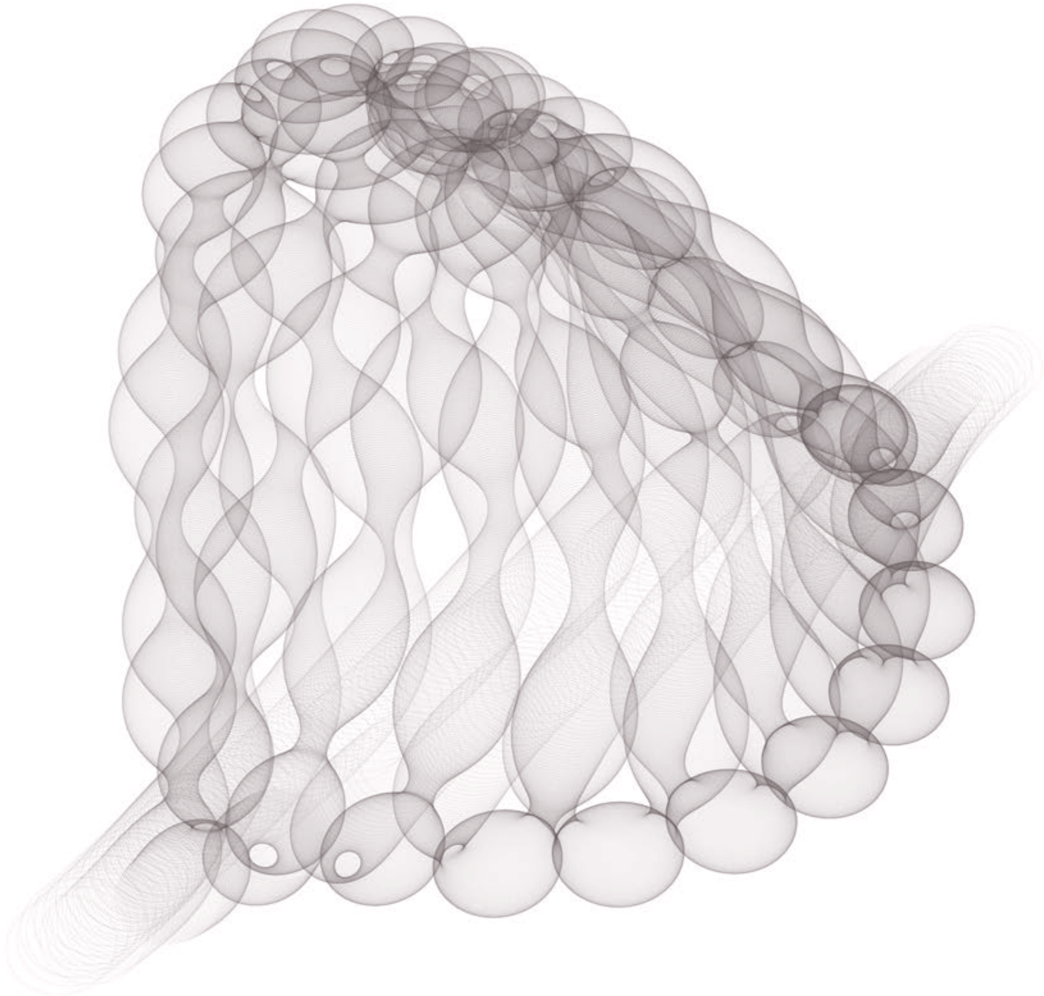
(Franz Kafka, 1915)

When an individual chooses a particular tool to design with, he or she is also choosing a specific technical constraint and bias. The choice of which tool to use determines which procedures can be followed. If the individual is unaware of this bias, the tool may be driving the designer, rather than the designer controlling the tool at hand. However, each tool has a philosophy embedded within it. This is especially the case for mathematics and many digital tools. As the author of this guide, I do not necessarily support or disagree with the philosophical bias of mathematics. My role is to design an understanding of mathematics and share my constructed reality. Note, my understanding of mathematics is self-constructed and designed, therefore it is also biased.

The world of mathematics does not think, feel or experience. It is a world of shapes, a world within itself; it is an autonomous reality – not a phenomenal one. It is autonomous, therefore architecture is autonomous when seen through its lens. It is not concerned with typology, materiality or even gravity.

Throughout this guide, many shapes appeared very spatial or even architectural. Remember, these are abstract line drawings defining the boundary of a geometry. They are only located within a Cartesian grid, not an actual site. They are lines, not detailed wall sections. They are diagrams: something between a dream and reality, theory and practice.

Finally, what we compute is not always what we see. A sine curve is a line that follows the periodic, trigonometric function sine along a distance defined by the parameter u. In the *modulating* transformation, we learned how the frequency of a curve can change. If the frequency of the curve increases to an extreme, the undulating curve will eventually appear to be a thick line or even a filled-in rectangle. If an individual was asked what they saw, he or she would not say an undulating sine curve – yet mathematics does not care what we see. It is autonomous. What it defines makes it what it is! This is similar to a dictionary, or to the field of linguistics – but governed here by a mathematical syntax. Mathematics is blinded by its own computational strength. Luckily, we can see, think, feel and experience.

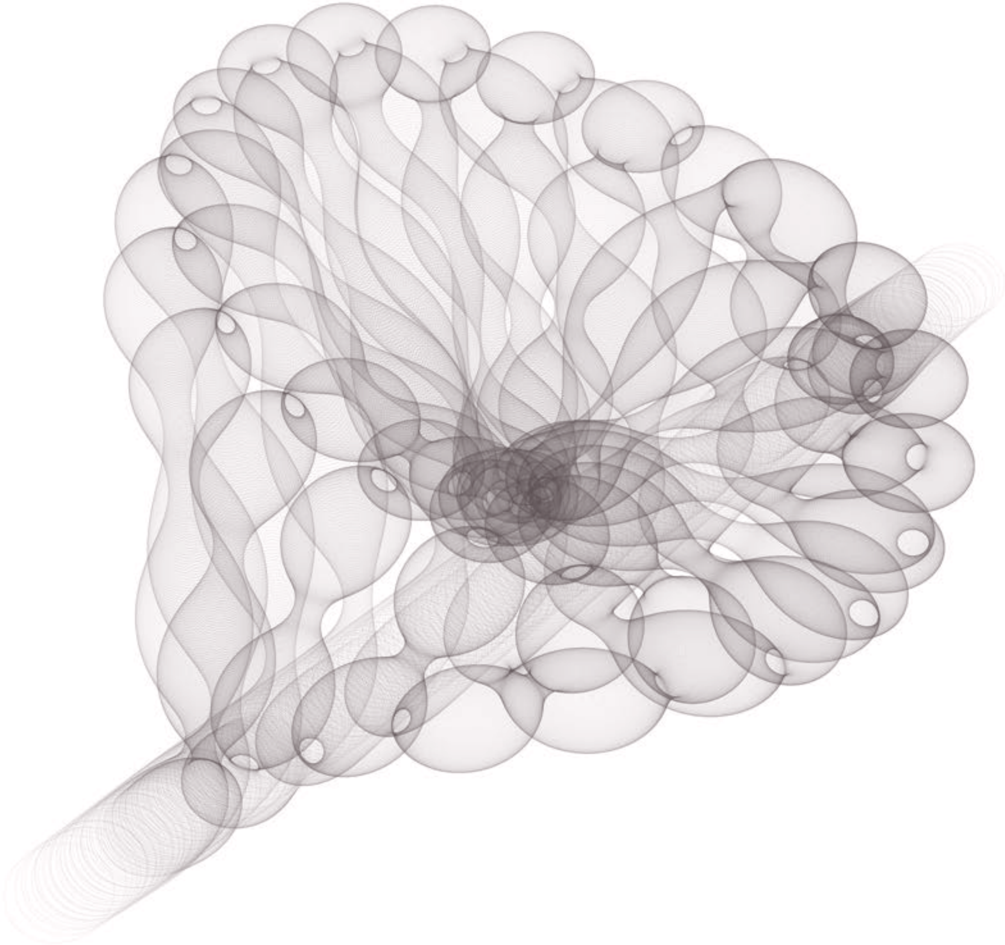


Disclaimer: This shape was generated by an ‘ugly’ equation. As a means to prevent mathematics from working harder than it needs to, ‘ugly’ equations are not shown in this guide.

A philosophical reading of mathematics

Within this guide, a basic transformation called *translating* was introduced. At first, it may appear to be trivial, however its philosophy is quite beautiful. It states that the location of a shape in space is part of that shape's inherent DNA. Therefore, if a shape moves in the x-direction, it transforms into a new shape. Two spheres of the same size in two different locations are not the same shape – according to mathematics. If a building was built in one city rather than another, it would operate differently. Similarly, if a building was rotated on the same site, its dialogue with solar orientation and passive systems would change. Mathematics believes the location of a shape is part of that shape. I suspect, therefore, that it is concerned with these architectural issues.

Mathematics is a formalist discipline. We can assume this is true, since it is primarily invested in shape definitions. Although mathematics can potentially define all the shapes in the world, I do not think it 'likes' all shapes equally. Perhaps, mathematics is inherently lazy – like many of us. If this is the case, it likes simple shapes, such as planes, cylinders, spheres and helicoids best. Shapes that are gestural and have complex asymmetrical relationships require longer, 'messy' or at times even 'ugly' equations. This is more work for mathematics, therefore it does not favour them as much. Similarly, if a design requires many equations to be fully defined, mathematics would prefer a different design, one that was defined by a single equation. Mathematics is a 'lazy' formal purist, a reductionist – perhaps even a modernist. It prefers shapes that express the inherent periodic nature of trigonometric functions. Why force a tool to do something it does not want to? Use a different tool that would prefer to do that task!



Disclaimer: This shape was generated by an equation 'too ugly' for this guide.

How was the guide developed?

As the author of this pedagogical guide it is my responsibility to confess that I am not a mathematician; I am a designer. As a result, my approach to researching mathematics is unorthodox. The question of whether or not every shape can be described by the trigonometric functions of sine and cosine is still an open problem. A mathematician might be invested in researching a proof to this question but I, like most designers, am not interested in a proof. Instead I am after the means and methods to instrumentalize trigonometric functions as a design tool. As a designer, I can learn far more by literally trying to make every shape in the world rather than trying to prove whether or not it can be done.

Imagine trying to write an algorithm to prove that every image can be drawn. A proof to this does not help an individual learn how to draw. In order to learn how to draw the individual must first draw! Words cannot easily express the feeling of pressing a brush pen down on a piece of paper. However, when an individual follows the rules of one- or two-point perspective, that individual is learning through a set of explicit instructions. Rules of perspective do not tell the individual what to draw, but rather guide the individual by giving him or her a means to a particular illusion of depth. Similarly, this pedagogical guide does not teach how to design with mathematics, but rather portrays a series of frameworks or a guide that can be used in many different ways.

When I began researching mathematics, I approached it a bit like learning how to draw. Through trial and error, I observed how specific operations yielded particular results. Initially, I could not explain why this was the case, as a mathematician would have been able to, but tacitly I knew what operations would cause a particular transformation. As I incrementally established a fundamental set of rules, I began asking more questions. Each topic within this guide builds on or challenges the prior one. In the end, this guide can be seen as a guide through my own thought process into mathematics. This pedagogical guide into mathematical transformations does not claim to be the only way to approach mathematics as a design tool. It is a particular, biased approach. It is my approach!

Bibliography

The following texts have influenced the research agenda, approach, or graphical aesthetic of this book.

Albers, Josef. *Interaction of Color: Text of the Original Edition with Revised Plate Section.* New Haven: Yale UP, 1975.

Arnheim, Rudolf. *Art and Visual Perception: A Psychology of the Creative Eye.* Berkeley: University of California, 1966.

Arnheim, Rudolf. *Visual Thinking.* Berkeley: University of California, 1969.

Ashby, W. Ross. *An Introduction to Cybernetics.* London: Chapman & Hall, 1956.

Barr, Stephen. *Experiments in Topology.* New York: Dover, 1989.

Brooks, Rodney Allen. *Cambrian Intelligence: The Early History of the New AI.* Cambridge, MA: MIT, 1999.

Choma, Joseph. 'T-H-I-C-K-E-N-I-N-G.' *Mathematics: From the Ideal to the Uncertain* (*The Cornell Journal of Architecture* 9, 2012): 191–199.

Corbusier, Le. *Towards a New Architecture.* New York: Dover Publications, 1986.

Critchlow, Keith. *Order in Space; a Design Source Book.* London: Thames & Hudson, 1969.

Demaine, Erik D., and Joseph O'Rourke. *Geometric Folding Algorithms: Linkages, Origami, Polyhedra.* Cambridge: Cambridge University Press, 2007.

Dewey, John. *Art as Experience.* New York: Minton, Balch & Company, 1934.

Dewey, John, and Arthur Fisher Bentley. *Knowing and the Known.* Boston: Beacon, 1949.

Dyson, Freeman J. 'Two Revolutions in Astronomy.' *Proceedings of the American Philosophical Society Held at Philadelphia for Promoting Useful Knowledge* 140.1 (1996): 1–9.

Engel, Heino. *Tragsysteme/Structure Systems.* Ostfildern: Hatje Cantz Verlag, 2009.

Foerster, Heinz von. *Understanding Understanding: Essays on Cybernetics and Cognition.* New York: Springer, 2003.

Francis, George K. *A Topological Picturebook.* New York: Springer-Verlag, 1987.

Fuller, R. Buckminster, and Richard J. Brenneman. *Fuller's Earth: A Day with Buckminster Fuller and the Kids.* New York, NY: New Press, 2009.

Ghyka, Matila C. *The Geometry of Art and Life.* New York: Dover Publications, 1977.

Glanville, Ranulph. 'Researching Design and Designing Research.' *Design Issues* 15.2 (1999): 80–91.

Kafka, Franz. *The Complete Stories.* New York: Schocken, dist. by Pantheon, 1988.

Kandinsky, Wassily. *Point and Line to Plane.* New York: Dover Publications, 1979.

Klee, Paul. *Pedagogical Sketchbook.* London: Faber and Faber Limited, 1953.

Krueger, Ted. *Designing Epistemology*. PhD Thesis, Architecture and Design. RMIT University, 2011.

Lakatos, Imre. 'Mathematics, Science and Epistemology.' *Philosophical Papers Volume 2*, ed. John Worrall and Gregory Currie. Cambridge: Cambridge University Press, 1978.

Langer, Ellen J. *Mindfulness.* Reading, MA: Addison-Wesley Publishing Company, 1989.

LeWitt, Sol. *Sol LeWitt – Isometric Drawings.* New York: Paula Cooper Gallery, 1982.

Lockwood, E. H. *A Book of Curves.* Cambridge: Cambridge University Press, 1961.

Loeb, Arthur L. *Color and Symmetry.* New York: Wiley-Interscience, 1971.

Loeb, Arthur L. *Concepts & Images: Visual Mathematics.* Boston: Birkhäuser, 1993.

Loos, Adolf. *Ornament and Crime: Selected Essays.* Riverside, CA: Ariadne, 1998.

Maeda, John. *Design by Numbers.* Cambridge, MA: MIT, 1999.

Maffei, Giorgio, and Emanuele de Donno. *Sol LeWitt, Artist's Books.* Mantova: Maurizio Corraini, 2010.

Minsky, Marvin Lee. *The Society of Mind.* New York: Simon and Schuster, 1988.

Negroponte, Nicholas. *The Architecture Machine.* Cambridge, MA: MIT, 1970.

Negroponte, Nicholas. *Soft Architecture Machines.* Cambridge, MA: MIT, 1975.

O'Hara, Morgan. *Live Transmissions: Attention and Drawing as Time-based Performance.* Bergamo: Lubrina, 2000.

Piaget, Jean, and Bärbel Inhelder. *The Psychology of the Child.* New York: Basic Books, 1969.

Polanyi, Michael. *The Tacit Dimension.* Garden City, NY: Doubleday, 1966.

Rowe, Colin. *The Mathematics of the Ideal Villa and Other Essays.* Cambridge, MA: MIT, 1979.

Schön, Donald A. *The Reflective Practitioner: How Professionals Think in Action.* New York: Basic Books, 1983.

Simon, Herbert A. *The Sciences of the Artificial.* Cambridge, MA: MIT, 1996.

Stiny, George, and James Gips. *Algorithmic Aesthetics: Computer Models for Criticism and Design in the Arts.* Berkeley: University of California, 1978.

Stiny, George. *Shape: Talking about Seeing and Doing.* Cambridge, MA: MIT, 2006.

Terzidis, Kostas. *Algorithmic Architecture.* Oxford; Burlington, MA: Architectural Press, 2006.

Terzidis, Kostas. 'Experiments on Morphing Systems.' *III Congreso Iberoamericano de Grafico Digital* (*SIGRADI Conference Proceedings*). Montevideo, Uruguay (1999): 149–151.

Thompson, D'Arcy Wentworth. *On Growth and Form.* Cambridge: Cambridge University Press, 1942.

Williams, Chris. 'Design by Algorithm.' *Digital Tectonics*, ed. Neil Leach, David Turnbull and Chris Williams. Chichester: Wiley-Academy, 2004: 78–85.

Williams, Robert. *The Geometry of Natural Structure: A Language of Form Source Book for Scientists and Designers.* San Francisco, CA: Eudaemon, 2009.

Acknowledgments

While a student at Massachusetts Institute of Technology, I became very interested in the inner computational workings behind digital tools. At some point, I became addicted to the curious autonomy of the field of mathematics. It is a bizarrely beautiful field, of which, even today, I have only scratched the surface.

Dennis Shelden was the first individual with whom I shared my research. As the work began to reach high levels of complexity, Dennis encouraged me to step back and understand the basic principles of the mathematics. Since that conversation, I have been obsessed with designing and understanding.

Embedded deep within this book is a philosophical framework rooted in the discourse of shape grammars. George Stiny challenges symbolic computing with visual calculation. When I first started working on this project, George told me to write fewer words and let the equations and shapes be the primary text. When flipping through this book, stop and look at the shapes and equations. They are one and the same.

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