Applied Mathematics and Computation 216 (2010) 3303–3306

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# Numerical solution of Troesch's problem by simple shooting method

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ARTICLE INFO

*Keywords:*
Troesch's problem
Shooting method
Variable transformation

ABSTRACT

This paper describes a simple and efficient approach to the Troesch's problem. In this approach, the hyperbolic nonlinear term in the equation is first converted into polynomial nonlinear terms by variable transformation, and a simple shooting method is then used directly to solve this transformed problem. The calculated results are in excellent agreement with those obtained by other analytical and numerical methods.



## 1. Introduction

Troesch's problem, defined by

$$\begin{aligned} &y'' = n\sinh(ny),\\ &y(0) = 0, \quad y(1) = 1, \end{aligned} \tag{1}$$

where $n$ is a positive constant, arises in an investigation of the confinement of a plasma column by radiation pressure [1] and also in the theory of gas porous electrodes [2,3]. It has been shown that $y(x)$ has a singularity located approximately at [4,5]

$$x_s = \frac{1}{n}\ln\left(\frac{8}{y'(0)}\right), \tag{2}$$

which implies that the singularity lies within the integration range if $y'(0) > 8e^{-n}$. This results in the problem being very difficult to solve by the shooting method and this difficulty increases as $n$ increases. Although, several iterative approximate methods such as Adomian decomposition method [6,7], variational iteration method [8], and modified homotopy perturbation method [9] fail to solve this problem for $n > 1$, other iterative or numerical methods such as differential transform method [10], multipoint shooting method combined with continuation and perturbation technique [11], invariant imbedding method [12], inverse shooting method [13] and simple shooting method combined with modified Newton's method, overflow trap or parameter mapping technique [14–16] have been successfully applied to this problem for $n > 5$ and yielded results varying in accuracy.

In this paper, we proposed a very simple numerical method in which the hyperbolic type nonlinearity in the problem is first converted into polynomial type nonlinearities by variable transformation, and a simple shooting method is then used to solve this transformed problem in a straightforward manner. This approach is reliable and efficient without requiring any specific technique such as overflow trap, modified Newton's method or parameter mapping technique. The calculated results for a wide range of $n$ are highly accurate as compared with those obtained by other analytical and numerical methods. It is expected that this approach can be extended to other inherently unstable two-point boundary value problems with hyperbolic nonlinearity.

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doi:10.1016/j.amc.2010.04.056