

Validation Report for the Equivalent Aggregation Theorem

The Equivalent Aggregation Theorem postulates that "Aggregation after Accumulation" employed by FL-DPCR is equivalent, in terms of aggregation, to the traditional FedAvg under differential privacy. It implies that FL-DPCR and DPFedAvg obtain exactly the same learning effect when FL-DPCR adopts SimpleMech as the DPCR model, which holds even when part of the clients are selected.

To verify this theorem, we executed the following commands in directory ".../FLDPCR" to run by DPFedAvg and FL-DPCR with SimpleMech:

```
python main.py '{"global_config':{'seed':1}, 'fed_config':{'C':0.4}, 'dpcr_model':{'name':'DPFedAvg'}}"
python main.py '{"global_config':{'seed':1}, 'fed_config':{'C':0.4}, 'dpcr_model':{'name':'SimpleMech'}}"
```

The above command uses dpcr_model as the only variable. They set the same random seed and the percentage of participants selected at 40%, which ensures the other conditions are consistent. Specifically, DPFedAvg utilizes the traditional Federated Aggregation, while FL-DPCR with SimpleMech uses "Aggregation after Accumulation". Refer to the train_federated_model function in server.py, as shown below:

```
# average each updated model parameters of the selected clients and update the global model
# DPFedAvg uses "traditional Federated Aggregation"; FL-DPCR algorithms use "Aggregation after Accumulation"
if self.dpcr_model['name'] == 'DPFedAvg':
    self.federatedAggregation(sampled_client_indices, mixing_coefficients)
else:
    self.aggregateAfterAccumulation(sampled_client_indices, mixing_coefficients)
```

The experimental results, depicted in Figure 1, illustrate that both DPFedAvg and FL-DPCR with SimpleMech consistently maintain identical test accuracy throughout the training process (The actual difference in test accuracy between them is only 0.03%, which we believe is due to the precision of floating-point numbers). This result emphatically verifies the correctness of Equivalent Aggregation Theorem.

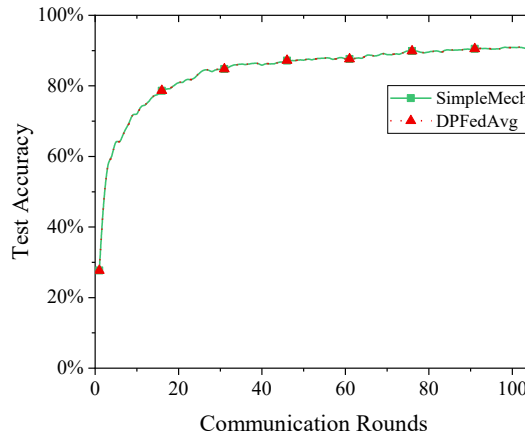


Fig 1. Test Accuracy of DPFedAvg and FL-DPCR with SimpleMech