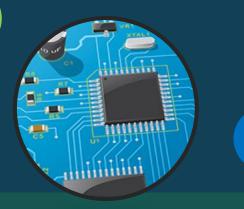


# Floating-Point Processing and Instruction Encoding







Topic # 11
Fall 2020

[ref]: Irvine, Kip R. Assembly Language for x86 Processors



## **Contents**

Floating-Point Binary Representation

Floating-Point Unit

x86 Instruction Encoding



## Floating-Point Binary Representation

- IEEE Floating-Point Binary Reals
- The Exponent
- Normalized Binary Floating-Point Numbers
- Creating the IEEE Representation
- Converting Decimal Fractions to Binary Reals



# **IEEE Floating-Point Binary Reals**

## **Types**

## **Single Precision**

32 bits: 1 bit for the sign, 8 bits for the exponent, and 23 bits for the fractional part of the significand.

#### **Double Precision**

64 bits: 1 bit for the sign, 11 bits for the exponent, and 52 bits for the fractional part of the significand.

#### **Double Extended Precision**

80 bits: 1 bit for the sign, 16 bits for the exponent, and 63 bits for the fractional part of the significand.



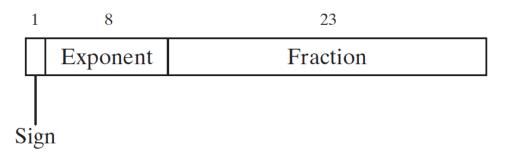
## **Single Precision**

# Approximate normalized range:

 $2^{-126}$  to  $2^{127}$ 

## Also called a short real

Figure 12–1 Single-precision format.





# Components of a Single-Precision Real

## Sign

1 = negative, 0 = positive

## **Significand**

decimal digits to the left & right of decimal point weighted positional notation

## **Example:**

$$123.154 = (1 \times 10^{2}) + (2 \times 10^{1}) + (3 \times 10^{0}) + (1 \times 10^{-1}) + (5 \times 10^{-2}) + (4 \times 10^{-3})$$

#### **Exponent**

unsigned integer

integer bias (127 for single precision)



# Components of a Single-Precision Real

## 11.1011

$$= (1 \times 2^{1}) + (1 \times 2^{0}) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) + (1 \times 2^{-4})$$

## **Sum of fractions**

$$.1011 = \frac{1}{2} + \frac{0}{4} + \frac{1}{8} + \frac{1}{16} = \frac{11}{16}$$

- The Decimal Numerator (11) represents the binary bit pattern 1011
- If e is the number of significant bits to the right of the binary point, the decimal denominator is 2e
- In our example, **e = 4**, so 2e = 16



## **Decimal Fractions vs Binary Floating-Point**

11.1011

$$= (1 \times 2^{1}) + (1 \times 2^{0}) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) + (1 \times 2^{-4})$$

$$.\,\mathbf{1011} = \frac{1}{2} + \frac{0}{4} + \frac{1}{8} + \frac{1}{16} = \frac{\mathbf{11}}{\mathbf{16}}$$

Table 12-2 Examples: Translating Binary Floating-Point to Fractions.

Binary Floating-Point	Base-10 Fraction
11.11	3 3/4
101.0011	5 3/16
1101.100101	13 37/64
0.00101	5/32
1.011	1 3/8
0.0000000000000000000000000000000000000	1/8388608

Table 12-3 Binary and Decimal Fractions.

Binary	Decimal Fraction	Decimal Value
.1	1/2	. 5
.01	1/4	.25
.001	1/8	.125
.0001	1/16	.0625
.00001	1/32	.03125



# The Exponent

- Sample Exponents represented in Binary
- Add 127 to actual exponent to produce the biased exponent

Table 12-4 Sample Exponents Represented in Binary.

Exponent (E)	Biased (E + 127)	Binary
+5	132	10000100
0	127	01111111
-10	117	01110101
+127	254	11111110
<b>-126</b>	1	00000001
-1	126	01111110



## **Normalizing Binary Floating-Point Numbers**

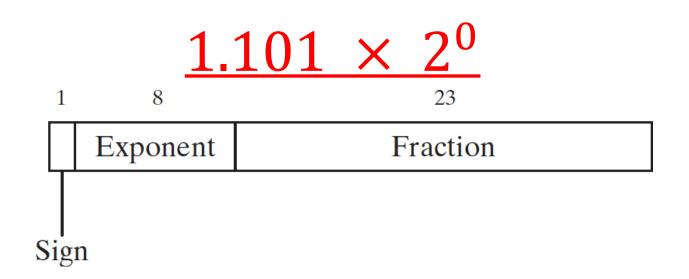
- Mantissa is normalized, when a single 1 appears to the left of the binary point
- Un-normalized: shift binary point until exponent is zero
- Examples

Denormalized	Normalized
1110.1	1.1101 x 2 <sup>3</sup>
.000101	1.01 x 2 <sup>-4</sup>
1010001.	1.010001 x 2 <sup>6</sup>



## **Creating IEEE Presentation**

Once the Sign Bit, Exponent and Significand fields are Normalized and Encoded, it's easy to generate a complete binary IEEE Short Real



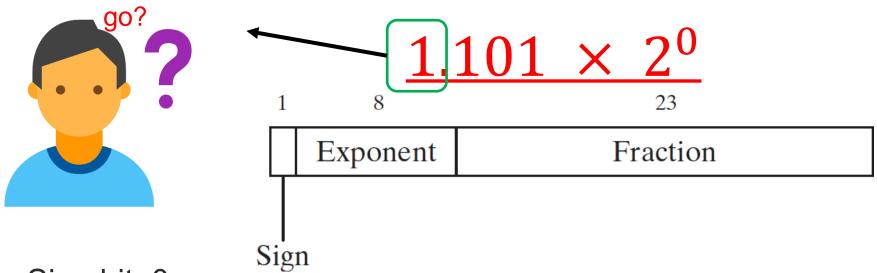
- Sign bit: 0
- Exponent: 01111111 (biased- exponent )
- Fraction: 1010000000000000000000



## **Creating IEEE Presentation**

Once the Sign Bit, Exponent and Significand fields are Normalized and Encoded, it's easy to generate a complete binary IEEE Short Real





- Sign bit: 0
- Exponent: 01111111 (biased- exponent )
- Fraction: 1010000000000000000000



## **Examples**

Table 12-5 Examples of Single Precision Bit Encodings.

Binary Value	Biased Exponent	Sign, Exponent, Fraction	
-1.11	127	1 0111111 11000000000000000000000	
+1101.101	130		
00101	124		
+100111.0	132	0 10000100 0011100000000000000000	
+.0000001101011	120	0 01111000 1010110000000000000000	



## Real-Number Encodings

- Normalized Finite Numbers
  - all the nonzero finite values that can be encoded in a normalized real number between zero and infinity
- Positive and Negative Infinity
- NaN (Not a Number)
  - Bit Pattern that is not a valid FP value
  - Two Types of NaN:
    - Quiet NaN
    - Signaling NaN



## Real-Number Encodings

- Normalized Finite Numbers
  - All the nonzero finite values that can be encoded in a normalized real number between zero and infinity
- Positive and Negative Infinity
- NaN (not a number)
  - Bit Pattern that is not a valid FP value

#### Two Types:

- Quiet Nan
- Signaling Nan

Table 12-6 Specific Single-Precision Encodings.

Value	Sign, Exponent, Significand		
Positive zero	0	00000000	000000000000000000000000000000000000000
Negative zero	1	00000000	000000000000000000000000000000000000000
Positive infinity	0	11111111	000000000000000000000000000000000000000
Negative infinity	1	11111111	000000000000000000000000000000000000000
QNaN	х	11111111	1xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
SNaN	x	11111111	0xxxxxxxxxxxxxxxxxxxxxxxx

<sup>&</sup>lt;sup>a</sup> SNaN significand field begins with 0, but at least one of the remaining bits must be 1.



## **Converting Fractions to Binary Reals**

- Express as a sum of fractions having denominators that are powers of 2
- Examples: Decimal 0.5 as fraction 5/10
- Decimal 5 is binary 0101, and decimal 10 is binary 1010

#### **Binary long division**

.1

 Table 12-7
 Examples of Decimal Fractions and Binary Reals.

Decimal Fraction	Factored As	Binary Real
1/2	1/2	.1
1/4	1/4	.01
3/4	1/2 + 1/4	.11
1/8	1/8	.001
7/8	1/2 + 1/4 + 1/8	.111
3/8	1/4 + 1/8	.011
1/16	1/16	.0001
3/16	1/8 + 1/16	.0011
5/16	1/4 + 1/16	.0101



## **Converting Single-Precision to Decimal**

- 1. If the MSB is 1, the Number is Negative; otherwise, it is Positive
- 2. The next 8 Bits represent the Exponent
  - Subtract Binary 01111111 (Decimal 127), producing the Unbiased exponent
  - Convert the Unbiased Exponent to Decimal.
- 3. The next 23 Bits represent the Significand
  - Notate a "1.", followed by the significand bits
  - Trailing Zeros can be ignored
  - Create a Floating-point Binary Number, using the Significand, the Significand determined in step 1, and the Exponent calculated in step 2
- 4. Denormalize the binary number produced in step 3, i.e. Shift the binary point to the number of places equal to the value of the exponent
  - Shift right if the exponent is positive, or left if the exponent is negative
- 5. From left to right, use Weighted Positional Notation to form the decimal sum of the powers of 2 represented by the floating-point binary number



$$1. xxxxxx \times 2^{+x}$$

1. The number is Positive



$$1. xxxxxx \times 2^{+3}$$

10000010 - 01111111 = 00000011

- 1. The number is Positive
- 2. The unbiased exponent is Binary 00000011, or Decimal 3



1. 01011 
$$\times$$
 2<sup>+3</sup>

- 1. The number is Positive
- 2. The unbiased exponent is Binary 00000011, or Decimal 3
- 3. Combining the Sign, Exponent and Significand, the Binary Number is +1.01011 X 2<sup>3</sup>



$$1.01011 \times 2^{+3}$$

- 1. The number is positive
- 2. The unbiased exponent is binary 00000011, or decimal 3
- 3. Combining the Sign, Exponent and Significand, the Binary Number is +1.01011 X 2<sup>3</sup>
- 4. The un-normalized Binary Number is +1010.11



$$1.01011 \times 2^{+3}$$

- 1. The number is positive
- 2. The unbiased exponent is binary 00000011, or decimal 3
- 3. Combining the Sign, Exponent and Significand, the Binary Number is +1.01011 X 2<sup>3</sup>
- 4. The un-normalized Binary Number is +1010.11
- 5. The Decimal value is +10 3/4, or +10.75



## Contents

- Floating-Point Binary Representation
- Floating-Point Unit
- x86 Instruction Encoding



# **Floating Point Unit**

- FPU Register Stack
- Rounding
- Floating-Point Exceptions
- Floating-Point Instruction Set
- Arithmetic Instructions
- Comparing Floating-Point Values
- Reading and Writing Floating-Point Values
- Exception Synchronization
- Mixed-Mode Arithmetic
- Masking and Unmasking Exceptions



- The FPU does not use the general-purpose registers (EAX, EBX, etc.)
- Instead, it has its own set of registers called a Register Stack
- It loads values from Memory into the Register Stack, performs calculations, and stores stack values into Memory

FPU instructions evaluate Mathematical Expressions in Postfix Format



FPU instructions evaluate Mathematical Expressions in Postfix Format

**Infix Format** 

$$(A + B) \times C$$



FPU instructions evaluate Mathematical Expressions in Postfix Format

**Infix Format** 

$$(A + B) \times C$$

$$(A + B) \times C$$



FPU instructions evaluate Mathematical Expressions in Postfix Format

**Infix Format** 

$$(A + B) \times C$$



FPU instructions evaluate Mathematical Expressions in Postfix Format

**Infix Format** 

$$(A + B) \times C$$

$$AB+C\times$$



FPU instructions evaluate mathematical expressions in postfix format

**Infix Format** 

$$(A + B) \times C$$

$$AB+C\times$$

$$(5 \times 6) + 4$$



FPU instructions evaluate mathematical expressions in postfix format

**Infix Format** 

$$(A + B) \times C$$

$$AB+C\times$$

$$(5 \times 6) + 4$$

$$56 \times 4 +$$



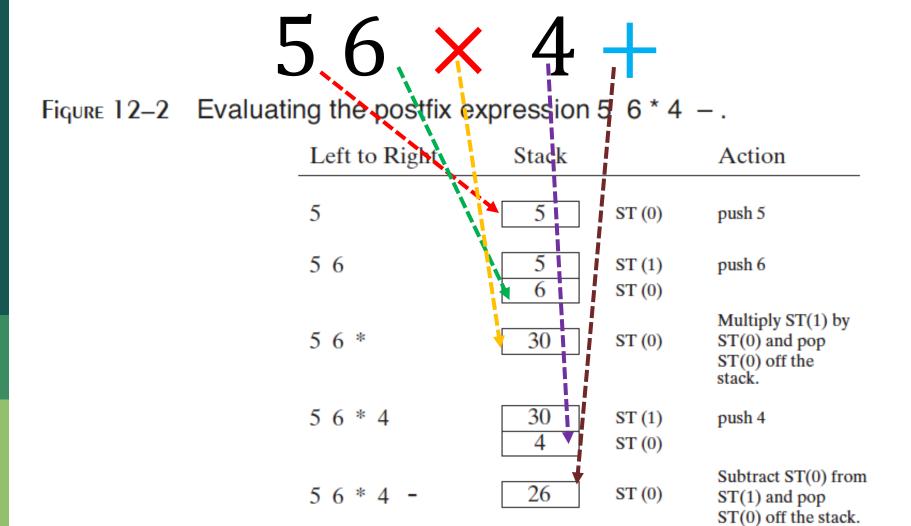




Table 12-8 Infix to Postfix Examples.

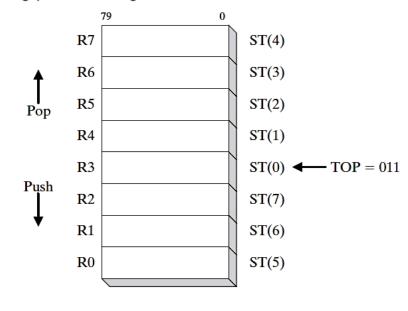
Infix	Postfix
A + B	A B +
(A - B) / D	
(A + B) * (C + D)	
((A + B) / C) * (E - F)	



 Eight individually addressable 80-bit data registers named R0 through R7

Figure 12-3 Floating-point data register stack.

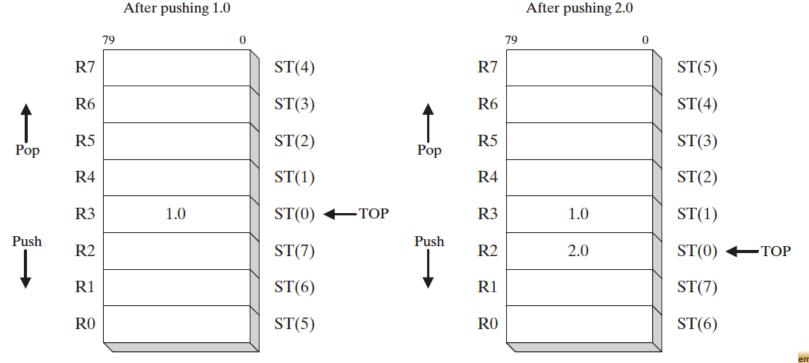
 Three-bit field named TOP in the FPU status word identifies the register number that is currently the top of stack





- A push operation (also called load) decrements TOP by 1 and copies an operand into the register identified as ST(0)
- Overwriting existing data in the register stack, a floatingpoint exception is generated

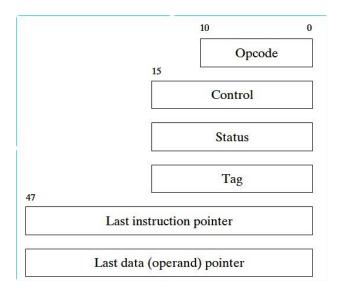
Figure 12–4 FPU stack after pushing 1.0 and 2.0.





# **Special Purpose Registers**

- Opcode register: stores opcode of last noncontrol instruction executed
- Control register: controls precision and rounding method for calculations
- Status register: top-of-stack pointer, condition codes, exception warnings
- Tag register: indicates content type of each register in the register stack
- Last instruction pointer register: pointer to last non-control executed instruction
- Last data (operand) pointer register: points to data operand used by last executed instruction





## **FPU Control word**

The FPU control word contains two bits named the RC field that specify which rounding method to use.

The field values are as follows:

- 00 binary: Round to nearest even (default)
- 01 binary: Round down toward negative infinity
- 10 binary: Round up toward positive infinity
- 11 binary: Round toward zero (truncate)

Table 12-9 Example: Rounding +1.0111.

Method	Precise Result	Rounded	
Round to nearest even	1.0111	1.100	
Round down toward −∞	1.0111	1.011	
Round toward +∞	1.0111	1.100	
Round toward zero	1.0111	1.011	



## **FPU Exception**

- Six types of exception conditions
  - Invalid operation (#I)
  - Divide by zero (#Z)
  - De-normalized operand (#D)
  - Numeric overflow (#O)
  - Numeric underflow (#U)
  - Inexact precision (#P)



#### For 'exceptions' details.

http://www.umiacs.umd.edu/~resnik/ling645 \_sp2002/cmu\_manual/node19.html

- Each has a corresponding mask bit
  - if set when an exception occurs, the exception is handled automatically by FPU
  - if clear when an exception occurs, a software exception handler is invoked



## Floating-Point Instruction Set

Instruction mnemonics begin with letter F
Second letter identifies data type of memory operand;

B = bcd

I = integer

no letter: floating point

## **Examples**

FLBD: load binary coded decimal

FISTP: store integer and pop stack

FMUL: multiply floating-point operands



## Floating-Point Instruction Set

#### **Operands**

- zero, one, or two
- no immediate operands
- no general-purpose registers (EAX, EBX, ...)
- integers must be loaded from memory onto the stack and converted to floating-point before being used in calculations
- if an instruction has two operands, one must be a FPU register



## **Floating Point Data Types**

## Table 12-11 Intrinsic Data Types.

Туре	Usage		
QWORD	64-bit integer		
TBYTE	80-bit (10-byte) integer		
REAL4	32-bit (4-byte) IEEE short real		
REAL8	64-bit (8-byte) IEEE long real		
REAL10	80-bit (10-byte) IEEE extended real		



## **Load Floating-Point Value**

#### .data

bigVal REAL10 1.212342342234234243E+864 .code

fld bigVal; load variable into stack

The fld (load floating-point value) instruction copies a floating-point operand to the top of the FPU stack [known as ST(0)].

Table 12-11 Intrinsic Data Types.

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## **Load Floating-Point Value**

.data

bigVal REAL10 1.212342342234234243E+864

.code

fld bigVal; load variable into stack

The fld (load floating-point value) instruction copies a floating-point operand to the top of the

FPU stack [known as ST(0)].

Example The following example loads two direct operands on the FPU stack:

.data

dblOne REAL8 234.56 dblTwo REAL8 10.1

.code

fld dblOne; ST(0) = dblOne

fld dblTwo; ST(0) = dblTwo, ST(1) = dblOne

ST(0)	234.56
ST(1)	234.56
	· · · · · · · · · · · · · · · · · · ·



## **Store Floating-Point Value**

#### **FST**

copies floating point operand from the top of the FPU stack into memory

#### **FSTP**

pops the stack after copying

- FST m32fp
- FST m80fp
- FST m64fp
- FST ST(i)



## **Arithmetic Instructions**

## Same operand types as FLD and FST

Table 12-12 Basic Floating-Point Arithmetic Instructions.

FCHS	Change sign		
FADD	Add source to destination		
FSUB	Subtract source from destination		
FSUBR	Subtract destination from source		
FMUL	Multiply source by destination		
FDIV	Divide destination by source		
FDIVR	Divide source by destination		



#### **Arithmetic Instructions: FCHS and FABS**

The FCHS (change sign) instruction reverses the sign of the floating-point value in ST(0)

The FABS (absolute value) instruction clears the sign of the number in ST(0) to create its absolute value

Neither instruction has operands:

- FCHS
- FABS



#### **Arithmetic Instructions: FADD**

The FADD (add) instruction has the following

formats:

- FADD
- FADD m32fp
- FADD m64fp
- FADD ST(0), ST(i)
- FADD ST(i), ST(0)

If no operands are used with FADD, ST(0) is added to ST(1).

The result is temporarily stored in ST(1). ST(0) is then popped from the stack, leaving the result on the top of the stack.

fadd	Before:	ST(1)	234.56
		ST(0)	10.1
	After:	ST(0)	244.66



#### **Arithmetic Instructions: FADD**

The FADD (add) instruction has the following

formats:

- **FADD**
- FADD m32fp
- FADD m64fp
- FADD ST(0), ST(i)
- FADD ST(i), ST(0)

Memory **Operand** 

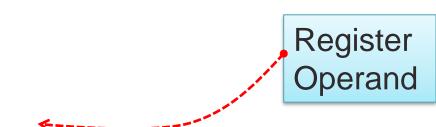
In memory operand, ST(0) will act as second operands. And memory location is added with ST(0).

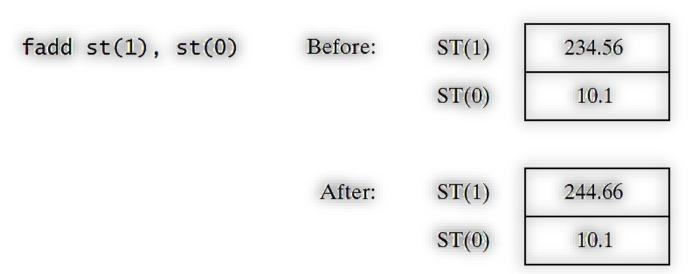


#### **Arithmetic Instructions: FADD**

## The FADD (add) instruction has the following formats:

- FADD
- FADD m32fp
- FADD m64fp
- FADD ST(0), ST(i)
- FADD ST(i), ST(0)







#### **Arithmetic Instructions: FSUB**

The FSUB instruction subtracts a source operand from a destination operand, storing the difference in the destination operand

No Operand FSUB Memory **Operand**  FSUB m32fp FSUB m64fp Register **Operand** • FSUB ST(0), ST(i) • FSUB ST(i), ST(0)



#### **Arithmetic Instructions: FMUL**

#### **FMUL:**

Multiplies source by destination, stores product in destination

## fmul mySingle; ST(0) \*= mySingle

#### **FDIV**:

Divides destination by source, then pops the stack

The no-operand form of FDIV divides ST(1) by ST(0)

The no-operand versions of FMUL and FDIV pop the stack after multiplying or dividing.



#### **Arithmetic Instructions: FMUL**

#### **FDIV:**

Divides destination by source, then pops the stack

The no-operand form of FDIV divides ST(1) by ST(0)

ST(0) is popped from the stack, leaving the dividend on the top of the stack

```
.data
```

dblOne REAL8 1234.56

dblTwo REAL8 10.0

dblQuot REAL8?

#### .code

fld dblOne; load into ST(0)

fdiv dblTwo; divide ST(0) by dblTwo

fstp dblQuot; store ST(0) to dblQuot



# FCOM instruction Operands:

Instruction	Description		
FCOM	Compare ST(0) to ST(1)		
FCOM m32fp	Compare ST(0) to m32fp		
FCOM m64fp	Compare ST(0) to m64fp		
FCOM ST(i)	Compare ST(0) to ST(i)		



# FCOM instruction Operands:

Instruction	Description		
FCOM	Compare ST(0) to ST(1)		
FCOM m32fp	Compare ST(0) to m32fp		
FCOM m64fp	Compare ST(0) to m64fp		
FCOM ST(i)	Compare ST(0) to ST(i)		

Condition Codes: Three FPU condition code flags, C3, C2, and C0, indicate the results of comparing floating-point values

Condition	C3 (Zero Flag)	C2 (Parity Flag)	C0 (Carry Flag)	Conditional Jump to Use
ST(0) > SRC	0	0	0	JA, JNBE
ST(0) < SRC	0	0	1	JB, JNAE
ST(0) = SRC	1	0	0	JE, JZ
Unordered <sup>a</sup>	1	1	1	(None)

[56]



# FCOM instruction Operands:

Instruction		Description			
FCOM		Compare	ST(0)	to	ST(1)
FCOM	m32fp	Compare	ST(0)	to	m32fp
FCOM	m64fp	Compare	ST(0)	to	m64fp
FCOM	ST(i)	Compare	ST(0)	to	ST(i)

The primary challenge after comparing two values and setting FPU condition codes is to find a way to branch to a label based on the conditions

#### Two steps are involved:

- Use the FNSTSW instruction to move the FPU status word into AX
- Use the SAHF instruction to copy AH into the EFLAGS register
- Use JA, JB, etc to do the branching



Calculate the absolute value of the difference between two floating-point values

```
.data
epsilon REAL8 1.0E-12
                                   ; difference value
val2 REAL8 0.0
                                   ; value to compare
                                   ; considered equal to val2
val3 REAL8 1.001E-13
.code
; if( val2 == val3 ), display "Values are equal".
    fld epsilon
                                   ; load epsilon
    fld val2
                                   ; load val2
                                  ; subtract
    fsub val3
    fabs
    fcomi ST(0),ST(1)
    ja skip
    mWrite <"Values are equal",0dh,0ah>
skip:
```



## Example

```
double X = 1.2;
double Y = 3.0;
int N = 0;
if( X < Y )
N = 1;
```

Write Assembly code for the above program?



## **Example**

```
double X = 1.2;
.data
                                 double Y = 3.0;
X REAL8 1.2
                                 int N = 0;
Y REAL8 3.0
                                 if(X < Y)
N DWORD 0
                                 N = 1;
.code
; if (X < Y)
; N = 1
fld X
              ; ST(0) = X
fcomp Y
                     ; compare ST(0) to Y
fnstsw ax
                     ; move status word into AX
                     ; copy AH into EFLAGS
sahf
jnb L1
                     ; X not < Y? skip
mov N,1
                     ; N = 1
L1:
```



The primary challenge after comparing two values and setting FPU condition codes is to find a way to branch to a label based on the conditions

#### Two steps are involved:

- Use the FNSTSW instruction to move the FPU status word into AX.
- Use the SAHF (store AH into flag) instruction to copy AH into the EFLAGS register.
- Use JA, JB, etc to do the branching.

Fortunately, the FCOMI instruction does steps 1 and 2 for you.

fcomi ST(0), ST(1); Floating Point Compare, move results to

inb Label1

**FFI AGS** 



## Floating-Point I/O

#### Irvine32 library procedures

#### ReadFloat

Reads FP value from keyboard, pushes it on the FPU stack

#### **WriteFloat**

Writes value from ST(0) to the console window in exponential format

#### **ShowFPUStack**

Displays contents of FPU stack



## **Exception Synchronization**

Main CPU and FPU can execute instructions concurrently

- if an unmasked exception occurs, the current FPU instruction is interrupted and the FPU signals an exception
- But the main CPU does not check for pending FPU exceptions. It might use a memory value that the interrupted FPU instruction was supposed to set.
- Example:

```
.data
intVal DWORD 25
.code
fild intVal  ; load integer into ST(0)
inc  intVal ; increment the integer
```



## **Exception Synchronization**

For safety, insert a fwait instruction, which tells the CPU to wait for the FPU's exception handler to finish:

```
.data
intVal DWORD 25
.code
fild intVal  ; load integer into ST(0)
fwait    ; wait for pending exceptions
inc    intVal ; increment the integer
```



#### **Example**

expression: valD = -valA + (valB \* valC)

#### .data

valA REAL8 1.5

valB REAL8 2.5

valC REAL8 3.0

valD REAL8 ? ; will be +6.0

#### .code

fld valA ; ST(0) = valA

fchs; change sign of ST(0)

fld valB ; load valB into ST(0)

**fmul valC** ; ST(0) \*= valC

fadd ; ST(0) += ST(1)

fstp valD ; store ST(0) to valD



#### **Mixed Mode Arithmetic**

Combining integers and reals. Integer arithmetic instructions such as ADD and MUL cannot handle reals FPU has instructions that promote integers to reals and load the values onto the floating point stack

Example: Z = N + X

.data N SDWORD 20 X REAL8 3.5 Z REAL8 ?

.code

fild N ; load integer into ST(0)

fwait ; wait for exceptions

fadd X; add mem to ST(0)

fstp Z ; store ST(0) to mem



fldcw ctrlWord

## **Masking and Unmasking Exceptions**

- Exceptions are masked by default
- Divide by zero just generates infinity, without halting the program
- If you unmask an exception processor executes an appropriate exception handler
- Unmask the divide by zero exception by clearing bit 2:

: load it back into FPU

## Questions?

## THANK YOU!