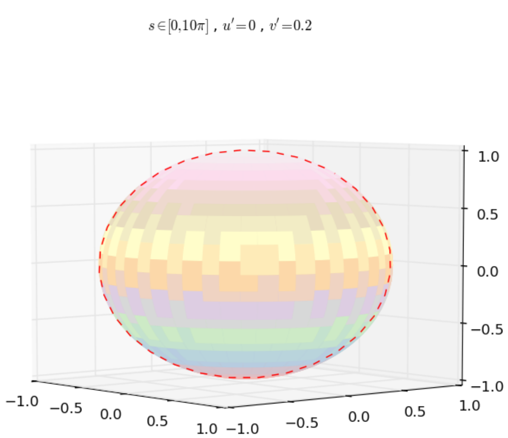
1 Results & Interpretation

1.1 Geodesic Solver

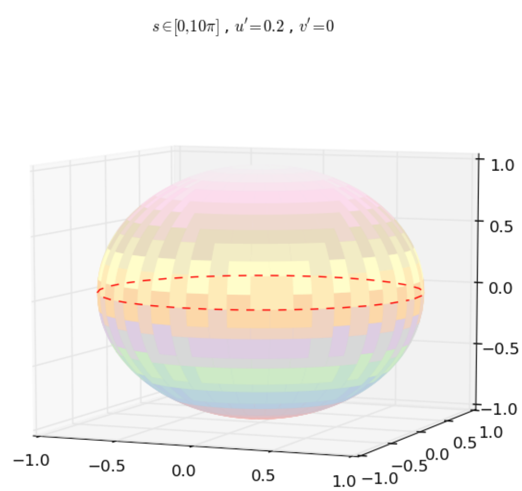
1.1.1 Sphere

The transformation from spherical to cartesian coordinates is given as

$$
\begin{aligned}
x &= u \sin(v) \cos(w) \\
y &= u \sin(v) \sin(w) \\
z &= u \cos(v)
\end{aligned}
$$

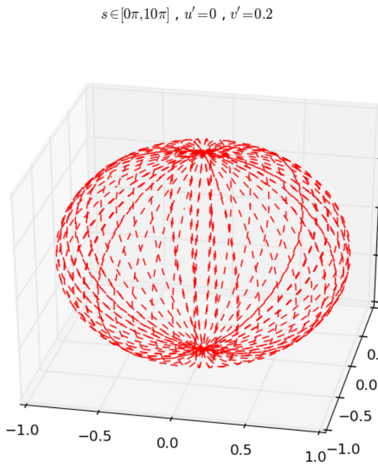


(a) $u' = 0, v' = 0.2$.

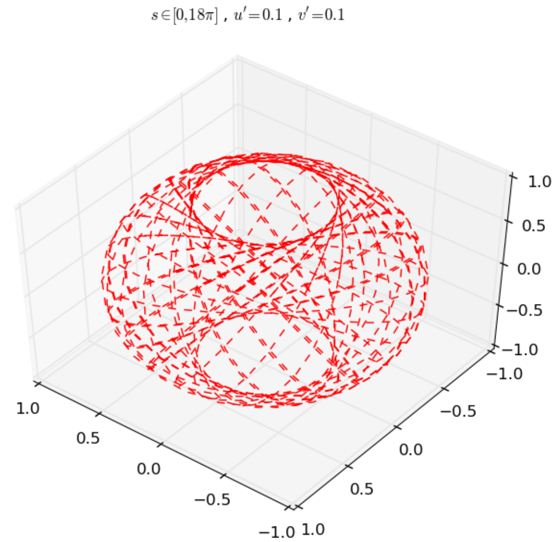


(b) $u' = 0.2, v' = 0$.

Figure 1: Geodesic curves on a sphere



(a) $u' = 0, v' = 0.2$.



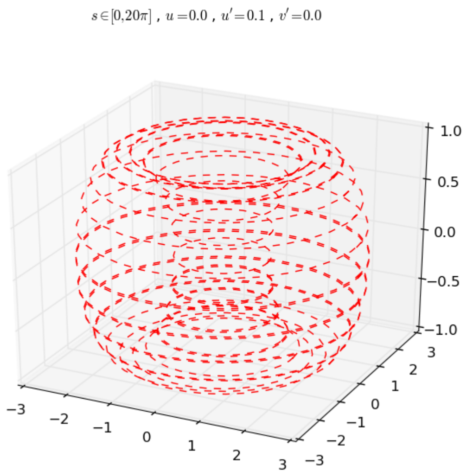
(b) $u' = 0.1, v' = 0.1$.

Figure 2: Geodesic curves on a sphere.

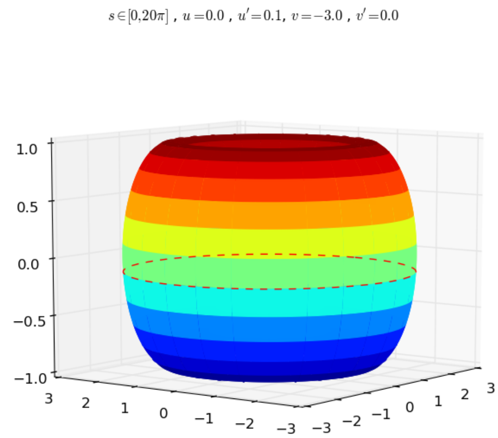
1.1.2 Torus

The transformation from toridal to cartesian coordinates is given as

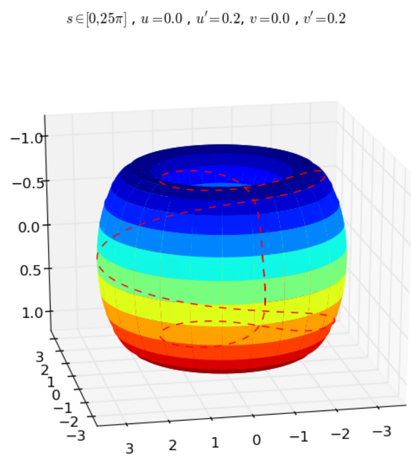
$$
\begin{aligned}
x &= (c + a\cos(u))\cos(v) \\
y &= (c + a\cos(u))\sin(v) \\
z &= \sin(u)
\end{aligned}
$$



(a) $u' = 0.1, v' = .0$.

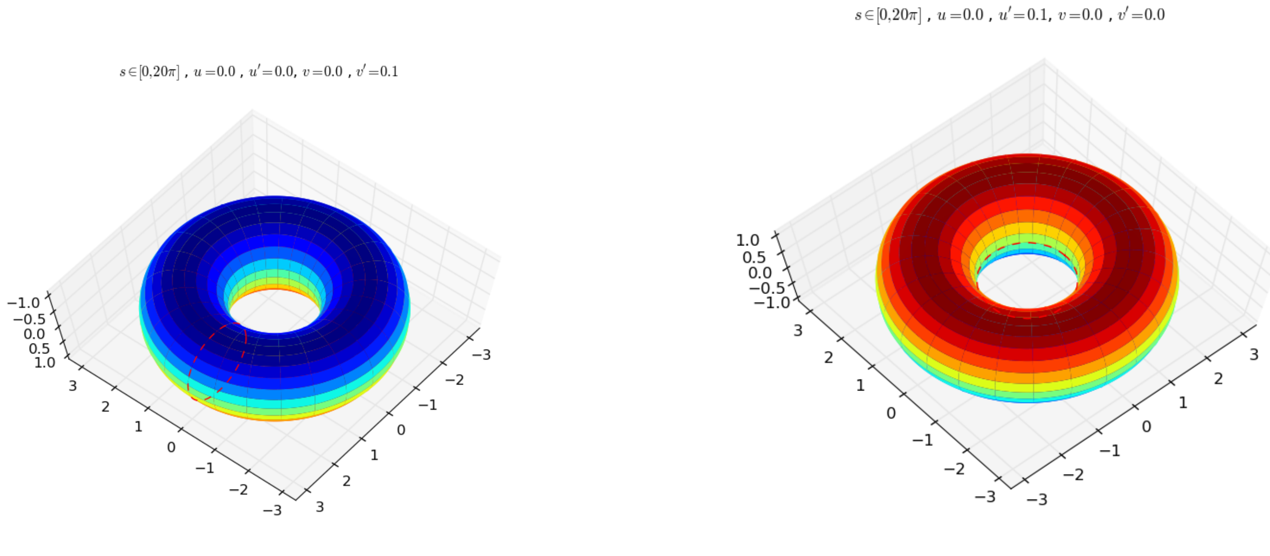


(b) $u' = 0, v' = 0.1$.



(c) $u' = 0.2, v' = 0.2$.

Figure 3: Geodesic curves on a torus.



(a) $u' = 0.1, v' = 0.1$.

(b) $u' = 0.1, v' = 0$.

Figure 4: Geodesic curves on a torus.

1.1.3 Cylindrical Catenoid

$$
\begin{aligned}
x &= \cos(u) - v\sin(u) \\
y &= \sin(u) + v\cos(u) \\
z &= v
\end{aligned}
$$

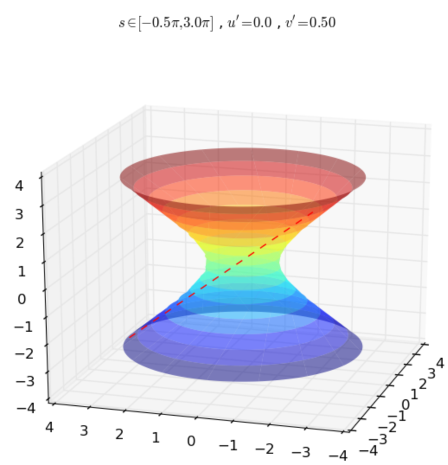
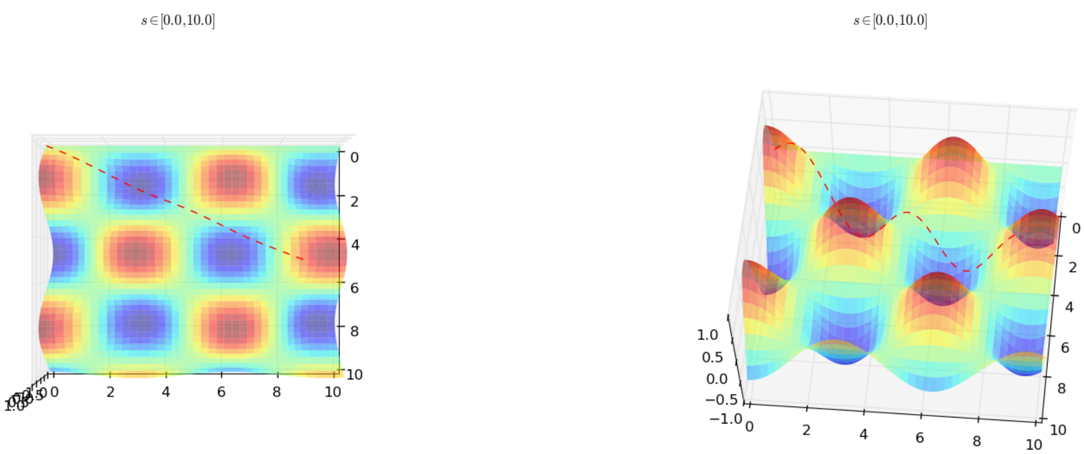


Figure 5: Geodesic curves on a cylindrical catenoid.

1.1.4 Egg Carton Surface

$$
\begin{aligned}
x &= u \\
y &= v \\
z &= \sin(u)\cos(v)
\end{aligned}
$$



(a) $u' = 0.1, v' = 0.1.$ (b) $u' = 0.1, v' = 0.1.$

Figure 6: Geodesic curves on an egg carton surface.

1.1.5 Mobius Strip

$$
\begin{aligned}
x &= \left[1 + \frac{v}{2}\cos\left(\frac{u}{2}\right)\right]\cos(u) \\
y &= \left[1 + \frac{v}{2}\cos\left(\frac{u}{2}\right)\right]\sin(u) \\
z &= \frac{v}{2}\sin\left(\frac{u}{2}\right)
\end{aligned}
$$

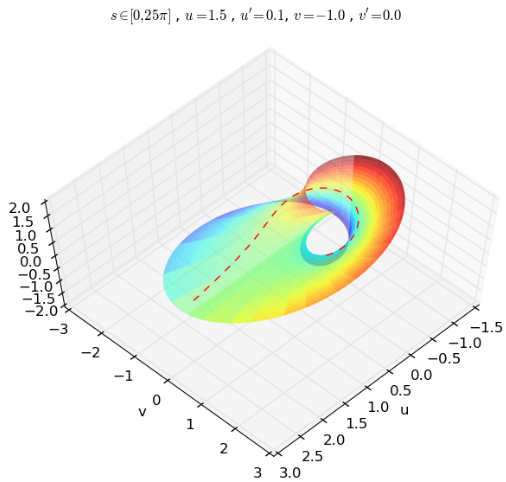


Figure 7: Geodesic curve on a Mobius strip.

1.1.6 3D Kerr Metric

The Kerr metric is given as

$$ds^2 = -\left(1 - \frac{2GMr}{r^2 + a^2\cos^2(\theta)}\right)dt^2 + \left(\frac{r^2 + a^2\cos^2(\theta)}{r^2 - 2GMr + a^2}\right)dr^2 + \left(r^2 + a^2\cos(\theta)\right)d\theta^2$$
$$+\left(r^2 + a^2 + \frac{2GMra^2}{r^2 + a^2\cos^2(\theta)}\right)\sin^2(\theta)d\phi^2 - \left(\frac{4GMra\sin^2(\theta)}{r^2 + a^2\cos^2(\theta)}\right)d\phi\, dt$$

For $a = 0$, this reduces to the Schwarzchild metric. We have run the geodesic solver for this case, and used the Schwarzchild radius to determine the coeffiencts G and M (r?), which amounts to determining the geodesics near a black hole.

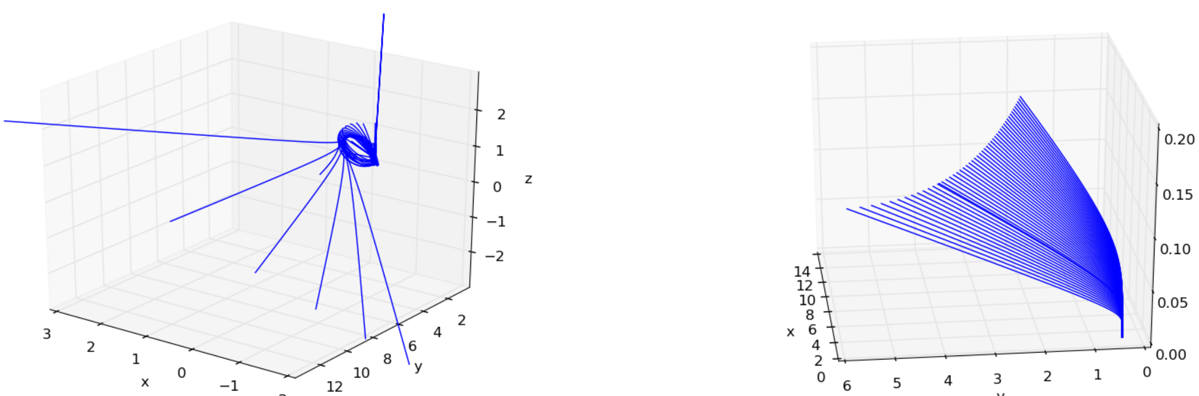


Figure 8: Geodesic curves on an egg carton surface.