

# Introduction to Nodal Method For Reactor Analysis

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# Overview

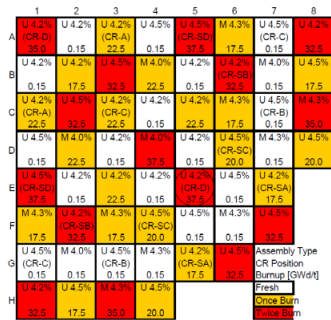
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- 4 References

# Introduction

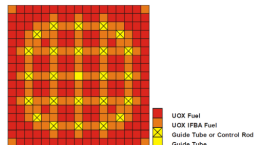
- Since it was proposed more than 40 years ago, nodal method has become a standard tool for many modern reactor simulators to perform routine reactor core calculations and safety analysis for commercial nuclear power plants.
- Nodal method reduces the number of unknowns by factor of hundreds or even thousands that allows reactor simulators to solve reactor problems rapidly compared to traditional Finite Difference Method (FDM).
- It also enables reactor spatial-kinetic calculations, instead of point kinetic, to be accomplished in today's computers with acceptable running time.

# Introduction

## Typical PWR Core



CR-A Control Rod Bank A  
CR-B Control Rod Bank B  
CR-C Control Rod Bank C  
CR-D Control Rod Bank D  
CR-SA Shutdown Rod Bank A  
CR-SB Shutdown Rod Bank B  
CR-SC Shutdown Rod Bank C  
CR-SD Shutdown Rod Bank D  
O Ejected Rod



Number of nodes for  
FDM

$$17 \times 17 \times 45 \times 180 \times 2 = 4,681,800$$

Number of nodes for  
Nodal Method

$$2 \times 2 \times 45 \times 36 \times 2 = 12,960$$

Images from [1]

Table: IAEA2D Problem

Code Methods	Number of mesh or nodes	Running Time (second)	K-eff
FDM	10,816	6.7	1.029541
Nodal	169	0.02	1.029600

Error on effective multiplication factor is 5.9 pcm

Table: IAEA3D Problem

Code Methods	Number of mesh or nodes	Running Time (second)	K-eff
FDM	2,055,040	1094	1.029013
Nodal	3,211	0.2	1.029082

Error on effective multiplication factor is 6.9 pcm

## IAEA3D Axially Averaged Power Distribution

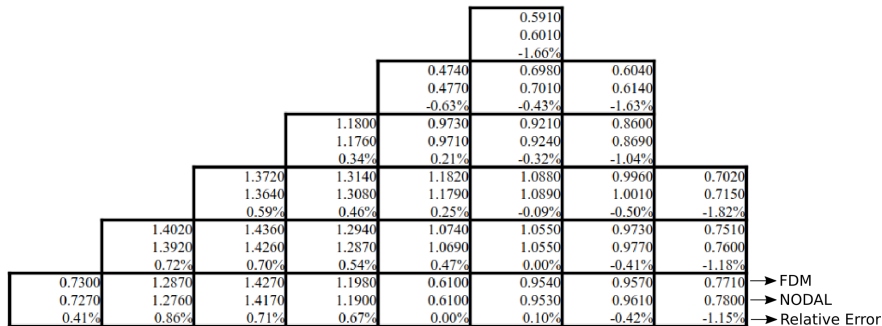


Table: UO<sub>2</sub>-MOX PWR 3D Transient Problem

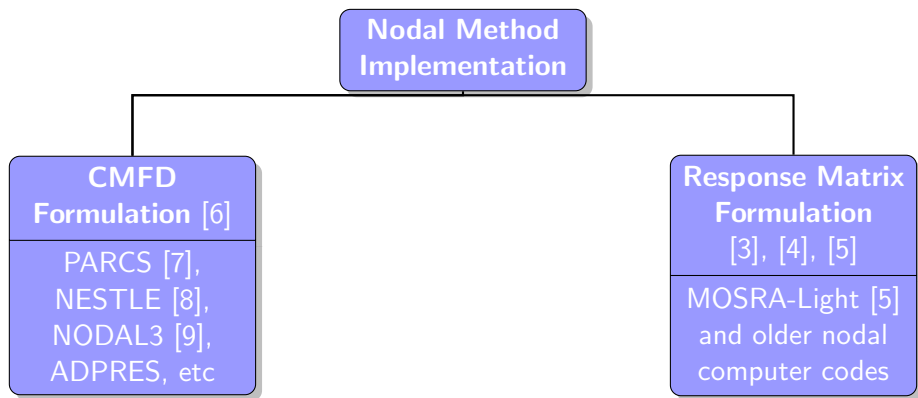
Codes	Peak time (s)	Peak Power (%)	Reactivity Peak (\$)	Integral Power (%.s)
Serpent-SCF [2]	0.35	179±26	1.18±0.02	27.7
ADPRES 2G	0.35	173	1.12	30.4
PARCS 2G [1]	0.34	142	1.12	27.2
PARCS 4G [1]	0.33	152	1.12	27.8
PARCS 8G [1]	0.32	172	1.14	29.1

## CPU Time (mins)

**Serpent-SCF:** 9,600,00 (about 18 years). The simulation was done on a cluster with 1280 processors (@2.6GHz) and it took about 120 hours

**ADPRES:** 5

# Nodal Method Implementation





## The Neutron Diffusion Equation

$$\begin{aligned} \frac{\partial}{\partial x} J_{gx}(x, y, z) + \frac{\partial}{\partial y} J_{gy}(x, y, z) + \frac{\partial}{\partial z} J_{gz}(x, y, z) + \Sigma_{r,g}(x, y, z) \phi_g(x, y, z) \\ = \frac{\chi_g}{k_{eff}} \sum_{g'=1}^G \nu \Sigma_{f,g'}(x, y, z) \phi_{g'}(x, y, z) + \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{s,g' \rightarrow g}(x, y, z) \phi_{g'}(x, y, z) \end{aligned} \quad (1)$$

## Fick's Law

$$J_{gx}(x, y, z) = -D_g(x, y, z) \frac{\partial}{\partial x} \phi_g^k(x, y, z) \quad (2)$$

# Coarse Mesh Finite Difference

Integrating over node  $k$  which is defined as

$$\left(-\frac{1}{2}\Delta_x^k \leq x \leq \frac{1}{2}\Delta_x^k\right), \left(-\frac{1}{2}\Delta_y^k \leq y \leq \frac{1}{2}\Delta_y^k\right), \left(-\frac{1}{2}\Delta_z^k \leq z \leq \frac{1}{2}\Delta_z^k\right) \quad (3)$$

We obtain discretized form of the neutron diffusion equation

$$\frac{1}{\Delta_x^k} (J_{gx+}^k - J_{gx-}^k) + \frac{1}{\Delta_y^k} (J_{gy+}^k - J_{gy-}^k) + \frac{1}{\Delta_z^k} (J_{gz+}^k - J_{gz-}^k) + \Sigma_{r,g}^k \bar{\phi}_g^k = \bar{Q}_g^k \quad (4)$$

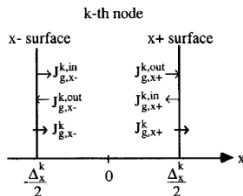
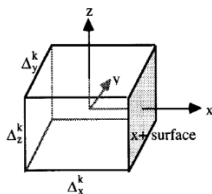


Image from [5]

Where in Eq. (4)

$$\bar{\phi}_g^k = \frac{1}{V_k} \int_{-\Delta x/2}^{+\Delta x/2} dx \int_{-\Delta y/2}^{+\Delta y/2} dy \int_{-\Delta z/2}^{+\Delta z/2} dz \phi_g^k(x, y, z)$$

$$\bar{Q}_g^k = \frac{1}{V_k} \int_{-\Delta x/2}^{+\Delta x/2} dx \int_{-\Delta y/2}^{+\Delta y/2} dy \int_{-\Delta z/2}^{+\Delta z/2} dz Q_g^k(x, y, z)$$

$$J_{gx\pm}^k = \frac{1}{\Delta_y^k \Delta_z^k} \int_{-\Delta y/2}^{+\Delta y/2} dy \int_{-\Delta z/2}^{+\Delta z/2} dz J_{gx}^k \left( x = \frac{\pm \Delta_x^k}{2}, y, z \right)$$

$$V^k = \Delta_x^k \Delta_y^k \Delta_z^k$$

# Coarse Mesh Finite Difference

## Traditional Finite Difference Method (FDM)

$$J_{gx\pm}^k = \mp \tilde{D}_{gx\pm}^k \left( \phi_g^{k\pm 1} - \phi_g^k \right) \quad (5)$$

## Coarse Mesh Finite Difference (CMFD)

$$J_{gx\pm}^k = \mp \tilde{D}_{gx\pm}^k \left( \phi_g^{k\pm 1} - \phi_g^k \right) - \hat{D}_{gx\pm}^k \left( \phi_g^{k\pm 1} - \phi_g^k \right) \quad (6)$$

$$\tilde{D}_{gx\pm}^k = \frac{2D_{gx}^{k\pm 1}D_{gx}^k}{D_{gx}^{k\pm 1}\Delta_x^k + D_{gx}^k\Delta_x^{k\pm 1}} \quad (7)$$

# Nodal Update

Transverse integrated diffusion equation can be obtained, for example, by integrating Eq. (1) over  $y$  and  $z$

$$\frac{d}{dx} \bar{J}_{gx}^k(x) + \Sigma_{r,g}^k \bar{\phi}_{gx}^k(x) = \bar{Q}_{gx}^k(x) - \frac{1}{\Delta_y^k} L_{gy}^k(x) - \frac{1}{\Delta_z^k} L_{gz}^k(x) \quad (8)$$

where

$$\begin{aligned} \bar{J}_{gx}^k(x) &= \frac{1}{\Delta_y^k \Delta_z^k} \int_{-\Delta_y/2}^{+\Delta_y/2} dy \int_{-\Delta_z/2}^{+\Delta_z/2} dz J_{gx}^k(x, y, z) \\ \bar{\phi}_{gx}^k(x) &= \frac{1}{\Delta_y^k \Delta_z^k} \int_{-\Delta_y/2}^{+\Delta_y/2} dy \int_{-\Delta_z/2}^{+\Delta_z/2} dz \phi_g^k(x, y, z) \\ L_{gy}^k(x) &= \frac{1}{\Delta_z^k} \int_{-\Delta_z/2}^{+\Delta_z/2} dz J_{gy}^k(x, y, z) \Bigg|_{y=-\frac{\Delta_y}{2}}^{y=\frac{\Delta_y}{2}} \\ L_{gz}^k(x) &= \frac{1}{\Delta_y^k} \int_{-\Delta_y/2}^{+\Delta_y/2} dy J_{gz}^k(x, y, z) \Bigg|_{z=-\frac{\Delta_z}{2}}^{z=\frac{\Delta_z}{2}} \end{aligned} \quad (9)$$

# Nodal Update

The one-dimensional flux is approximated by

$$\bar{\phi}_{gx}^k(x) \cong \bar{\phi}_{g,0}^k f_0(x) + \sum_{n=1}^4 a_{gx,n}^k f_n(x), \text{ in which } \left( -\frac{\Delta x}{2} \leq x \leq \frac{\Delta x}{2} \right) \quad (10)$$

The basis functions shall satisfy

$$\frac{1}{\Delta x} \int_{-\Delta x/2}^{+\Delta x/2} dx f_n(x) = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

- In Analytic Nodal Method (ANM), however, instead of approximating one-dimensional flux by polynomials, Eq. (8) is analytically solved
- This is possible because Eq. (8) material within the node is homogeneous
- However, this puts limit on ANM to solve reactor problems only with two-group energy

# Nodal Method

- Substituting Eq. (10) to one-dimensional form of Eq. (2), one would obtain nodal solutions (not FDM solutions) for net currents (i.e  $J_{gx\pm}^k$ ,  $J_{gy\pm}^k$ , and  $J_{gz\pm}^k$ )
- Using these corrected nodal net currents, now  $\hat{D}_{gx\pm}^k$ ,  $\hat{D}_{gy\pm}^k$ , and  $\hat{D}_{gz\pm}^k$  in Eq. (6) can be updated
- To do so, we must determine nodal expansion coefficients  $a_{gx\pm n}^k$ ,  $a_{gy\pm n}^k$  and  $a_{gz\pm n}^k$  for  $n = 1, 2, 3, 4$ . This can be done by solving **Two Nodes Problem**, hence we need to determine 8G nodal expansion coefficients for both nodes (4 nodal expansion coefficients for each node and each group).
- Therefore, we need 8G equations to obtain the 8G nodal expansion coefficients.
- This process involves non-linear iteration

# Nodal Method

- These 8G equations are
  - 2G neutron balance equations  $\rightarrow$  substituting Eq. (10) to the first term on LHS of the Eq. (8), and integrating over  $x$
  - 2G first moment equations  $\rightarrow$  substituting Eq. (10) to Eq. (8), weighting by  $f_1(x)$ , and integrating over  $x$
  - 2G second moment equations  $\rightarrow$  substituting Eq. (10) to Eq. (8), weighting by  $f_2(x)$ , and integrating over  $x$
  - G flux continuity equations  $\rightarrow$  solve Eq. (10) for  $x = -\frac{\Delta_x^k}{2}$  and  $x = \frac{\Delta_x^k}{2}$
  - G net current continuity equations  $\rightarrow$  substituting Eq. (10) to Eq. (2) and solve for  $x = -\frac{\Delta_x^k}{2}$  and  $x = \frac{\Delta_x^k}{2}$
- These procedures are also carried out for  $y$  and  $z$  directions

LEFT NODE	RIGHT NODE
$\phi_+^L$	$\phi_-^R$
$J_+^L$	$J_-^R$

Image from [13]



# Nodal Expansion Method (NEM)

NEM uses following basis functions [8]

$$f_0(x)=1$$

$$f_1(x)=\xi=\frac{x}{\Delta_x^k}$$

$$f_2(x)=3\xi^2-\frac{1}{4}$$

$$f_3(x)=\xi\left(\xi+\frac{1}{2}\right)\left(\xi-\frac{1}{2}\right)$$

$$f_4(x)=\left(\xi^2-\frac{1}{20}\right)\left(\xi+\frac{1}{2}\right)\left(\xi-\frac{1}{2}\right)$$

- This will results  $8G \times 8G$  matrix to obtain  $8G$  nodal expansion coefficients in the two-nodes problem
- This basis functions were initially used when nodal method was implemented using for matrix response formulation
- Not quite efficient because  $8G \times 8G$  matrix must be solved for **each node and each direction**
- This method was implemented in NESTLE

# Polynomial Nodal Method (PNM) [10]

In PNM, the Eq. (8) is mapped from  $(-\frac{1}{2}\Delta_x^k \leq x \leq \frac{1}{2}\Delta_x^k)$  to  $(-1 \leq u \leq 1)$

$$-\frac{d}{du^2}\phi_g^k(u) + \sum_{g'=1}^G \left(B_{gg'}^k\right)^2 \phi_{g'}^k(u) = -\frac{(\Delta x)^2}{4D_g^k} L_{gx}^k(u)$$

where:

$$u = \frac{2x}{\Delta_x^k} \text{ and } u \in [-1, 1]$$

$$\left(B_{gg'}^k\right)^2 = \frac{(\Delta x)^2}{4D_g^k} \left( \Sigma_{rg}^k \delta_{gg'} - \frac{\chi_g}{k_{\text{eff}}} \nu \Sigma_{f,g'}^k - \Sigma_{s,g' \rightarrow g}^k \right)$$

$$L_{gx}^k(u) = \frac{1}{\Delta_y^k} L_{gy}^k(u) + \frac{1}{\Delta_z^k} L_{gz}^k(u)$$

# Polynomial Nodal Method (PNM)

PNM uses following basis functions

$$f_0(u)=1$$

$$f_1(u)=u$$

$$f_2(u)=\frac{1}{2}(3u^2-1)$$

$$f_3(u)=\frac{1}{2}(5u^3-3u)$$

$$f_4(u)=\frac{1}{8}(35u^4-30u^2+3)$$

- These basis functions are Legendre polynomials
- Legendre polynomials are orthogonal polynomials which enable the  $8G$  matrix is decoupled into  $G$  and  $2G$  matrices
- This makes the Polynomial Nodal Method (PNM) calculation is more efficient
- PNM has the same degree of accuracy as NEM

# Semi-Analytic Nodal Method (SANM) [11]

SANM uses the same basis functions as PNM, except for

$$f_3(u) = \frac{\sinh(\alpha_{gx}^k u) - m_{gx1}^k(\sinh)f_1(u)}{\sinh(\alpha_{gx}^k) - m_{gx1}^k(\sinh)}$$
$$f_4(u) = \frac{\cosh(\alpha_{gx}^k u) - m_{gx0}^k(\cosh)f_0(u) - m_{gx2}^k(\cosh)f_2(u)}{\cosh(\alpha_{gx}^k) - m_{gx0}^k(\cosh) - m_{gx2}^k(\cosh)}$$

where

$$\alpha_{gx}^k = \sqrt{\frac{\sigma_{fg}^k}{D_k^g} \frac{\Delta x}{2}}$$

$$m_{gx1}^k(\sinh) = \frac{1}{N_1} \int_{-1}^1 \sinh(\alpha_{gx}^k u) f_1(u) du$$

$$m_{gxi}^k(\cosh) = \frac{1}{N_i} \int_{-1}^1 \cosh(\alpha_{gx}^k u) f_i(u) du, \text{ for } i=0,2$$

$$N_i = \frac{2}{2i+1}, \text{ for } i=0,1,2$$

# Semi-Analytic Nodal Method (SANM) [11]

- These basis functions in SANM are also Legendre polynomials but in the trigonometric function forms for  $f_3$  and  $f_4$
- SANM accuracy is close to ANM
- SANM can be used to solve multi-group reactor problems

**Table:** Critical Boron Concentration in PPM for the NEACRP PWR Benchmark Problems

Methods	Cases					
	A1	A2	B1	B2	C1	C2
ANM	561.2	1156.6	1248.0	1183.8	1128.3	1156.6
SANM	560.5	1156.0	1247.4	1184.6	1127.7	1156.0
PNM	561.7	1159.6	1251.7	1188.1	1131.8	1159.6

Note: ANM results from PANTHER

# ADPRES Features

- Input is straightforward, modular and in a free-format form
- Solves both static and transient core problems with or without TH feedback
- Performs forward, adjoint and fixed-source calculations
- Performs calculations using branched cross sections data.
- Critical boron concentration search
- Rod ejection simulation or Reactivity Initiated Accident (RIA)
- Solves multi-group of neutron energy
- Solves calculations with Assembly Discontinuity Factors (ADFs)
- Thermal-hydraulics solutions are obtained by solving mass and energy conservation equations in an enclosed channel
- Three nodal kernels are available: FDM, PNM, and SANM

# General Rules

Some general rules for ADPRES inputs:

- Input deck is in free-format form with maximum 200 columns
- Comments are marked by !.

## Example

```
! sigtr   siga   nu*sigf sigf   chi   sigs_g1  sigs_g2
0.222222  0.010   0.000  0.000   1.0   0.1922   0.020
0.833333  0.080   0.135  0.135   0.0   0.000   0.7533   ! MAT1 : Outer Fuel
```

## General Rules (continued)

- ADPRES input is modular, where it is broken into several cards. Cards' keywords shall be uppercase and marked by %.

### Example

```
%MODE  
FORWARD
```

```
%XSEC      ! Cross section card  
2 4        ! Number of groups and number of materials
```

```
%GEOM      ! Geometry card  
12 12 2    !nx, ny, nz  
...
```

- Numbers can be repeated using \* mark.

### Example

```
10.0 5*20.0 !is equivalent to 10.0 20.0 20.0 20.0 20.0 20.0
```



## %MODE and %XSEC cards

```
%MODE
FORWARD
```

```
%XSEC
```

```
2 5 ! Number of groups and number of materials
! sigtr siga nu*sigf sigf chi sigs_g1 sigs_g2
0.222222 0.010 0.000 0.000 1.0 0.1922 0.020
0.833333 0.080 0.135 0.135 0.0 0.000 0.7533 ! MAT1 : Outer Fuel
0.222222 0.010 0.000 0.000 1.0 0.1922 0.020
0.833333 0.085 0.135 0.135 0.0 0.000 0.7483 ! MAT2 : Inner Fuel
0.222222 0.0100 0.000 0.000 1.0 0.1922 0.020
0.833333 0.1300 0.135 0.135 0.0 0.000 0.7033 ! MAT3 : Inner Fuel + Control Rod
0.166667 0.000 0.000 0.000 0.0 0.1267 0.040
1.111111 0.010 0.000 0.000 0.0 0.000 1.1011 ! MAT4 : Reflector
0.166667 0.000 0.000 0.000 0.0 0.000 0.040
1.111111 0.055 0.000 0.000 0.0 0.000 0.000 ! MAT5 : Reflector + Control Rod
```

Two Group Cross Sections for Each Composition

Material	$D_2$	$\Sigma_{a2}$	$\nu\Sigma_{f2}$	$\Sigma_{a2 \leftarrow 1}$
Fuel1	1.500	0.010	0.000	0.020
	0.400	0.085	0.135	
Fuel1+Rod	1.500	0.010	0.000	0.020
	0.400	0.130	0.135	
Fuel2	1.500	0.010	0.000	0.020
	0.400	0.080	0.135	
Reflector	2.000	0.000	0.000	0.040
	0.300	0.010	0.000	
Reflector+Rod	2.000	0.000	0.000	0.040
	0.300	0.055	0.000	

Image from [12]

# ADPRES Input

## %GEOM cards

```
%GEOM
9 9 19      !nx, ny, nz
10.0 8*20.0 !x-direction assembly size in cm
1 8*2       !x-direction assembly divided into 2 (10 cm each)
8*20.0 10.0 !y-direction assembly size in cm
8*2 1       !y-direction assembly divided into 2 (10 cm each)
19*20.0     !z-direction assembly in cm
19*1        !z-direction nodal is not divided
4           !np number of planar type
1 13*2 4*3 4 !planar assignment (from bottom to top)
! Planar_type_1 (Bottom Reflector)
....
....
! Planar_type_3 (Fuel+Partial Control Rods)
3 2 2 2 2 3 2 2 1 4
2 2 2 2 2 2 2 2 1 4
2 2 3 2 2 2 1 1 4
2 2 2 2 2 2 1 4 4
3 2 2 2 3 1 1 4 0
2 2 2 2 1 1 4 4 0
2 2 1 1 1 4 4 0 0
1 1 1 4 4 4 0 0 0
4 4 4 4 0 0 0 0 0
...
! Boundary conditions (east), (west), (north), (south), (bottom), (top)
1 2 2 1 1 1
```

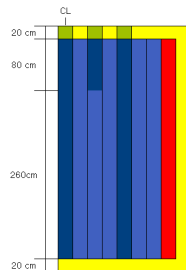
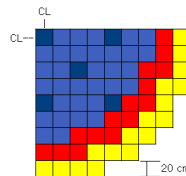
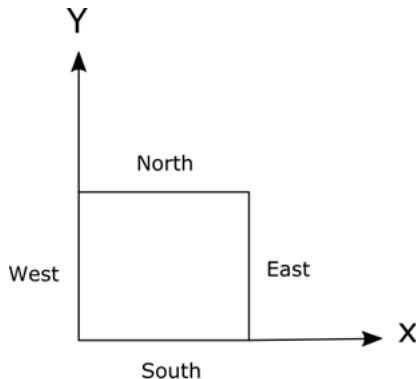
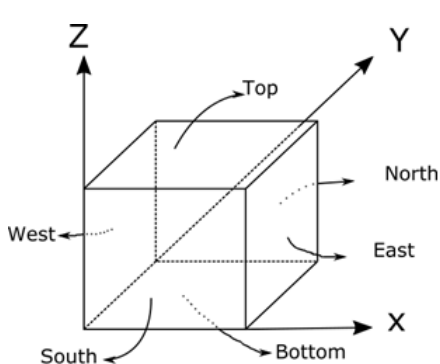


Image from [12]

# ADPRES Input

## Point of Origin for Geometry Modelling in ADPRES



## Output

### CALCULATION RESULTS

Itr	k-eff	Fis.Src Error	Inner Error
1	0.981424	5.47871E-01	8.55259E+03
2	1.001319	2.41976E-01	8.56379E+00
3	1.009804	1.67297E-01	6.27992E-01
4	1.013500	1.33833E-01	1.44559E-01
...FISSION SOURCE EXTRAPOLATED...			
5	1.026980	2.65156E+00	1.31348E-01
6	1.024050	2.69425E-01	1.90647E+00
7	1.024688	9.31087E-02	4.43462E+01
8	1.024358	3.99972E-02	1.10592E+00
9	1.024191	3.47063E-02	1.96477E-01
...FISSION SOURCE EXTRAPOLATED...			
10	1.023838	1.12907E+00	5.87407E-02
11	1.024188	1.12316E-01	9.62168E-01
12	1.025744	1.03113E-01	4.24920E-01
13	1.026299	8.09335E-02	1.86532E-01
...			
128	1.029082	9.48481E-06	2.04727E-05
129	1.029082	7.46358E-06	9.66768E-06

MULTIPLICATION EFFECTIVE (K-EFF) = 1.029082

## Output

### Radial Power Distribution

	1	2	3	4	5	6	7	8	9
9	0.727	1.276	1.417	1.190	0.610	0.953	0.961	0.780	
8	1.276	1.392	1.426	1.287	1.069	1.055	0.977	0.760	
7	1.417	1.426	1.364	1.308	1.179	1.089	1.001	0.715	
6	1.190	1.287	1.308	1.176	0.971	0.924	0.869		
5	0.610	1.069	1.179	0.971	0.477	0.701	0.614		
4	0.953	1.055	1.089	0.924	0.701	0.601			
3	0.961	0.977	1.001	0.869	0.614				
2	0.780	0.760	0.715						
1									

MAX POS.            Maximum Value  
( 7, 2)            1.426

# ADPRES Output

## Output

### Axial Power Density Distribution

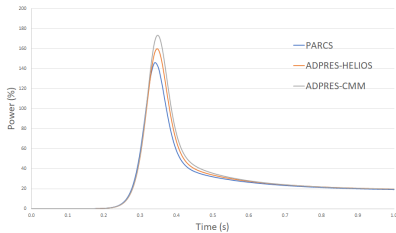
=====

Plane Number	Power	Height
-----		
19 (TOP)	0.000	380.00
18	0.195	360.00
17	0.353	340.00
16	0.538	320.00
15	0.743	300.00
14	0.975	280.00
13	1.182	260.00
12	1.349	240.00
11	1.470	220.00
10	1.539	200.00
9	1.554	180.00
8	1.514	160.00
7	1.420	140.00
6	1.276	120.00
5	1.087	100.00
4	0.858	80.00
3	0.599	60.00
2	0.348	40.00
1 (BOTTOM)	0.000	20.00

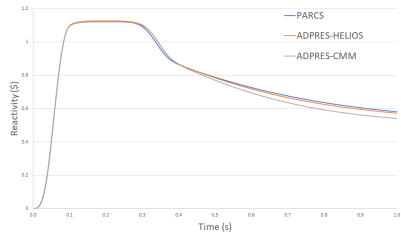
MAX POS.            Maximum Value  
( 9)                1.554

# UO2-MOX PWR Transient Benchmark Results

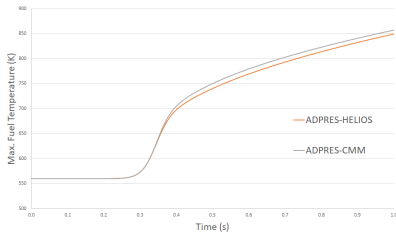
Transient Core Power From HZP



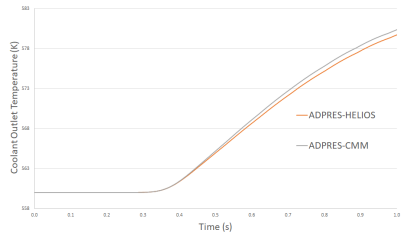
Transient Reactivity



Maximum Fuel Centerline Temperature



Coolant Outlet Temperature



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