## * Error Detection and Correction: Hamming codes

— When the digital information in the binary form is transmitted from one circuit or system to another circuit or system an error may occur. This means a signal corresponding to 0 may change to 1 or vice-versa due to presence of noise.

— To maintain the data integrity between transmitter and receiver, extra bit or more than one bit are added in the data. These extra bits allow the detection and sometimes correction of error in the data.

— The data along with the extra bit/bits forms the code.

— Codes which allow only error detection are called error detecting codes and codes which allow error detection and correction are called error detecting and correcting codes.

## * Error-Detecting codes:

— When a digital information is transmitted, it may not be received correctly by the receiver.

Eg: Consider BCD code corresponding to decimal-9

i.e 1001

Case-1:— This is transmitted and received as 1011.

— Since 1011 is an invalid BCD code, it may be detected by the receiver.

Case-2: If it is received as 0001 which is a valid BCD for decimal 1, the receiver will interpret as decimal-1 and the error is not detected.

## ★ Parity Bit

- For detection of error an extra bit known as parity bit is attached to each code word to make the number of 1's in the code even (Even parity) or odd (Odd parity).

| BCD code | | | | BCD code with even parity | | | | | BCD code with odd parity | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D | C | B | A | P | D | C | B | A | P | D | C | B | A |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |

- The parity bit 1 or 0 is attached to the code to be transmitted at the transmitter end and the parity of the received (n+1)-bit word is checked at the receiving end.
- If there is only one error, the erroneous code is detected at the receiving end by parity check.
- If odd number of bits are transmitted erroneously, then also the parity check will detect the incorrect code

But if there are even number of bits received incorrectly, this method will not detect the error.

: Write the ASCII code of the word "COMPUTER" Using even parity.

:

| | P | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|---|---|---|
| C | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| O | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| M | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| P | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| U | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| T | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| E | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| R | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |

* Error Correcting Codes: Hamming Code

- Hamming Code is called Error detecting and correcting code.
- The code uses number of parity bits (depending on the number of information bits) located at certain positions in the code group.

Assigning Values to Parity bits: (For a 7-bit code)

— How to determine 1 or 0 value to each parity bit

Assignment of P1: Parity bit $P_1$ checks bit locations 1, 3, 5 and 7 and assigns P1 according to even or odd parity

Assignment of P2: 2, 3, 6 and 7

Assignment of P4: 4, 5, 6 and 7

b: Encode the binary word 1011 into 7-bit even parity hamming code.

| $D_7$ | $D_6$ | $D_5$ | $P_4$ | $D_3$ | $P_2$ | $P_1$ |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | | 1 | | |

$P_1$ — ? 1 1 1 → 1

$P_2$ — ? 1 0 1 → 0

$P_4$ — ? 1 0 1 → 0

∴ The Hamming Code is: 1 0 1 0 1 0 1

5: Assume that the even parity hamming code 0110011 is transmitted and that 0100011 is received. Determine bit location where error has occurred using received code.

Sol: Received code:

| $D_7$ | $D_6$ | $D_5$ | $P_4$ | $D_3$ | $P_2$ | $P_1$ |
|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 1 | 1 |

$P_1 - 1\ 0\ 0\ 0 \longrightarrow 1$ (LSB)

$P_2 - 1\ 0\ 1\ 0 \longrightarrow 0$

$P_4 - 0\ 0\ 1\ 0 \longrightarrow 1$

The resultant word is: $1\ 0\ 1 = 4+1 = 5$

$\therefore$ Error is at location 5

$\therefore$ The correct code is: 0110011

6b: The Hamming Code 101101101 is received. Correct it if any errors. There are four parity bits and odd parity is used.

Received codeword

Sol:

| $D_9$ | $P_8$ | $D_7$ | $D_6$ | $D_5$ | $P_4$ | $D_3$ | $P_2$ | $P_1$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |

$P_1 \longrightarrow 11011 \longrightarrow 1$ LSB

$P_2 \longrightarrow 0111 \longrightarrow 0$

$P_4 \longrightarrow 1011 \longrightarrow 0$

$P_8 \longrightarrow 01 \longrightarrow 0$

Error word

~~Parity Code~~ $= 0001 = (1)_{10}$

$\therefore$