

1. Submit the class notebook.

2. We want to see the numerical accuracy of the derivatives from finite differences that we introduced during class. For this purpose you are required to calculate numerically the position versus time of an object of mass  $m = 0.5$  kg, subject to the force of a spring  $F = -k \cdot x$  where  $k = 5$  N/m where the initial position is  $x_0 = -1$  m by increasing the number of timesteps. The workflow should be (1) calculate the  $a_i$  for each timestep  $i$ , (2) calculate the velocity  $v_i$  using numerical differentiation, and (3) calculate the new position  $x_{i+1}$ .

a) Use the forward difference to plot the position of  $x_i$  using different time steps  $dt$  until you find differences with the analytic solutions. Represent this difference against the exact harmonic oscillator motion and show the evolution of this difference over time.

b) Use the leapfrog method to improve the numerical accuracy of the calculation. This method consists in using velocities  $v_i = v(t_i + dt/2)$  calculated at  $t = t_i + dt/2$ , rather than  $t = t_i$ .