假设:　　高斯判别分析

$y \sim Bernoulli(\phi) = p^{\phi} p^{1-\phi}$

$x|y=0 \sim N(\mu_0, \Sigma)$

$x|y=1 \sim N(\mu_1, \Sigma)$

我们目的 将 $\phi, \mu_0, \mu_1, \Sigma$ 参数求出.

$p(y) = \phi^{y}(1-\phi)^{1-y}$

$$p(x|y=0) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu_0)^T\Sigma^{-1}(x-\mu_0)\right)$$

$$p(x|y=1) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)\right)$$

$$l(\phi, \mu_0, \mu_1, \Sigma)$$

$$= \log \prod_{i=1}^{m} p(x^{(i)}, y^{(i)}; \phi, \mu_0, \mu_1, \Sigma)$$

$$= \log \prod_{i=1}^{m} p(x^{(i)} | y^{(i)}; \mu_0, \mu_1, \Sigma)\, p(y^{(i)}; \phi)$$

$$\phi = \frac{1}{m}\sum_{i=1}^{m} 1\{y^{(i)}=1\}$$

$$\mu_0 = \frac{\sum_{i=1}^{m} 1\{y^{(i)}=0\} x^{(i)}}{\sum_{i=1}^{m} 1\{y^{(i)}=0\}}$$

$$\mu_1 = \frac{\sum_{i=1}^{m} 1\{y^{(i)}=1\} x^{(i)}}{\sum_{i=1}^{m} 1\{y^{(i)}=1\}}$$

$$\Sigma = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)} - \mu_{y^{(i)}})(x^{(i)} - \mu_{y^{(i)}})^T$$

将 $\phi, \Sigma, \mu_0, \mu_1$ 代回来模型 就可以求得样本的概率