

Programming Paradigms

Lecture 3

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Statements, Kernel Language, Abstract Machine

Reminder of last lecture

■ Kernel language

- %o linguistic abstraction
- %o data types
- %o variables and partial values
- %o unification

Overview

- Some Oz concepts
 - %o Pattern matching
 - %o Tail recursion
 - %o Lazy evaluation
- Kernel language
 - %o statements and expressions
- Kernel language semantics
 - %o Use *operational semantics*
 - Aid programmer in reasoning and understanding
 - %o The model is a sort of an *abstract machine*, but leaves out details about registers and explicit memory address
 - Aid implementer to do an efficient execution on a real machine

Pattern-Matching on Numbers

```
fun {Fact N}  
  
  case N  
  of 0 then 1  
  [] N then N*{Fact (N-1)} end  
  
end
```

Pattern Matching on Structures

```
fun {Depth T}

case T

of leaf(value:_)
  then 1

[] node(left:L right:R value:_)
  then 1 + {Max {Depth L} {Depth R} }

end

end
```

Compared to Conditional

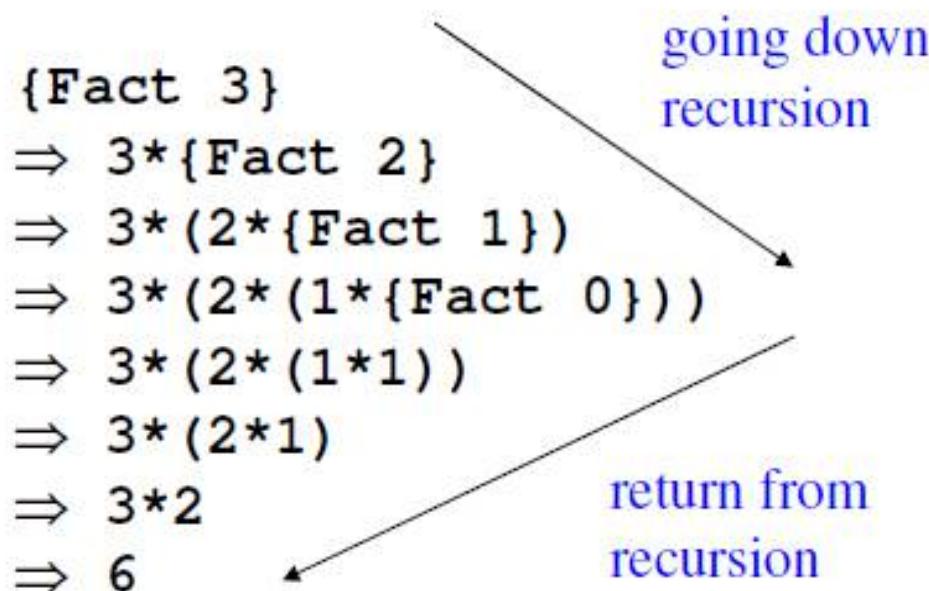
```
fun {SumList Xs}
    case Xs
        of nil  then 0
        [] X|Xr then X + {SumList Xr} end
    end
```

Using only Conditional

```
fun {SumList Xs}
    if {Label Xs}=='nil' then 0
    elseif {Label Xs}=='|' andthen {Width Xs}==2
        then Xs.1 +{SumList Xs.2}
    end
end
```

Linear Recursion

```
fun {Fact N}
  case N
    of 0  then 1
    [] N then N * {Fact (N-1)} end
end
```

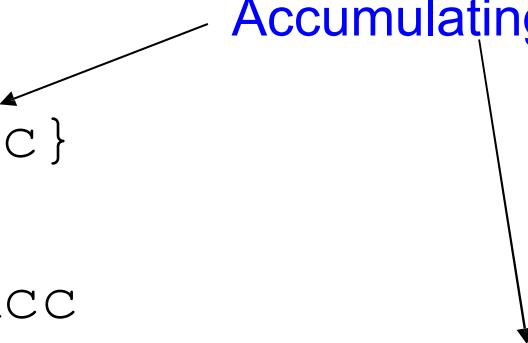


Accumulating Parameter

```
fun {Fact N } {FactT N 1} end
```

```
fun {FactT N Acc}  
  case N  
    of 0 then Acc  
    [] N then {FactT (N-1) N*Acc} end  
end
```

Accumulating Parameter



Accumulating Parameter

```
{Fact 3}  
⇒ {FactT 3 1}  
⇒ {FactT 2 3*1}  
⇒ {FactT 2 3})  
⇒ {FactT 1 2*3}  
⇒ {FactT 1 6}  
⇒ {FactT 0 1*6}  
⇒ {FactT 0 6}  
⇒ 6
```

going down
recursion and accumulating
result in parameter

Accumulating Parameter = Tail Recursion = Loop!

Tail Recursion = Loop

```
fun {FactT N Acc}  
  case N  
    of 0  then Acc  
    [] N then N=N-1  
         Acc=N*Acc  
  end  
end
```

Acc=N*Acc
~~{FactT N Acc}~~

jump
Last call = Tail call

Lazy Evaluation

Infinite list of numbers!

```
fun lazy {Ints N} N|{Ints N+1} end  
  
          {Ints 2}  
⇒ 2|{Ints 3}  
⇒ 2| (3|{Ints 4})  
⇒ 2| (3| (4|{Ints 5}))  
⇒ 2| (3| (4| (5|{Ints 6})))  
⇒ 2| (3| (4| (5| (6|{Ints 7}))))  
      :
```

What if we were to compute : {SumList {Ints 2}} ?

Taking first N elements of List

```
fun {Take L N}
  if N<=0 then nil
  else case L of
    nil then nil
    [] X|Xs then X|{Take Xs (N-1)} end end
end
```

```
{Take [a b c d] 2}
⇒ a|{Take [b c d] 1}
⇒ a|b|{Take [c d] 0}
⇒ a|b|nil
```

```
{Take {Ints 2} 2}
⇒ ?
```

Eager Evaluation

```
{Take {Ints 2} 2}
⇒ {Take 2|{Ints 3} 2}
⇒ {Take 2|(3|{Ints 4}) 2}
⇒ {Take 2|(3|(4|{Ints 5}))) 2}
⇒ {Take 2|(3|(4|(5|{Ints 6})))) 2}
⇒ {Take 2|(3|(4|(5|(6|{Ints 7})))) 2}
:
```

Loop as Infinite list
eagerly evaluated!

Lazy Evaluation

Evaluate the lazy argument only as needed

```
{Take {Ints 2} 2}  
⇒ {Take 2|{Ints 3} 2}  
⇒ 2|{Take {Ints 3} 1}  
⇒ 2|{Take 3|{Ints 4} 1}  
⇒ 2|{3|{Take {Ints 4} 0}}  
⇒ 2|{3|nil}
```

terminates despite infinite list

Kernel Concepts

- Single-assignment store
- Environment
- Semantic statement
- Execution state and Computation
- Statements Execution for:
 - %_o skip and sequential composition
 - %_o variable declaration
 - %_o store manipulation
 - %_o conditional

Procedure Declarations

■ Kernel language

$\langle x \rangle = \text{proc} \{ \$ \langle y_1 \rangle \dots \langle y_n \rangle \} \langle s \rangle \text{end}$
is a legal statement

- binds $\langle x \rangle$ to procedure value
- declares (introduces a procedure)

■ Familiar syntactic variant

`proc { $\langle x \rangle \langle y_1 \rangle \dots \langle y_n \rangle \rangle \langle s \rangle \text{end}$`

introduces (declares) the procedure $\langle x \rangle$

■ A procedure declaration is a value, whereas a procedure application is a statement!

What Is a Procedure?

- It is a **value** of the **procedure type**.

- Java: methods with `void` as return type

declare

X = proc { \$ Y } → \$ is the nesting operator

{Browse 2*Y}

end

{X 3} → 6

{Browse X} → <P/1 X>

- But how to return a result (as parameter) anyway?
 - Idea: use an unbound variable
 - Why: we can supply its value after we have computed it!

Operations on Procedures

- Three basic operations:

- Defining them (with `proc` statement)
- Calling them (with `{ }` notation)
- Testing if a value is a procedure
 - `{IsProcedure P}` returns true if `P` is a procedure, and false otherwise

```
declare
```

```
X = proc {$ Y}  
          {Browse 2*Y}  
      end  
{Browse {IsProcedure X}}
```

Towards Computation Model

- Step One: Make the language small
 - % Transform the language of function on partial values to a small kernel language
 - Kernel language
 - % procedures no functions
 - % records no tuple syntax
 - % local declarations no list syntax
 - % nested calls no nested constructions

From Function to Procedure

```
fun {Sum Xs}  
  case Xs  
  of nil then 0  
  [] X|Xr then X+{Sum Xr}  
  end  
end
```

- Introduce an output parameter for procedure

```
proc {SumP Xs N}  
  case Xs  
  of nil then N=0  
  [] X|Xr then N=X+{Sum P Xr}  
  end  
end
```

Why we need `local` statements?

```
proc {SumP Xs N}
  case Xs
    of nil then N=0
    [] X|Xr then
      local M in {SumP Xr M} N=X+M end
    end
end
```

- Local declaration of variables supported.
- Needed to allow kernel language to be based entirely on procedures

How N was actually transmitted?

- Having the call $\{\text{SumP } [1 \ 2 \ 3] \ \text{C}\}$, the identifier x_S is bound to $[1 \ 2 \ 3]$ and c is unbound.
- At the callee of sumP , whenever N is being bound, so will be c .
- This way of passing parameters is called **call by reference**.
- Procedures output are passed as references to unbound variables, which are bound inside the procedure.

Local Declarations

`local x in ... end`

Introduces the variable identifier `x`

`%o` visible between `in` and `end`

`%o` called scope of the variable/declaration

Creates a new store variable

Links environment identifier to store variable

Abbreviations for Declarations

- Kernel language

- %_o just one variable introduced at a time

- %_o no assignment when first declared

- Oz language syntax supports:

- %_o several variables at a time

- %_o variables can be also assigned (initialized) when introduced

Transforming Declarations Multiple Variables

```
local X Y in  
  <statement>  
end
```



```
local X in  
  local Y in  
    <statement>  
  end  
end
```

Transforming away Declarations' Initialization

```
local  
  x=<expression>  
in  
  <statement>  
end
```



```
local x in  
  x=<expression>  
  <statement>  
end
```

Transforming Expressions

- Replace function calls by procedure calls
- Use local declaration for intermediate values
- Order of replacements:
 - ❑ left to right
 - ❑ innermost first
 - ❑ it is different for record construction: outermost first
 - ❑ Left associativity: $1+2+3$ means $((1+2)+3)$
 - ❑ Right associativity: $a|b|X$ means $(a|(b|X))$, so build the first ' $|$ ', then the second ' $|$ '

Function Call to Procedure Call

X = { F Y }



{ F Y X }

Replacing Nested Calls

{P {F X Y} Z}



local U1 in
{F X Y U1}
{P U1 Z}
end

Replacing Nested Calls

{P {F {G X} Y} Z} 

local U2 in
local U1 in
 {G X U1}
 {F U1 Y U2}
end
{P U2 Z}
end

Replacing Conditionals

```
if X>Y then          local B in
...                  B = (X>Y)
else                if B then
...                  ...
end                  else
...                  ...
end                  end
end
```



Expressions to Statements

```
X = if B then  
    ...  
else  
    ...  
end
```

```
if B then  
    X = ...  
else  
    X = ...  
end
```



Functions to Procedures: Length (0)

```
fun {Length Xs}
    case Xs
        of nil then 0
        [] X|Xr then 1+{Length Xr}
    end
end
```

Functions to Procedures: Length (1)

```
proc {Length Xs N}  
    N=case Xs  
        of nil then 0  
        [] X|Xr then 1+{Length Xr}  
    end  
end
```

- Make it a procedure

Functions to Procedures: Length (2)

```
proc {Length Xs N}  
    case Xs  
        of nil then N=0  
        [] X|Xr then N=1+{Length Xr}  
    end  
end
```

- Expressions to statements

Functions to Procedures: Length (3)

```
proc {Length Xs N}
  case Xs
  of nil then N=0
  [] X|Xr then
    local U in
      {Length Xr U}
    N=1+U
  end
end
end
```

- Replace function call by its corresponding proc call.

Functions to Procedures: Length (4)

```
proc {Length Xs N}
  case Xs
  of nil then N=0
  [] X|Xr then
    local U in
      {Length Xr U}
      {Number.'+' 1 U N}
    end
  end
end
```

- Replace operation (+, dot-access, <, >, ...): procedure!

Kernel Language Statement Syntax

$\langle s \rangle$ denotes a statement

$\langle s \rangle ::=$	skip	<i>empty statement</i>
	$\langle x \rangle = \langle y \rangle$	<i>variable-variable binding</i>
	$\langle x \rangle = \langle v \rangle$	<i>variable-value binding</i>
	$\langle s_1 \rangle \langle s_2 \rangle$	<i>sequential composition</i>
	$\text{local } \langle x \rangle \text{ in } \langle s_1 \rangle \text{ end}$	<i>declaration</i>
	$\text{if } \langle x \rangle \text{ then } \langle s_1 \rangle \text{ else } \langle s_2 \rangle \text{ end}$	<i>conditional</i>
	$\{ \langle x \rangle \langle y_1 \rangle \dots \langle y_n \rangle \}$	<i>procedure application</i>
	$\text{case } \langle x \rangle \text{ of } \langle \text{pattern} \rangle \text{ then } \langle s_1 \rangle \text{ else } \langle s_2 \rangle \text{ end}$	<i>pattern matching</i>

$\langle v \rangle ::= \dots$ *value expression*

$\langle \text{pattern} \rangle ::= \dots$

Abstract Machine

- *Environment* maps variable identifiers to store entities
- *Semantic statement* is a pair of:
 - %₀₀ statement
 - %₀₀ environment
- *Execution state* is a pair of:
 - %₀₀ stack of semantic statements
 - %₀₀ single assignment store
- *Computation* is a sequence of execution states
- An **abstract machine** performs a computation

Single Assignment Store

- Single assignment store σ
 - set of store variables
 - partitioned into
 - sets of variables that are equivalent but unbound
 - variables bound to a value (number, record or procedure)
- Example store $\{x_1, x_2=x_3, x_4=a|x_2\}$
 - x_1 unbound
 - x_2, x_3 equal and unbound
 - x_4 bound to partial value $a|x_2$

Environment

- Environment E
 - maps variable identifiers to entities in store σ
 - written as set of pairs $X \rightarrow x$
 - identifier X
 - store variable x
- Example of environment: $\{ X \rightarrow x, Y \rightarrow y \}$
 - maps identifier X to store variable x
 - maps identifier Y to store variable y

Environment and Store

- Given: environment E , store σ
- Looking up value for identifier X :
 - find store variable in environment using $E(X)$
 - take value from σ for $E(X)$
- Example:

$\sigma = \{x_1, x_2=x_3, x_4=a x_2\}$	$E = \{X \rightarrow x_1, Y \rightarrow x_4\}$
$E(X) = x_1$	where no information in σ on x_1
$E(Y) = x_4$	where σ binds x_4 to $a x_2$

Calculating with Environments

- Program execution looks up values
 - assume store σ
 - given identifier $\langle x \rangle$
 - $E(\langle x \rangle)$ is the value of $\langle x \rangle$ in store σ
- Program execution modifies environments
 - for example: declaration
 - add mappings for new identifiers
 - overwrite existing mappings
 - restrict mappings on sets of identifiers

Environment Adjunction

- Given: Environment E

then $E + \{\langle x \rangle_1 \rightarrow x_1, \dots, \langle x \rangle_n \rightarrow x_n\}$

is a new environment E' with mappings added:

- always take store entity from new mappings
- might overwrite (or shadow) old mappings

Environment Projection

- Given: Environment E

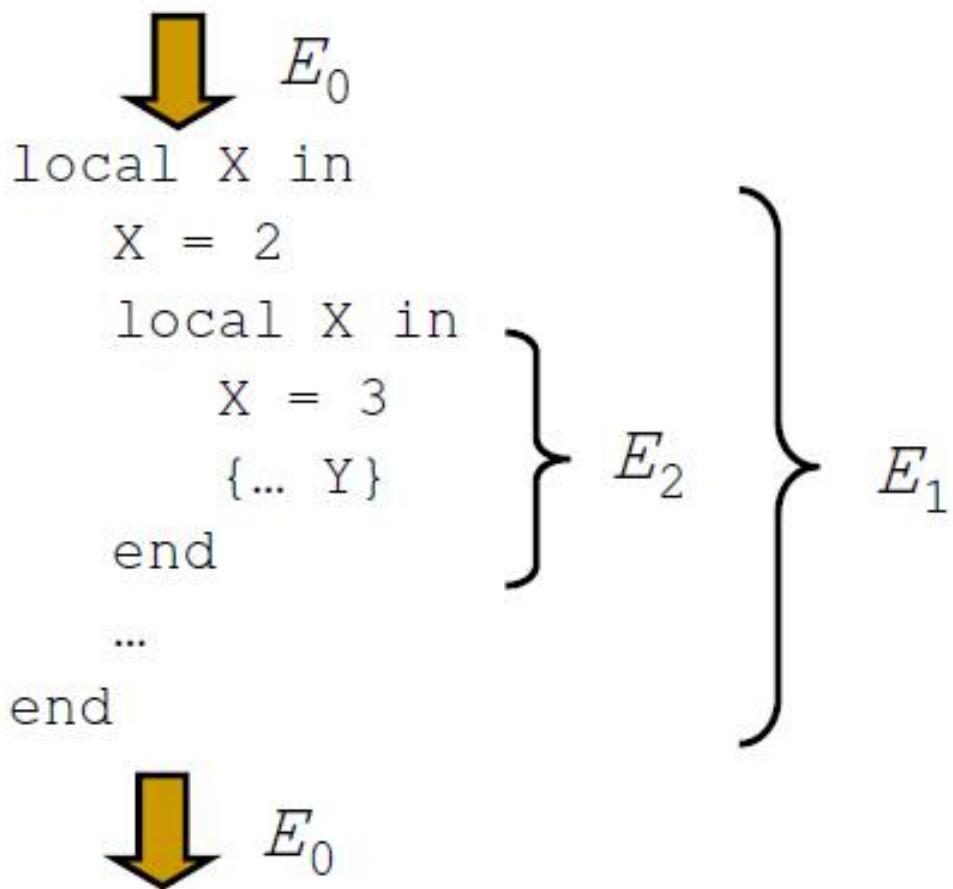
$$E \mid \{\langle x \rangle_1, \dots, \langle x \rangle_n\}$$

is a new environment E' where only mappings for $\{\langle x \rangle_1, \dots, \langle x \rangle_n\}$ are retained from E

Adjunction Example

- $E_0 = \{\langle Y \rangle \rightarrow 1\}$
- $E_1 = E_0 + \{\langle X \rangle \rightarrow 2\}$
 - corresponds to $\{\langle X \rangle \rightarrow 2, \langle Y \rangle \rightarrow 1\}$
 - $E_1(\langle X \rangle) = 2$
- $E_2 = E_1 + \{\langle X \rangle \rightarrow 3\}$
 - corresponds to $\{\langle X \rangle \rightarrow 3, \langle Y \rangle \rightarrow 1\}$
 - $E_2(\langle X \rangle) = 3$

Why Adjunction?



Semantic Statements

- Semantic statement $(\langle s \rangle, E)$
 - pair of (statement, environment)
- To actually execute statement:
 - environment to map identifiers
 - modified with execution of each statement
 - each statement has its own environment
 - store to find values
 - all statements modify same store
 - single store

Stacks of Statements

- Execution maintains stack of semantic statements $ST = [(\langle s \rangle_1, E_1), \dots, (\langle s \rangle_n, E_n)]$
 - always topmost statement $(\langle s \rangle_1, E_1)$ executes first
 - $\langle s \rangle$ is statement
 - E denotes the environment mapping
 - rest of stack: remaining work to be done
- Also called: *semantic stack*

Execution State

- *Execution state* (ST, σ)
 - pair of (semantic stack, store)
- *Computation*
$$(ST_1, \sigma_1) \Rightarrow (ST_2, \sigma_2) \Rightarrow (ST_3, \sigma_3) \Rightarrow \dots$$
 - sequence of execution states

Program Execution

- Initial execution state
 - ([($\langle s \rangle, \emptyset$)], \emptyset)
 - empty store \emptyset
 - stack with semantic statement $[(\langle s \rangle, \emptyset)]$
 - single statement $\langle s \rangle$, empty environment \emptyset
- At each execution step
 - pop topmost element of semantic stack
 - execute according to statement
- If semantic stack is empty, then execution stops

Semantic Stack States

- Semantic stack can be in following states
 - %_o *terminated* stack is empty
 - %_o *Runnable* can do execution step
 - %_o *suspended* stack not empty, no execution step possible
- Statements
 - %_o *non-suspending* can always execute
 - %_o *suspending* need values from store dataflow behavior

Summary up to now

- Single assignment store σ
- Environments E
 - adjunction, projection $E + \{...\}$ $E | \{...\}$
- Semantic statements $(\langle s \rangle, E)$
- Semantic stacks $[(\langle s \rangle, E) \dots]$
- Execution state (ST, σ)
- Computation = sequence of execution states
- Program execution
 - runnable, terminated, suspended
- Statements
 - suspending, non-suspending

Statement Execution

- Simple statements

- %_{oo} skip and sequential composition

- %_{oo} variable declaration

- %_{oo} store manipulation

- %_{oo} Conditional (`if` statement)

- Computing with procedures (next lecture)

- %_{oo} lexical scoping

- %_{oo} closures

- %_{oo} procedures as values

- %_{oo} procedure call

Simple Statements

$\langle s \rangle$ denotes a statement

$\langle s \rangle ::=$

- skip
- $\langle x \rangle = \langle y \rangle$
- $\langle x \rangle = \langle v \rangle$
- $\langle s_1 \rangle \langle s_2 \rangle$
- local $\langle x \rangle$ in $\langle s_1 \rangle$ end
- if $\langle x \rangle$ then $\langle s_1 \rangle$ else $\langle s_2 \rangle$ end

empty statement
variable-variable binding
variable-value binding
sequential composition
declaration
conditional

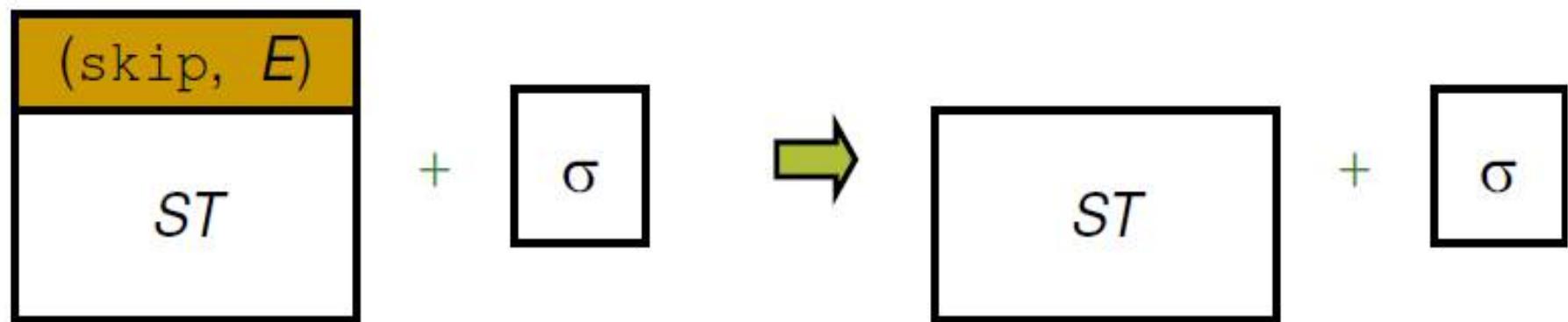
$\langle v \rangle ::= \dots$

value expression
(no procedures here)

Executing skip

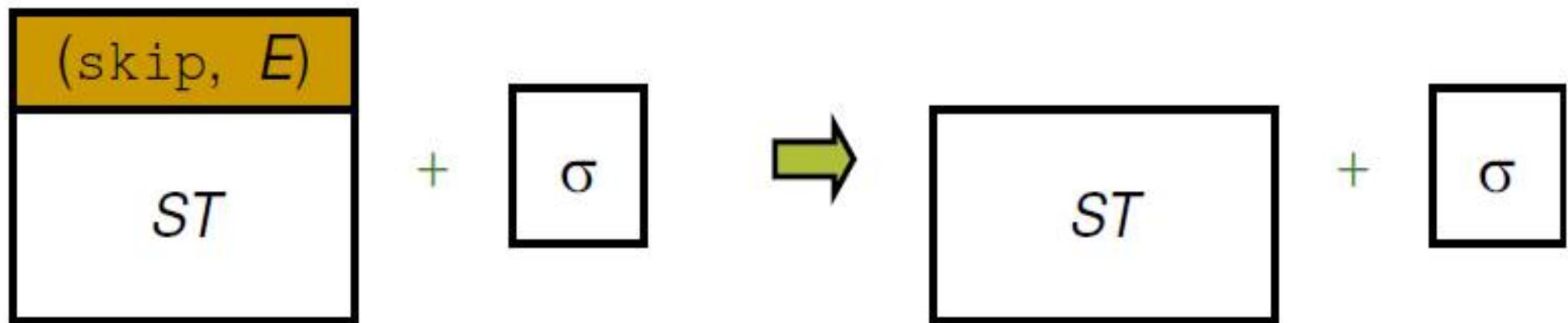
- Execution of semantic statement
 (skip, E)
- Do nothing
 - means: continue with next statement
 - non-suspending statement

Executing skip



- No effect on store σ
- Non-suspending statement

Executing skip

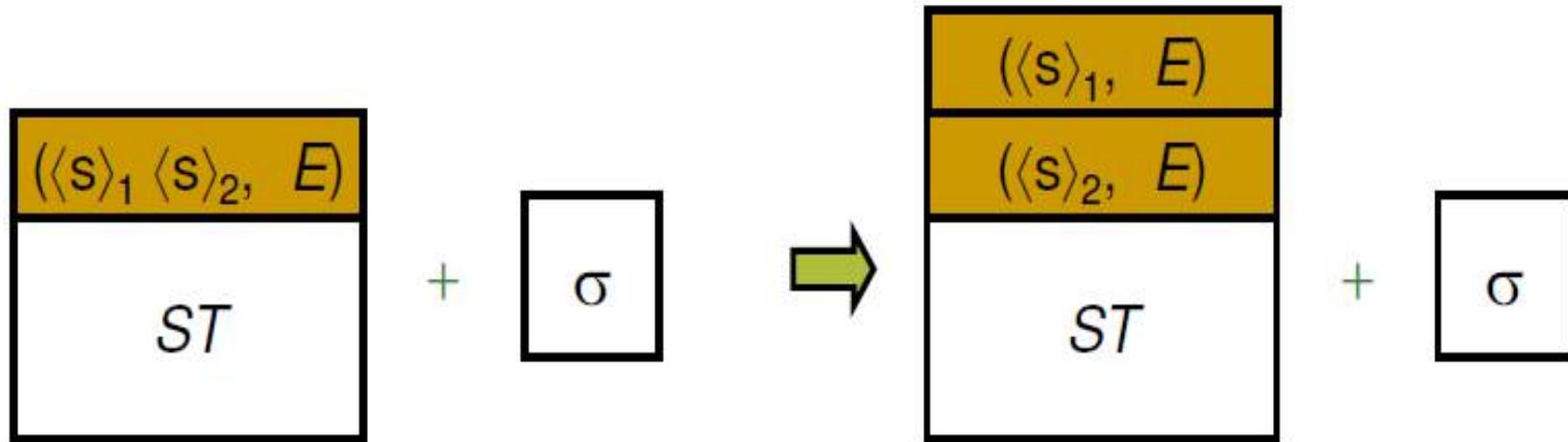


- Remember: topmost statement is always popped!

Executing Sequential Composition

- Semantic statement is
 $(\langle s \rangle_1 \langle s \rangle_2, E)$
- Push in following order
 - $\langle s \rangle_2$ executes after
 - $\langle s \rangle_1$ executes next
- Statement is non-suspending

Sequential Composition

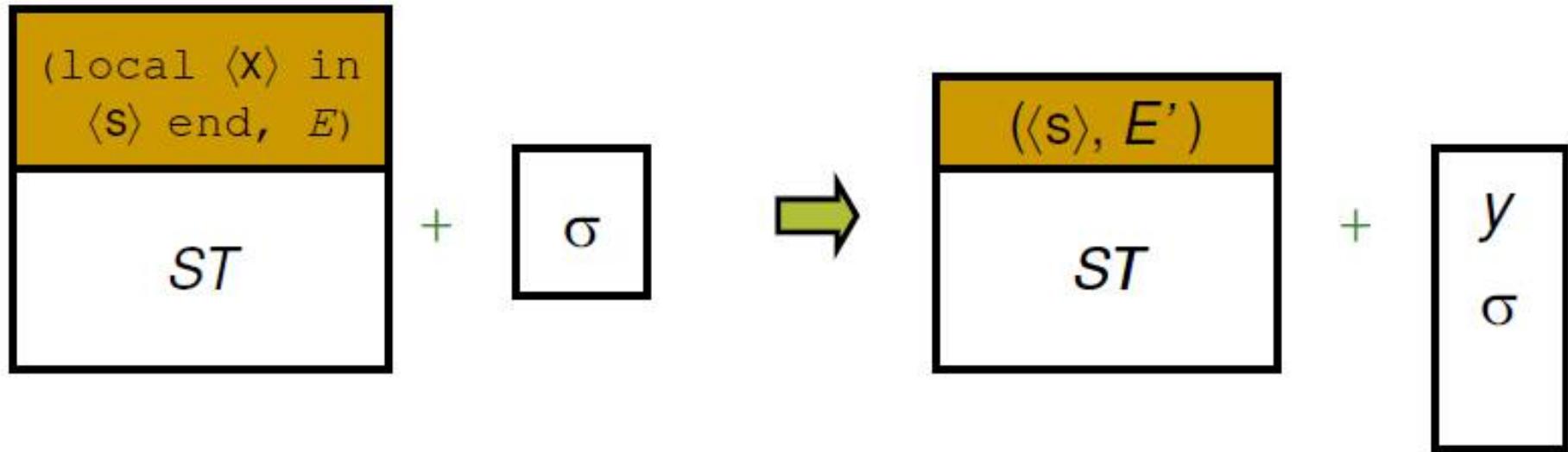


- Decompose statement sequences
 - environment is given to both statements

Executing `local`

- Semantic statement is
 $(\text{local } \langle x \rangle \text{ in } \langle s \rangle \text{ end}, E)$
- Execute as follows:
 - create new variable y in store
 - create new environment $E' = E + \{\langle x \rangle \rightarrow y\}$
 - push $(\langle s \rangle, E')$
- Statement is non-suspending

Executing local

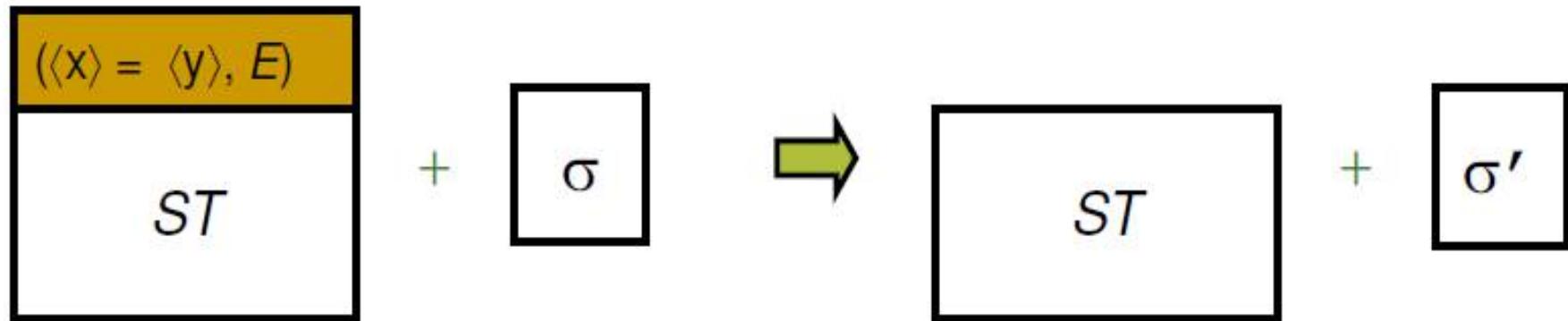


- With $E = E + \{\langle x \rangle \rightarrow y\}$

Variable-Variable Equality

- Semantic statement is
 $(\langle x \rangle = \langle y \rangle, E)$
- Execute as follows
 - bind $E(\langle x \rangle)$ and $E(\langle y \rangle)$ in store
- Statement is non-suspending

Executing Variable-Variable Equality



- σ' is obtained from σ by binding $E(\langle x \rangle)$ and $E(\langle y \rangle)$ in store

Variable-Value Equality

- Semantic statement is

$$(\langle x \rangle = \langle v \rangle, E)$$

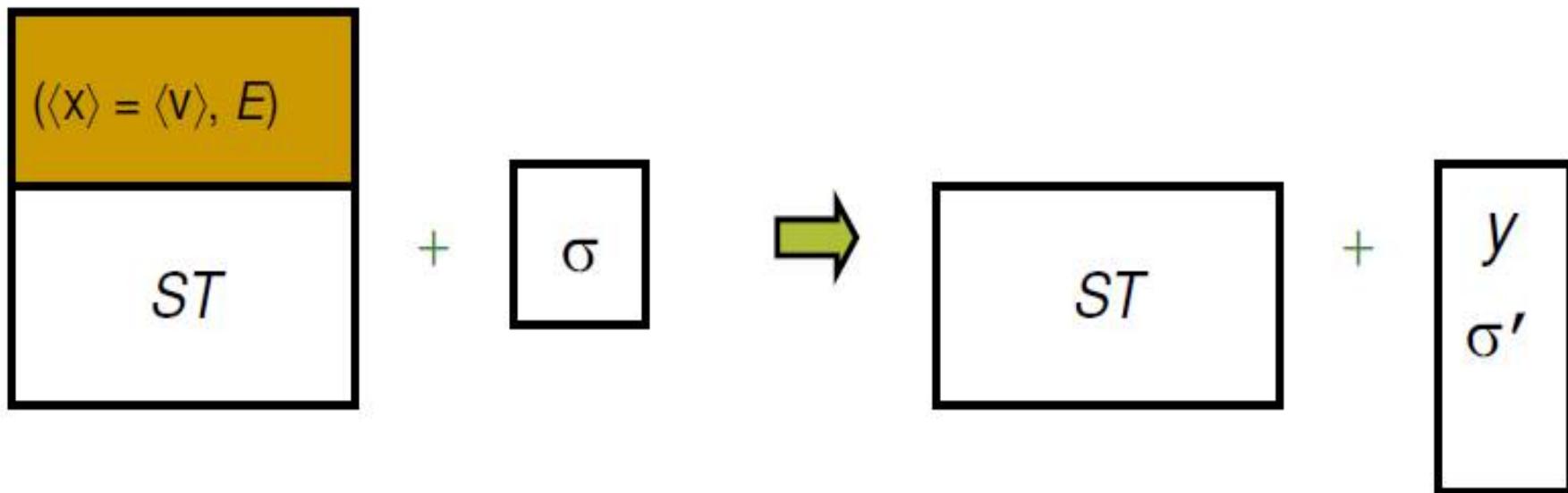
where $\langle v \rangle$ is a number or a record (procedures will be discussed later)

- Execute as follows

- create a variable y in store and let y refers to value $\langle v \rangle$
 - any identifier $\langle z \rangle$ from $\langle v \rangle$ is replaced by $E(\langle z \rangle)$
 - bind $E(\langle x \rangle)$ and y in store

- Statement is non-suspending

Executing Variable-Value Equality



- y refers to value $\langle v \rangle$
- Store σ is modified into σ' such that:
 - any identifier $\langle z \rangle$ from $\langle v \rangle$ is replaced by $E(\langle z \rangle)$
 - bind $E(\langle x \rangle)$ and y in store σ

Suspending Statements

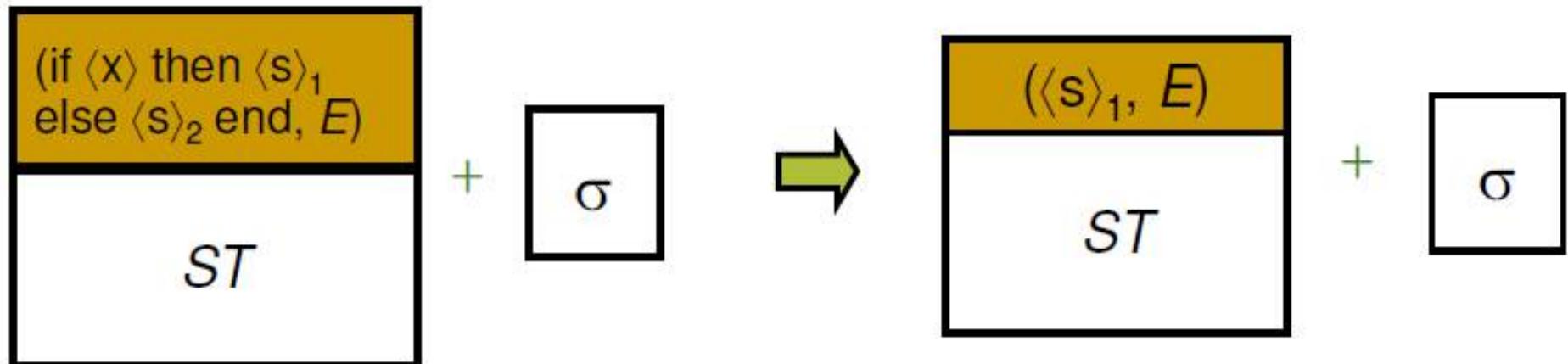
- All statements so far can always execute
 - non-suspending (or immediate)
- Conditional?
 - requires condition $\langle x \rangle$ to be bound variable
 - *activation condition*: $\langle x \rangle$ is bound (determined)

Executing `if`

- Semantic statement is
 $(\text{if } \langle x \rangle \text{ then } \langle s \rangle_1 \text{ else } \langle s \rangle_2 \text{ end}, E)$
- If the activation condition “`bound(<x>)`” is true
 - if $E(\langle x \rangle)$ bound to true push $\langle s \rangle_1$
 - if $E(\langle x \rangle)$ bound to false push $\langle s \rangle_2$
 - otherwise, raise error
- Otherwise, suspend the `if` statement...

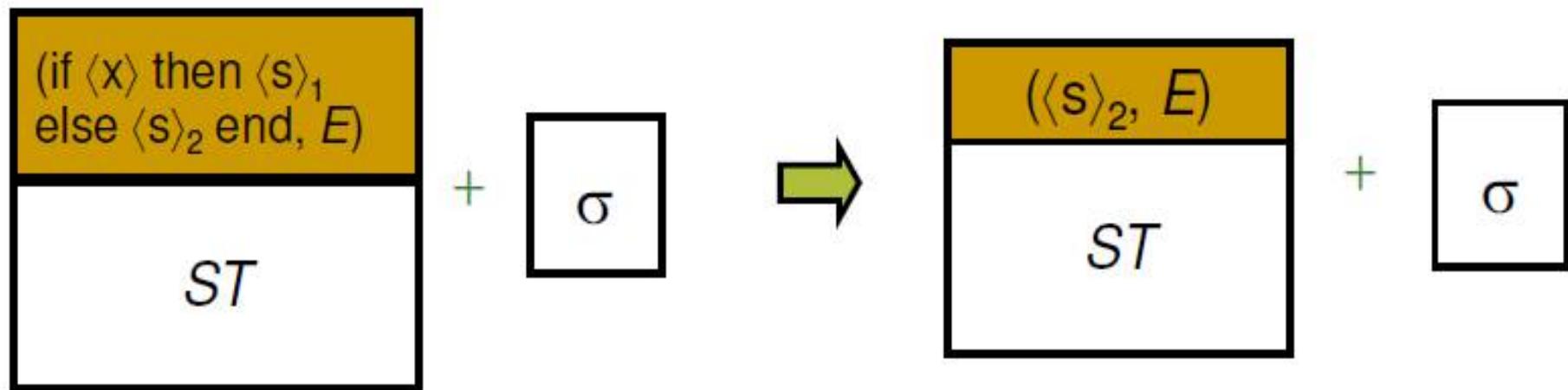
Executing if

- If the activation condition “ $\text{bound}(\langle x \rangle)$ ” is true
 - if $E(\langle x \rangle)$ bound to true



Executing if

- If the activation condition “ $\text{bound}(\langle x \rangle)$ ” is true
 - if $E(\langle x \rangle)$ bound to false



An Example

```
local X in
    local B in
        B=true
        if B then X=1 else skip end
    end
end
```

- We can reason that x will be bound to 1

Example: Initial State

```
( [ (local X in
    local B in
        B=true
            if B then X=1 else skip end
        end
    end, ∅) ] ,
∅)
```

- Start with empty store and empty environment

Example: local

```
( [ (local B in
    B=true
        if B then X=1 else skip end
    end,
    {x → x}) ] ,
{x})
```

- Create new store variable x
- Continue with new environment

Example: local

```
( [ (B=true  
      if B then X=1 else skip end  
      ,  
      {B → b, X → x} ) ] ,  
      {b,x})
```

- Create new store variable b
- Continue with new environment

Example: Sequential Composition

```
( [ (B=true, {B → b, x → x}) ,  
  (if B then x=1  
   else skip end, {B → b, x → x}) ] ,  
 {b,x})
```

- Decompose to two statements
- Stack has now two semantic statements

Example: Variable-Value Assignment

```
( [ (if B then X=1  
      else skip end, {B → b, X → x}) ] ,  
{b=true, X})
```

- Environment maps B to b
- Bind b to $true$

Example: if

```
( [ (x=1, {B → b, x → x}) ] ,  
{b=true, X})
```

- Environment maps B to b
- Bind b to $true$
- Because the activation condition “ $\text{bound}(\langle x \rangle)$ ” is $true$, continue with then branch of if statement

Example: Variable-Value Assignment

```
( [ ] ,  
  {b=true, x=1} )
```

- Environment maps x to x
- Binds x to 1
- Computation terminates as stack is empty

Summary up to now

- Semantic statement execute by
 - ❑ popping itself always
 - ❑ creating environment local
 - ❑ manipulating store local, =
 - ❑ pushing new statements local, if sequential composition
 - Semantic statement can suspend
 - ❑ activation condition (if statement)
 - ❑ read store

Pattern Matching

- Semantic statement is

```
(case <x>
  of <lit>(<feat>1:<y>1 ... <feat>n:<y>n) then <s>1
  else <s>2 end, E)
```

- It is a suspending statement
- Activation condition is: “bound(<x>)”
- If activation condition is false, then suspend!

Pattern Matching

- Semantic statement is

(**case** $\langle x \rangle$
 of $\langle \text{lit} \rangle(\langle \text{feat} \rangle_1:\langle y \rangle_1 \dots \langle \text{feat} \rangle_n:\langle y \rangle_n)$ **then** $\langle s \rangle_1$
 else $\langle s \rangle_2$ **end**, E)

- If $E(\langle x \rangle)$ matches the pattern, that is,

- label of $E(\langle x \rangle)$ is $\langle \text{lit} \rangle$ and
 - its arity is $[\langle \text{feat} \rangle_1 \dots \langle \text{feat} \rangle_n]$,

then push

$(\langle s \rangle_1,$
 $E + \{\langle y \rangle_1 \rightarrow E(\langle x \rangle). \langle \text{feat} \rangle_1,$
 $\dots,$
 $\langle y \rangle_n \rightarrow E(\langle x \rangle). \langle \text{feat} \rangle_n\})$

Pattern Matching

- Semantic statement is

(**case** $\langle x \rangle$
 of $\langle \text{lit} \rangle (\langle \text{feat} \rangle_1 : \langle y \rangle_1 \dots \langle \text{feat} \rangle_n : \langle y \rangle_n)$ **then** $\langle s \rangle_1$
 else $\langle s \rangle_2$ **end**, E)

- If $E(\langle x \rangle)$ does not match pattern, push
 $(\langle s \rangle_2, E)$

Pattern Matching

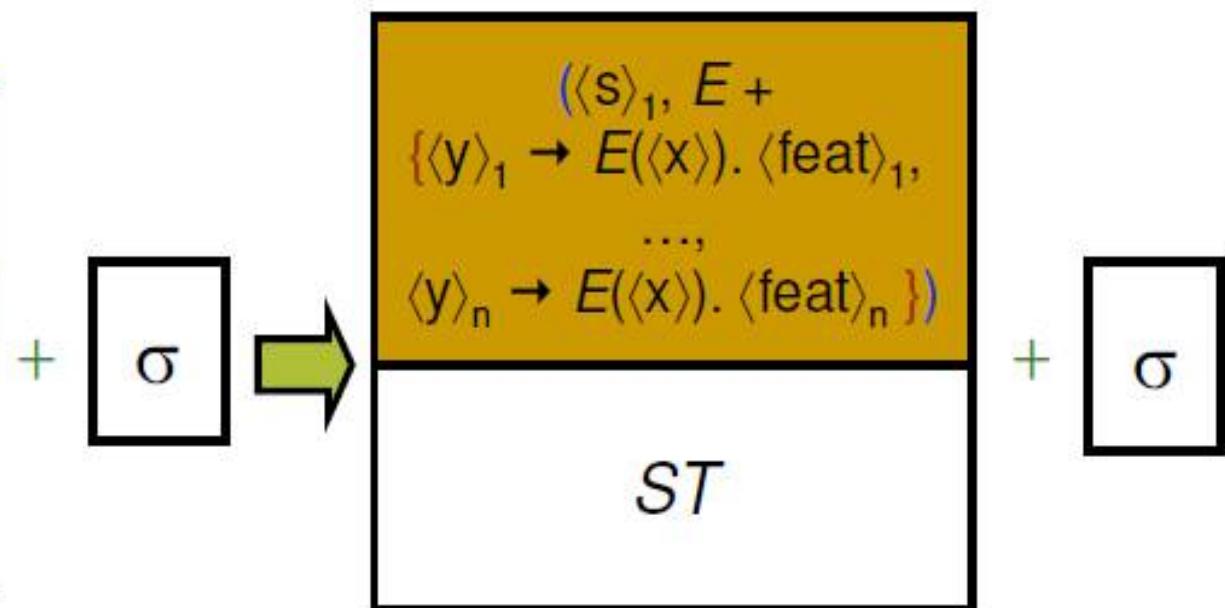
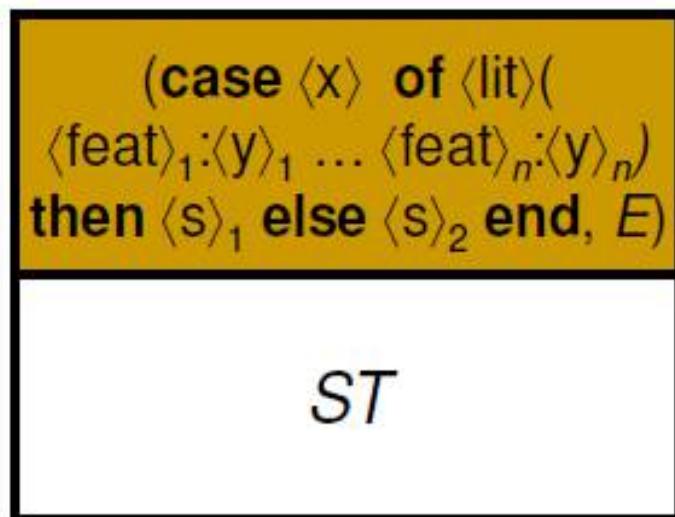
- Semantic statement is

```
(case ⟨x⟩  
  of ⟨lit⟩(⟨feat⟩1:⟨y⟩1 ... ⟨feat⟩n:⟨y⟩n) then ⟨s⟩1  
  else ⟨s⟩2 end, E)
```

- It does not introduce new variables in the store
- Identifiers ⟨y⟩₁ ... ⟨y⟩_n are visible only in ⟨s⟩₁

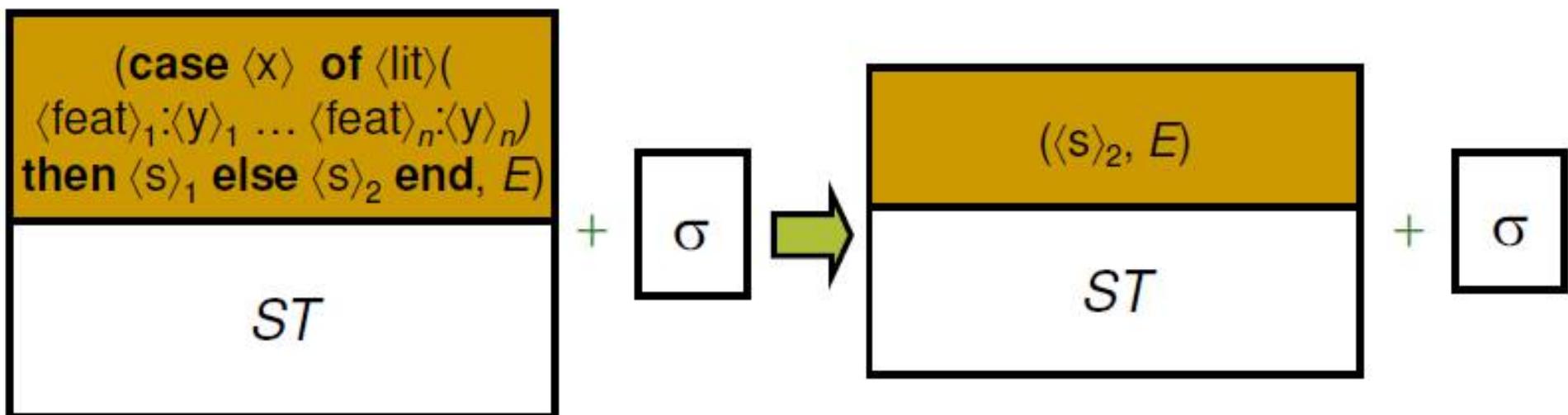
Executing case

- If the activation condition “ $\text{bound}(\langle x \rangle)$ ” is true
 - if $E(\langle x \rangle)$ matches the pattern



Executing case

- If the activation condition “ $\text{bound}(\langle x \rangle)$ ” is true
 - if $E(\langle x \rangle)$ does not match the pattern



Example: case Statement

```
( [ (case X of
    f(X1 X2) then Y = g(X2 X1)
    else Y = c
    end,
    {X →v1, Y →v2})], % Env
    {v1=f (v3 v4), v2, v3=a, v4=b} % Store
)
```

- We declared X, Y, X1, X2 as local identifiers and X=f (v3 v4), X1=a and X2=b
- What is the value of Y after executing case?

Example: case Statement

```
( [ (Y = g(X2 X1),  
     {X →v1, Y →v2, X1 →v3, X2 →v4})  
   ],  
   {v1=f(v3 v4), v2, v3=a, v4=b}  
)
```

- The activation condition “bound($\langle x \rangle$)” is true
- Remember that $X1=a$, $X2=b$

Example: case Statement

```
( [ ] ,  
  { v1=f (v3  v4) ,  
    v2=g (v4  v3) , v3=a , v4=b }  
)
```

- Remember Y refers to v2 , so

$$\text{Y} = \text{g}(\text{b} \ \text{a})$$

Summary

- Kernel language
 - linguistic abstraction
 - data types
 - variables and partial values
 - statements and expressions
- Computing with procedures (next lecture)
 - lexical scoping
 - closures
 - procedures as values
 - procedure call