## Homework 8, due 11:59pm Monday April 4

## STATS/DATASCI 531, Winter 2022

This is a group assignment for which you will collaborate with your final project group. You should submit one homework report for your group. This report should be written in Rmd, and you should submit both the Rmd source and a version compiled to HTML. For each question, you should give both a letter answer (one or more letters, depending on the form of the question) and some text explaining your answer. Most of the questions do not need any coding, but you can show your code when appropriate.

The goal is to think about some issues that might arise in final projects, and indeed some of the questions alert you to common errors from previous 531 final projects.

If you poke around, you may find solutions to these questions. That is not against the rules, following the usual source attribution requirements for the course that all sources are acceptable as long as they are properly acknowledged. The requirement, as in other homeworks, is that you explicitly list all sources you consulted and you make clear what you learned from them. For group work, it is your responsibility to make sure that every source consulted by every group member is listed at the end of the report and referenced properly where used within the report.

It may be simplest for this homework if you avoid consulting solutions online. If you do obtain answers online, you should find ways to go beyond the sources to make your own contribution.

These questions were developed for a course on Simulation-based Inference for Epidemiological Dynamics. That is a vague acknwledgement of a large but unspecified intellectual debt which is okay in this context but not sufficient as an explanation of sources for a homework report.

Question 1. From a scientific perspective, conclusions should not depend on the units we choose. However, we must get the details straight to correctly describe a POMP model and its pomp representation. Suppose our data are two years of weekly aggregated case reports of a disease and we have a continuous time model solved numerically using an Euler timestep of size dt. Which one of the following is a correct explanation of our options for properly implementing this in a pomp object called po?

- (A) The measurement times, time(po), should be in units of weeks, such as 1, 2, ..., 104. The latent process can be modeled using arbitrary time units, say days or weeks or years. The units of dt should match the time units of the **latent** process.
- (B) The measurement times, time(po), should be in units of weeks, such as 1, 2, ..., 104. The latent process can be modeled using arbitrary time units, say days or weeks or years. The units of dt should be in weeks (in practice, usually a fraction of a week) to match the units of the **measurement** times.
- (C) The measurement times do not have to be in units of weeks. For example, we could use time(po)=1/52, 2/52, ..., 2. The latent process and dt should use the same units of time as the measurement times.
- (D) The measurement times do not have to be in units of weeks. For example, we could use time(po)=1/52, 2/52, ..., 2. The latent process can also use arbitrary units of time, which do not necessarily match the units of the measurement times. The units of dt should match the units used for the **latent** process.
- (E) The measurement times do not have to be in units of weeks. For example, we could use time(po)=1/52,  $2/52, \ldots, 2$ . The latent process can also use arbitrary units of time, which do not necessarily match the units of the measurement times. The units of dt should match the units used for the **measurement** times.

Question 2. Suppose you obtain the following error message when you build your pomp model using Csnippets.

```
Error: error in building shared-object library from C snippets: in 'Cbuilder': compilation error:
cannot compile shared-object library '/var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_be9007eb030e47cb34264e3e779b6da9.so':
status = 1
compiler messages:
clang -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG
-I'/Users/ionides/Library/R/x86_64/4.1/library/pomp/include'
                                                          -fPIC -Wall -g -02
-I'/Users/ionides/sbied/questions' -I/usr/local/include
-c /var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_be9007eb030e47cb34264e3e779b6da9.c
-o /var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_be9007eb030e47cb34264e3e779b6da9.o
/var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_be9007eb030e47cb34264e3e779b6da9.c:39:5:
error: called object type 'int' is not a function or function pointer
   W = 0;
/var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/5
In addition: Warning message:
In system2(command = R.home("bin/R"), args = c("CMD", "SHLIB", "-c", :
running command 'PKG_CPPFLAGS="-I'/Users/ionides/Library/R/x86_64/4.1/library/pomp/include'
-I'/Users/ionides/sbied/questions'" '/Library/Frameworks/R.framework/Resources/bin/R'
CMD SHLIB -c -o /var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_be9007eb030e47cb34264e3e779b6da9.so
/var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_be9007eb030e47cb34264e3e779b6da9.c
2>&1' had status 1
```

Which one of the following is the most plausible cause for this error?

- (A) Using R syntax within a C function that has the same name as an R function.
- (B) A parameter is missing from the paramnames argument to pomp.
- (C) Indexing past the end of an array because C labels indices starting at 0.
- (D) Using beta as a parameter name when it is a declared C function.
- (E) A missing semicolon at the end of a line.

Question 3. Suppose you obtain the following error message when you build your pomp model using Csnippets.

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```

```
-o /var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_b675d99e691eda865610f570058ea3be.o
/var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_b675d99e691eda865610f570058ea3be.c:33:16:
error: use of undeclared identifier 'pop'; did you mean 'pow'?
   double m = pop/(S_0+I_0+R_0);
               pow
/Applications/
In addition: Warning message:
In system2(command = R.home("bin/R"), args = c("CMD", "SHLIB", "-c", :
 running command 'PKG_CPPFLAGS="-I'/Users/ionides/Library/R/x86_64/4.1/library/pomp/include'
-I'/Users/ionides/sbied/questions'" '/Library/Frameworks/R.framework/Resources/bin/R' CMD SHLIB
-c -o /var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_b675d99e691eda865610f570058ea3be.so
/var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
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```

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**Question 4.** Suppose you obtain the following error message when you build your pomp model using Csnippets.

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5324/pomp_77886fb66d95b4b9904440d86a4425b3.so': status = 1
compiler messages:
clang -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG
-I'/Users/ionides/Library/R/x86_64/4.1/library/pomp/include'
-I'/Users/ionides/sbied/questions' -I/usr/local/include
                                                          -fPIC -Wall -g -02
-c /var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_77886fb66d95b4b9904440d86a4425b3.c
-o /var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_77886fb66d95b4b9904440d86a4425b3.o
/var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
5324/pomp_77886fb66d95b4b9904440d86a4425b3.c:39:36:
error: too many arguments to function call, expected 2, have 3
     rep = nearbyint(rnorm(1,mean,sd));
In addition: Warning message:
In system2(command = R.home("bin/R"), args = c("CMD", "SHLIB", "-c", :
running command 'PKG_CPPFLAGS="-I'/Users/ionides/Library/R/x86_64/4.1/library/pomp/include'
-I'/Users/ionides/sbied/questions'" '/Library/Frameworks/R.framework/Resources/bin/R'
CMD SHLIB -c -o /var/folders/fv/pt62sh2d6_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/
```

5324/pomp\_77886fb66d95b4b9904440d86a4425b3.so /var/folders/fv/pt62sh2d6\_gf9fp3t7b466vr0000gr/T//RtmpD16GmG/ 5324/pomp\_77886fb66d95b4b9904440d86a4425b3.c 2>&1' had status 1

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- (E) A missing semicolon at the end of a line.

**Question 5.** Let  $V_n$  be a Markov process and let  $W_n = h(V_n)$  for some function h. Which of the following statements are true?

- i)  $W_n$  is a Markov process for all choices of h.
- ii)  $W_n$  is a Markov process for some choices of h.
- iii)  $W_n$  is not a Markov process for any choice of h.
- iv) If  $V_n = (X_n, Y_n)$  where  $X_n$  and  $Y_n$  are a POMP model, and  $h(X_n, Y_n) = X_n$  then  $W_n$  is a Markov process.
- v) If  $V_n = (X_n, Y_n)$  where  $X_n$  and  $Y_n$  are a POMP model, and  $h(X_n, Y_n) = Y_n$  then  $W_n$  is a Markov process.
- (A) i,iv,v
- (B) ii,iv
- (C) ii,v
- (D) iii
- (E) None of the above

Question 6. Suppose that 10 replications of a particle filter, each using  $10^3$  particles, runs in 15 minutes with no parallelization. To look for a more precise likelihood evaulation, you consider running 20 replicates, each with  $10^4$  particles. How many minutes will this take, if you distribute the calculation across 4 cores?

- (A) 50
- (B) 60
- (C) 75
- (D) 120
- (E) 300

Question 7. A particle filter is repeated 5 times to evaluate the likelihood at a proposed maximum likelihood estimate, each time with  $10^4$  particles. Suppose the log likelihood estimates are -2446.0, -2444.0, -2443.0, -2442.0, -2440.0. Which of the following is an appropriate estimate for the log likelihood at this parameter value and its standard error.

- (A) Estimate = -2443.0, with standard error 1.0
- (B) Estimate = -2443.0, with standard error 2.2
- (C) Estimate = -2443.0, with standard error 5.0
- (D) Estimate = -2441.4, with standard error 2.2
- (E) Estimate = -2441.4, with standard error 1.4

Question 8. What is the log likelihood (to the nearest unit) of the Dacca cholera data for the POMP model constructed in pomp via

```
d <- dacca(deltaI=0.08)
```

with cholera mortality rate 8% and other parameters fixed at the default values.

- (A) -3764
- (B) -3765
- (C) -3766
- (D) -3767
- (E) -3768

Question 9. Effective sample size (ESS) is one of the main tools for diagnosing the success of a particle filter. If you plot an object of class  $pfilterd\_pomp$  (created by applying pfilter to a pomp object), the ESS is displayed. Suppose one or more time points have low ESS (say, less than 10) even when using a fairly large number of particles (say,  $10^4$ ). What is the proper interpretation?

- (A) There is a problem with data, perhaps an error recording an observation.
- (B) There is a problem with the model which means that it cannot explain something in the data.
- (C) The model and data have no major problems, but the model happens to be problematic for the particle filter algorithm.
- (D) At least one of A, B and C.
- (E) Either A or B or both, but not C. If the model fits the data well, the particle filter is guaranteed to work well.

Question 10. When carrying out inference by iterated particle filtering, the likelihood increases for the first 10 iterations or so, and then steadily decreases. Testing the inference procedure on simulated data, this does not happen and the likelihood increases steadily toward convergence. Which one of the following is the best explanation for this?

- (A) One or more random walk standard deviation is too large.
- (B) One or more random walk standard deviations is too small.
- (C) The model is misspecified, so it does not fit the data adequately.
- (D) A combination of the parameters is weakly identified, leading to a ridge in the likelihood surface.
- (E) Too few particles are being used.

Question 11. People sometimes confuse likelihood profiles with likelihood slices. Suppose you read a figure which claims to construct a profile confidence interval for a parameter  $\rho$  in a POMP model with four unknown parameters. Suppose that the code producing the plot is available to you as an Rmarkdown file. Which one of the following confirms that the plot is, or is not, a properly constructed profile confidence interval.

- (A) The CI is constructed by obtaining the interval of rho values whose log likelihood is within 1.92 of the maximum on a smoothed curve of likelihood values plotted against  $\rho$ .
- (B) The code involves evaluation of the likelihood but not maximization.
- (C) The points along the  $\rho$  axis are not equally spaced.
- (D) The smoothed line shown in the plot is close to quadratic.
- (E) Both A and D together.

Question 12. For each of the following, say whether the statement is true or false.

- (A) A profile likelihood must lie above every slice.
- (B) Confidence intervals can be read from likelihood slices.
- (C) A poor man's profile must lie above the true profile.
- (D) A poor man's profile must lie below the true profile.

Question 13. The iterated filtering convergence diagnostics plot in Fig. 1 comes from a student project. What is the best interpretation?

- (A) Everything seems to be working fine. The likelihood is climbing. The replicated searches are giving consistent runs. The spread of convergence points for  $\sigma_{\nu}$  and  $H_0$  indicates weak identifability, which is a statistical fact worth noticing but not a weakness of the model.
- (B) The consistently climbing likelihood is promising, but the failure of  $\sigma_{\nu}$  and  $H_0$  to converge needs attention. Additional searching is needed, experimenting with larger values of the random walk perturbation standard deviation for these parameters to make sure the parameter space is properly searched.
- (C) The consistently climbing likelihood is promising, but the failure of  $\sigma_{\nu}$  and  $H_0$  to converge needs attention. Additional searching is needed, experimenting with **smaller** values of the random walk perturbation standard deviation for these parameters to make sure the parameter space is properly searched.

## MIF2 convergence diagnostics

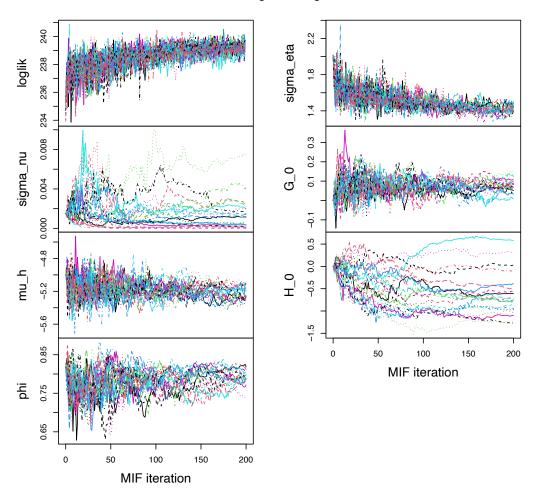


Figure 1: Iterated filtering convergence diagnotic plot for Question 13

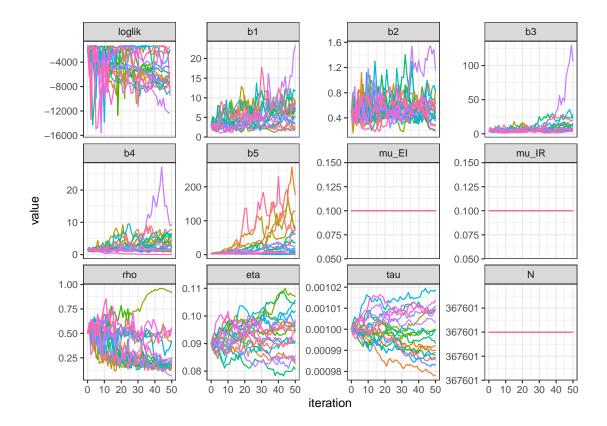


Figure 2: Iterated filtering convergence diagnotic plot for Question 14

- (D) The consistently climbing likelihood is promising, but the failure of  $\sigma_{\nu}$  and  $H_0$  to converge needs attention. This indicates weak identifiability which cannot be solved by improving the searching algorithm. Instead, we should change the model, or fix one or more parameters at scientifically plausible values, to resolve the identifiability issue before proceeding.
- (E) Although the log likelihood seems to be climbing during the search, until the convergence problems with  $\sigma_{\nu}$  and  $H_0$  have been addressed we should not be confident about the successful optimization of the likelihood function or the other parameter estimates.

Question 14. The iterated filtering convergence diagnostics plot in Fig. 2 comes from a student project, calculated using  $10^3$  particles. Which of the following is the best interpretation of this diagnostic plot?

- (A) Everything seems to be working fine. There is a clear consensus from the different searches concerning the highest likelihood that can be found. Therefore, the search is doing a good job of maximization. Occasional searches get lost, such as the purple line with a low likelihood, but that is not a problem.
- (B) The seaches obtain likelihood values spread over thousands of log units. We would like to see consistent convergence within a few log units. We should use more particles and/or more iterations to achieve this.
- (C) The seaches obtain likelihood values spread over thousands of log units. We would like to see consistent convergence within a few log units. We should compare the best likelihoods obtained with simple statistical models, such as an auto-regressive moving average model, to look for evidence of model misspecification.

- (D) The seaches obtain likelihood values spread over thousands of log units. We would like to see consistent convergence within a few log units. We should look at the effective sample size plot for the best fit we have found yet, to see whether there are problems with the particle filtering.
- (E) All of B, C and D.

Question 15. In the following call to mif2, which of the statements below are true? You may assume that obj is a pomp object with parameters alpha, Beta, gamma, and delta.

```
obj %>%
  mif2(
    Nmif=100,
    partrans=parameter_trans(log=c("Beta","alpha","delta")),
    paramnames=c("Beta","alpha","delta"),
    rw.sd=rw.sd(Beta=0.05,alpha=ivp(0.02),gamma=0.05),
    cooling.fraction.50=0.1
) -> obj
```

- (A) 50 IF2 iterations will be performed.
- (B) Beta and alpha are estimated on the log scale.
- (C) gamma is not estimated.
- (D) delta is not estimated.
- (E) The magnitude of the perturbation on Beta at the end of the run will be  $0.05 \times 0.1^{100} = 5 \times 10^{-102}$ .
- (F) The magnitude of the perturbation on gamma at the end of the run will be  $0.05 \times 0.1^{100/50} = 5 \times 10^{-4}$ .
- (G) alpha is an initial-value parameter; it will be perturbed only at the beginning of the time series.
- (H) After the call, obj is an object of class 'mif2d\_pomp'.

Question 16. Assume that obj is the result of the call in Question 15, we consider the result of the following call.

```
obj %>%
mif2(
   rw.sd=rw.sd(Beta=0.05,alpha=ivp(0.02)),
   cooling.fraction.50=0.2
)
```

Explain whether each of the following are true or false.

- (A) 100 more IF2 iterations will be performed.
- (B) The settings of the previous calculation are re-used, with the exception of rw.sd and cooling.fraction.50.
- (C) The starting point of the new calculation is the end point of the old one.
- (D) Beta and alpha are estimated on the log scale.
- (E) gamma is not estimated.
- (F) delta is not estimated.
- (G) The cooling occurs more quickly than in the previous call.