CSE 3504: Homework 6

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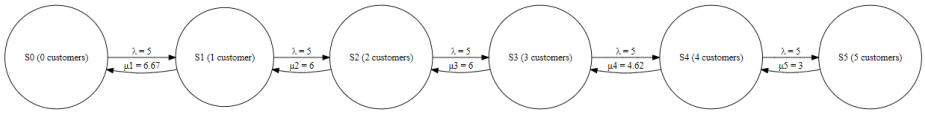
Problem 1:

1a.



1b.

$$\lambda P_0 = \mu_1 P_1$$

$$(\lambda + \mu_1) P_1 = \lambda P_0 + \mu_2 P_2$$

$$(\lambda + \mu_2) P_2 = \lambda P_1 + \mu_3 P_3$$

$$(\lambda + \mu_3) P_3 = \lambda P_2 + \mu_4 P_4$$

$$(\lambda + \mu_4) P_4 = \lambda P_3 + \mu_5 P_5$$

$$\mu_5 P_5 = \lambda P_4$$

$$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1$$

1c.

$$Q = \begin{bmatrix} -5 & 5 & 0 & 0 & 0 & 0 \\ 6.67 & -11.67 & 5 & 0 & 0 & 0 \\ 0 & 6 & -11 & 5 & 0 & 0 \\ 0 & 0 & 6 & -11 & 5 & 0 \\ 0 & 0 & 0 & 4.62 & -9.62 & 5 \\ 0 & 0 & 0 & 0 & 3 & -3 \end{bmatrix}$$

1d.

```
lambda <- 5
mu <- c(6.67, 6, 6, 4.62, 3)

P0 <- 1 / (1 + lambda/mu[1] + (lambda^2)/(mu[1]*mu[2]) + (lambda^3)/(mu[1]*mu[2]*mu[3]
           (lambda^4)/(mu[1]*mu[2]*mu[3]*mu[4]) + (lambda^5)/(mu[1]*mu[2]*mu[3]*mu[4]

P <- numeric(6)
P[1] <- P0
for (i in 2:6) {
  P[i] <- P[i-1] * lambda / mu[i-1]
}

P_idle <- P[1]
P_reject <- P[6]
L <- sum((0:5) * P)

cat("Probability the hairstylist is idle:", P_idle, "\n")
cat("Probability an incoming request is turned away:", P_reject, "\n")
cat("Average number of customers in the system:", L, "\n")
```

Results:

- $P_0 = 0.2274143$
- $P_5 = 0.2135385$
- $L = 2.389943$

Problem 2:

2a.

$$\rho = \frac{\lambda}{\mu} = \frac{20}{30} = 0.6667$$

$$\rho = 0.6667 \,(\text{or } 66.67\%)$$

2b.

$$L = \frac{\rho}{1-\rho} = \frac{0.6667}{1-0.6667} = 2$$

2c.

$$L_q = \rho \cdot L = 0.6667 \cdot 2 = 1.3334$$

2d.

$$W = \frac{L}{\lambda} = \frac{2}{20} = 0.1 \,\text{hours} = 6 \,\text{minutes}$$

2e.

$$W_q = \frac{L_q}{\lambda} = \frac{1.3334}{20} = 0.06667 \,\text{hours} = 4 \,\text{minutes}$$

2f.

$$P_{n \geq 5} = \rho^5 = 0.6667^5 \approx 0.1317$$

Problem 3:

3a.

$$L = \sum_{n=0}^{4} n \cdot p_n = (0 \cdot \frac{1}{6}) + (1 \cdot \frac{4}{16}) + (2 \cdot \frac{6}{16}) + (3 \cdot \frac{4}{16}) + (4 \cdot \frac{1}{16}) = 2$$

$$L = 2$$

3b.

$$\text{Customers served} = (1 \cdot \frac{4}{16}) + (2 \cdot \frac{6}{16}) + (2 \cdot \frac{4}{16}) + (2 \cdot \frac{1}{16}) = 1.625$$

$$L_q = L - \text{Customers served} = 2 - 1.625 = 0.375$$

$$L_q = 0.375$$

3c.

$$\text{Customers served} = 1.625$$

3d.

$$W = \frac{L}{\lambda} = \frac{2}{2} = 1 \,\text{hour}$$

$$W_q = \frac{L_q}{\lambda} = \frac{0.375}{2} = 0.1875 \,\text{hours} = 11.25 \,\text{minutes}$$

$$W = 1 \,\text{hour}$$

$$W_q = 11.25 \,\text{minutes}$$

3e.

$$\text{Service Time} = W - W_q = 1 - 0.1875 = 0.8125\,\text{hours} = 48.75\,\text{minutes}$$

$$\text{Service Time} = 48.75\,\text{minutes}$$