

CSE 3504: Project 2

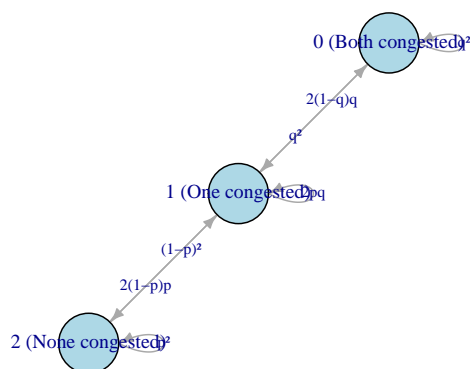
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Problem 1:

1a.

State Transition Diagram for DTMC



1b.

$$P = \begin{bmatrix} q^2 & 2(1-q)q & (1-q)^2 \\ q^2 & 2pq & (1-p)^2 \\ p^2 & 2(1-p)p & (1-p)^2 \end{bmatrix}$$

1c.

$$\pi P = \pi, \quad \text{where} \quad \sum_i \pi_i = 1$$

$$p = 0.5, q = 0.5$$

$$P = \begin{bmatrix} 0.25 & 0.5 & 0.25 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.5 & 0.25 \end{bmatrix}$$

R code:

```
P <- matrix(c(0.25, 0.5, 0.25,
              0.25, 0.5, 0.25,
              0.25, 0.5, 0.25),
            nrow = 3, byrow = TRUE)
```

```
n <- nrow(P)
A <- t(P) - diag(n)
A[n, ] <- 1
b <- c(rep(0, n-1), 1)
steady_state <- solve(A, b)
```

steady_state

Results:

$$\pi_0 = 0.25, \quad \pi_1 = 0.5, \quad \pi_2 = 0.25$$

Thus:

- π_0 : Probability that no route is congested is 0.25.
- π_1 : Probability that one route is congested is 0.5.
- π_2 : Probability that both routes are congested is 0.25.

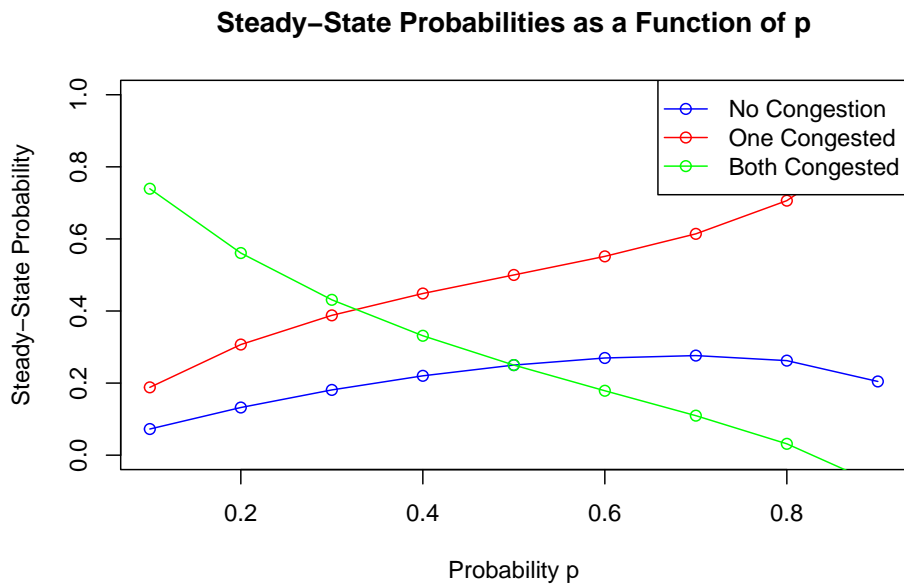
1d.

```
steady_state_probabilities <- function(p, q) {
  P <- matrix(c(q^2, 2*(1-q)*q, (1-q)^2,
                q^2, 2*p*q, (1-p)^2,
                p^2, 2*(1-p)*p, (1-p)^2),
            nrow = 3, byrow = TRUE)
  n <- nrow(P)
  A <- t(P) - diag(n)
  A[n, ] <- 1
  b <- c(rep(0, n-1), 1)
  steady_state <- solve(A, b)
  return(steady_state)
}
```

```
p_values <- seq(0.1, 0.9, by = 0.1)
q <- 0.5
results <- sapply(p_values, function(p) steady_state_probabilities(p, q))
```

```
plot(p_values, results[,1], type = "o", col = "blue", ylim = c(0, 1),
     xlab = "Probability p", ylab = "Steady-State Probability",
     main = "Steady-State Probabilities as a Function of p")
lines(p_values, results[,2], type = "o", col = "red")
lines(p_values, results[,3], type = "o", col = "green")
legend("topright", legend = c("No Congestion", "One Congested", "Both Congested"),
     col = c("blue", "red", "green"), lty = 1, pch = 1)
```

Resulting plot:



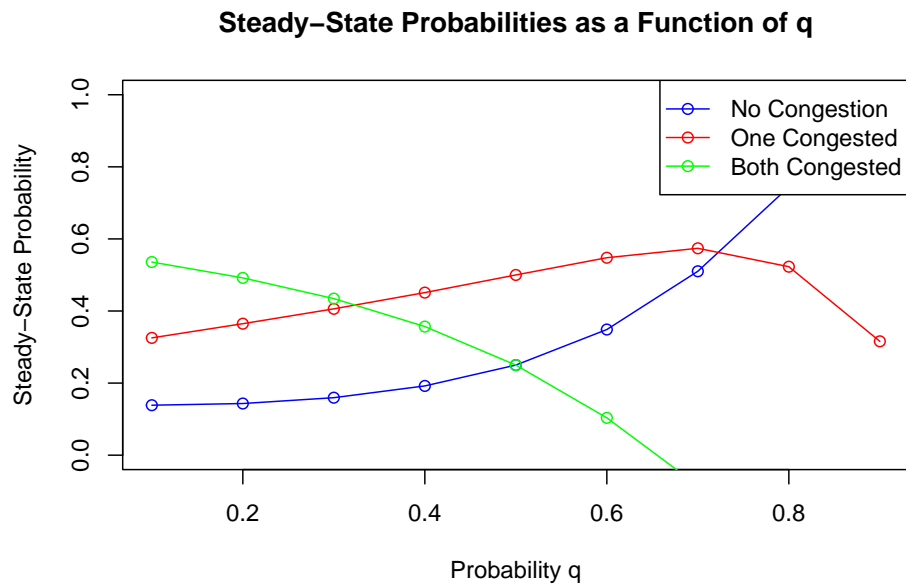
1e.

```
steady_state_probabilities <- function(p, q) {
  P <- matrix(c(q^2, 2*(1-q)*q, (1-q)^2,
               q^2, 2*p*q, (1-p)^2,
               p^2, 2*(1-p)*p, (1-p)^2),
             nrow = 3, byrow = TRUE)
  n <- nrow(P)
  A <- t(P) - diag(n)
  A[n, ] <- 1
  b <- c(rep(0, n-1), 1)
  steady_state <- solve(A, b)
  return(steady_state)
}

q_values <- seq(0.1, 0.9, by = 0.1)
p <- 0.5
results <- sapply(q_values, function(q) steady_state_probabilities(p, q))

plot(q_values, results[,1], type = "o", col = "blue", ylim = c(0, 1),
     xlab = "Probability q", ylab = "Steady-State Probability",
     main = "Steady-State Probabilities as a Function of q")
lines(q_values, results[,2], type = "o", col = "red")
lines(q_values, results[,3], type = "o", col = "green")
legend("topright", legend = c("No Congestion", "One Congested", "Both Congested"),
     col = c("blue", "red", "green"), lty = 1, pch = 1)
```

Resulting plot:



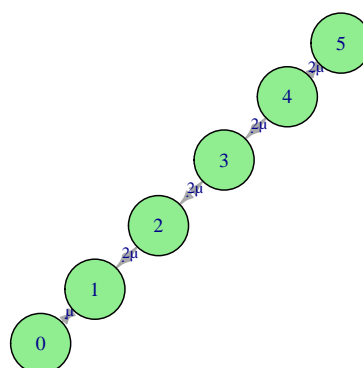
1f.

As p increases, the probability of congestion increases. When it decreases, both one and two congested routes decrease. This means that the higher p there is, there is an overall reduction in congestion. However, as q increases, the probability of both routes being congested increases, while the probabilities of no congestion and one congested route decreases. A higher q then means more congested states.

Problem 2:

2a.

State Transition Diagram for CTMC



2b.

$$Q = \begin{bmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 \\ \mu & -(\lambda + \mu) & \lambda & 0 & 0 & 0 \\ 0 & 2\mu & -(\lambda + 2\mu) & \lambda & 0 & 0 \\ 0 & 0 & 2\mu & -(\lambda + 2\mu) & \lambda & 0 \\ 0 & 0 & 0 & 2\mu & -(\lambda + 2\mu) & \lambda \\ 0 & 0 & 0 & 0 & 2\mu & -2\mu \end{bmatrix}$$

2c.

R code:

```
lambda <- 6
mu <- 4

Q <- matrix(c(
  -lambda, lambda, 0, 0, 0, 0,
  mu, -(lambda + mu), lambda, 0, 0, 0,
  0, 2*mu, -(lambda + 2*mu), lambda, 0, 0,
  0, 0, 2*mu, -(lambda + 2*mu), lambda, 0,
  0, 0, 0, 2*mu, -(lambda + 2*mu), lambda,
  0, 0, 0, 0, 2*mu, -2*mu
), nrow = 6, byrow = TRUE)

n <- nrow(Q)
A <- t(Q)
A[n, ] <- 1
b <- c(rep(0, n-1), 1) # Right-hand side
steady_state <- solve(A, b)

p_both_busy <- sum(steady_state[3:6])
p_turned_away <- steady_state[6]
```

The results are:

$$P(\text{both servers busy}) = 0.55, \quad P(\text{turned away}) = 0.085$$