CSE3504 Homework 1

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1 Problem 1:

1.1 Let S = 1, 2, ,,,, 100. Define E_2 as the event that a number is divisible by 2, and E_3 as event that the number is divisible by 3.

- The cardinality of event E_2 is $100/2 = 50$ and event E_3 is $100/3 = 33$.
- Even numbers divisible by 3 are also divisible by 2, thus half of all numbers divisible by 3 are also divisible by 2. This means the cardinality between the intersection of E_2 and E_3 is $100/3 * 1/2 = 100/6 = 16$.

2 Problem 2:

2.1 Two teams A and B play a soccer match, and we are interested in the winner. The sample space can be defined as: S = a, b, d where "a" shows the outcome that A wins, "b" shows the outcome that B wins, and "d" shows the outcome that they draw. Suppose that we know that the probability that A wins is P(a) = 0.5 and the probability of a draw is P(d) = 0.25.

- The probability that B wins is $P(b) = 1 - P(a) - P(d) = 1 - .5 - .25 = .25$.
- The probability that B wins or a draw occurs is $P(bord) = 1 - P(a) = .5$.

3 Problem 3:

3.1 Three factories make .20, .30, and .50 of the computer chips for a company. The probability of a defective chip is 0.04, 0.03, and 0.02 for the three factories.

- The probability that a chip is defective is $.2 * .04 + .3 * .03 + .5 * .02 = .027$.

- If a chip is defective, the chances it came from factory one is $(.04 * .2)/.027 = .296$.

4 Problem 4:

4.1 A password consists of six characters. These characters are chosen from the 10 digits and 26 letters of the alphabet. Passwords are also case sensitive.

- There are $62^6 = 56800235584$ different combinations of passwords (case sensitive).
- Using the non-replacement formula $N!/(N-k)!$ you get $62!/(62-56)! = factorial(62)/factorial(56) = 44261653680$.
- A hacker guessing 100 million passwords per second would take $62^6/10^8 = 568s$.
- To choose a password with a letter and a number you would have to first select from 52 letters. There are a total of 62^6 passwords with no constraints and 52^6 passwords with no digits (there are 10 total digits and 5 remaining characters). Thus, the number of valid passwords would be $52 * (62^5 - 52^5) = 27868297600$.
- $27868297600/100,000,000 = 278s$.

5 Problem 5:

5.1 A hash table contains slots, and a hash function assigns values to these slots using a hash function. A collision is said to occur if more than one value hashes into any particular slot.

- $P(nocollision) = 100/100 * 99/100 * 98/100 * 97/100 * 96/100 * 95/100 * 94/100*93/100*92/100*91/100 = .6281$ thus $P(collision) = 1-P(nocollision) = 1 - .6281 = .3719$ or in R $pbirthday(n = 10, classes = 100) = .3719$.
- Simply guess and check. $P(nocollision) = 100/100 * 99/100 * 98/100 * 97/100*96/100 = .902 \geq .9$ thus six values will drop the percentage below 90 percent. In R this would be $qbirthday(prob = 0.10, classes = 100) = 6$

6 Problem 6:

6.1 A family has n children, $n \geq 2$. We pick one of them at random and find out that she is a girl. What is the probability that all their children are girls, given at least one of them is a girl?

Given that at least one is a girl, and the minimum amount of children is two, the highest probability of all children being girls is when n is at its lowest (2). At n = 2, there is a 50 percent chance all children are girls as it is a simple coin flip for one child. At n = 3, this percentage drops by half to 25 percent as it is now two coin flips. This trend continues and can be moduled by the equation $chance = .5 * (1/2)^{n-1}, n \geq 2$.

7 Problem 7:

7.1 A manufacturing process produces integrated circuit chips. Over the long run the fraction of bad chips produced by the process is around .20. Thoroughly testing a chip to determine whether it is good or bad is rather expensive, so a cheap test is tried. All good chips will pass the cheap test, but so will .10, of the bad chips

- $P(Pass) = P(Pass|Good) * P(Good) + P(Pass|Bad) * P(Bad) = 1 * .8 + .1 * .2 = .82$. Now plugging into the formula $P(Good|Pass) = (P(Pass|Good) * P(Good))/P(Pass) = (1 * .8)/(.82) = .9756$.
- $1 - .9756 = .0244$ thus 2.44 percent.

8 Problem 8:

8.1 There are 20 black cell phones and 30 white cell phones in a store. An employee takes 10 phones at random. Find the probability that:

- To find the probability of selecting four black phones you use the Hypergeometric Distribution Formula $P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$. where $N = 50, K = 20, n = 10, k = k$. Plugging this values in you get $P(X = 4) = \frac{\binom{20}{4}\binom{30}{6}}{\binom{50}{10}}$ which can be calculated in R using the command $dhyper(4, 20, 30, 10) = .28$ or 28 percent.

- To find the probability of less than three black phones you use the same formula as above, however, you sum each result. In R this would be $sum(dhyper(0, 20, 30, 10), dhyper(1, 20, 30, 10), dhyper(2, 20, 30, 10)) = .139$ or 13.9 percent.