CSE 3504: Project 2

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Problem 1:

1a.

1. **System States:**
   - State 0: Both copier 1 and copier 2 are operational.
   - State 1: Copier 1 is operational, and copier 2 is down.
   - State 2: Copier 2 is operational, and copier 1 is down.
   - State 3: Both copiers are down, and the repairman is working on copier 1.
   - State 4: Both copiers are down, and the repairman is working on copier 2.
2. **Transition Dynamics:**
   - The system transitions between states due to:
     - Failures, governed by failure rates γ_1, γ_2.
     - Repairs, governed by repair rates β_1, β_2.
   - The repairman prioritizes copier 1 over copier 2 when both are down:
     - If copier 2 is being repaired and copier 1 fails, the repairman switches to copier 1.
3. **State Transition Diagram:**
   - From State 0:
     - Transition to State 1 at rate γ_2.
     - Transition to State 2 at rate γ_1.
   - From State 1:
     - Transition to State 0 at rate β_2.
     - Transition to State 3 at rate γ_1.
   - From State 2:
     - Transition to State 0 at rate β_1.
     - Transition to State 4 at rate γ_2.
   - From State 3:
     - Transition to State 1 at rate β_1.

- From State 4:
  - Transition to State 2 at rate β_2.

4. **Priority Rule:**
   - Repair work on copier 2 is interrupted if copier 1 fails, transitioning the system to State 3.

1b.

State 0: Both copiers are operational.
State 1: Copier 1 is operational, copier 2 is down.
State 2: Copier 2 is operational, copier 1 is down.
State 3: Both copiers are down, repairman is working on copier 1.
State 4: Both copiers are down, repairman is working on copier 2.

$$Q = \begin{bmatrix} -(\gamma_1+\gamma_2) & \gamma_2 & \gamma_1 & 0 & 0 \\ \beta_2 & -(\beta_2+\gamma_1) & 0 & \gamma_1 & 0 \\ \beta_1 & 0 & -(\beta_1+\gamma_2) & 0 & \gamma_2 \\ 0 & \beta_1 & 0 & -\beta_1 & 0 \\ 0 & 0 & \beta_2 & 0 & -\beta_2 \end{bmatrix}$$

1c.

$$\gamma_1 = 1, \quad \gamma_2 = 3, \quad \beta_1 = 2, \quad \beta_2 = 4$$

$$p = [p_0, p_1, p_2, p_3, p_4] = [0.3627, 0.3040, 0.0853, 0.1840, 0.0640]$$

- $p_0 = 0.3627$: Both copiers are operational 36.27% of the time.
- $p_1 = 0.3040$: Copier 1 is operational, and copier 2 is down 30.40% of the time.
- $p_2 = 0.0853$: Copier 2 is operational, and copier 1 is down 8.53% of the time.
- $p_3 = 0.1840$: Both copiers are down, and the repairman is working on copier 1, 18.40% of the time.
- $p_4 = 0.0640$: Both copiers are down, and the repairman is working on copier 2, 6.40% of the time.

Long-Run Proportions:

- **Copier 1 Operational:**

$$P(\text{Copier 1 Operational}) = p_0 + p_1 = 0.3627 + 0.3040 = 0.6667\,(66.67\%).$$

- **Copier 2 Operational:**

$$P(\text{Copier 2 Operational}) = p_0 + p_2 = 0.3627 + 0.0853 = 0.4480\,(44.80\%).$$

- **Repairman Busy:**

$$P(\text{Repairman Busy}) = p_3 + p_4 = 0.1840 + 0.0640 = 0.2480\,(24.80\%).$$

Problem 2:

Configuration (a):

$$R_s = R_m \times R_c$$
$$R_s = 0.9 \times 0.9 = 0.81$$

Configuration (b):

- Reliability of each subsystem:

$$R_{\text{sub}} = R_m \times R_c = 0.9 \times 0.9 = 0.81$$

- Reliability of the parallel system:

$$R_s = 1 - (1 - R_{\text{sub}})^2$$
$$R_s = 1 - (1 - 0.81)^2 = 1 - (0.19)^2 = 1 - 0.0361 = 0.9639$$

Conclusion

Configuration (b) is better because its reliability (0.9639) is significantly higher than that of configuration (a) (0.81). This improvement is due to the redundancy provided by the parallel subsystems, which ensures that the system functions as long as at least one subsystem is operational.