CSE3504 Homework

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Problem 1

(a)

$$\mu_X = \sum_{x=0}^{2} x \cdot p(X) = 2.8$$
$$\text{Var}(X) = \sum_{x=0}^{2} x^2 p(X) - \mu_X^2 = 1.66$$
$$\sigma_X = \sqrt{\text{Var}(X)} = 1.288$$

(b)

$$\mu_Y = \sum_{y=0}^{2} y \cdot p(Y) = 1.7$$
$$\text{Var}(Y) = \sum_{y=0}^{2} y^2 \cdot p(Y) - \mu_Y^2 = 0.61$$
$$\sigma_Y = \sqrt{\text{Var}(Y)} = 0.781$$

(c)

$$\mu_C = 3 \cdot \mu_X = 8.4$$
$$\text{Var}(C) = 3^2 \cdot \text{Var}(X) = 14.94$$
$$\sigma_C = \sqrt{\text{Var}(C)} = 3.865$$

(d)

$$\mu_B = 10 \cdot \mu_Y = 7$$
$$\text{Var}(B) = 10^2 \cdot \text{Var}(Y) = 61$$
$$\sigma_B = \sqrt{\text{Var}(B)} = 7.81$$

(e)

$$\begin{aligned}\mu_Z &= \mu_X + \mu_Y = 3.5\\ \mathrm{Var}(Z) &= \mathrm{Var}(X) + \mathrm{Var}(Y) = 2.27\\ \sigma_Z &= \sqrt{\mathrm{Var}(Z)} = 1.507\end{aligned}$$

(f)

$$\begin{aligned}\mu_W &= \mu_C + \mu_B = 15.4\\ \mathrm{Var}(W) &= \mathrm{Var}(C) + \mathrm{Var}(B) = 75.94\\ \sigma_W &= \sqrt{\mathrm{Var}(W)} = 8.714\end{aligned}$$

(g)

$$\begin{aligned}\mu_D &= \mu_X - \mu_Y = 2.1\\ \mathrm{Var}(D) &= \mathrm{Var}(X) + \mathrm{Var}(Y) = 2.27\\ \sigma_D &= \sqrt{\mathrm{Var}(D)} = 1.507\end{aligned}$$

Problem 2

(a)

$$\mu_X = \int_1^\infty x \cdot \frac{3}{x^4}\,dx = \int_1^\infty 3x^{-3}\,dx = \frac{3}{2}$$

(b)

$$\int_1^\infty x^2 \cdot \frac{3}{x^4}\,dx = \int_1^\infty 3x^{-2}\,dx = 3$$

(c)

$$\mathrm{Var}(X) = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$$

Problem 3

(a) Use the `pbinom` function in R.

(b)

$$\begin{aligned}\mu &= 400 \cdot 0.2 = 80\\ \sigma &= \sqrt{400 \cdot 0.2 \cdot 0.8} = 8\end{aligned}$$

(c)

$$z = \frac{74.5 - 80}{8} = -0.6875$$
$$z = \frac{100.5 - 80}{8} = 2.5625$$

Problem 4

(a)

$$\begin{aligned} Z &= P_{\text{hole}} - P_{\text{peg}} \\ \mu_Z &= 0.253 - 0.25 = 0.003 \\ \sigma_Z &= \sqrt{0.002^2 + 0.006^2} = 0.00632 \\ P_{\text{fit}} &= 0.6825 \end{aligned}$$

(b)

$$\begin{aligned} \mu_Z &= 0.25553 - 0.25 = 0.00553 \\ \sigma_Z &= 0.00632 \\ P_{\text{fit}} &= 0.8092 \end{aligned}$$

Problem 5

$$\sum_{k=2}^{\infty} \frac{1}{2^k}(1-p)^{k-1}p = p\sum_{k=1}^{\infty} \frac{(1-p)^{k-1}}{2^k}$$

Problem 6

$$\begin{aligned} 1 &= -2\,E[X] + 3 \\ -2 &= -2\,E[X] \\ E[X] &= 1 \\ \text{Var}(X) &= E[X^2] - E[X]^2 = 2 \end{aligned}$$

Problem 7

$$
\begin{aligned}
E[T] &= 12 \cdot 62.5 = 750 \text{ milliseconds} \\
\text{Var}(T) &= 12 \cdot 5^2 = 300 \\
\sigma &= \sqrt{300} = 17.32 \text{ milliseconds} \\
z &= \frac{750 - 780}{17.32} = -1.73 \\
z &= \frac{825 - 750}{17.32} = 2.60 \\
P_{ttc} &= 0.9537
\end{aligned}
$$

Problem 8

(a)

$$p(X = 2) = \frac{1}{2}, \quad p(X = 3) = \frac{1}{3}, \quad p(X = 5) = \frac{1}{3}, \quad p(X = 8) = \frac{1}{2}$$

(b)

$$E[X] = 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{3} + 5 \cdot \frac{1}{3} + 8 \cdot \frac{1}{2} = 4.14$$

(c)

$$E[X^2] = 2^2 \cdot \frac{1}{2} + 3^2 \cdot \frac{1}{3} + 5^2 \cdot \frac{1}{3} + 8^2 \cdot \frac{1}{2} = 20.71$$

(d)

$$\text{Var}(X) = 20.71 - (4.14)^2 = 3.55$$