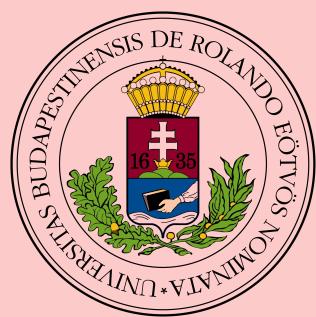
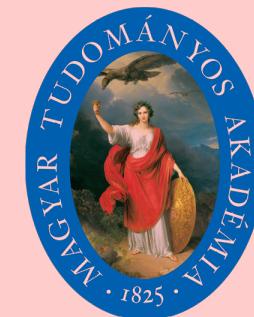


Pulsar Timing: An Overview



Tim Pennucci
Eötvös Loránd University
Budapest, Hungary

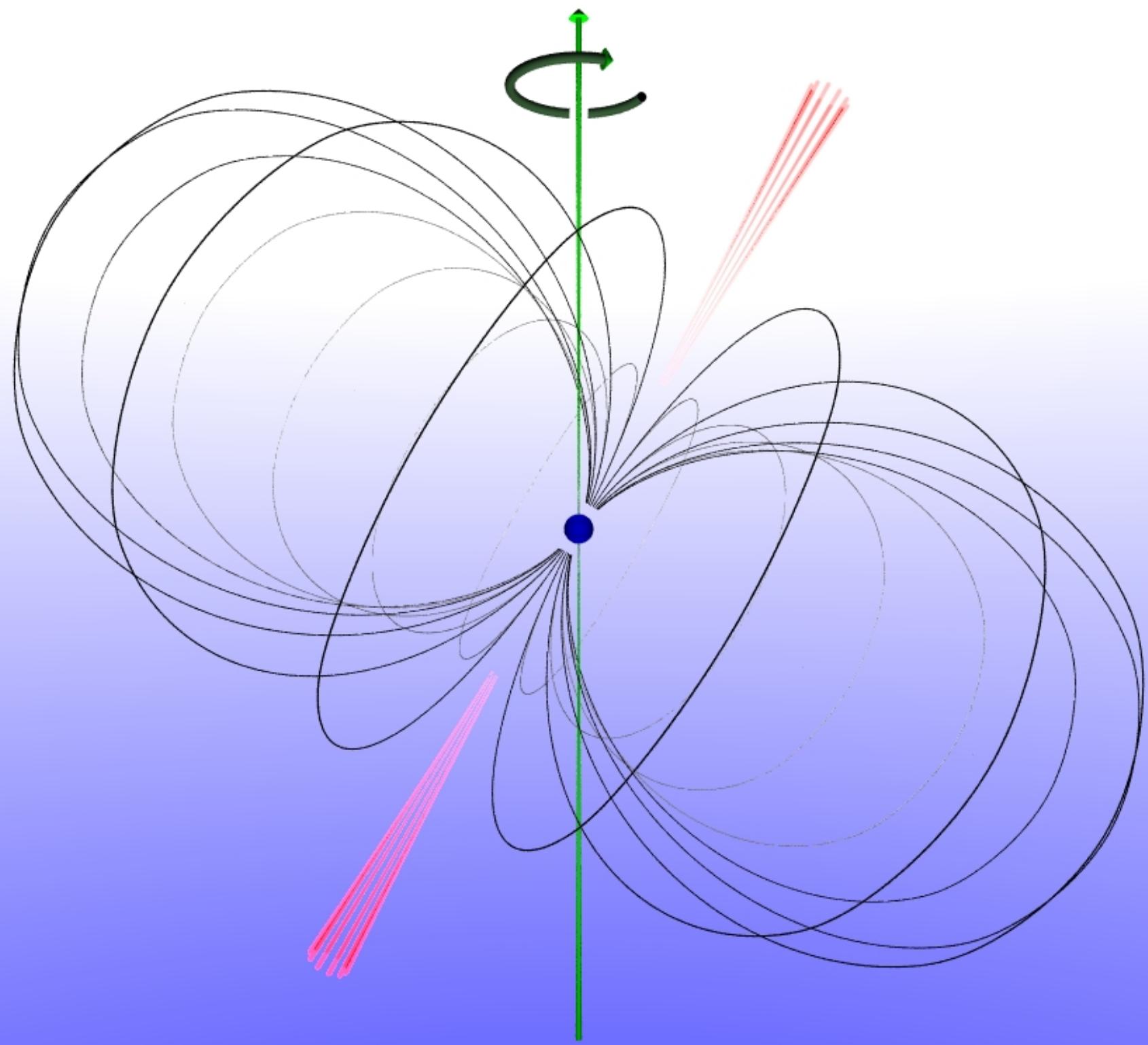


Resources:

- Many graphics/ideas/slides borrowed from David Nice
- Lorimer & Kramer, *Handbook of Pulsar Astronomy*
- Lyne & Graham-Smith, *Pulsar Astronomy*
- NANOGrav Glossary:
http://nanograv.org/assets/files/NANOGrav_Glossary.pdf
- References cited within

Questions to be addressed:

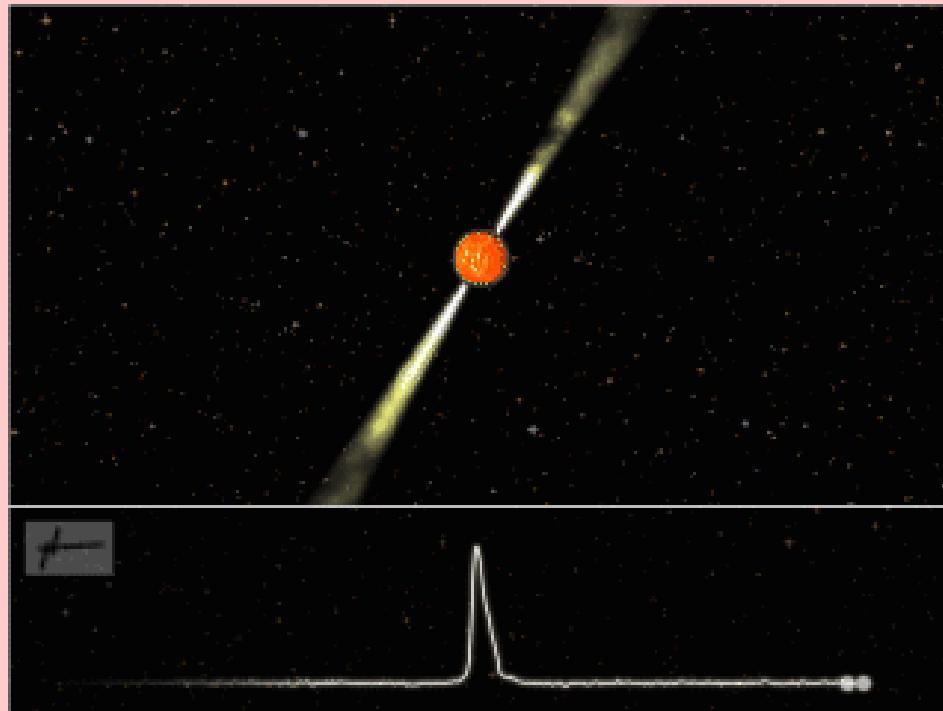
- What is pulsar timing?
- How are TOAs measured?
- What makes up a timing model?
- Super briefly: What are the pulsar timing “use cases”?



The pulsar “zoo”:

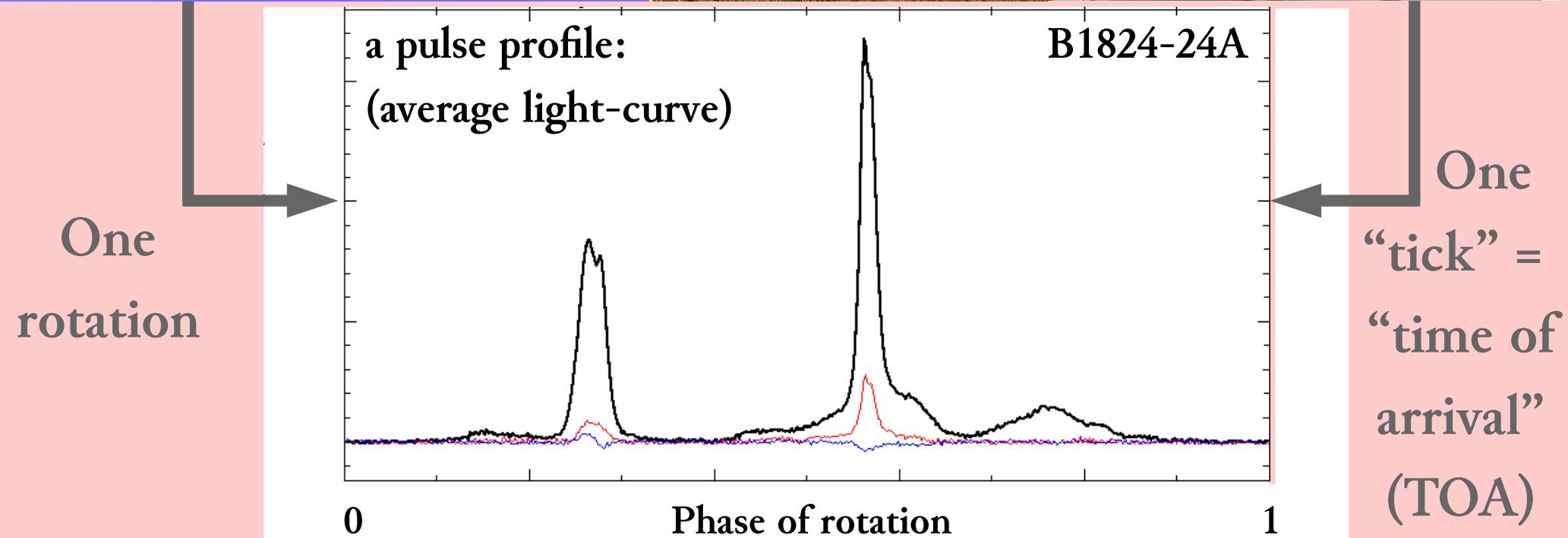
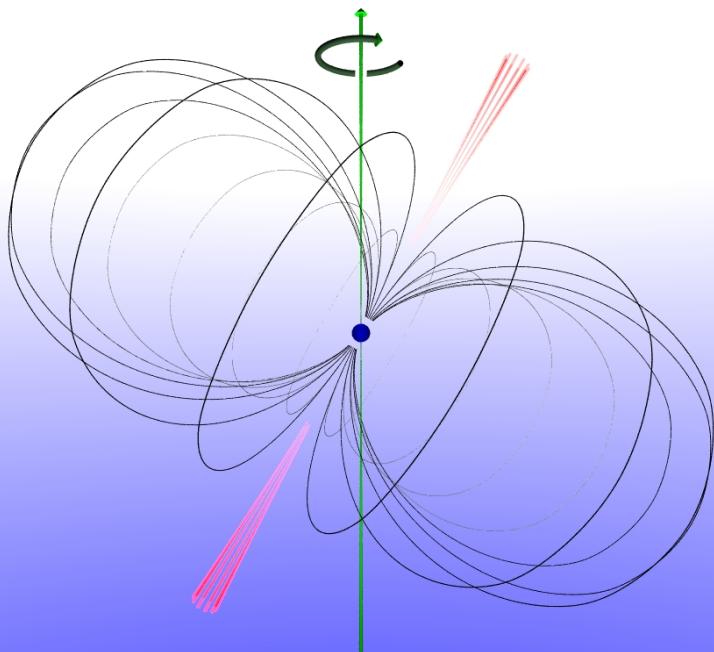
- “Classic” radio pulsar (“slow” pulsar, “canonical” pulsar)
- Millisecond pulsar (fully “recycled” pulsar)
- Young pulsar (e.g., SNR associations, Crab, Vela)
- Magnetar
- Black widow
- Redback
- Double neutron star
- X-ray pulsar
- AMXP
- XDIN
- Gamma-ray pulsar
- “gravitational-wave” pulsar?
- ...

The “lighthouse” model:



[Figure: J. vLeeuwen]

Pulsars are clocks!



Pulsars are clocks!

DEFINITION:

pulsar timing – *n.* the unambiguous accounting of each and every rotation of a neutron star

Example – PSR J1713+0747 over 21 years (Zhu et al., 2015):

- First Observed Pulse: 15 Aug. 1992 00:35:00.500421805 ~160ns
- Last Observed Pulse: 23 Nov. 2013 22:07:41.189191195 ~140ns

Pulsar underwent exactly 146,495,457,989 rotations over this time.
~219 Hz ~ 4.6 ms / rotation

Two fundamental *observables* for all pulsars:

- Spin Period (P)
- Spin Period derivative (“P-dot”)

Pulsars are clocks!

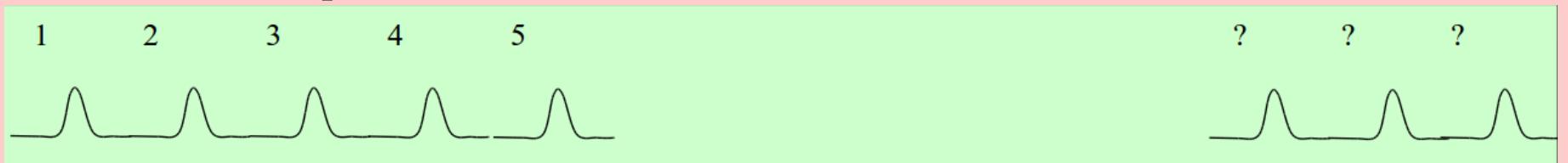
DEFINITION:

pulsar timing – *n.* the unambiguous accounting of each and every rotation of a neutron star

(1) Observe some pulses:



(2) Build a timing model to account for observed pulses:



(3) Predict arrival of later pulses to see if you have maintained “phase connection”:

(4) Repeat & improve timing solution!

But what is meant by “phase-connection”?

Real-life/anecdotal example with a timescale of days:

- You go on vacation to some remote place far from any timekeeping devices and arrive on a Monday (reference epoch).
- Surely, after a few days, your mind will still know the day of the week: Tuesday, then Wednesday, then Thursday, ... then Monday, ...
- But after some amount of time or days (rotations), will you know which day of the week it is? If you're off by a day, you've “lost phase connection”, you've missed a rotation.

But what is meant by “phase-connection”?

Real-life/anecdotal examples with timescales of hours:

- Your anxious mind wakes up a few minutes before, or, if your alarm fails to go off, wakes up a few minutes after your alarm clock is supposed to.
- You get hungry at the same times every day.
- Circadian sleep cycles can also be thought of in this sense (as modeling the daily day/night cycle).
- It follows that jet-lag is the loss of phase-connection!

But what is meant by “phase-connection”?

Real-life/anecdotal examples with timescales of seconds:

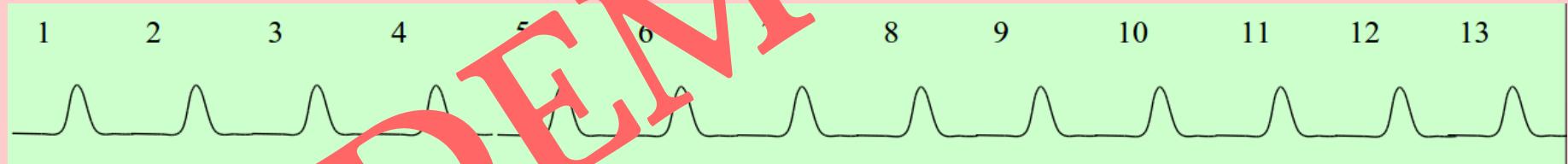
- You're on the dance floor, in step with the rhythm of the music, and, suddenly, the music cuts out.
 - Obviously, you must keep dancing.
- When the music returns some seconds later, are you still in step with the music? And if you are, are you *sure* you haven't missed a beat?

Pulsars are clocks!

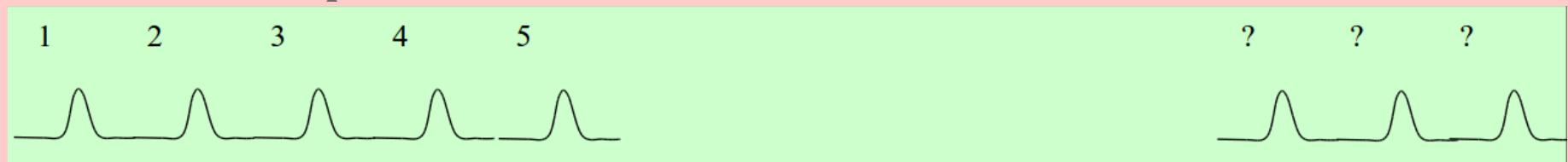
DEFINITION:

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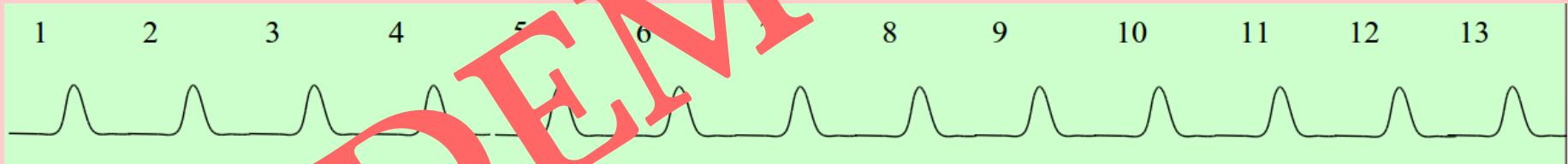
Pulsars are clocks!

DEFINITION:

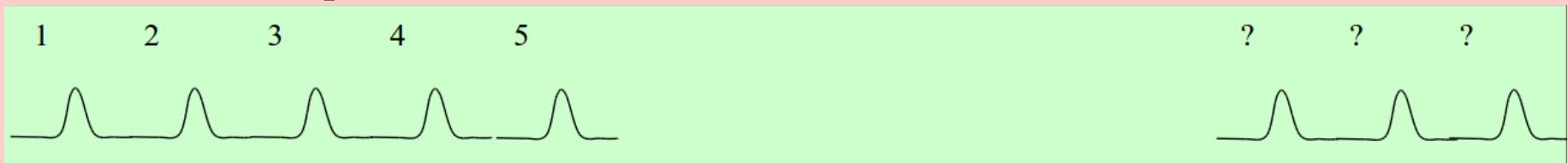
pulsar timing – *n.* the unambiguous accounting of each and every rotation of a neutron star

#2!

(1) Observe some pulses:

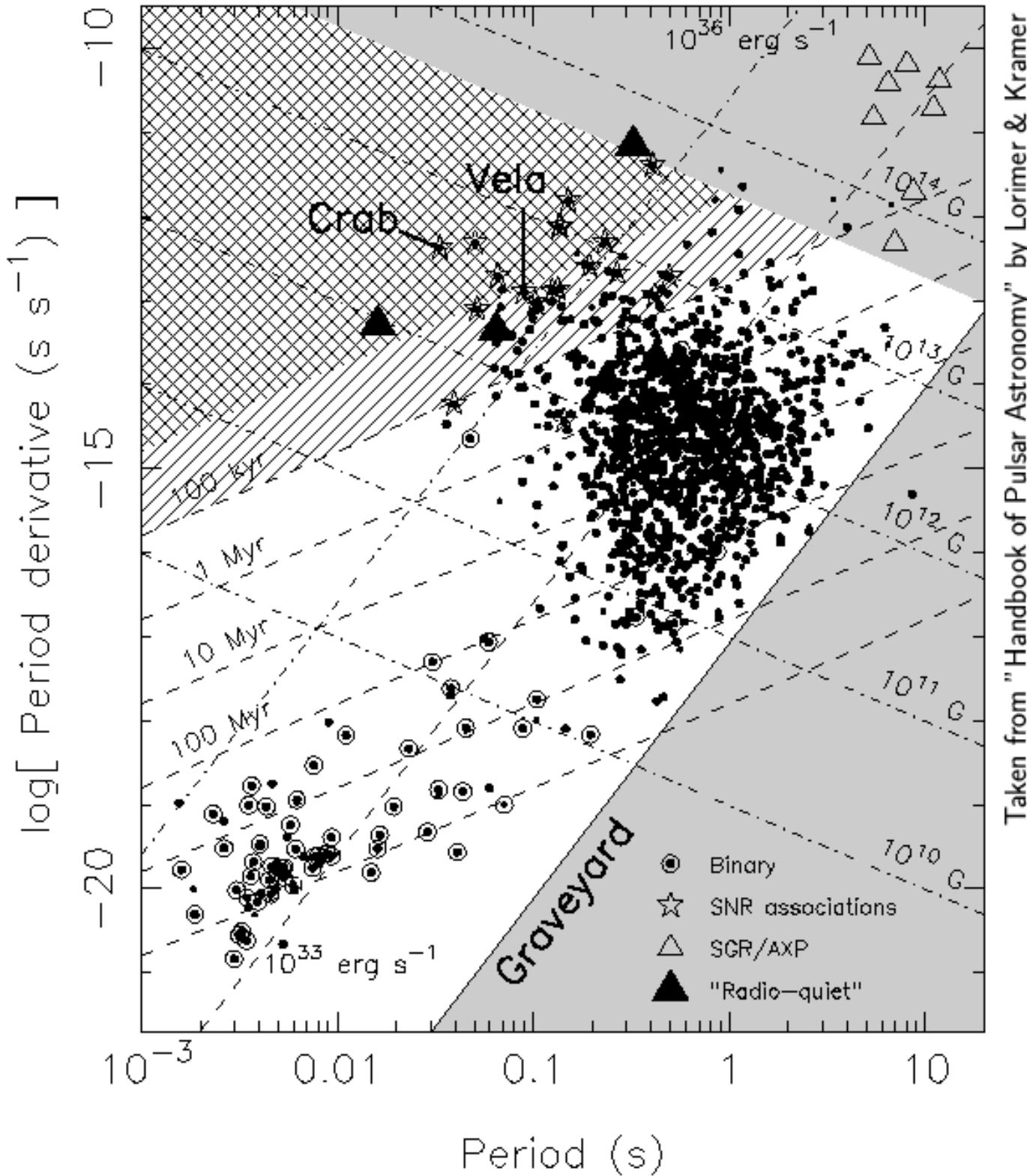


(2) Build a timing model to account for observed pulses:



(3) Predict arrival of later pulses to see if you have maintained “phase connection”:

(4) Repeat & improve timing solution!



→ >2500 known; all but a handful in the Milky Way Galaxy

→ Two fundamental observables:

- (1) spin period
- (2) period derivative

→ Basic assumption:

- (1) Magnetic dipole braking

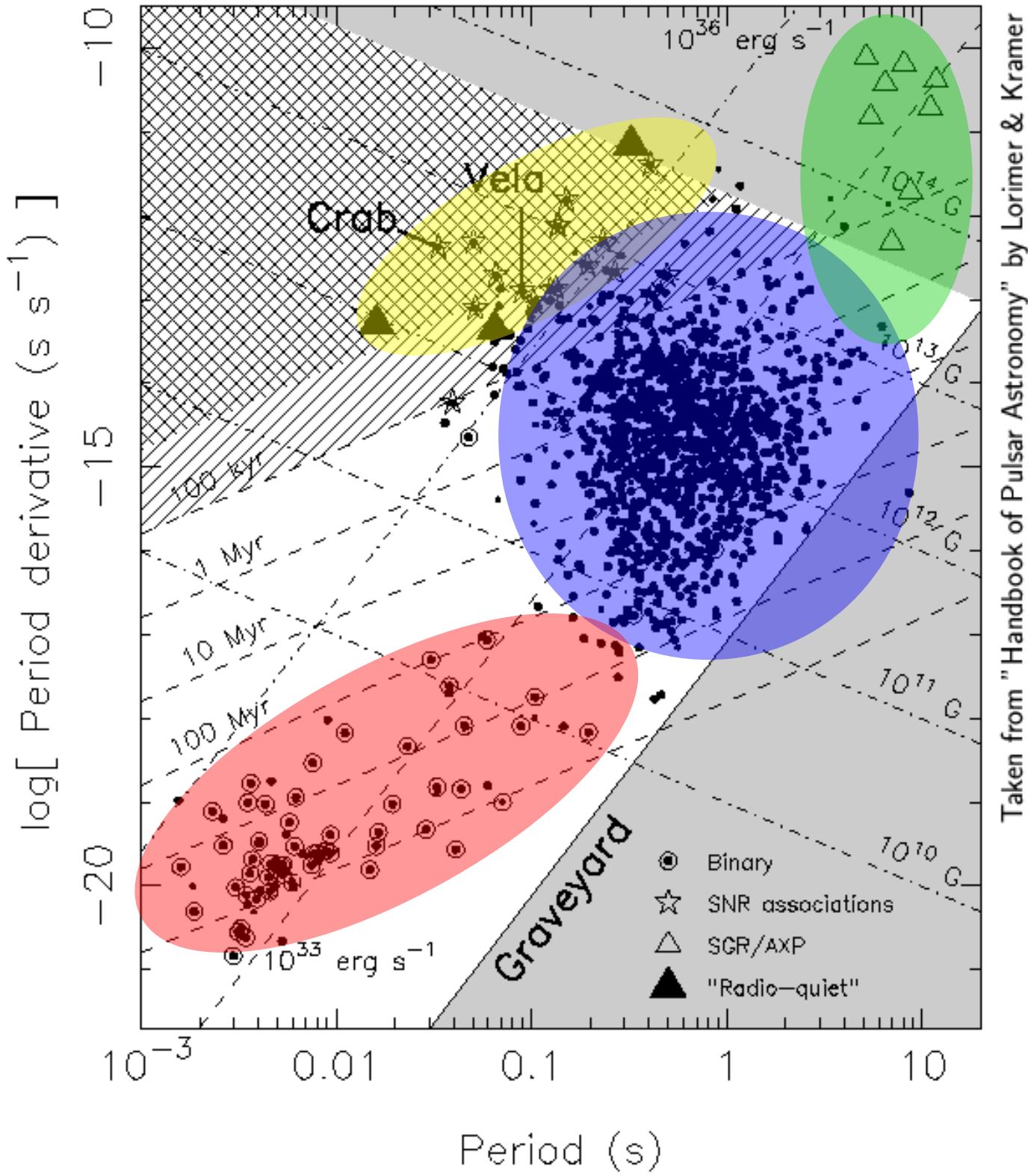
→ Derived parameters:

- (1) Spin down power
- (2) Magnetic field strength
- (3) Characteristic age

Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Broad
categories of pulsars:

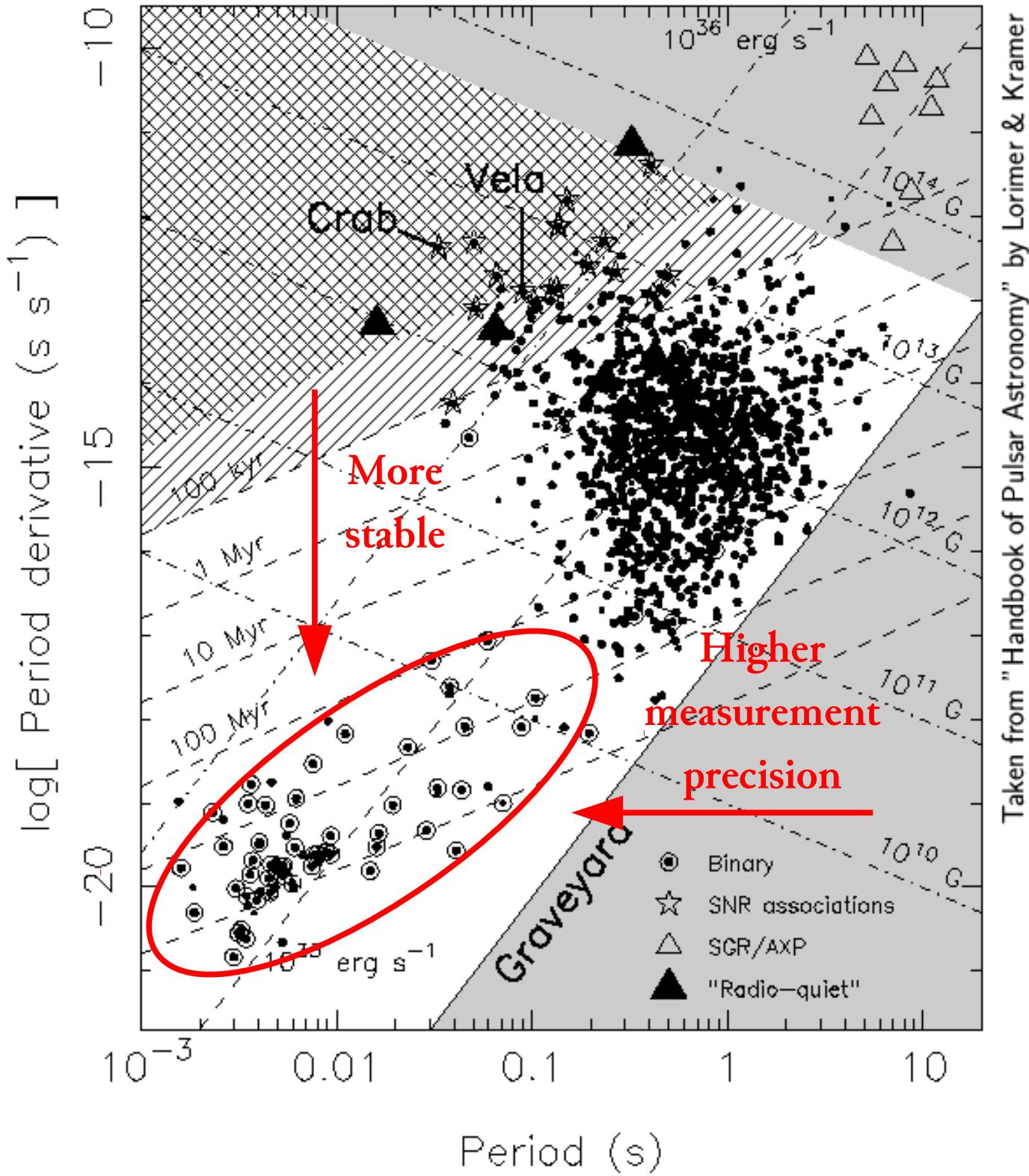
- Canonical
- Millisecond
- Young
- Magnetars



Why are MSPs the best clocks?

→ They are the fastest spinning pulsars, giving the highest measurement precision.

→ Slower pulsars with higher magnetic fields are less stable clocks.

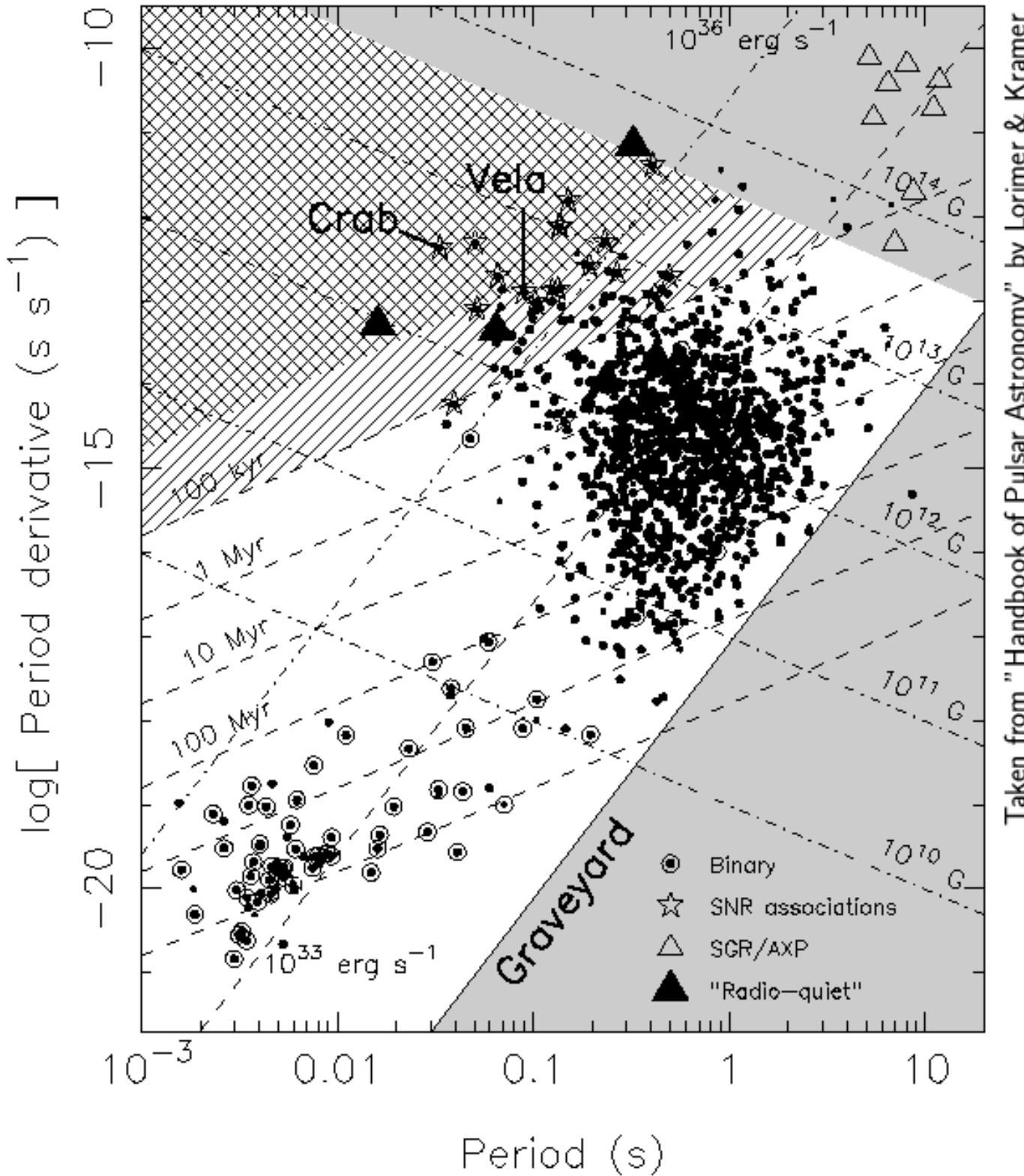


Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

POP QUIZ 1

What is this “graveyard”?

- A) A dearth of observed pulsars due to observational constraints (“selection bias”)
- B) A real deficit in the pulsar population
- C) The place where large main sequence stars end up after they die in a supernova explosion
- D) The place where neutron stars die once degeneracy pressure fails

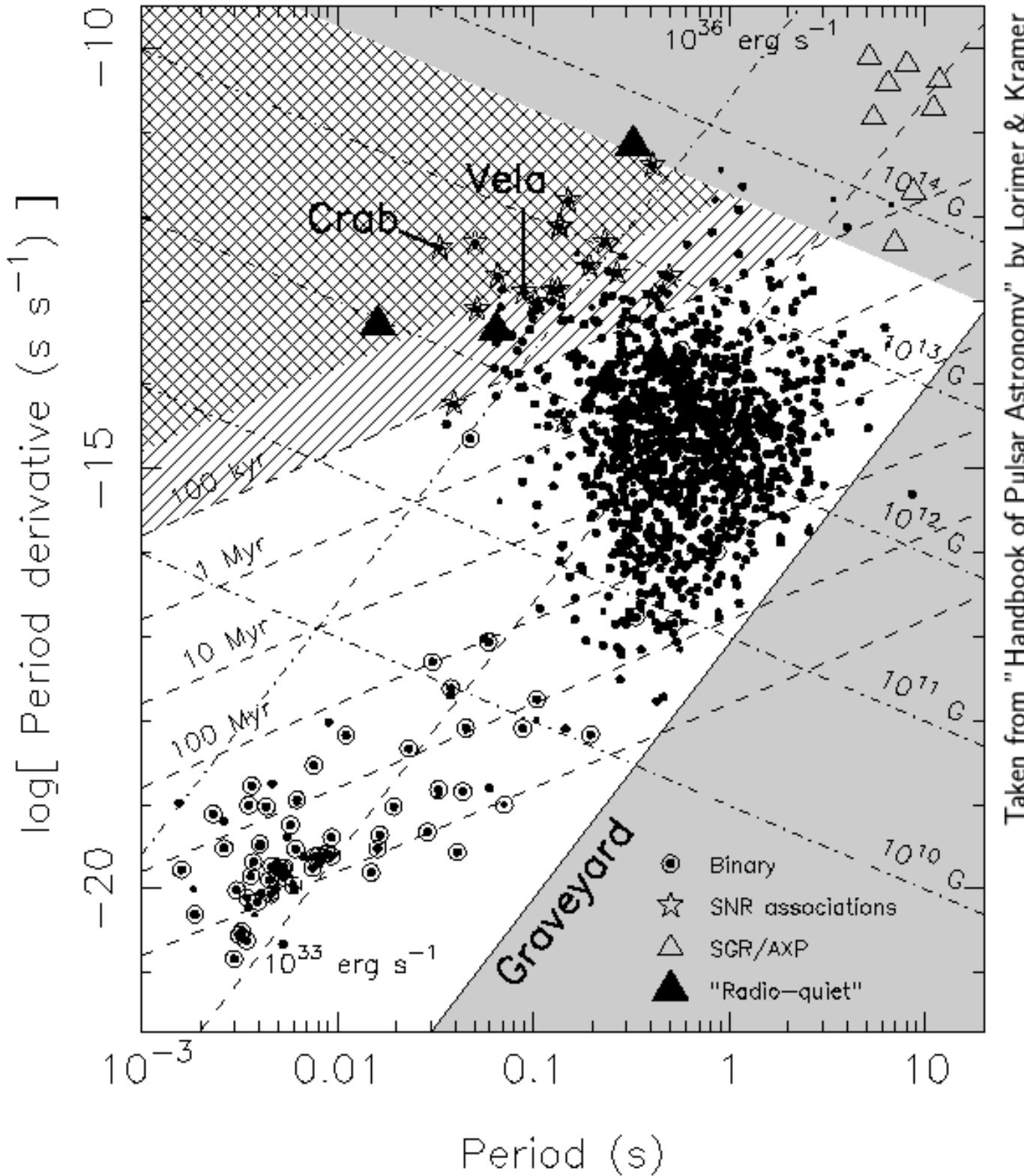


Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

POP QUIZ 1

What is this “graveyard”?

- A) A dearth of observed pulsars due to observational constraints (“selection bias”)
- B) A real deficit in the *radio* pulsar population
- C) The place where large main sequence stars end up after they die in a supernova explosion
- D) The place where neutron stars die once degeneracy pressure fails



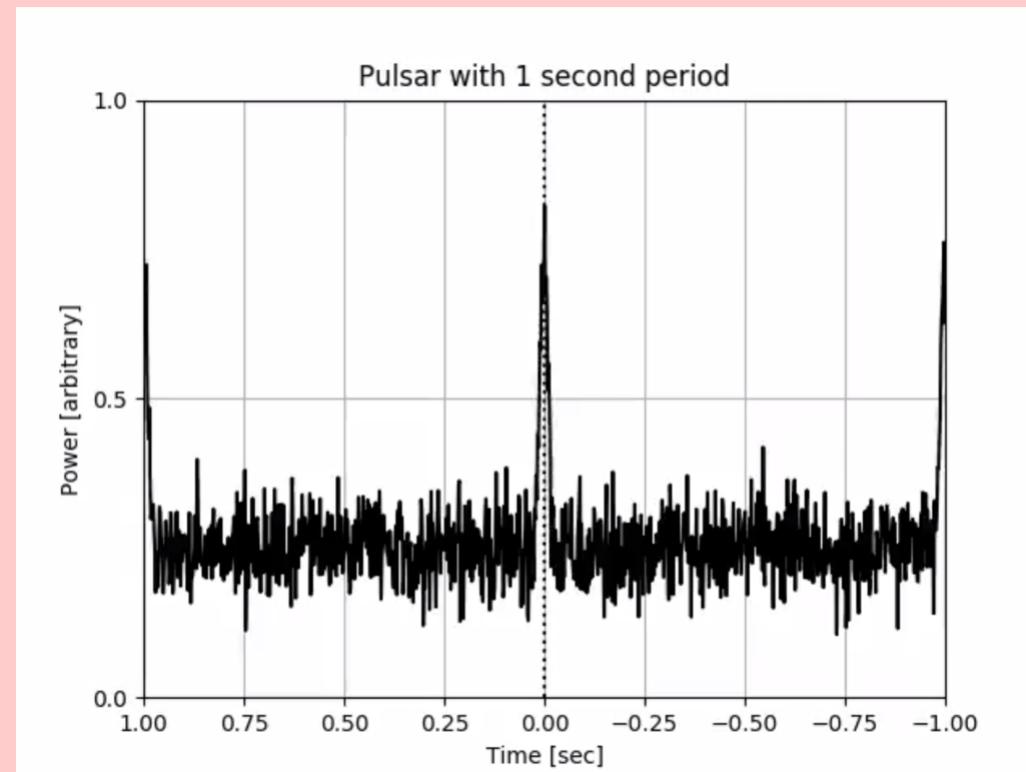
Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Time-of-arrival (TOA) Measurements

(1) Observe some pulses:

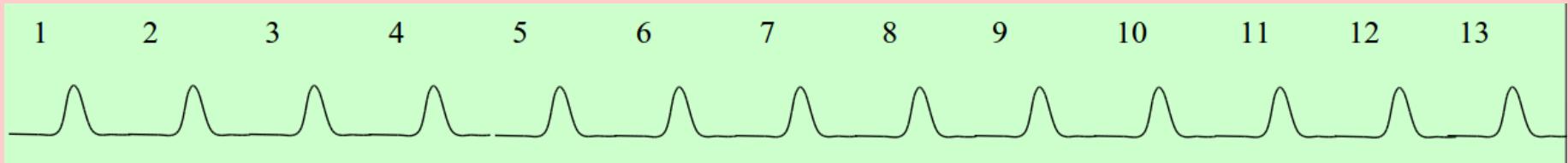


Pulse train movie:

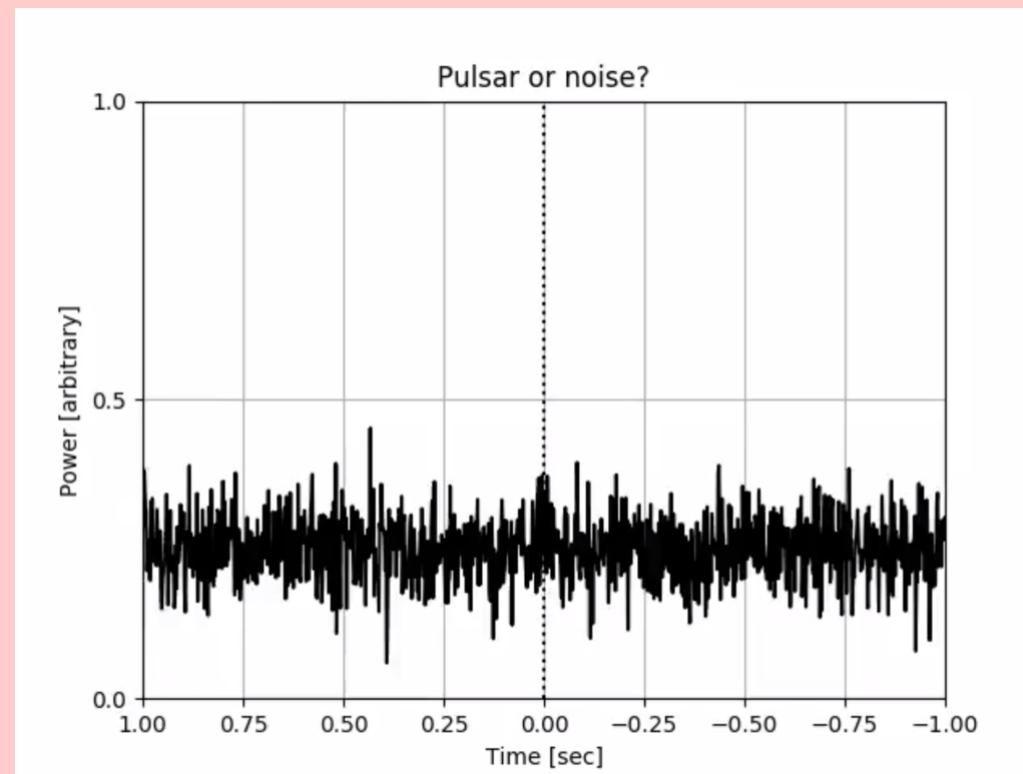


Time-of-arrival (TOA) Measurements

(1) Observe some pulses:

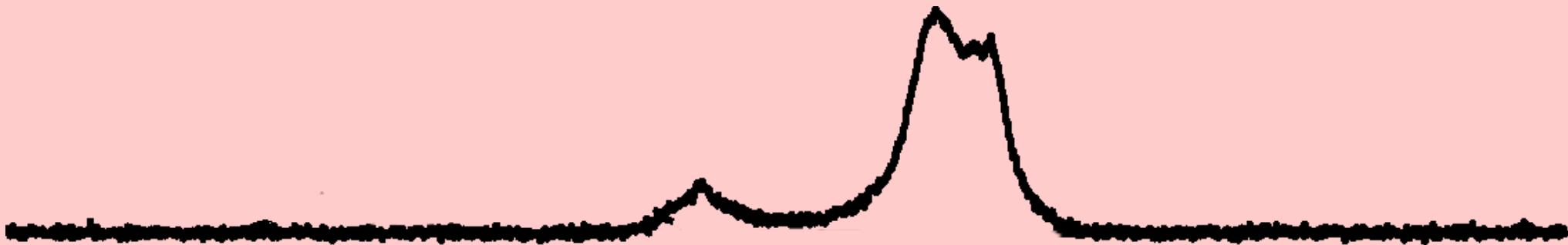


Pulse train movie:



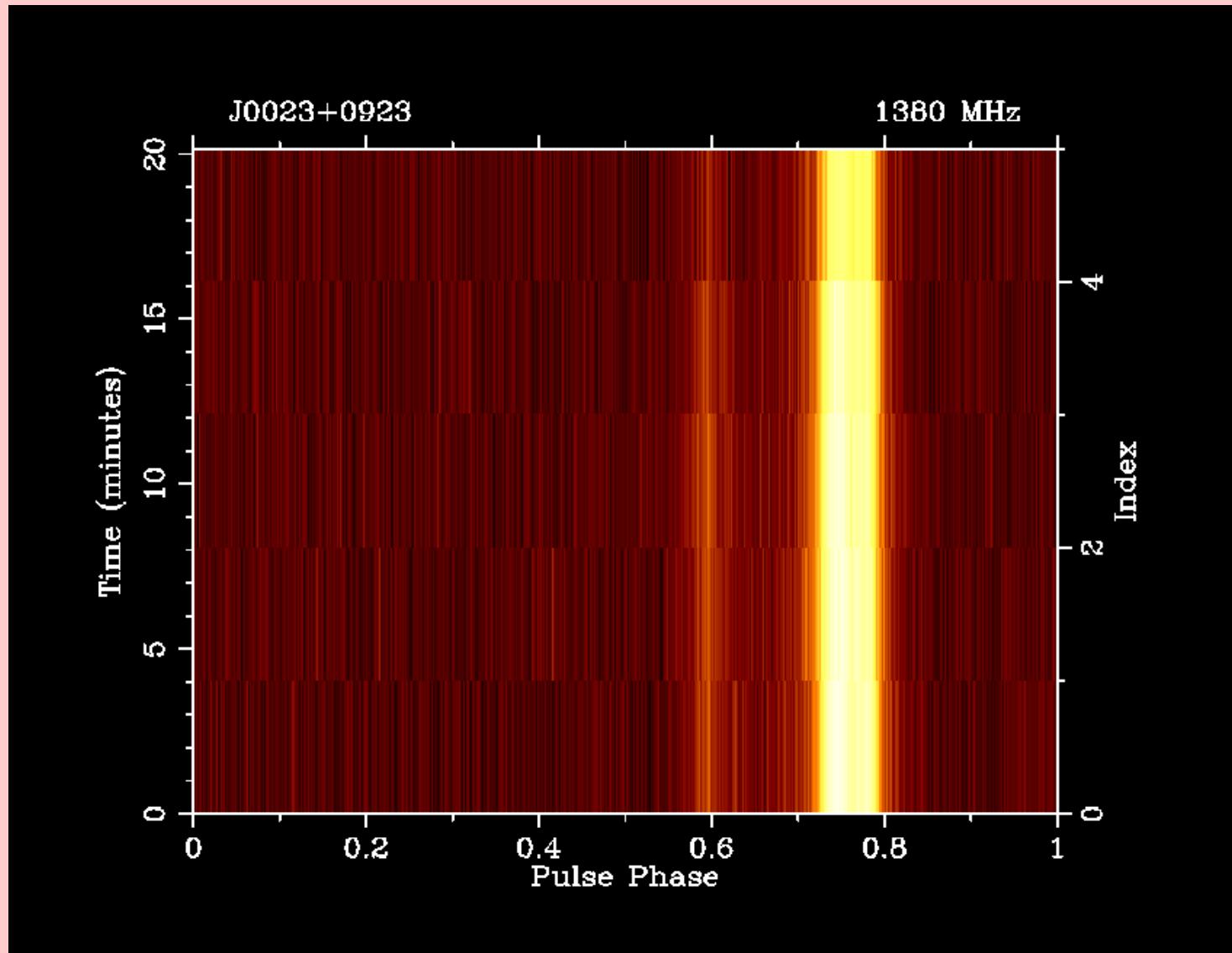
Time-of-arrival (TOA) Measurements

- Usually we measure TOAs from *folded* time-series data
- This means we take that very noisy time-series, with no apparent signal, slice it up *modulo* the pulse period, and average the pieces together to get a high signal-to-noise ratio (S/N) pulse profile:



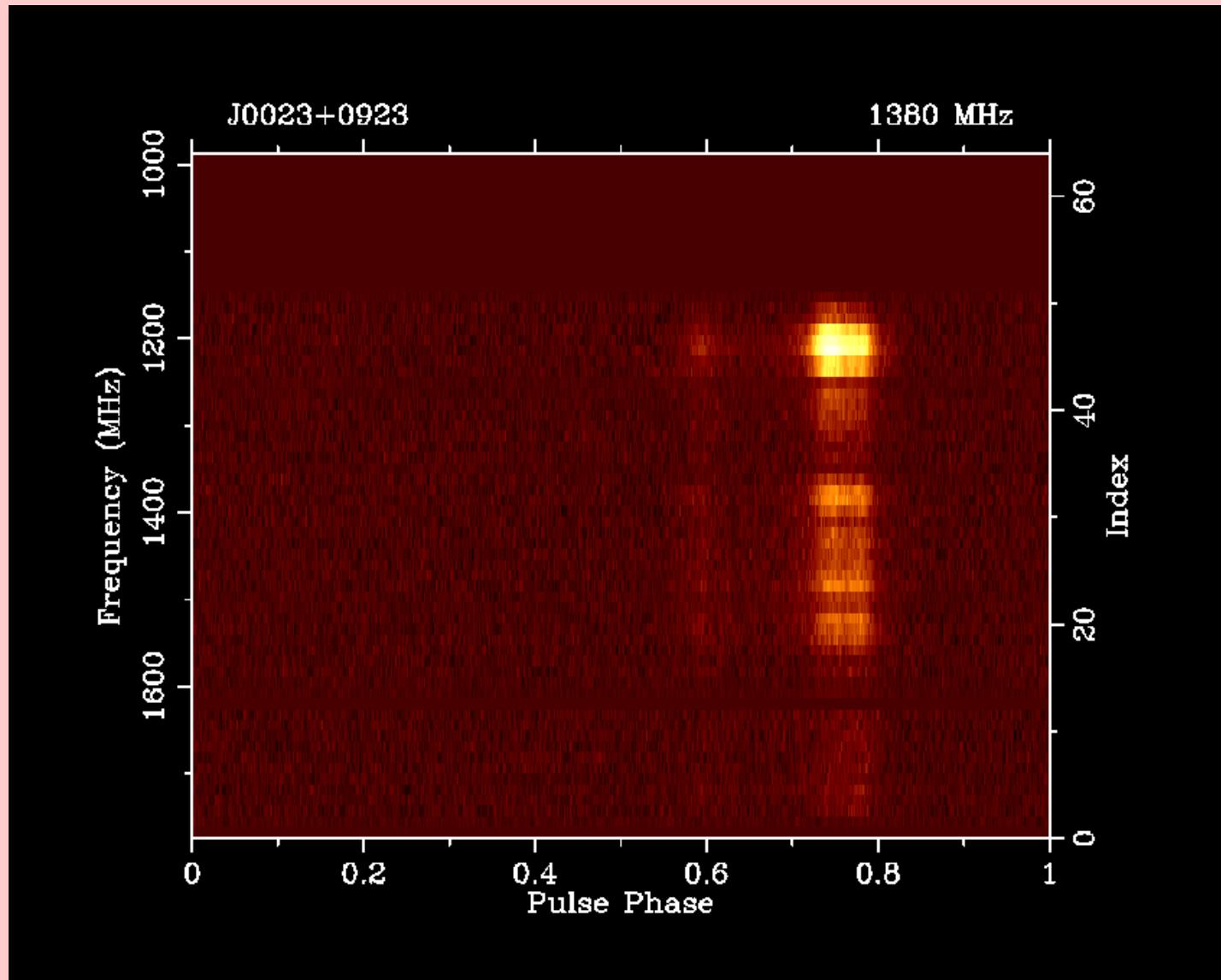
Time-of-arrival (TOA) Measurements

- We can form these folded profiles as a function of *time*, folding and averaging up all of the data every, e.g., 4 min:



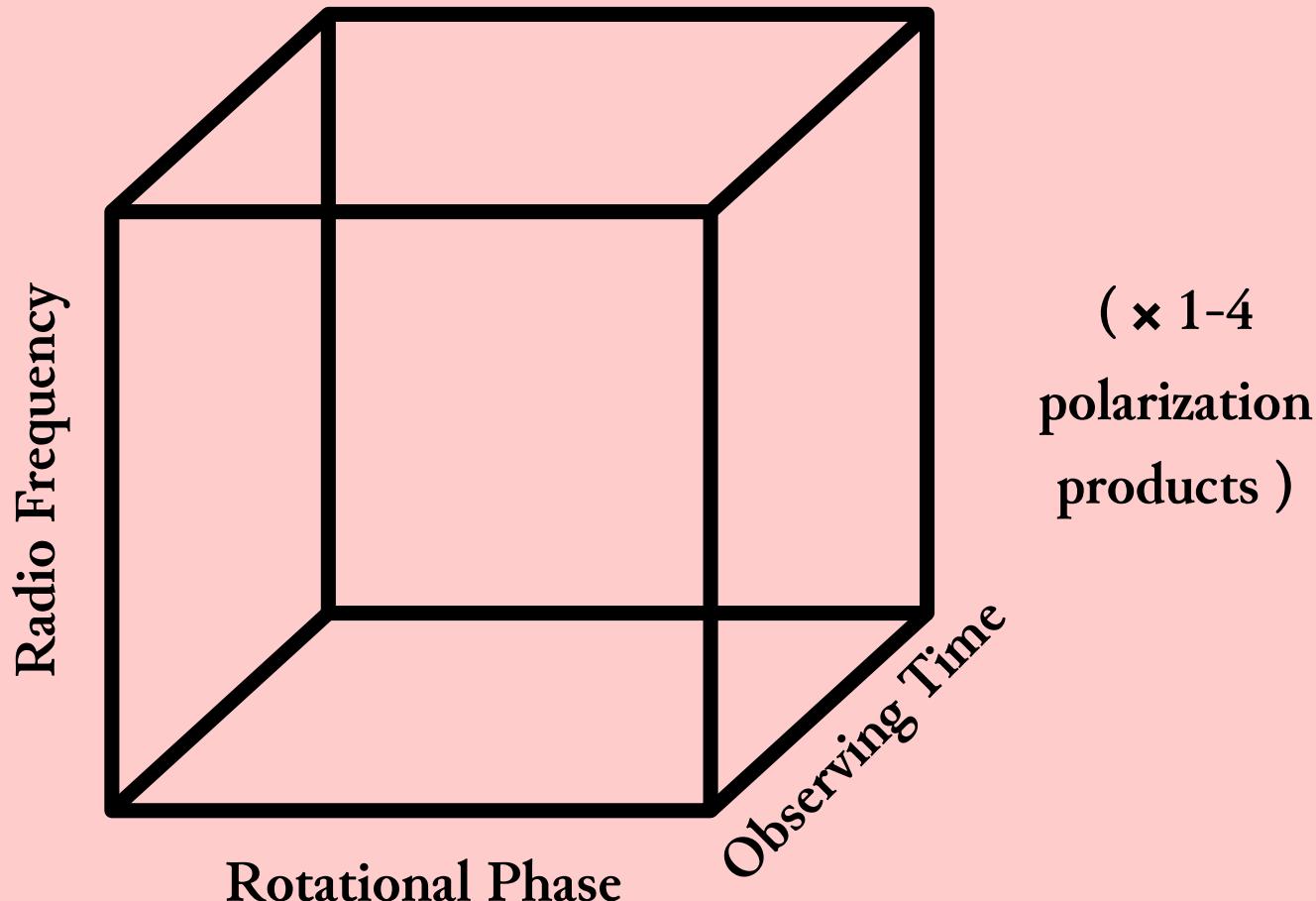
Time-of-arrival (TOA) Measurements

→ We can form these folded profiles as a function of frequency, since we observe across large bandwidths:



Time-of-arrival (TOA) Measurements

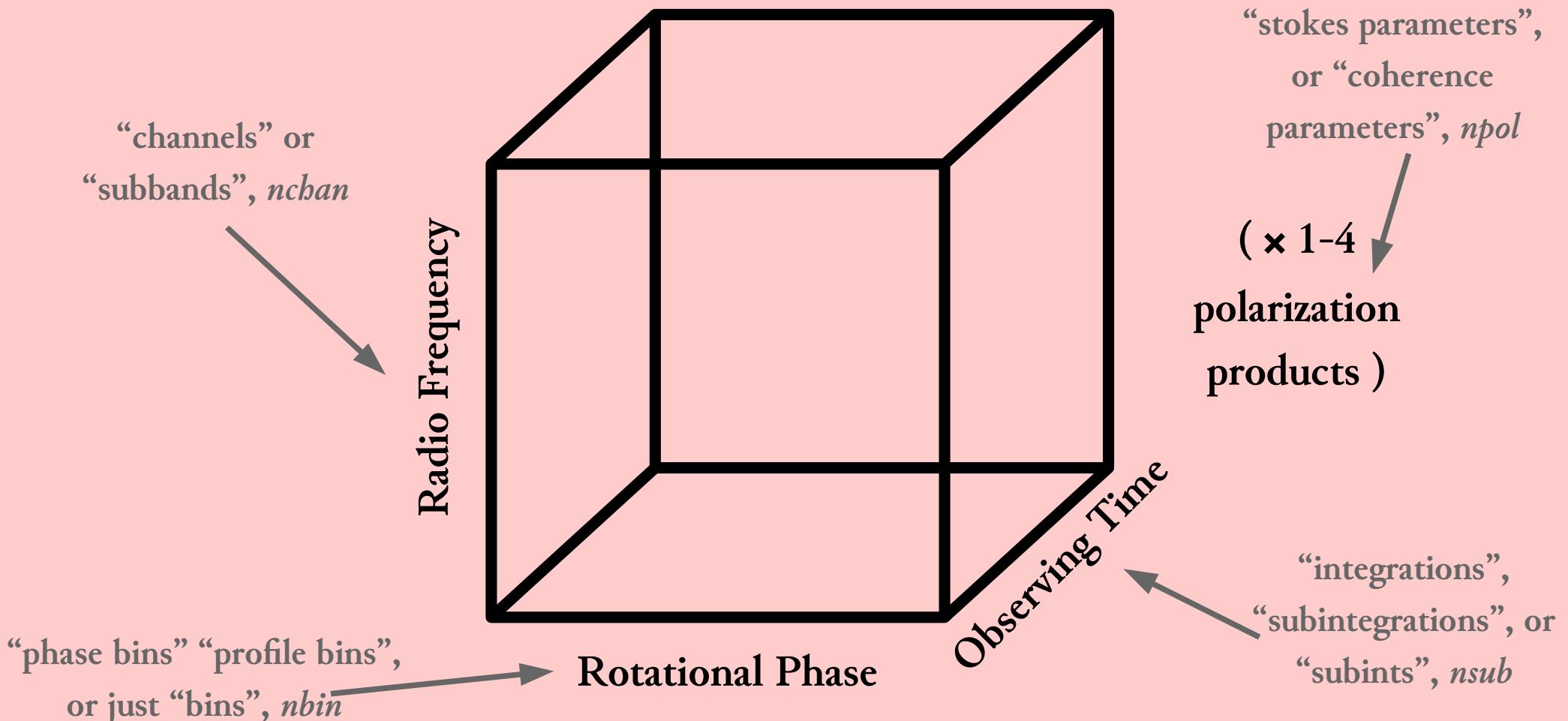
- Generally speaking, our pulsar timing data come in the format of one-to-four data cubes:



- Collection of flux density measurements as function of phase, frequency, time, and polarization...

Time-of-arrival (TOA) Measurements

→ Generally speaking, our pulsar timing data come in the format of one-to-four data cubes:



→ Collection of flux density measurements as function of phase, frequency, time, and polarization...

Time-of-arrival (TOA) Measurements

→ So how do we get TOAs from these data?

→ And how many?

One per subint? One per frequency channel? One per polarization?

Time-of-arrival (TOA) Measurements

→ So how do we get TOAs from these data?

→ And how many?

One per subint? One per frequency channel? One per polarization?



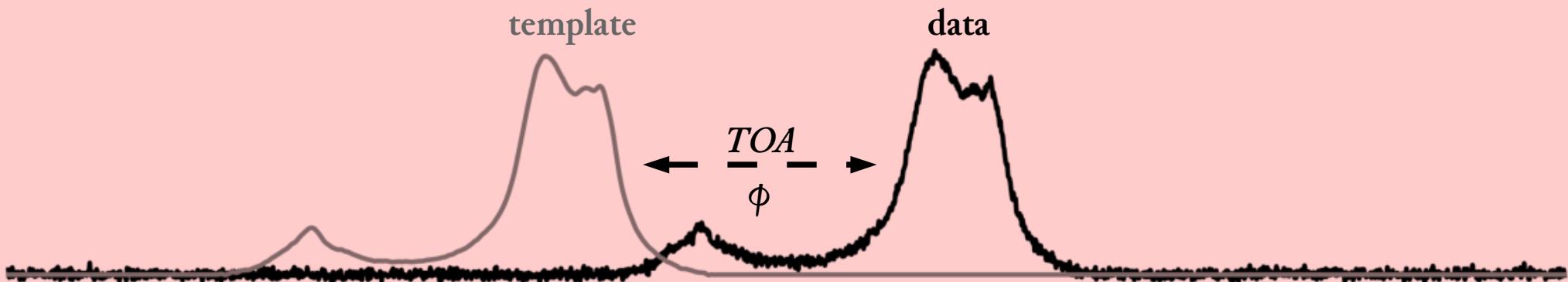
Depends on your needs:
short orbits, short time-
scale variations require a
higher cadence of TOAs;
PTAs only need “one” TOA
per observation

Will result in *very* many
TOAs; can help diagnose
ISM effects, jitter, profile
evolution etc.; but in
principle can be dealt with
using “wideband” methods

Should not be necessary, but
rather, should use all of the
pol'n info in measuring your
TOA(s); at the present, this is
almost never done – see
van Straten (2006), “matrix
template matching” (MTM)

Time-of-arrival (TOA) Measurements

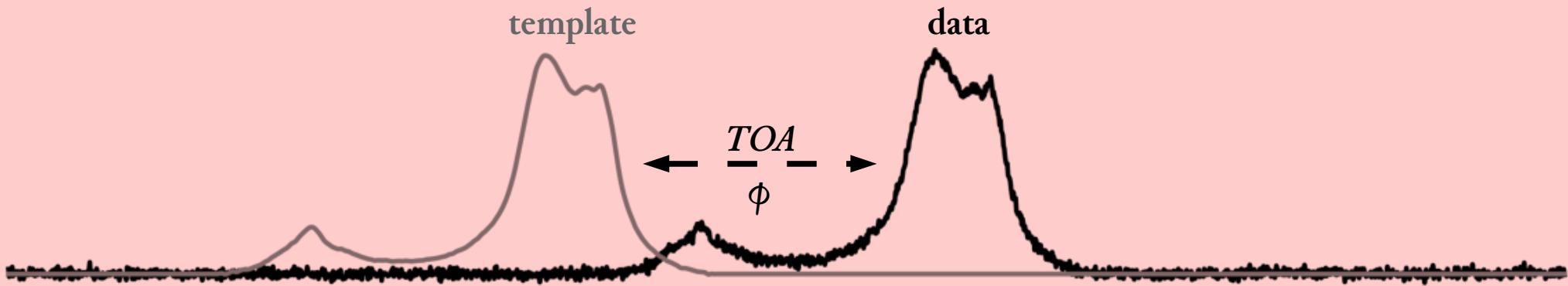
→ Basic idea is a *cross-correlation* with a template
(cf. “matched filter” or “template matching”)



→ The phase offset ϕ times the instantaneous spin period P_s of the pulsar gives a time offset Δt , which is added to the absolute start time of the data profile to obtain the TOA. That's it (almost).

POP QUIZ 2

What is the measurement of a TOA *ultimately a proxy* for in pulsar timing?



- A) The arrival time of one single pulse of radiation emitted in the N^{th} rotation of the neutron star
- B) The average arrival time of many pulses of radiation around the N^{th} rotation of the neutron star
- C) The time of passage of the same, fiducial longitude on the neutron star surface across the line-of-sight in the N^{th} rotation of the neutron star
- D) The time that the pulse of radiation left the neutron star

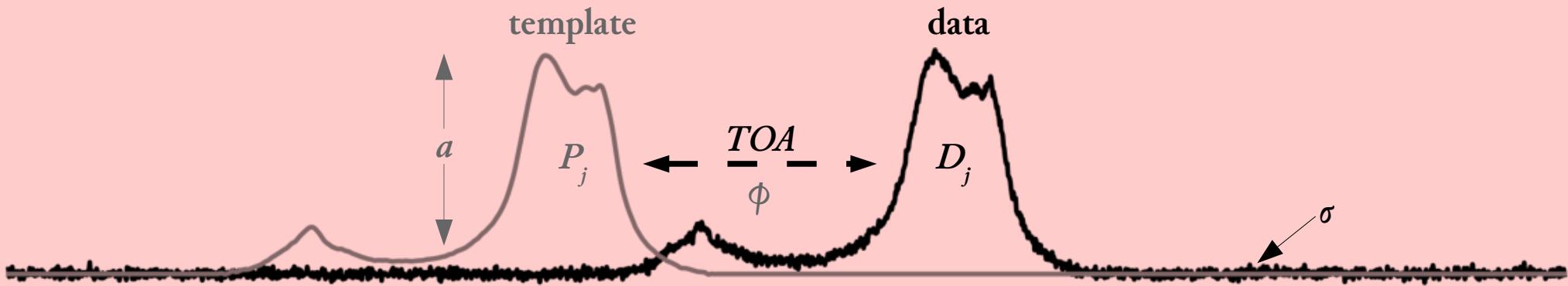
Time-of-arrival (TOA) Measurements

The two most important take-home messages about TOAs:

#2: Truly “next generation” methods are looking to eschew the use/measurement of TOAs altogether – Lentati et al. (201?)

#1: TOAs are *proxies* for what we actually care about modeling, the passage of time of a fixed point on the neutron star

The TOA likelihood



$$\chi^2(\phi, a) = \sum_k \frac{|d_k - ap_k e^{-2\pi i k \phi}|^2}{\sigma^2}$$

D_j = data profile

P_j = template profile

j = phase bin index

d_k = FourierTransform{ D_j }

p_k = FourierTransform{ P_j }

a = scaling amplitude

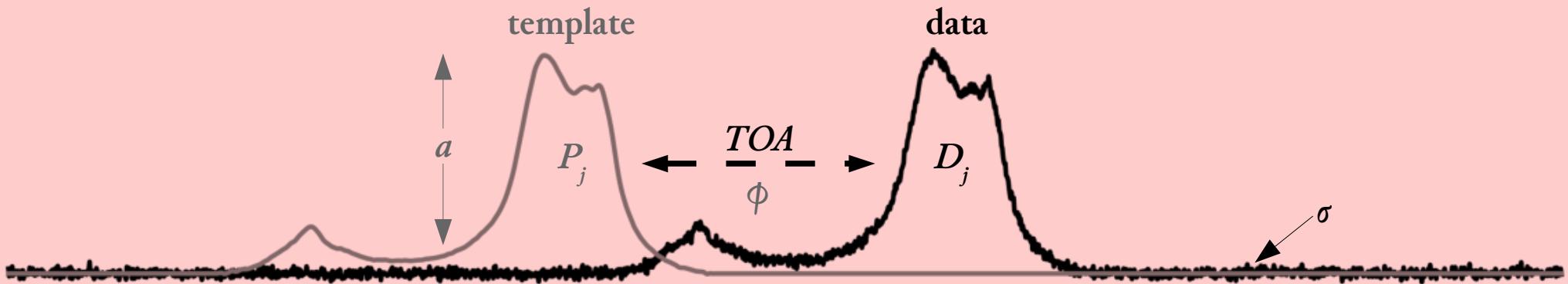
ϕ = phase offset

k = Fourier harmonic index

σ = standard deviation of the noise

χ^2 = “chi-squared” statistic, related to the “log-likelihood”

The TOA likelihood



$$\chi^2(\phi, a) = \sum_k \frac{|d_k - apk e^{-2\pi i k \phi}|^2}{\sigma^2}$$

- Turns out, in most cases, doing things in “the Fourier domain” is better because cross-correlations (in “the time domain”) take more computing time
- The $e^{-2\pi k \phi}$ term is called the “phasor” and is a result of the Fourier Shift Theorem
- This formulation was first (?) written down by Joe Taylor in an appendix of a 1992 paper, and was later referred to as “FFTFIT”
 - It is often called (e.g., in PSRCHIVE) a phase-gradient algorithm
- Non-linear in ϕ , so need an optimizer to find “best-fit” value, but easy to write down the parameter uncertainties
 - There are other techniques, this is the most common

TOA Uncertainty

- Formal TOA uncertainty is derivable from the χ^2 function from before
- But we expect the TOA “error” to scale according to the table
- *Lots* of things could cause the “*true*” TOA uncertainty (remember the proxy!) to be larger than we expect: pulse “jitter”, incorrect correction of the interstellar medium, a bad template profile (pulse profile evolution)

[table: D. Nice]

$$\sigma_{TOA} = \left(\frac{T_{sys}}{G} \right) \left(\frac{\eta}{S} \right) \frac{1}{\sqrt{\eta t B n_p}} \eta P$$

Typical* Values

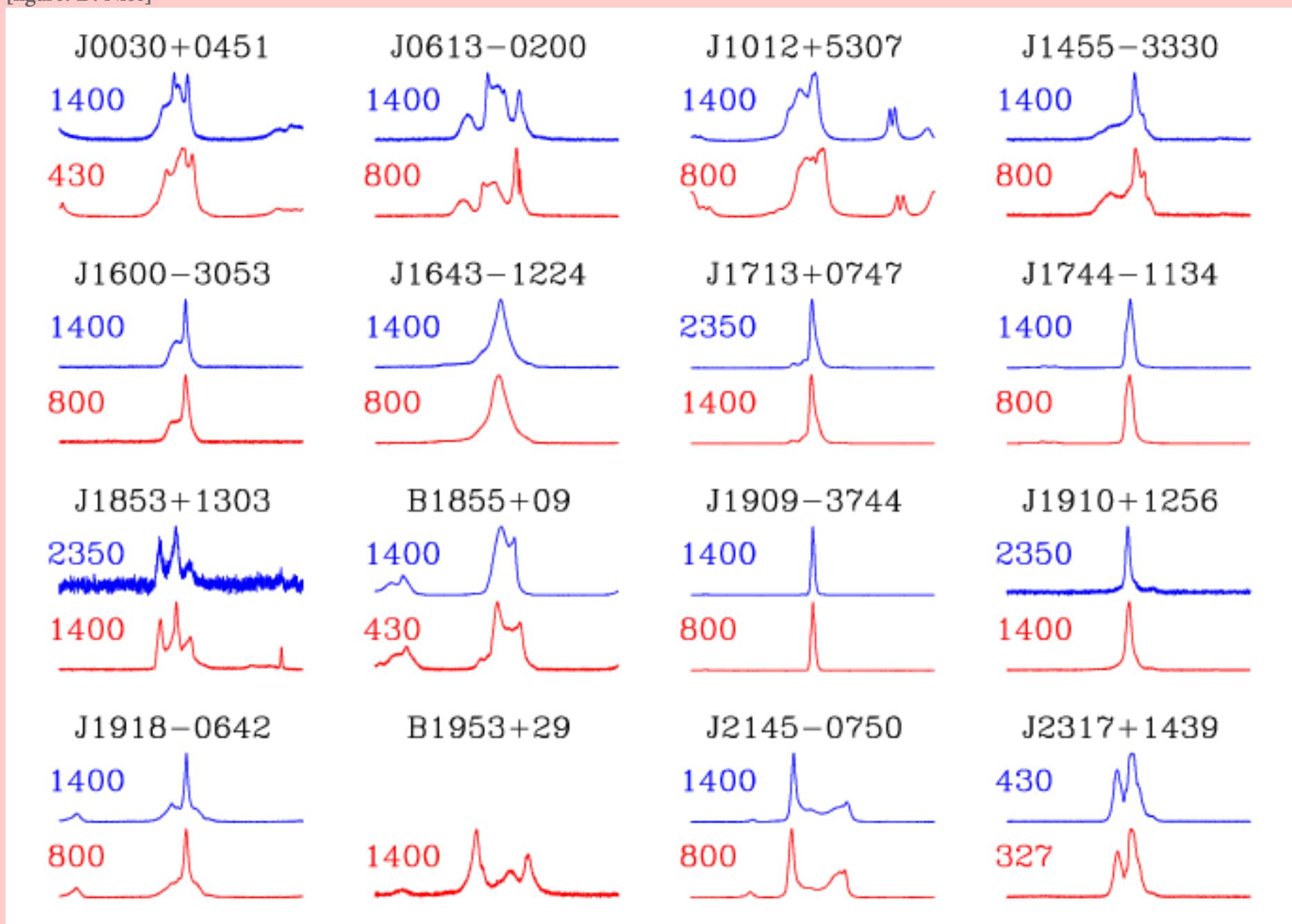
P	= 0.004 s	Pulse period
η	= 0.05	Duty cycle
T_{sys}	= 20K	System temperature
G	= 2 K/Jy	Telescope gain
S	= 0.001 Jy	Pulsar flux density
t	= 1000 s	Observation time
B	= 10^8 Hz	Bandwidth
n_p	= 2	Number of polarizations

$$\rightarrow \sigma_{TOA} = 200 \text{ ns}$$

*Not really typical. This would be a good strong pulsar observed with 100m telescope.

Millisecond Pulsar (MSP) Profiles

[figure: D. Nice]



What TOAs (used to) look like

Observatory	Radio Frequency	Pulse Time of Arrival	Measurement Uncertainty
a 3751 1518+49	370.000	50942.02369981804596	69.1
a 3751 1518+49	370.000	50942.02508871578912	74.9
a 3752 1518+49	370.000	50942.02710263928441	107.8
a 3752 1518+49	370.000	50942.02849153928888	68.4
a 3753 1518+49	370.000	50942.03050309034722	63.0
a 3753 1518+49	370.000	50942.03189199466585	71.4
a 3754 1518+49	370.000	50942.03389643284537	64.2
a 3754 1518+49	370.000	50942.03528532340819	57.4
a 3755 1518+49	370.000	50942.03728740139970	74.4
a 3755 1518+49	370.000	50942.03867629785610	65.1
a 3756 1518+49	370.000	50942.04067884384616	54.2
a 3756 1518+49	370.000	50942.04206774860490	87.3
a 3757 1518+49	370.000	50942.04406981298474	88.9
a 3757 1518+49	370.000	50942.04545870833792	71.8
a 3758 1518+49	370.000	50942.04748447411745	110.3
a 3758 1518+49	370.000	50942.04887336536594	78.6
a 3759 1518+49	370.000	50942.05089865820880	60.2
a 3759 1518+49	370.000	50942.05228755033977	131.1
a 3760 1518+49	370.000	50942.05428961858992	63.4
a 3760 1518+49	370.000	50942.05567851214494	93.2
a 3761 1518+49	370.000	50942.05768105475176	116.2
a 3761 1518+49	370.000	50942.05906994776154	75.0
a 3762 1518+49	370.000	50942.06108244410689	72.2
a 3762 1518+49	370.000	50942.06247133259781	76.9
a 3763 1518+49	370.000	50942.06450988581265	86.1
a 3763 1518+49	370.000	50942.06589877480622	61.9
a 3764 1518+49	370.000	50942.06790794988299	90.1
a 3764 1518+49	370.000	50942.06929683956486	67.2
a 3765 1518+49	370.000	50942.07129227137214	63.5
a 3765 1518+49	370.000	50942.07268116130441	139.5

[slide: D. Nice]

What TOAs (used to) look like

Observatory	Radio Frequency	Pulse Time of Arrival	Measurement Uncertainty
a 3751 1518+49	370.000	50942.02369981804596	69.1
a 3751 1518+49	370.000	50942.02508871578912	74.9
a 3752 1518+49	370.000	50942.02710263928441	107.8
a 3752 1518+49	370.000	50942.02849153928888	68.4
a 3753 1518+49	370.000	50942.03050309034722	63.0

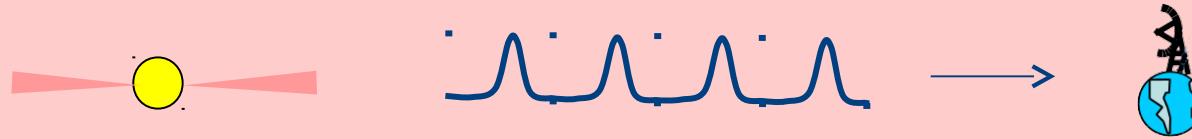
Many modern TOA files contain more details about the observation, as shown in the example below. The most critical parts are still the observatory, radio frequency, time of arrival, and measurement uncertainty.

```
puppi_56599_J2317+1439_0107.8y.x.ff 1153.437 56599.031436860011304 0.932 ao  
-fe L-wide -be PUPPI -f L-wide_PUPPI -bw 12.5 -tobs 1200.8 -tmplt J2317+1439.L-  
wide.PUPPI.8y.x.sum.sm -gof 0.999 -nbin 2048 -nch 8 -chan 50 -subint 0 -snr 100.96 -  
wt 5629 -proc 8y -pta NANOGRAV
```

a 3762 1518+49	370.000	50942.06108244410009	72.2	9-May-98
a 3762 1518+49	370.000	50942.06247133259781	76.9	9-May-98
a 3763 1518+49	370.000	50942.06450988581265	86.1	9-May-98
a 3763 1518+49	370.000	50942.06589877480622	61.9	9-May-98
a 3764 1518+49	370.000	50942.06790794988299	90.1	9-May-98
a 3764 1518+49	370.000	50942.06929683956486	67.2	9-May-98
a 3765 1518+49	370.000	50942.07129227137214	63.5	9-May-98
a 3765 1518+49	370.000	50942.07268116130441	139.5	9-May-98

Basic picture

[figure: D. Nice]



Recall: pulsars spin (P or F), and they spin down (P-dot or F-dot)

$$F(t) = F_0 + F\text{-dot} (t-t_0)$$

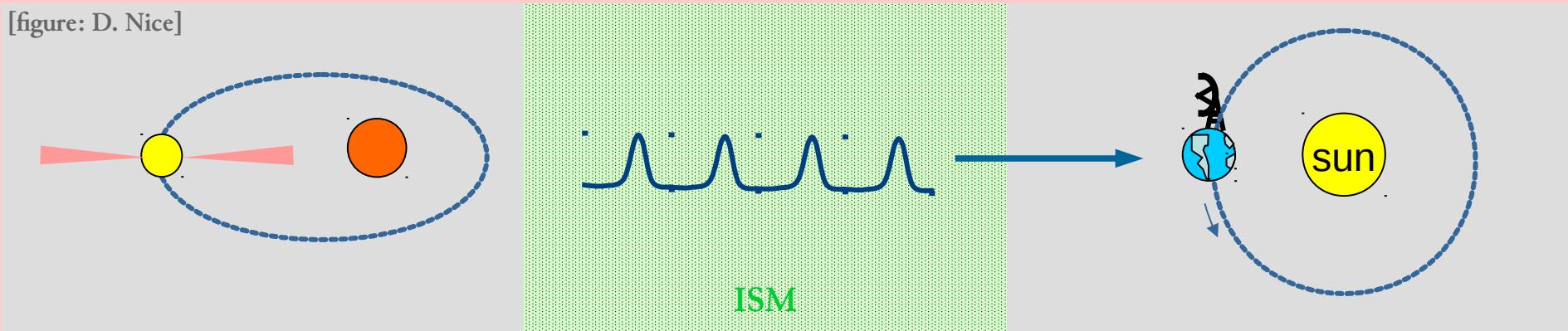
(integrate)
↓

$$\phi(t) = \phi_0 + F_0(t-t_0) + 1/2 F\text{-dot} (t-t_0)^2$$

$\phi(t)$ = phase as function of time = rotations as function of time =
timing model
(done!)

Basic picture

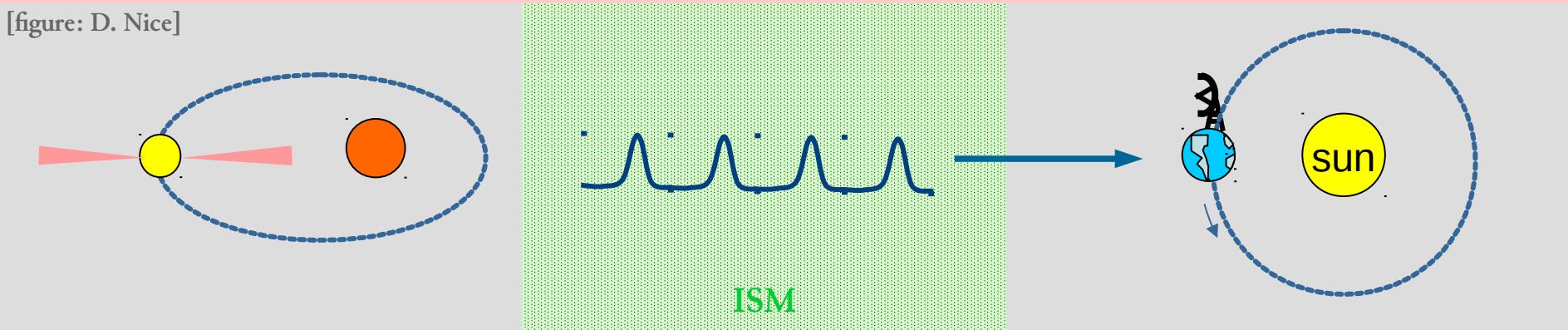
[figure: D. Nice]



- We are not in an inertial frame with respect to the rotating neutron star, so no pulsar timing model is that simple
- There are *many* delays that need to be applied to the TOAs to put them in an inertial frame (i.e., the solar system barycenter)
- If the pulsar is in a binary (most MSPs are), then the TOAs need to be corrected for the binary orbit, too

Basic picture

[figure: D. Nice]



→ Two SSB transformations to the TOAs that can be confusing:

1) *Time transfer:*

Observatory clock → GPS → UT → TDB

(think of TDB as the time some clock reads which is co-moving at the location of the SS barycenter, but free from gravity; the TOAs reflect the time on the TDB clock when they arrived *at the observatory*; this is like a time-zone correction)

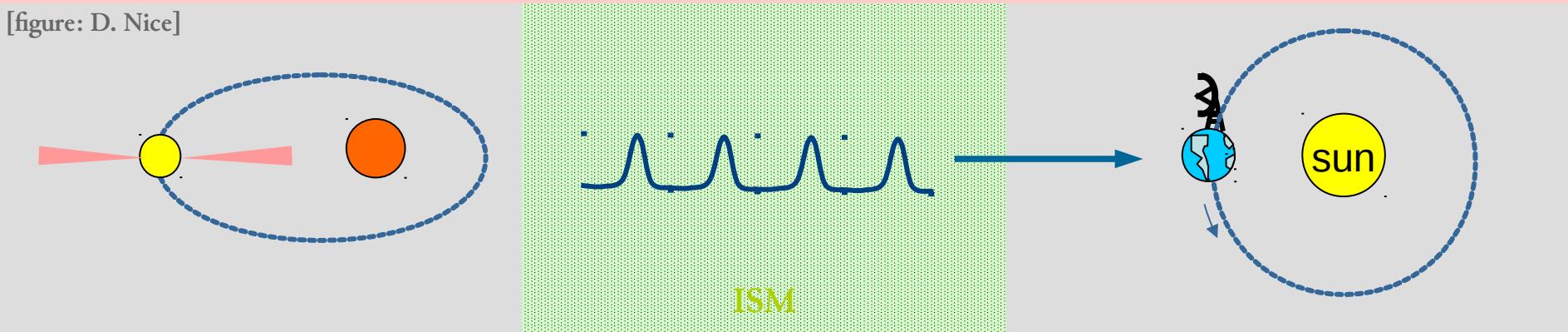
2) *Position transfer:*

Observatory → Geocenter → Barycenter

(this is now what the TOA would be if the observatory were located *at the barycenter*; this is really accounting for the propagation of the signal, not just a unit conversion)

Basic picture

[figure: D. Nice]



Pulsar:

- Spin & spin-down
- Orbital elements (classical & relativistic)
- Secular variations
- Timing noise

Interstellar medium:

- Dispersion
- Varying index of refraction / multi-path propagation
- “Twinkling”

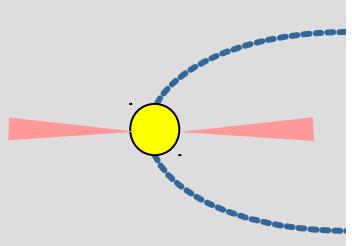
Solar System:

- Position
- Proper motion
- Parallax
- Dispersion from Solar wind***
- Clock corrections
- Earth's rotation/motion
- Planetary ephemeris***

Deterministic/Fit – Noise process/Modeled – Held fixed from external measurements

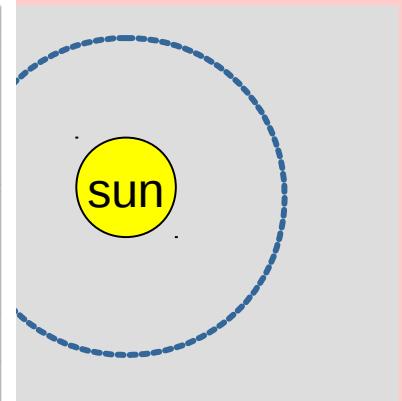
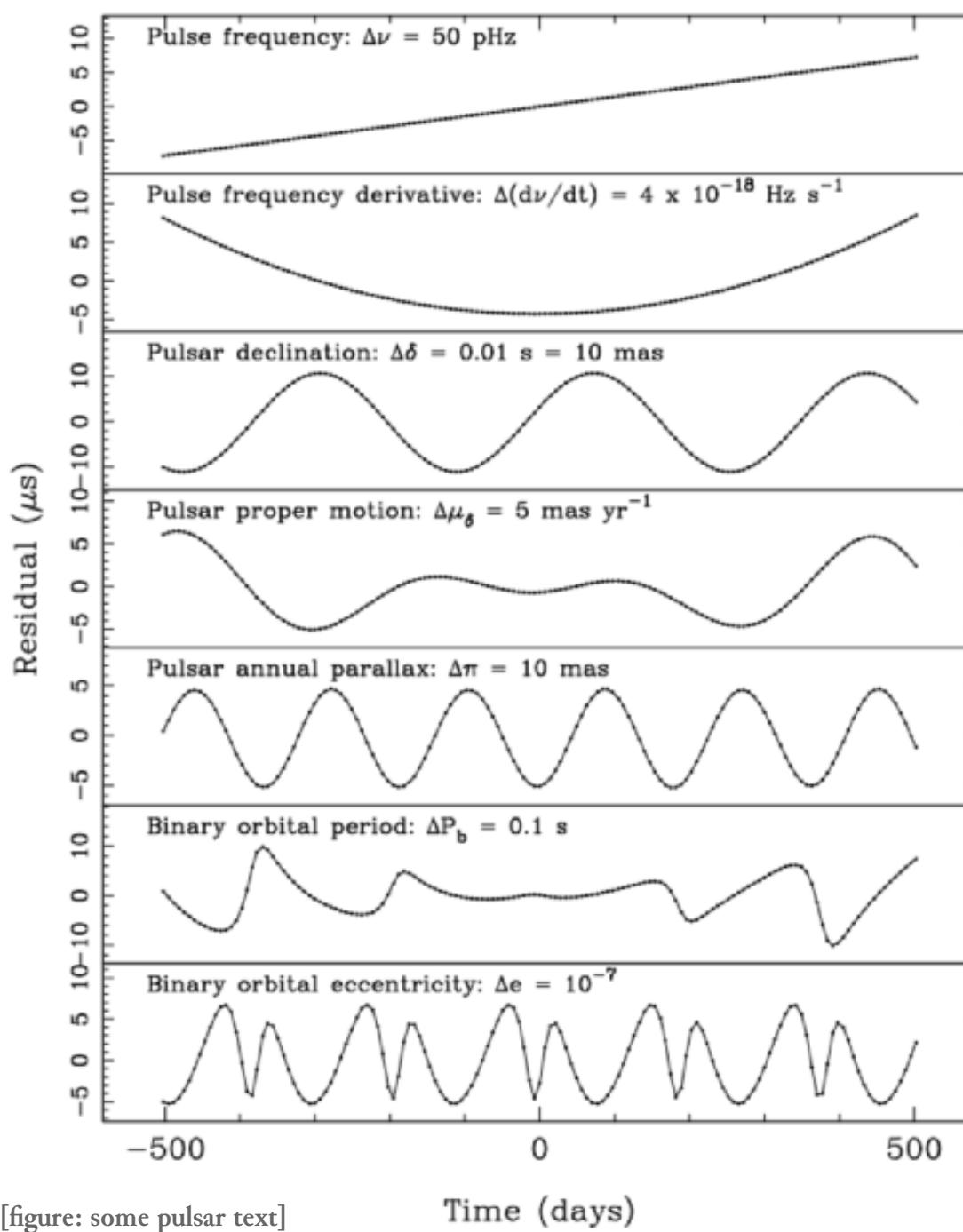
Basic picture

[figure: D. Nice]



- Pulsar:
- Spin & spin-down
- Orbital element & relativistic)
- Secular variations
- Timing noise

Deterministic



Solar System:

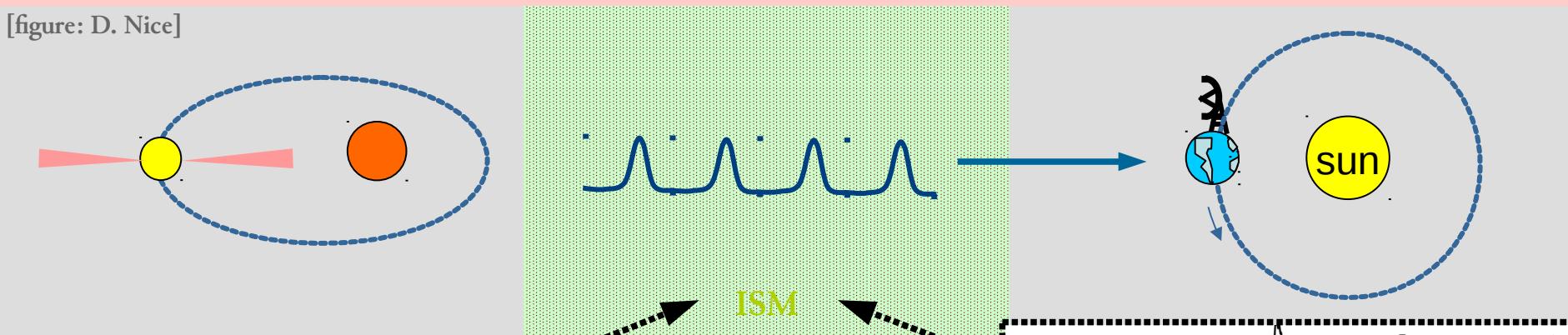
tion

from Solar wind***
ections
ation/motion
ephemeris***

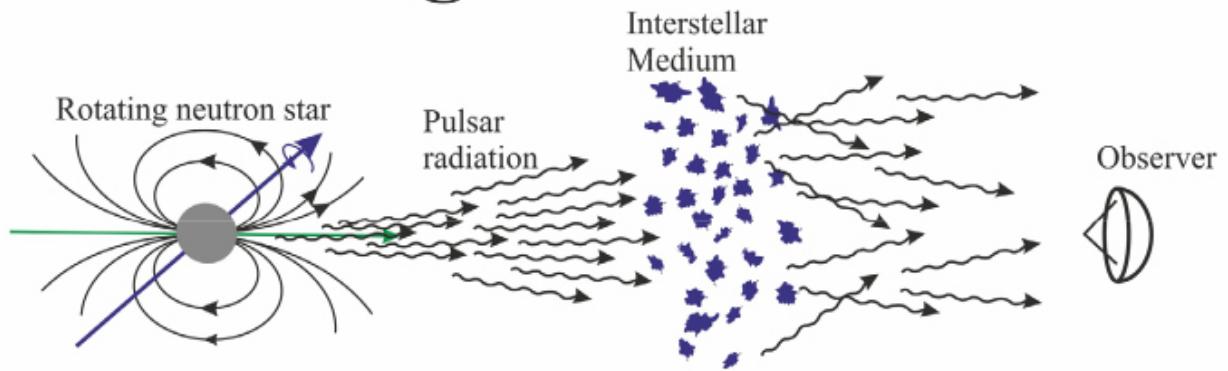
measurements

Basic picture

[figure: D. Nice]

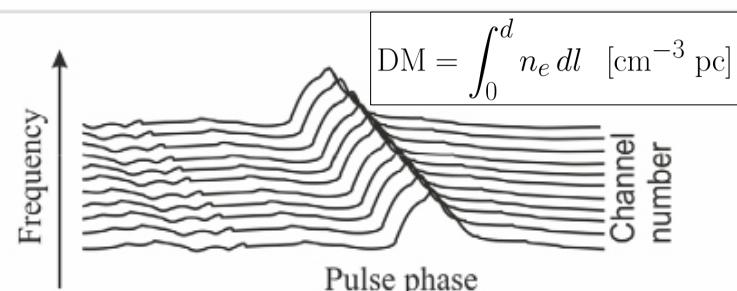


Scattering and scintillation

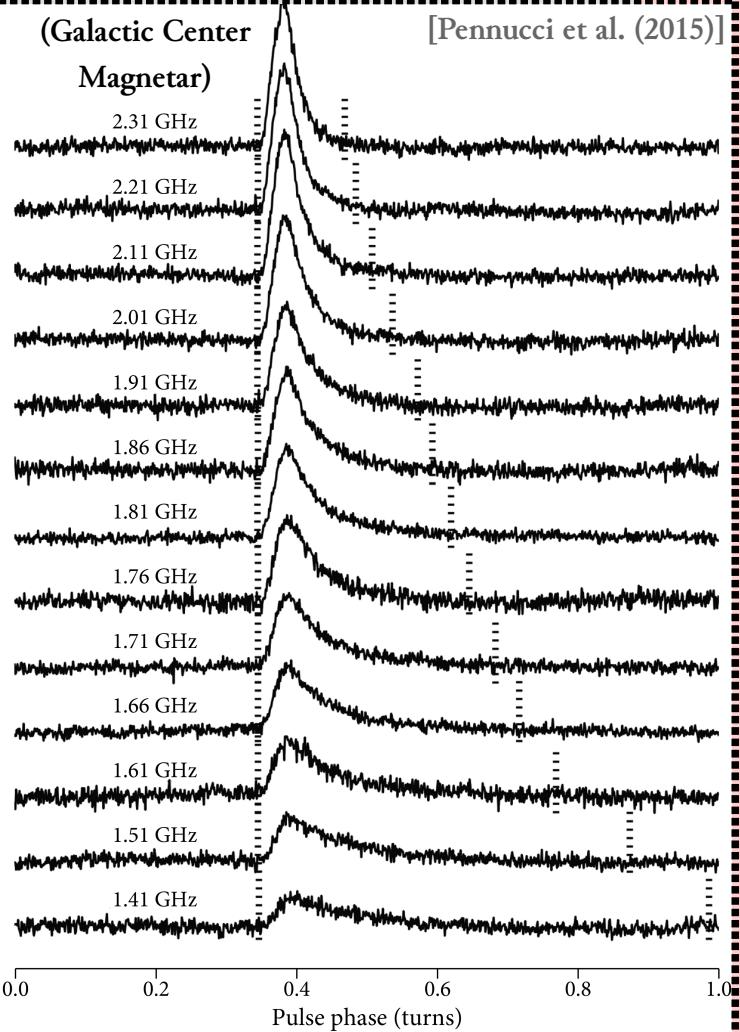


Dispersion

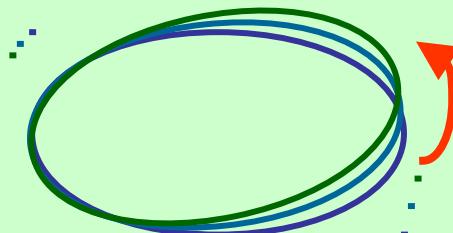
During multichannel pulsar observations we can note dispersive delays over a bandwidth



[Błaszkiewicz et al. (2016)]

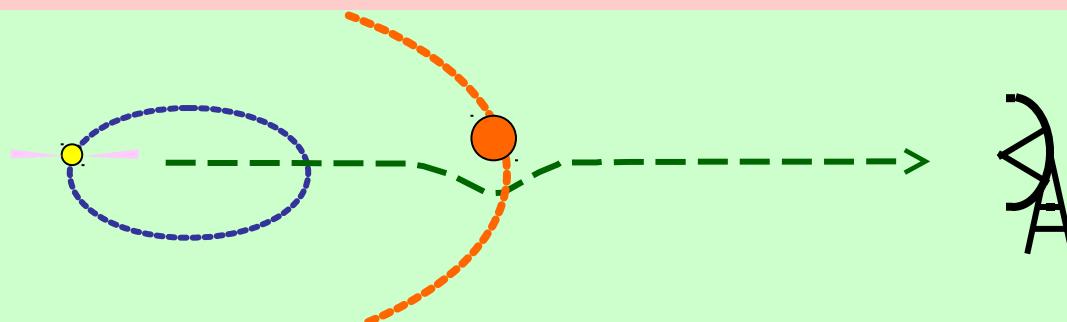


Precession



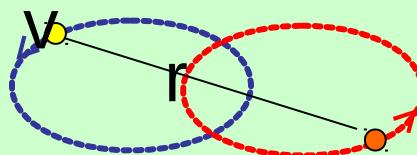
$$\omega = 3 \left(\frac{P_b}{2\pi} \right)^{-5/3} \frac{1}{1-e^2} \left[\frac{G}{c^3} (m_1 + m_2) \right]^{2/3}$$

Shapiro Delay



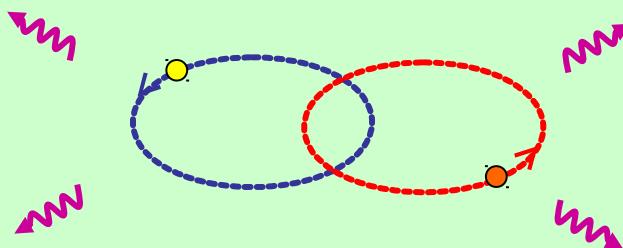
$$\Delta t = 2 \frac{G}{c^3} m_2 \ln [1 - \sin i \sin(\varphi - \varphi_0)]$$

Grav. Redshift/Time Dilation



$$\gamma = \frac{G^{2/3}}{c^2} \left(\frac{P_b}{2\pi} \right)^{1/3} e^{-\frac{m_2(m_1+2m_2)}{(m_1+m_2)^{4/3}}}$$

Gravitational Radiation



$$P_b = - \left(\frac{192\pi}{5} \right) \frac{G^{5/3}}{c^5} \left(\frac{P_b}{2\pi} \right)^{-5/3} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \frac{1}{(1-e^2)^{7/2}} \frac{m_1 m_2}{(m_1 + m_2)^{1/3}}$$

The making of a timing model

The making of a timing model

→ Transformation to a frame that has just the spinning down pulsar involves numerous corrections to the pulse time-of-arrival (TOA):

$$\Delta t = \Delta_C + \Delta_A + \Delta_{E\odot} + \Delta_{R\odot} + \Delta_{S\odot} - D/f^2 + \Delta_{VP} + \Delta_B$$

atmosphere
clock corrections delays dispersion secular motion binary motion

(most terms are functions of timing model parameters)

The making of a timing model

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atmosphere
clock corrections delays dispersion secular motion binary motion

→ Once “corrected” to the frame of the spinning down pulsar, calculate the predicted phase evolution:

$$\phi_i = \phi_0 + \nu(t_i - t_0) + \dot{\nu}(t_i - t_0)^2/2 + \dots$$

(the spin frequency and its derivative are parameters, with higher derivatives vanishing)

The making of a timing model

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→ Compare to the observed pulse phase to get a phase offset, divided by the instantaneous spin period to get a “timing residual”:

$$R_i = \frac{\phi_i - N_i}{\nu}$$

(remember: rotations come in integers!)

The making of a timing model

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$$R_i = \frac{\phi_i - N_i}{\nu}$$

→ Vary the (linearized) timing model in your favorite optimization routine to minimize the difference / maximize the likelihood:

$$\chi^2 = \sum_{i=1}^N \left(\frac{R_i}{\sigma_i} \right)^2$$

[equations: L&K05]

(this is where it gets tricky – enter tempo, tempo2, pint, enterprise, temponest, etc.)

A Timing Model

Fit and data set	
Pulsar name	J0437–4715
MJD range	53041.3–53767.3
Number of TOAs	603
Rms timing residual (μ s)	1.6
Weighted fit	<i>N</i>
Measured quantities	
Right ascension	04:37:15.78858(13)
Declination	−47:15:08.4685(15)
Pulse frequency (s^{-1})	173.687 946 306 02(7)
First derivative of pulse frequency (s^{-2})	$-1.7292(4) \times 10^{-15}$
Dispersion measure (cm^{-3}pc)	2.641 23(17)
Proper motion in Right ascension (mas yr^{-1})	120.9(3)
Proper motion in Declination (mas yr^{-1})	−71.0(3)
Parallax (mas)	8(3)
Orbital period (d)	5.741 046 4584(16)
Epoch of periastron (MJD)	511 94.620(6)
Projected semi-major axis of orbit (lt-s)	3.36670624(19)
Longitude of periastron ($^{\circ}$)	0.9(4)
Orbital eccentricity	0.000 018 99(11)
Set quantities	
Epoch of frequency determination (MJD)	511 94
Epoch of position determination (MJD)	511 94
Epoch of dispersion measure determination (MJD)	511 94
Sine of inclination angle	0.6788
First derivative of orbital period	3.64×10^{-12}
Periastron advance (deg yr^{-1})	0.016
Companion mass (M_{\odot})	0.236
Derived quantities	
$\log_{10}(\text{Characteristic age, yr})$	9.20
$\log_{10}(\text{Surface magnetic field strength, G})$	8.76
Assumptions	
Clock-correction procedure	TT(TAI)
Solar system ephemeris model	DE405
Binary model	DD
Model version number	5.00
<i>Note.</i> Figures in parentheses are twice the nominal 1σ TEMPO2 uncertainties in the least-significant digits quoted.	
[table: M. Vallisneri]	

(14 digits of precision!)

Sifting through residuals

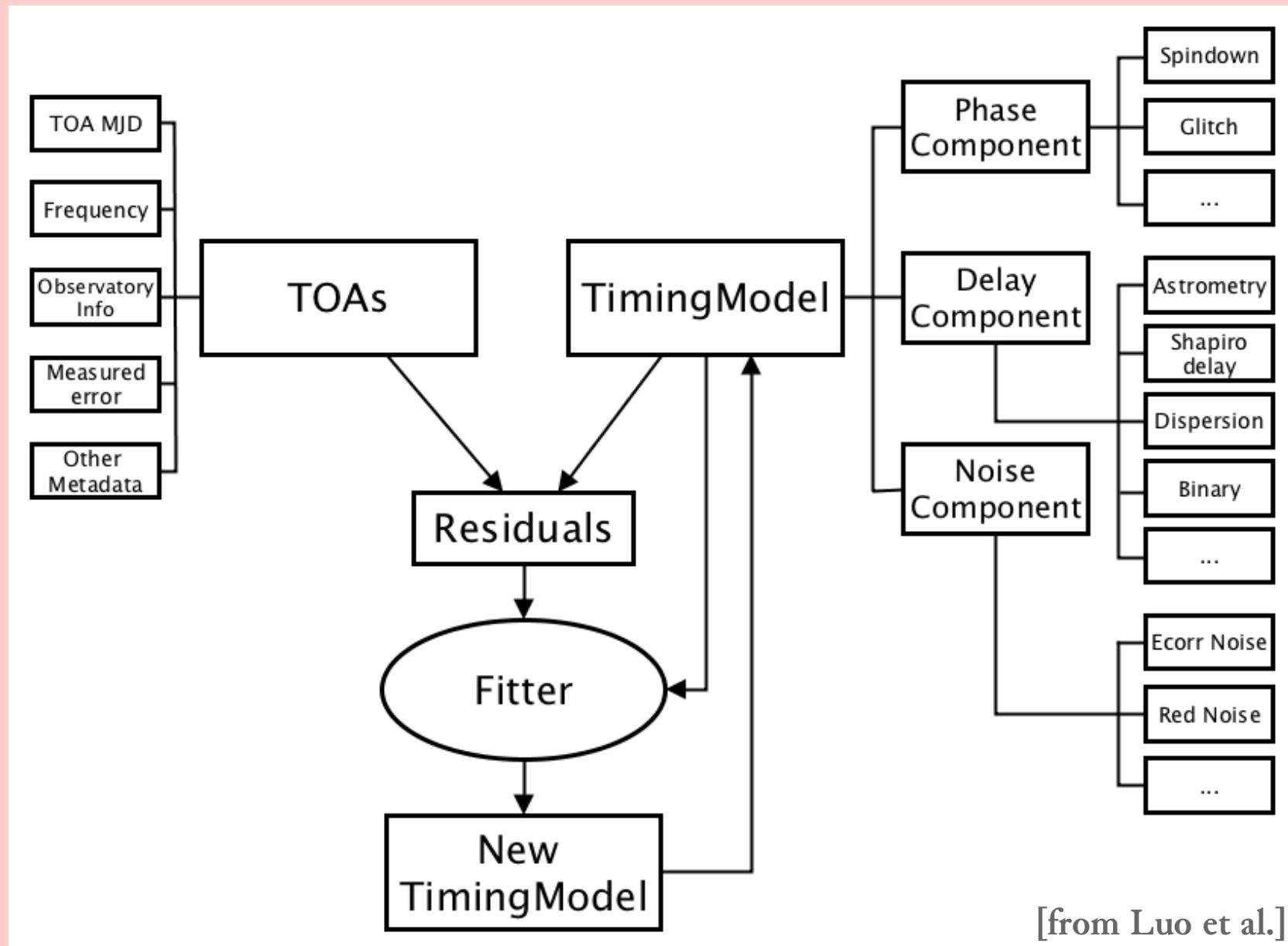
– What's left after subtracting the timing model from the data (TOAs) are “timing residuals” which are a combination of systematic and random errors arising from:

- 1) radiometer noise (white)
- 2) timing model errors (systematic/red)
- 3) unmitigated ISM effects (white/red)
- 4) pulse jitter noise (white)
- 5) timing noise / “clock drift” (red)
- 6) GWs! (systematic/red)

→ Use complex Bayesian machinery to model each as a noise source with different temporal, spectral, and spatial characteristics

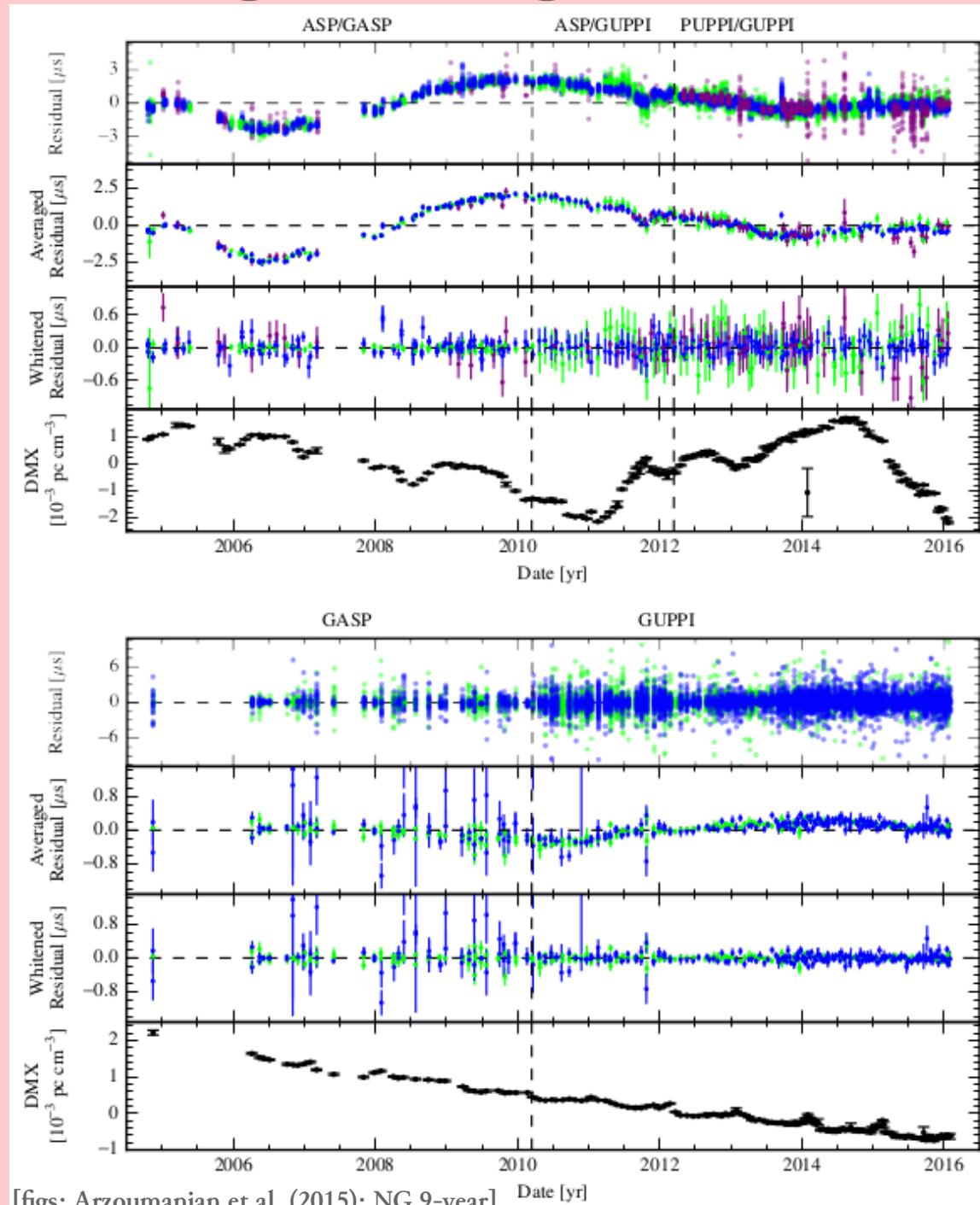
→ Ultimately, we want our residuals to be *Gaussian distributed* and *white*

Timing machinery schematic (PINT)

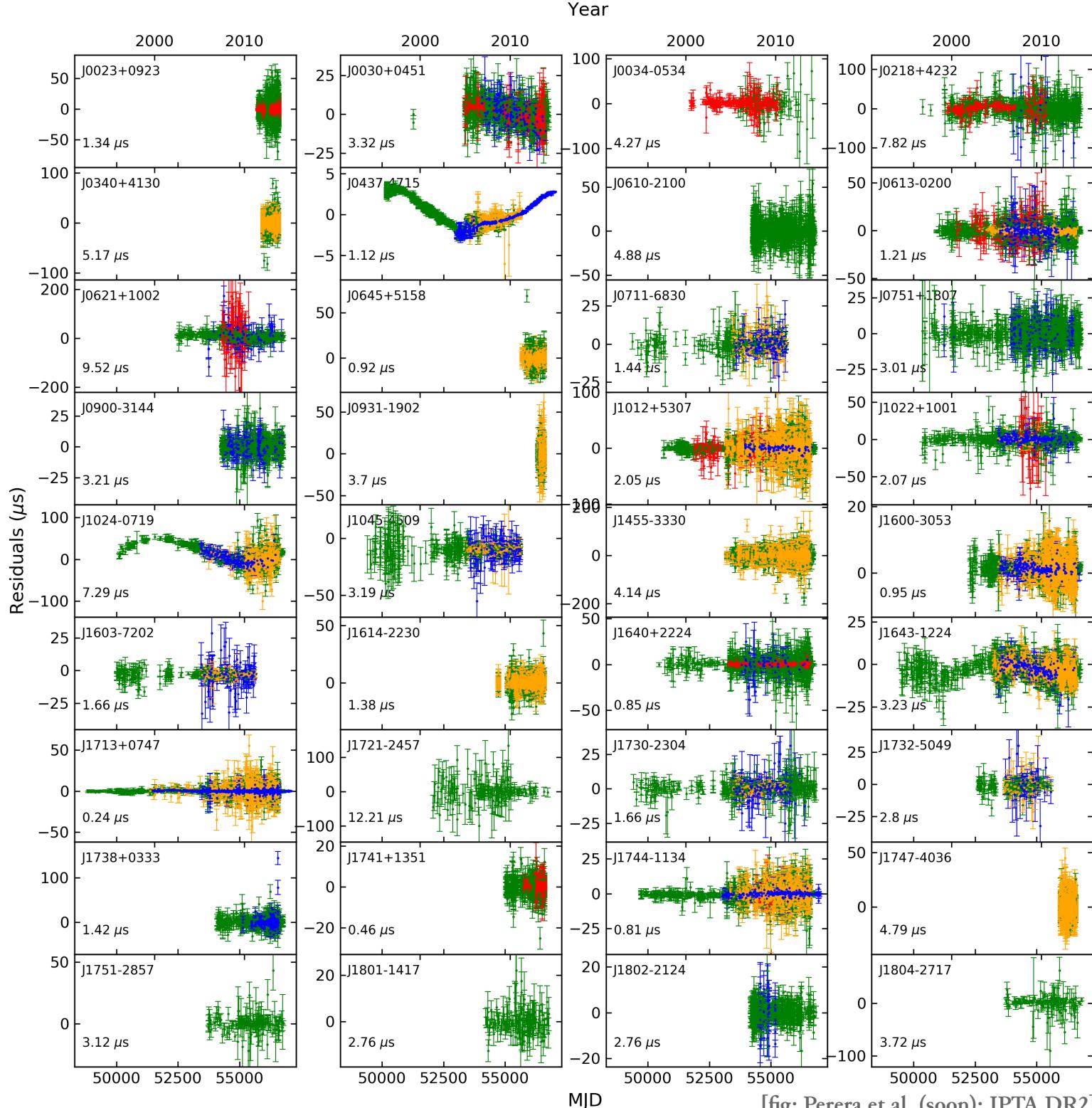


[from Luo et al.]

Sifting through residuals



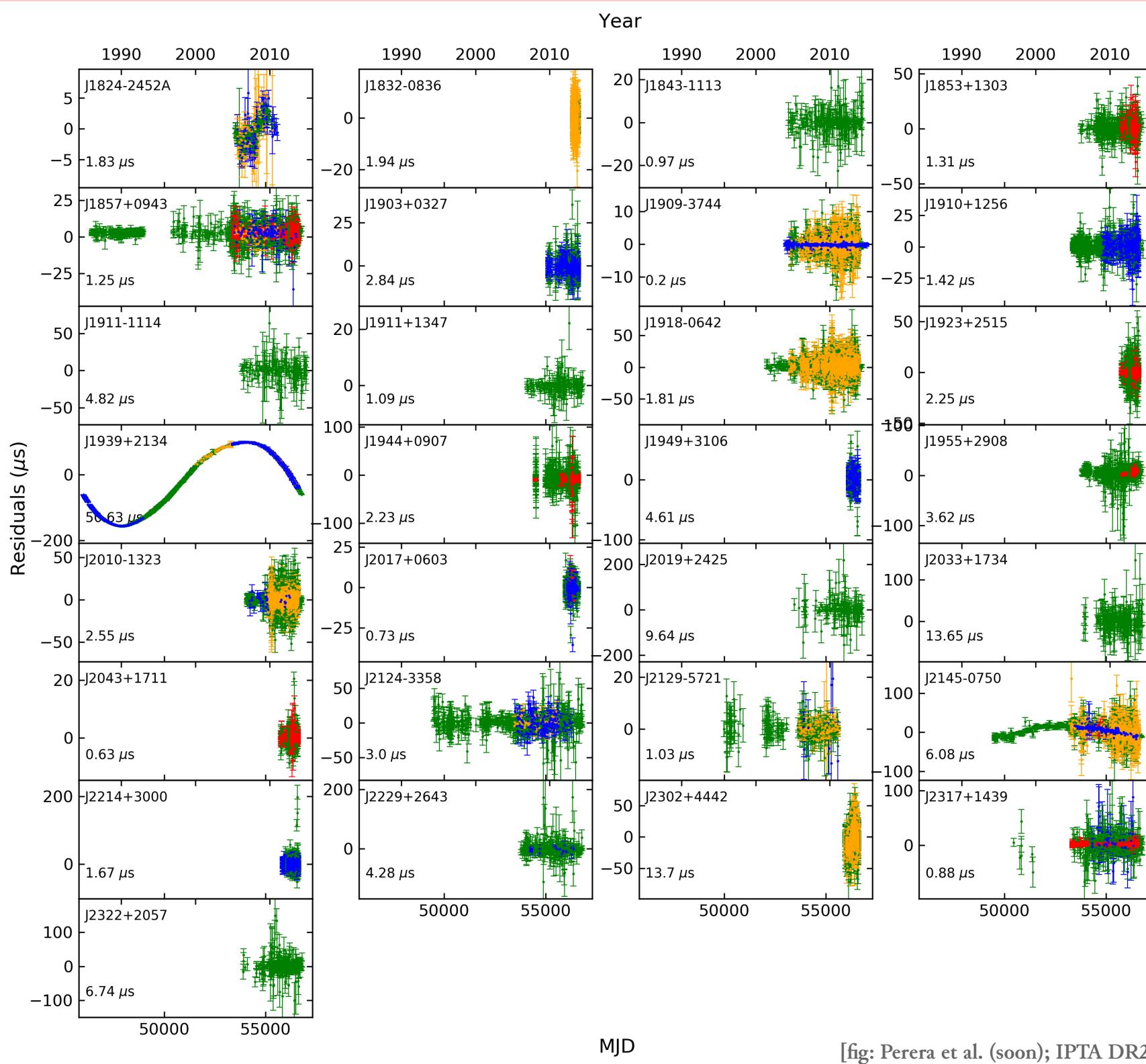
More residuals



[fig: Perera et al. (soon); IPTA DR2]

IPTA DR2!

More residuals



[fig: Perera et al. (soon); IPTA DR2]

IPTA DR2!

Why are residuals different from zero?

1) Systematic errors:

“All models are wrong, some models are useful.”

- A parameter has the wrong value
- A parameter is missing from the model
- The choice of model is a poor one

2) Random errors:

“The generation of random numbers is too important to be left to chance.”

and

“Any one who considers arithmetical methods of producing random digits is, of course, in a state of sin.”

- Stochastic uncertainties of various origins
- “True noise”, “irreducible uncertainty”, if there exists such a thing

Why are residuals different from zero?

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“All models are wrong, some models are useful.”

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“Classic”

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- Stochastic uncertainties of various origins
- “True noise”, “irreducible uncertainty”, if there exists such a thing

A note on “noise”:

“noise” = stochastic (i.e., unpredictable) variations

→ Not all noise is the same! ←

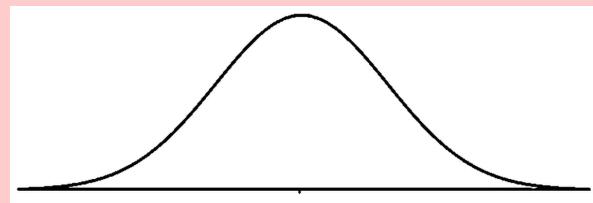
- While not classically predictable event-to-event, we can hope (*assume*) sources of noise each have *a definite statistical description*, and each of those may be individually separable in a way similar to discerning between two, classical deterministic signals.
- Physically describing sources of noise is even more challenging

A note on “noise”:

“noise” = stochastic (i.e., unpredictable) variations

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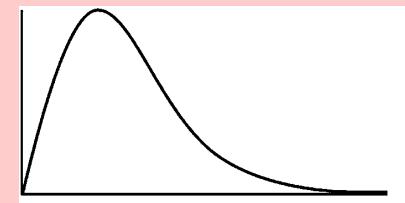
“Gaussian noise”



→ Distribution of samples is Gaussian, of course

→ Can be result of Central Limit Theorem = many independent sources of noise

“Non-Gaussian noise”



→ Distribution of samples is non-Gaussian, of course

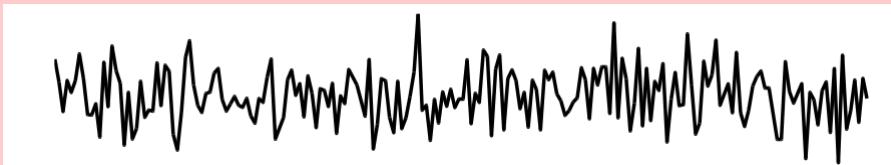
→ We hope to never encounter this, as many of our statistical methods assume Gaussianity, but it often shows up in real life

A note on “noise”:

“noise” = stochastic (i.e., unpredictable) variations

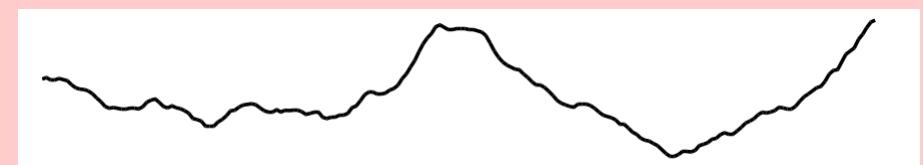
→ Not all noise is the same! ←

“White noise”



- Equal power at all frequencies
 - “Uncorrelated”
- Coin flips, from one to the next, are white noise
- Dice rolls, from one to the next, are white noise

“Red noise”



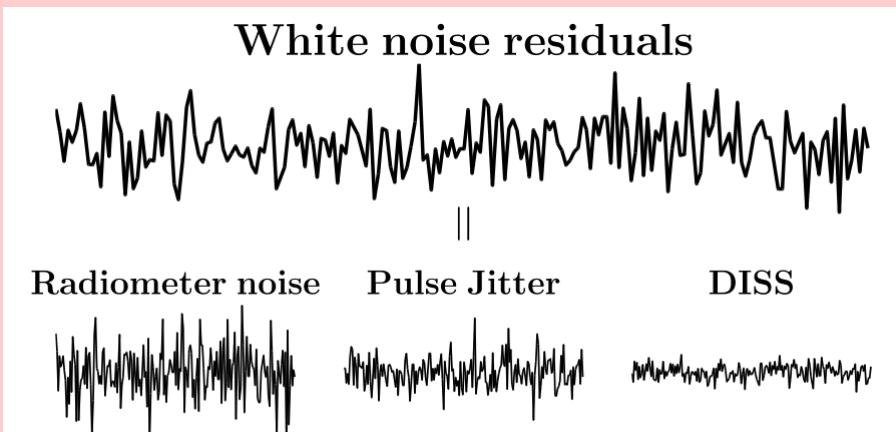
- More power at low frequencies
 - “Correlated” on larger scales
- Sum of heads (+1) and tails (-1) over a long game is red noise
- Sum of the rolls of dice over a long game is red noise
- A “random walk” is a red noise “process”

A note on “noise”:

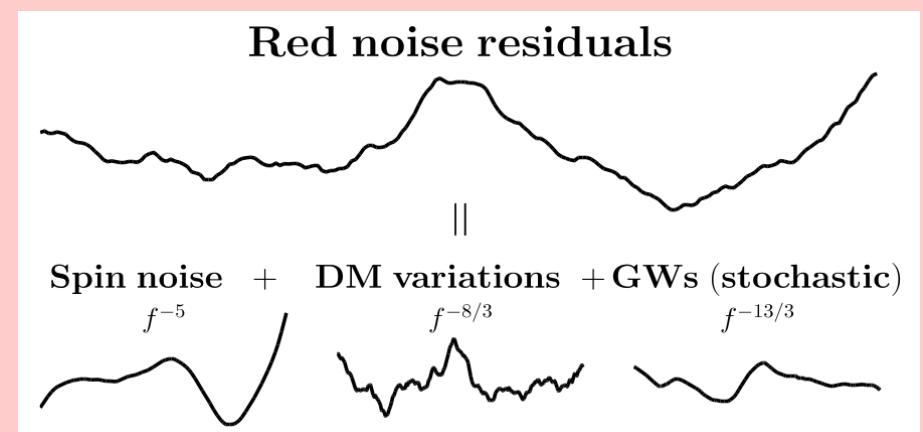
“noise” = stochastic (i.e., unpredictable) variations

→ Not all noise is the same! ←

“White noise”



“Red noise”



[figures: J. Cordes]

→ Lots of sources of white noise
in pulsar timing, each
characterized simply by their
amplitude

→ Lots of sources of red noise in
pulsar timing, each characterized
by at least their amplitude and
index.

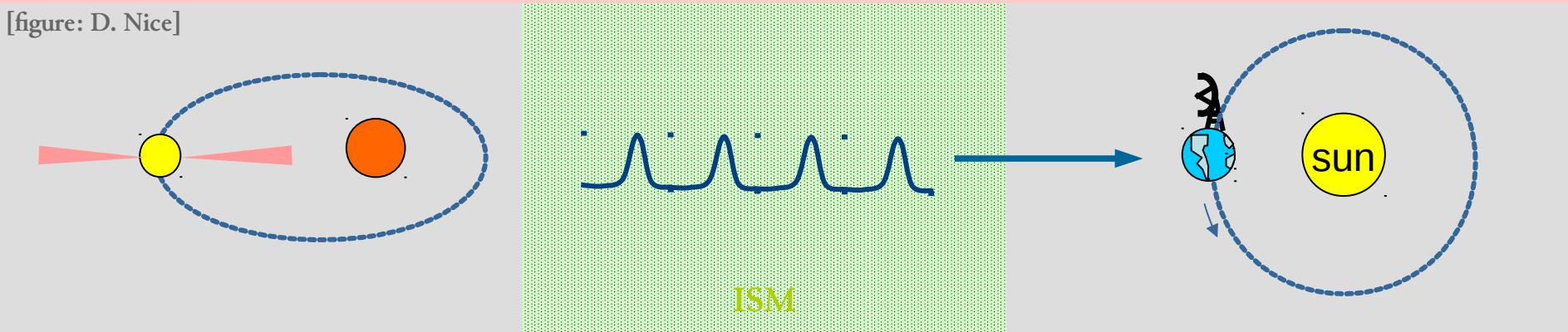
POP QUIZ 3

Timing noise is:

- A) Red
- B) White
- C) Both
- D) Neither, it's Blue

Basic picture

[figure: D. Nice]



Pulsar:

- Spin & spin-down
- Orbital elements (classical & relativistic)
- Secular variations
- Timing noise

Interstellar medium:

- Dispersion
- Varying index of refraction / multi-path propagation
- “Twinkling”

Solar System:

- Position
- Proper motion
- Parallax
- Dispersion from Solar wind***
- Clock corrections
- Earth's rotation/motion
- Planetary ephemeris***

Deterministic/Fit – Noise process/Modeled – Held fixed from external measurements

Dispersion Measure (DM)

$$\text{DM} = \int_0^d n_e \, dl \quad [\text{cm}^{-3} \text{ pc}]$$

→ DM is the “integrated column density of free electrons along the line of sight”

→ Induces time delay \propto (radio frequency) $^{-2}$

Dispersion Measure (DM)

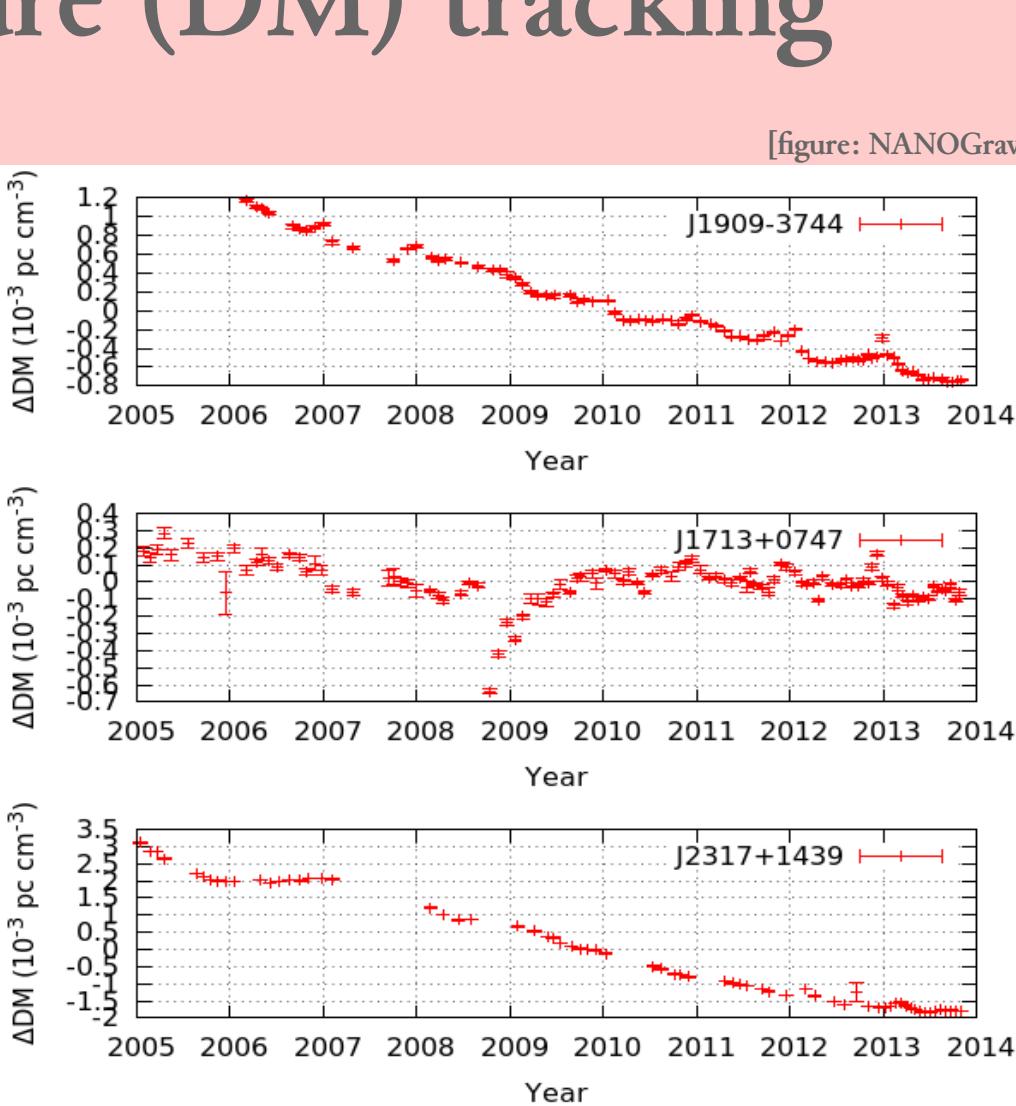
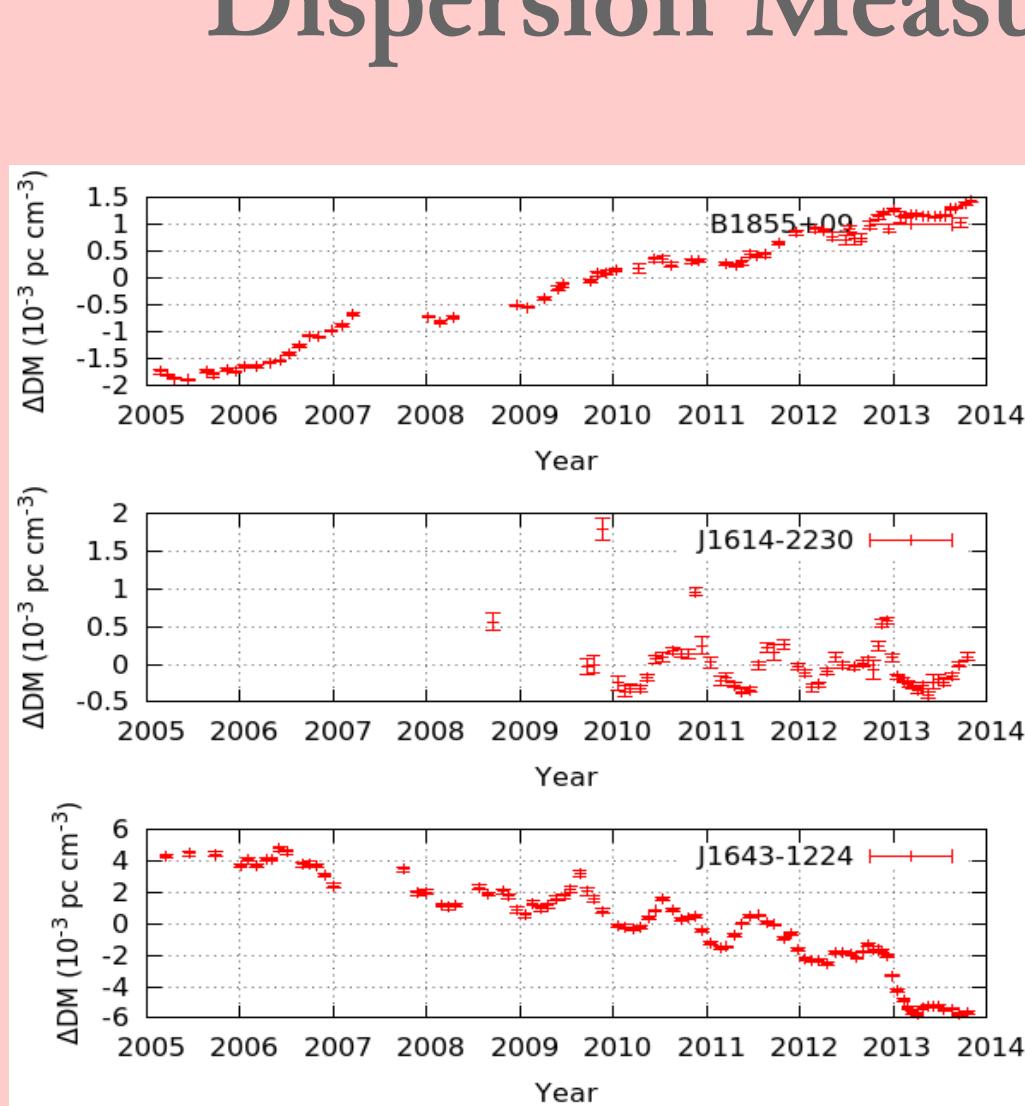
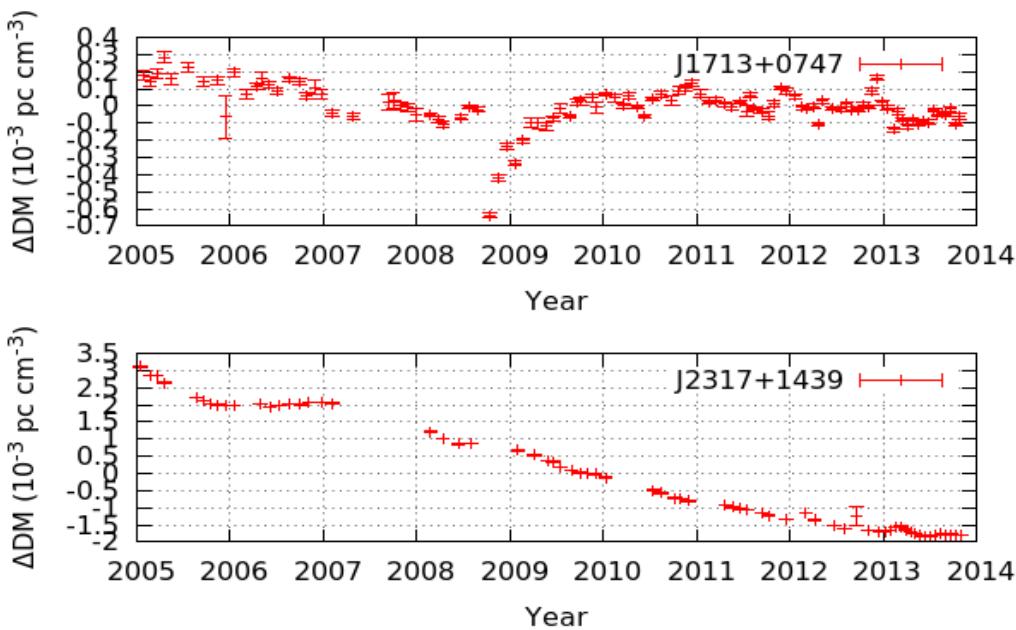
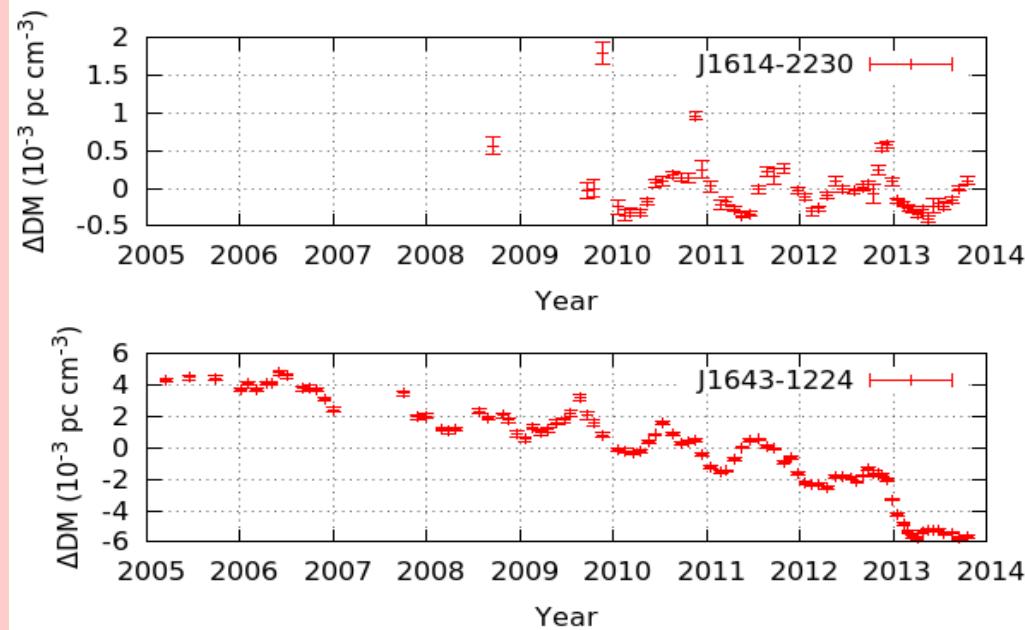
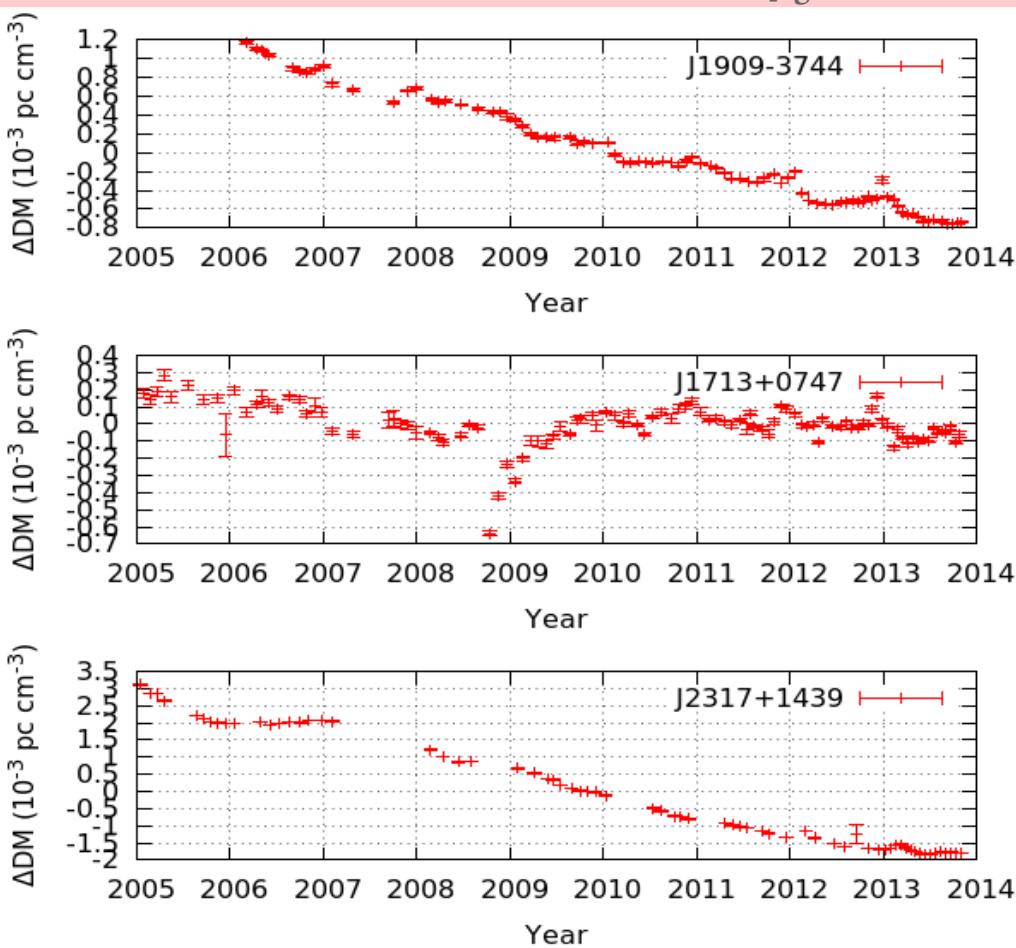
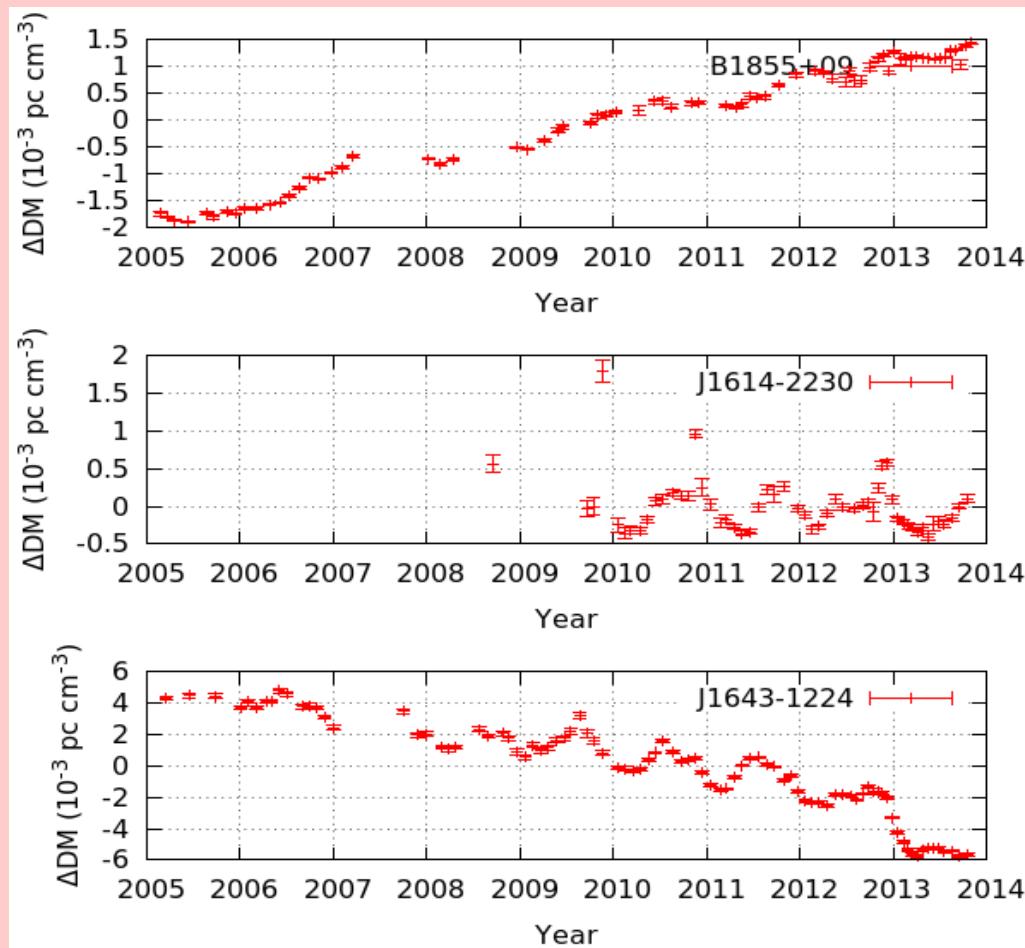
$$DM = \int_0^{\infty} n_e(t) ds(t, v) [cm^{-3} pc]$$

The equation shows the dispersion measure (DM) as a function of velocity (v). The integral is from 0 to infinity. The integrand is the electron density (n_e) at time t, multiplied by the differential distance element (ds) along the line of sight. A red arrow points to the term $n_e(t)$, another to the differential distance element ds , and a third to the units $[cm^{-3} pc]$.

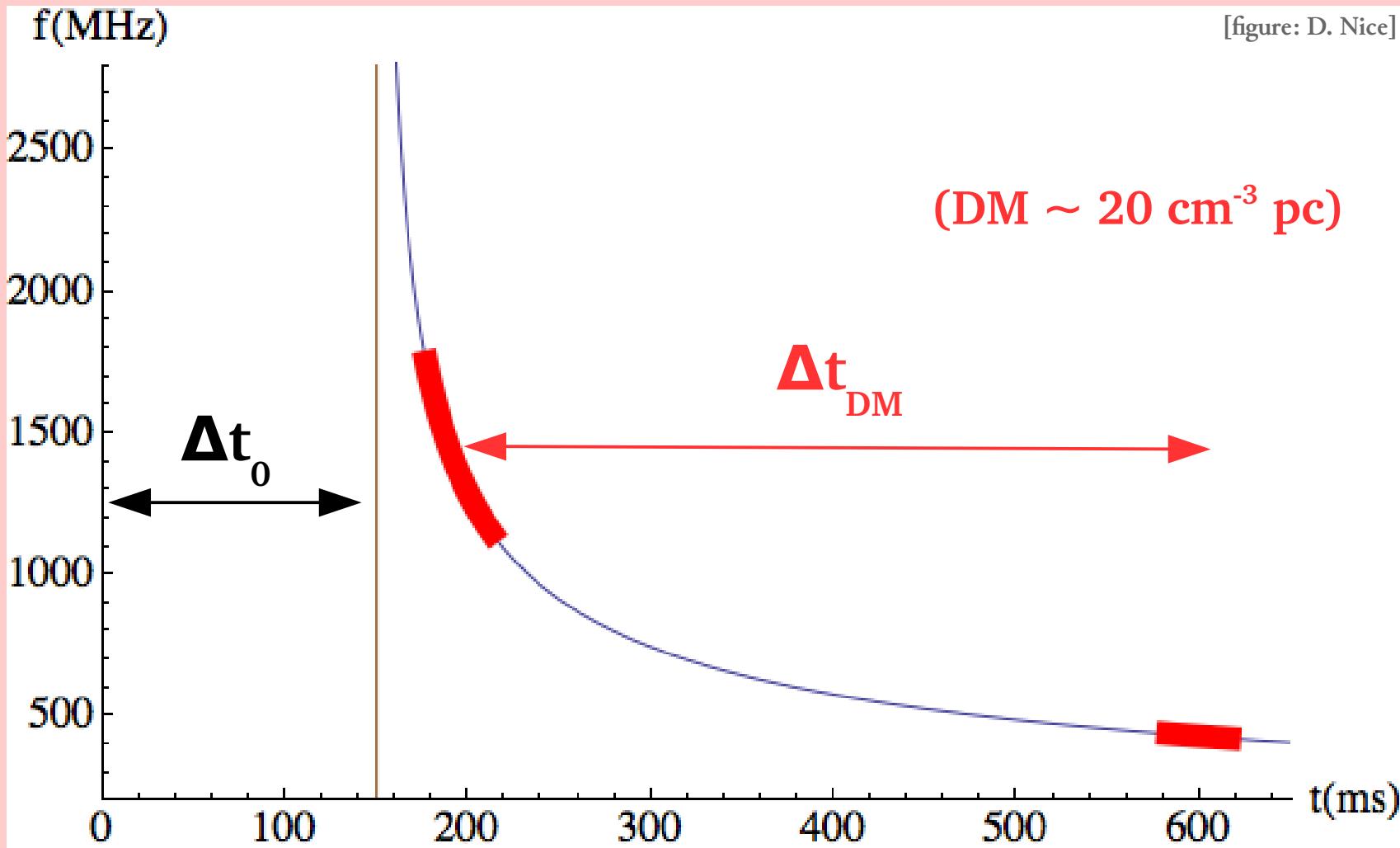
→ We can detect $DM(t)$ in our MSPs...
...part of “space weather”

Dispersion Measure (DM) tracking

[figure: NANOGrav]



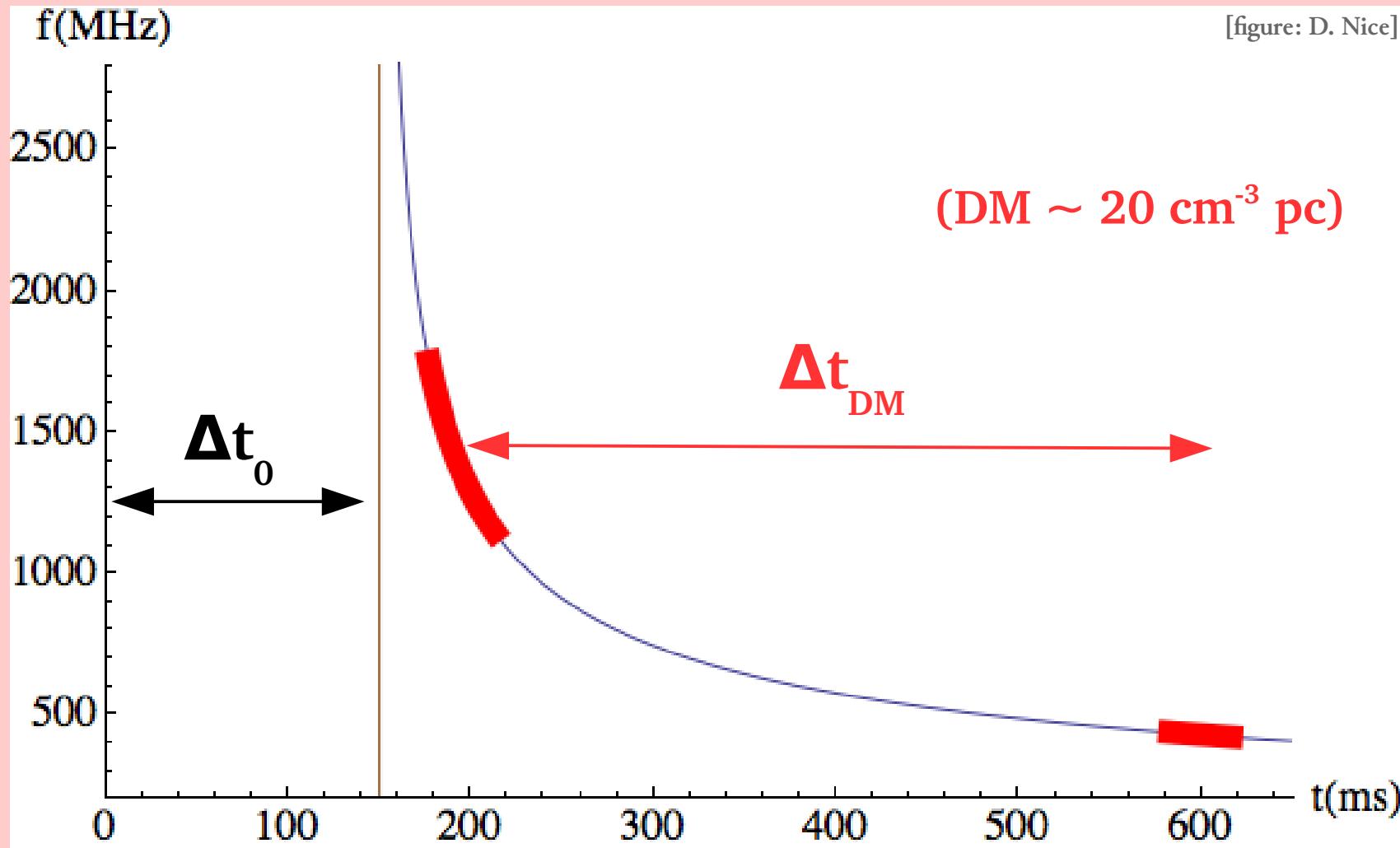
Dispersion Measure (DM) tracking



→ Wide bandwidth “lever” improves separation of DM from achromatic timing shifts

Dispersion Measure (DM) tracking

[figure: D. Nice]



→ Wide bandwidth “lever” improves separation of DM from achromatic timing shifts

How can we improve pulsar timing?

1) Improve precision of timing measurements (σ_{TOA})

a) Longer integration times (t_{obs})

b) Lower receiver temperature (T_{sys})

c) Wider instantaneous bandwidth (Δf)

d) Increase telescope area (A_{eff})

$$\sigma_{\text{TOA}} \propto \frac{T_{\text{sys}}}{A_{\text{eff}} \sqrt{t_{\text{obs}} \Delta f}} \times \frac{P \delta^{3/2}}{S_{\text{mean}}}$$

How can we improve pulsar timing?

1) Improve precision of timing measurements (σ_{TOA})

a) Longer integration times (t_{obs})

→ *Limited telescope time for increasing N pulsars*

b) Lower receiver temperature (T_{sys})

→ *Receivers near engineering limits*

c) Wider instantaneous bandwidth (Δf)

→ *New receivers now being developed/deployed*

d) Increase telescope area (A_{eff})

→ *New pulsar telescopes don't come around often*

But: IPTA, CHIME, MeerKAT, FAST...

$$\sigma_{\text{TOA}} \propto \frac{T_{\text{sys}}}{A_{\text{eff}} \sqrt{t_{\text{obs}} \Delta f}} \times \frac{P \delta^{3/2}}{S_{\text{mean}}}$$

The Next Generation of Broadband (Pulsar) Telescopes:

'mid' frequency telescopes:

Parkes (PPTA)

[Australia]



Effelsberg (EPTA)

[Germany/Europe]



MeerKat (MeerTime)

[South Africa/Int'l]



'low' frequency telescopes:

CHIME

[Canada]



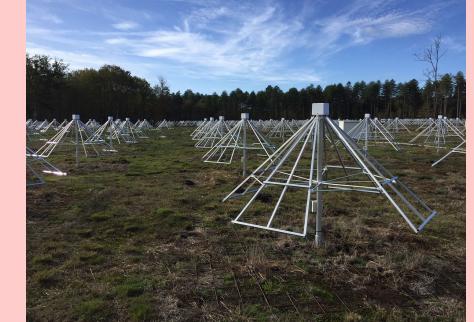
LOFAR

[Netherlands/Europe]



NenuFar

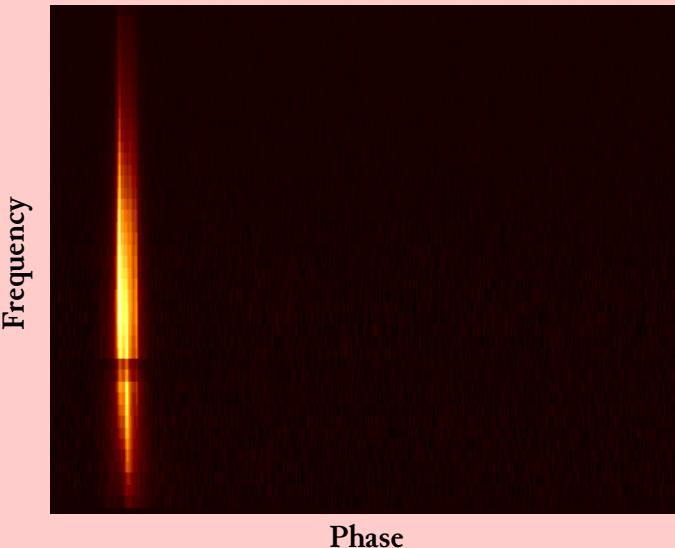
[France]



and more .. (e.g., GBT & AO(?) upgrades).

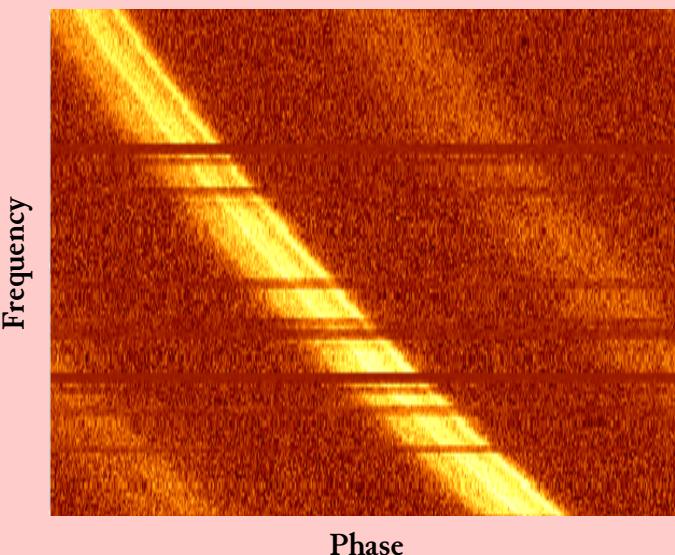
However, with great bandwidth comes great responsibility:

→ Across large fractional bandwidths, pulse profiles intrinsically evolve:



*(may also evolve
due to scatter
broadening...)*

→ Across large fractional bandwidths, you can measure dispersion (the DM)
from epoch to epoch:

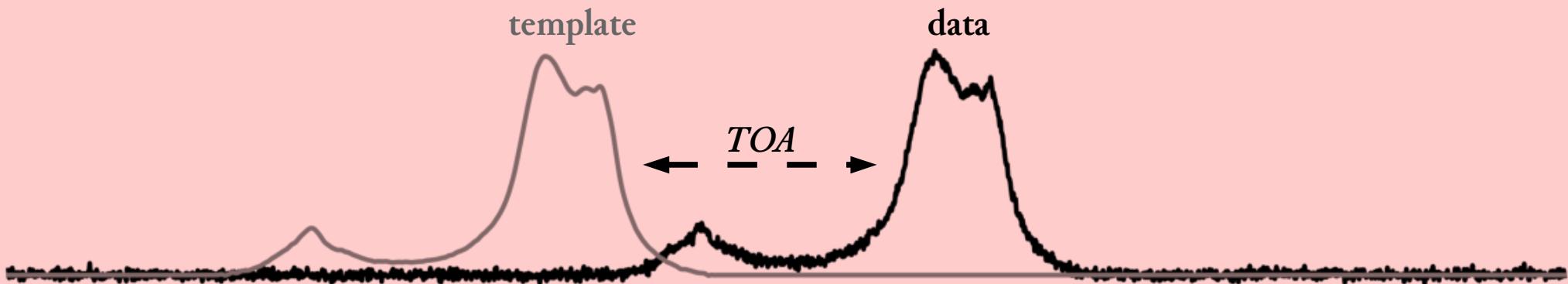


*(scale exaggerated here;
DM variation usually
induces ~phase bin level
difference)*

However, with great bandwidth comes great responsibility:

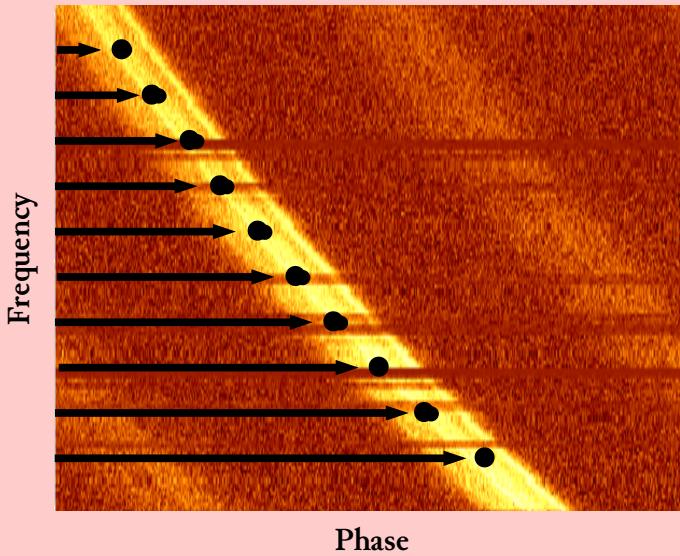
- TOAs are still the fundamental quantities for pulsar timing experiments
- TOAs are related to time offsets measured in via template-matching
- This was straightforward when it was acceptable to average over a narrow bandwidth, in which neither profile evolution nor $DM(t)$ were discernible

e.g.,



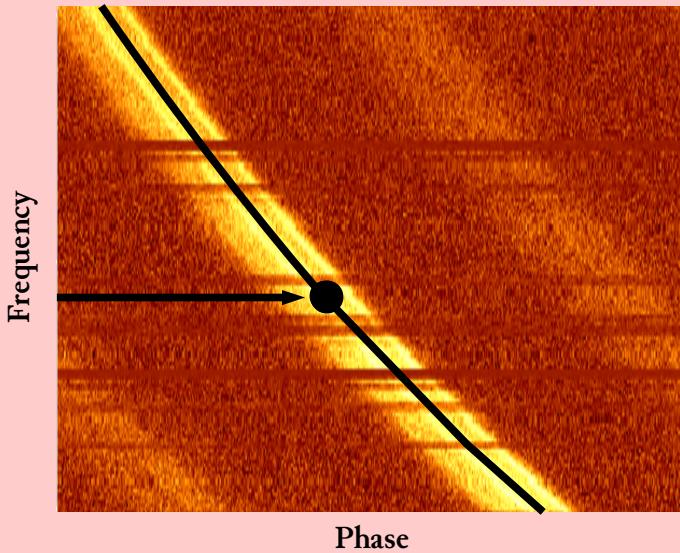
But! you sacrifice timing precision and can introduce bias by doing this!

“Conventional” or “Channelized” = 1 TOA per subintegration *per frequency subband*



(many TOAs! the same profile template is used across the band, mismatched to the data!)

“Wideband” (WB) = 1 TOA & 1 DM per subintegration

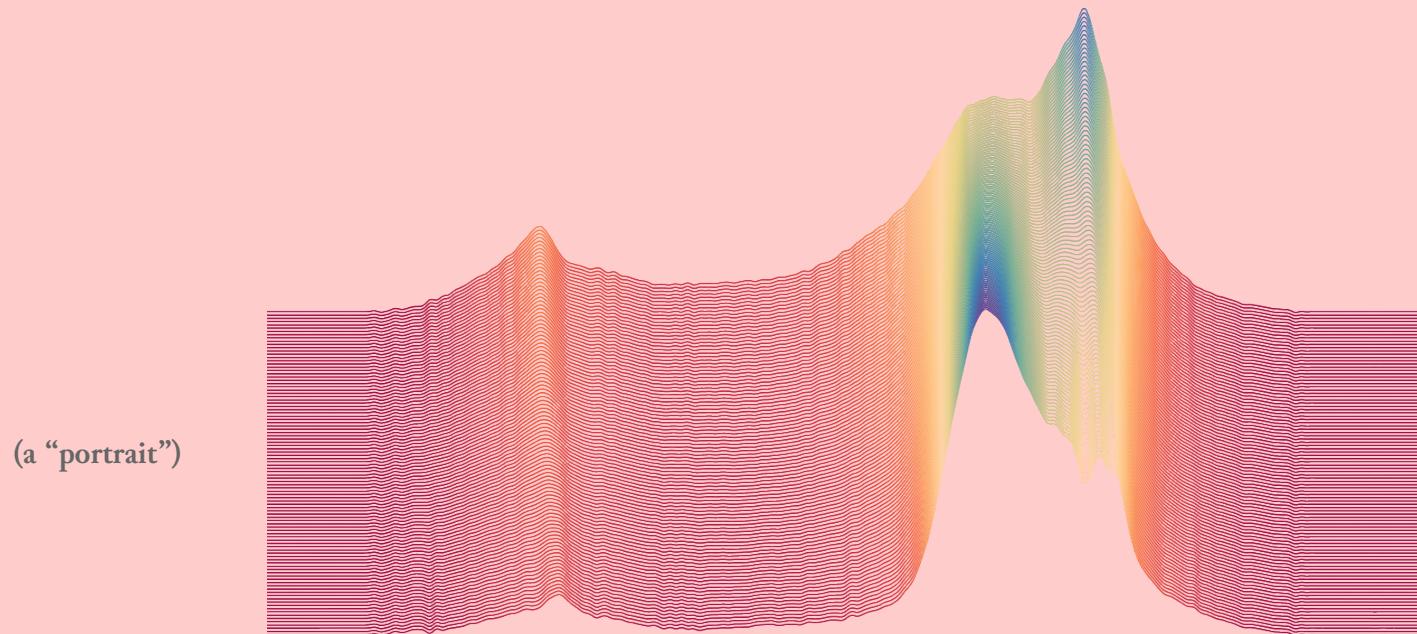


(one set of measurements using a model of profile evolution!)

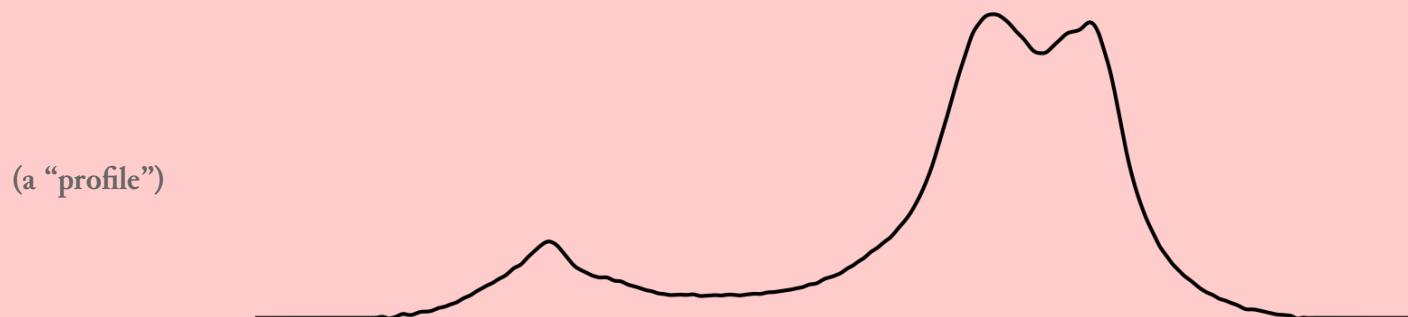
→ In this way, wideband datasets can be *much* smaller (10-30x)

Wideband TOA \neq Average of Conventional TOAs

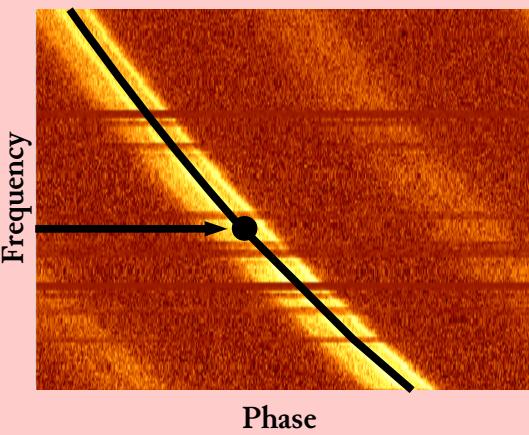
→ In summary: a fixed model of profile evolution substitutes for a single template profile & *ad hoc* timing model parameters
i.e., the wideband matched-template scheme uses something like this:



instead of this:



The “wideband” TOA likelihood



$$\chi^2(\phi_{ref}^\circ, \text{DM}, \tau, \alpha, a_n) = \sum_{n,k} \frac{|d_{nk} - a_n b_{nk} p_{nk} e^{-2\pi i k \phi_n}|^2}{\sigma'_n^2}$$

$$\phi_n(\phi_\circ, \text{DM}, \text{GM}) = \phi_\circ + \frac{K \times \text{DM}}{P_s} \left(\nu_n^{-2} - \nu_{\phi_\circ}^{-2} \right).$$

→ d and p now both have a frequency channel index n , which indexes the frequency channels ν_n !

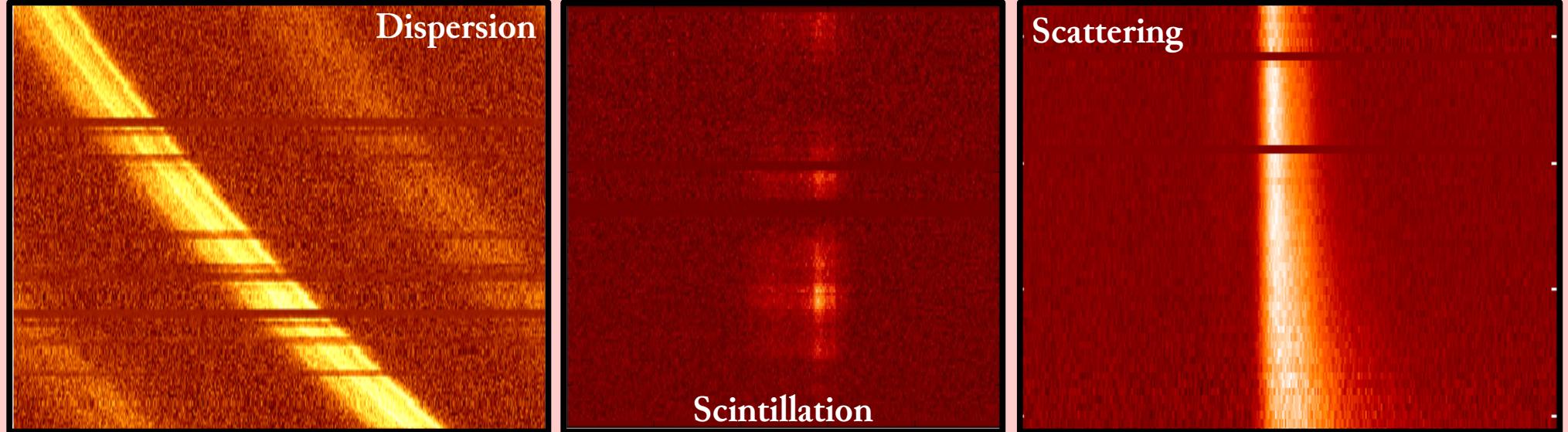
→ So does the phase offset term ϕ_n , but those offsets are constrained by the dispersive relation and is a function only of the achromatic offset ϕ_\circ (related to the TOA) and the DM!

a_n = scaling amplitude per channel (accounts for frequency-dependent intensity)

b_{nk} = Fourier Transform of ISM impulse response function (accounts for scattering)

→ Scattering usually looks like $\tau(\nu) = \tau_\circ (\nu/\nu_\circ)^\alpha$

Wideband timing method handles most common ISM effects:



Flowchart illustrating the data processing pipeline:

```

    graph TD
        DM[Dispersion Measure] --> Chi2[χ²(ϕᵣᵩ, DM, τ, α, aₙ)]
        Scint[Scintillation] --> Chi2
        Data[Data] --> Chi2
        TauAlpha[τ, α] --> Chi2
        Model[Model] --> Chi2
        PhiRef[ϕᵣᵩ, DM] --> Chi2
    
```

Annotations below the flowchart:

- TOA (Time of Arrival) is associated with the Dispersion Measure input.
- Scattering is associated with both the Scintillation input and the Noise input.
- channel index and Fourier harmonic are associated with the Data input.
- Noise is associated with the Data input.

$$\chi^2(\phi_{ref}^\circ, \text{DM}, \tau, \alpha, a_n) = \sum_{n,k} \frac{|d_{nk} - a_n b_{nk} p_{nk} e^{-2\pi i k \phi_n}|^2}{\sigma_n'^2}$$

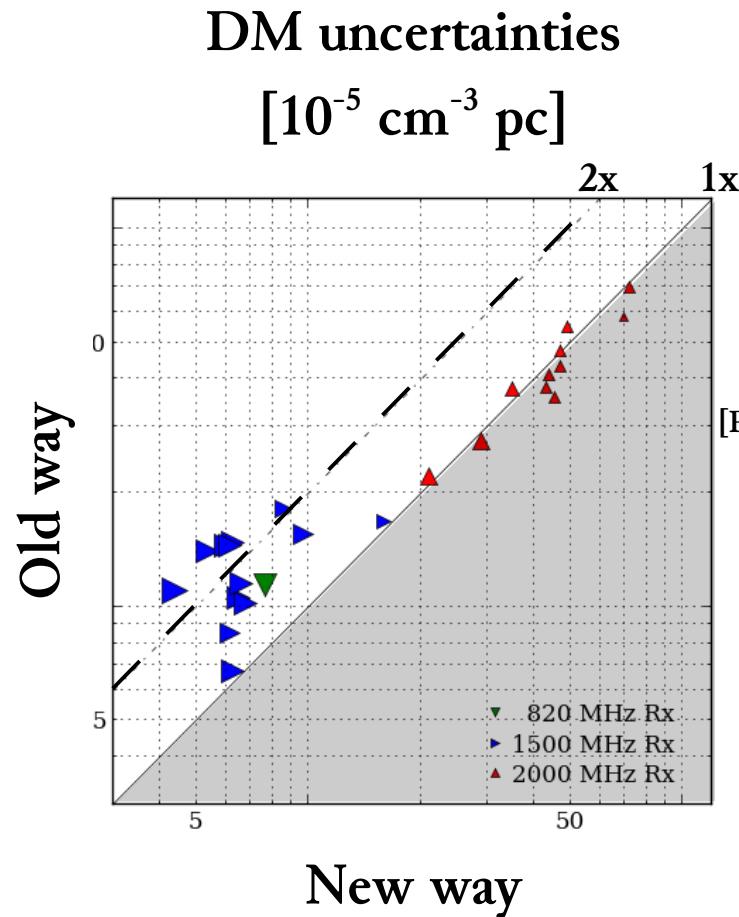
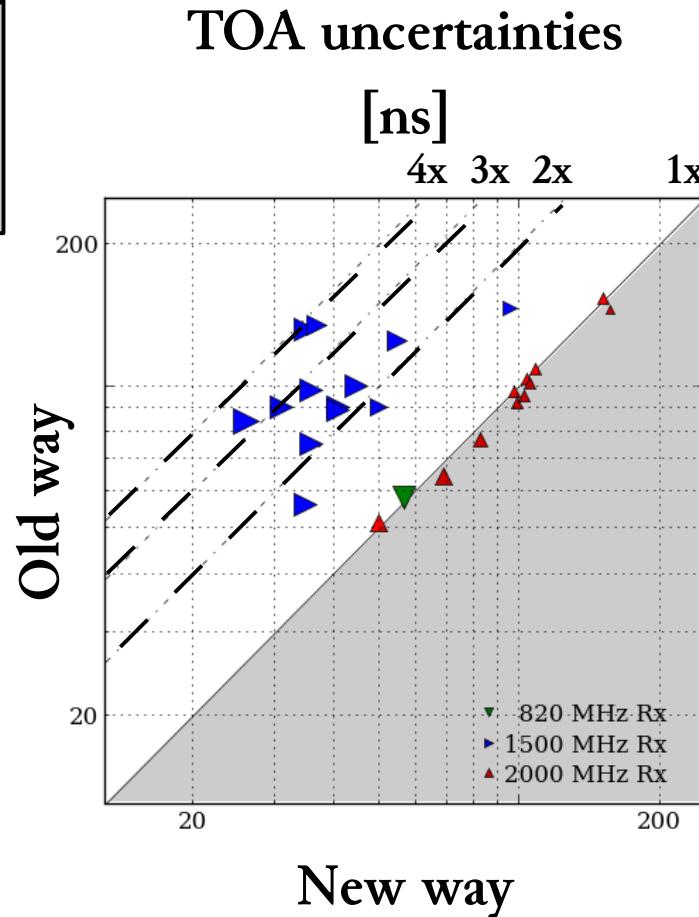
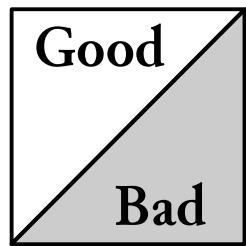
$$\phi_n(\phi_o, \text{DM}, \text{GM}) = \phi_o + \frac{K \times \text{DM}}{P_s} \left(\nu_n^{-2} - \nu_{\phi_o}^{-2} \right)$$

→ NB: Completely agnostic about choice of profile and its evolution (d_{nk}) as well as ISM PBF (b_{nk}) !!! ←

→ Scattering as function of time may be interesting in era of CHIME / new ultra-WB receivers

→ Now possible to also fit for ν^{-4} delay parameter as part of ϕ_n

Reduced Measurement Uncertainties

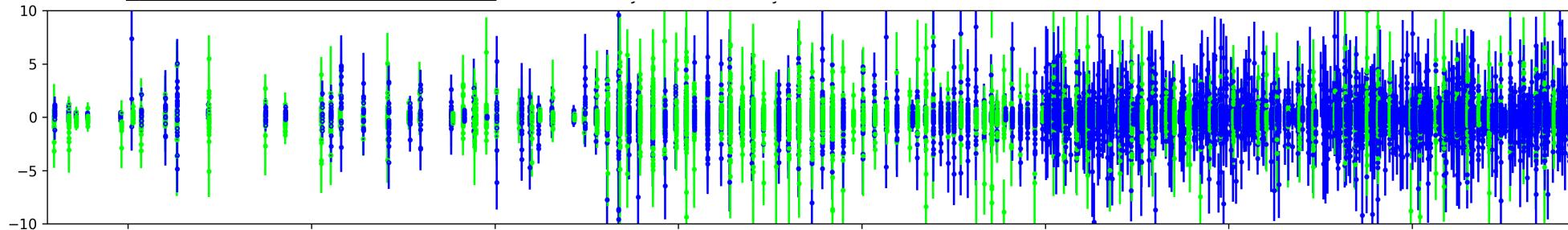


- By modeling pulse profile evolution, the uncertainties of the TOA and DM measurements are improved
- These leads to better timing precision

Example 12.5y data for J1909-3744 (GBT – GASP+GUPPI)

“conventional” = 23,128 TOAs

Residual [μs]

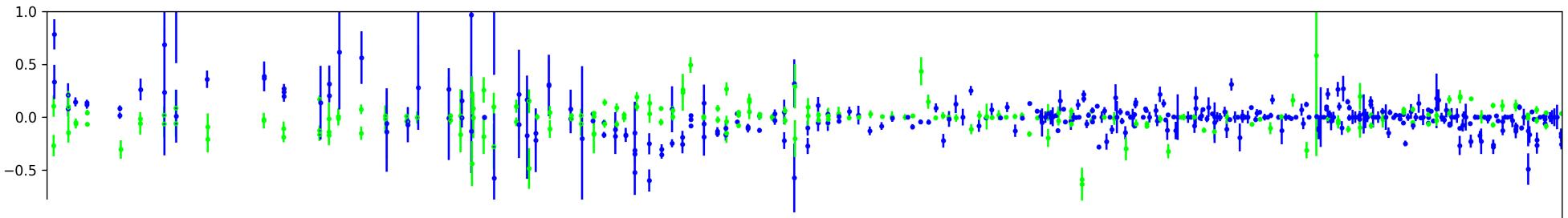


1500 MHz

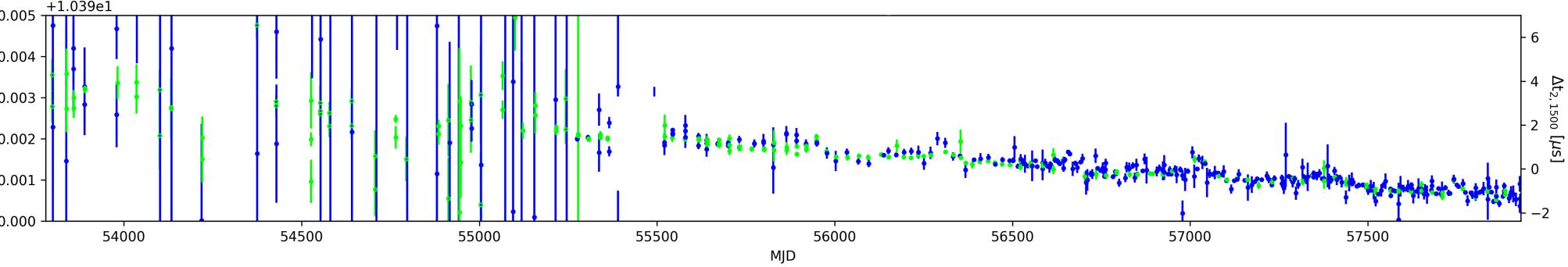
820 MHz

“wideband” = 558 TOAs + 557 DMs

Residual [μs]

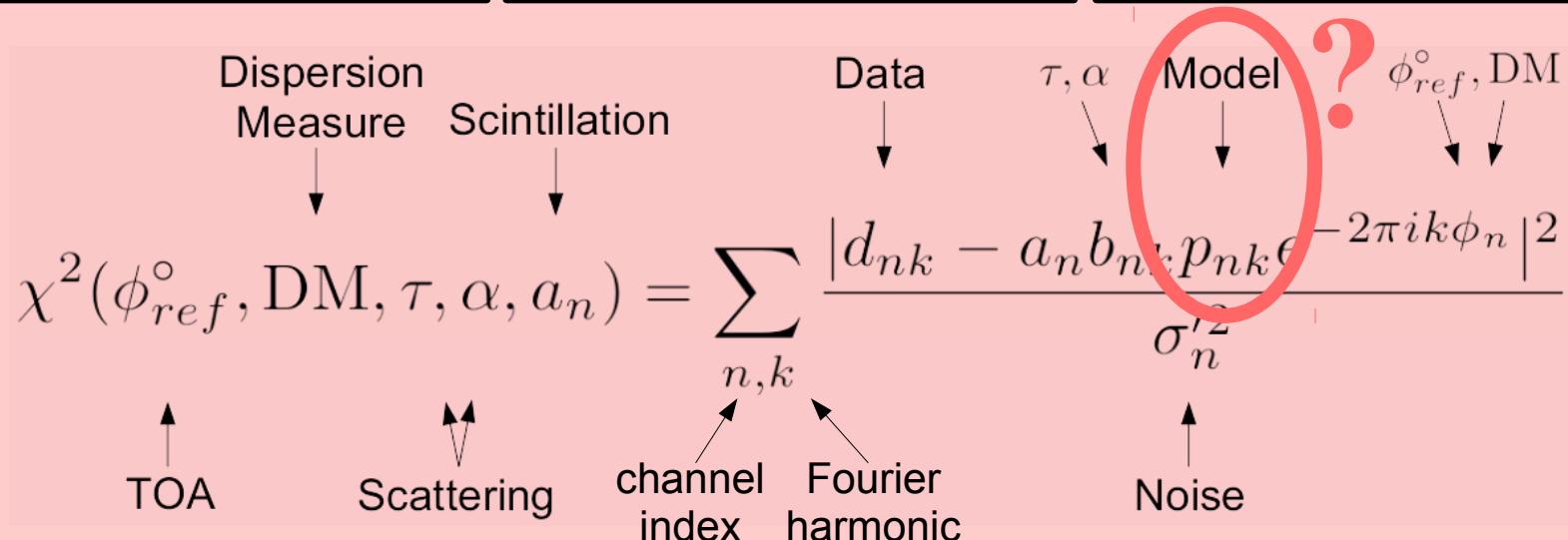
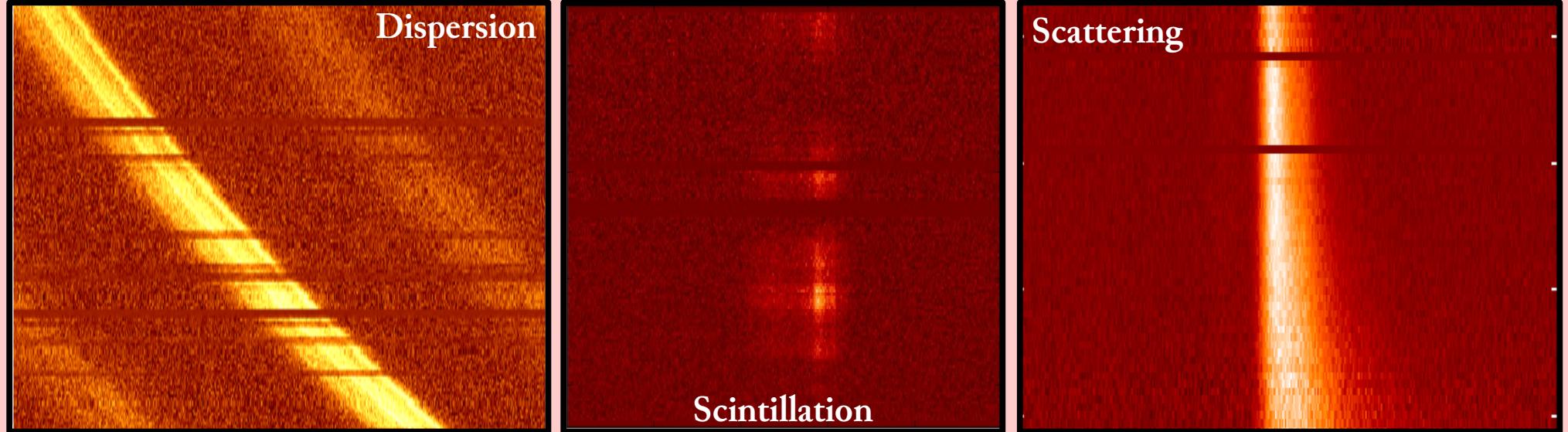


DM [$\text{cm}^{-3} \text{ pc}$]



- The task now is to modify the timing & GW analyses to incorporate these new DM measurements
- GW analyses run *much* faster with reduced data volume

Wideband timing method handles most common ISM effects:



- NB: Completely agnostic about choice of profile and its evolution (d_{nk}) as well as ISM PBF (b_{nk}) !!! ←
- Scattering as function of time may be interesting in era of CHIME / new ultra-WB receivers
- Now possible to also fit for v^{-4} delay parameter as part of ϕ_n

POP QUIZ 4

These “wideband” timing methods help to solve problems having to do with which of the following:

- A) Profile evolution
- B) The interstellar medium
- C) Data volume
- D) All of the above

“So how do I make a noise-free profile template?”

“So how do I make a noise-free profile template?”

Simplest:

- (1) Use basic analytic function (e.g., Gaussian shape) to describe the pulse shape

-or-

- (2) Average the data profiles together and smooth them

*“So how do I make a noise-free profile template
that evolves with frequency?”*

*“So how do I make a noise-free profile template
that evolves with frequency?”*

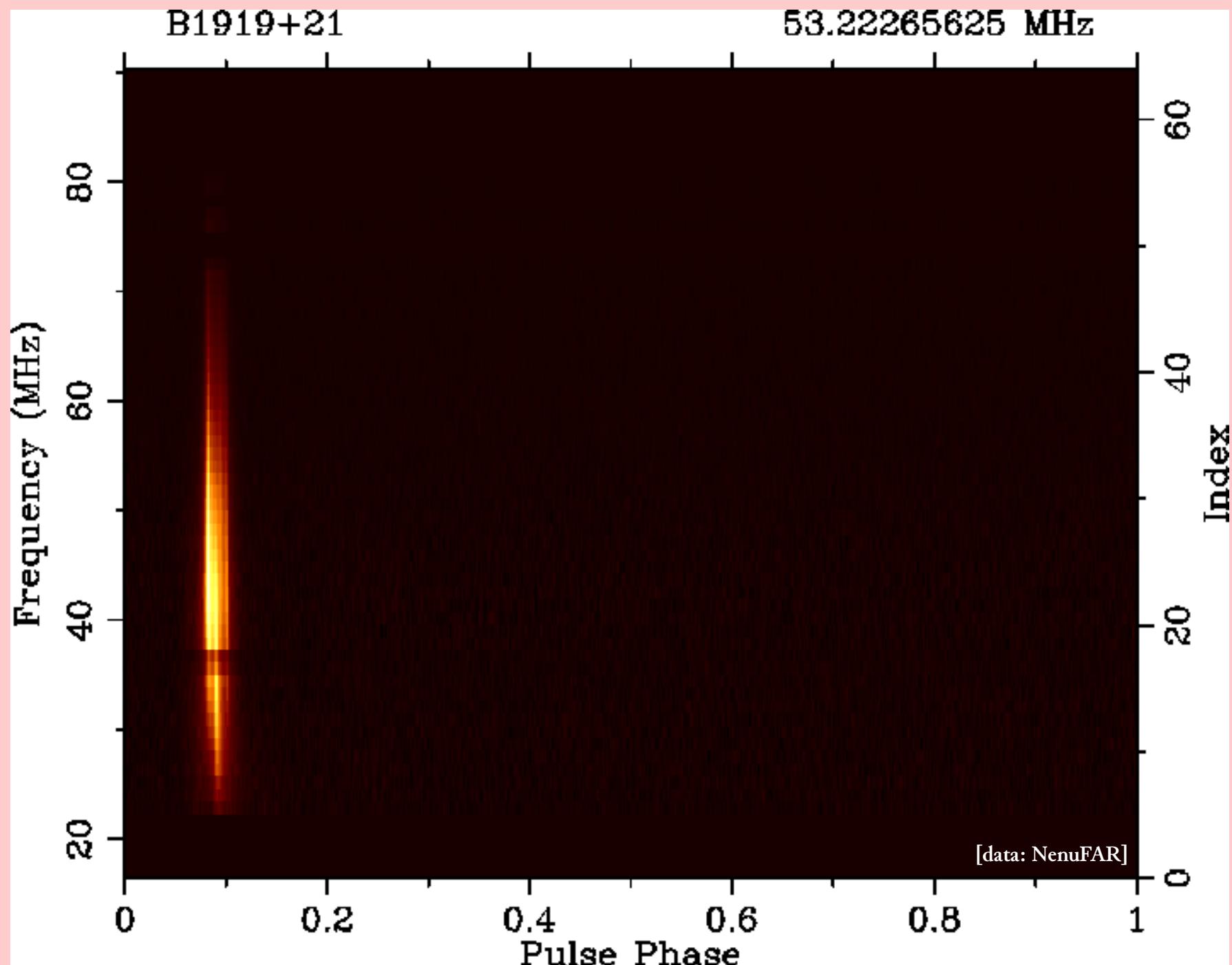
(1) Could use evolving Gaussians

-or-

(2) Come up with something data-driven, better suited for general purposes

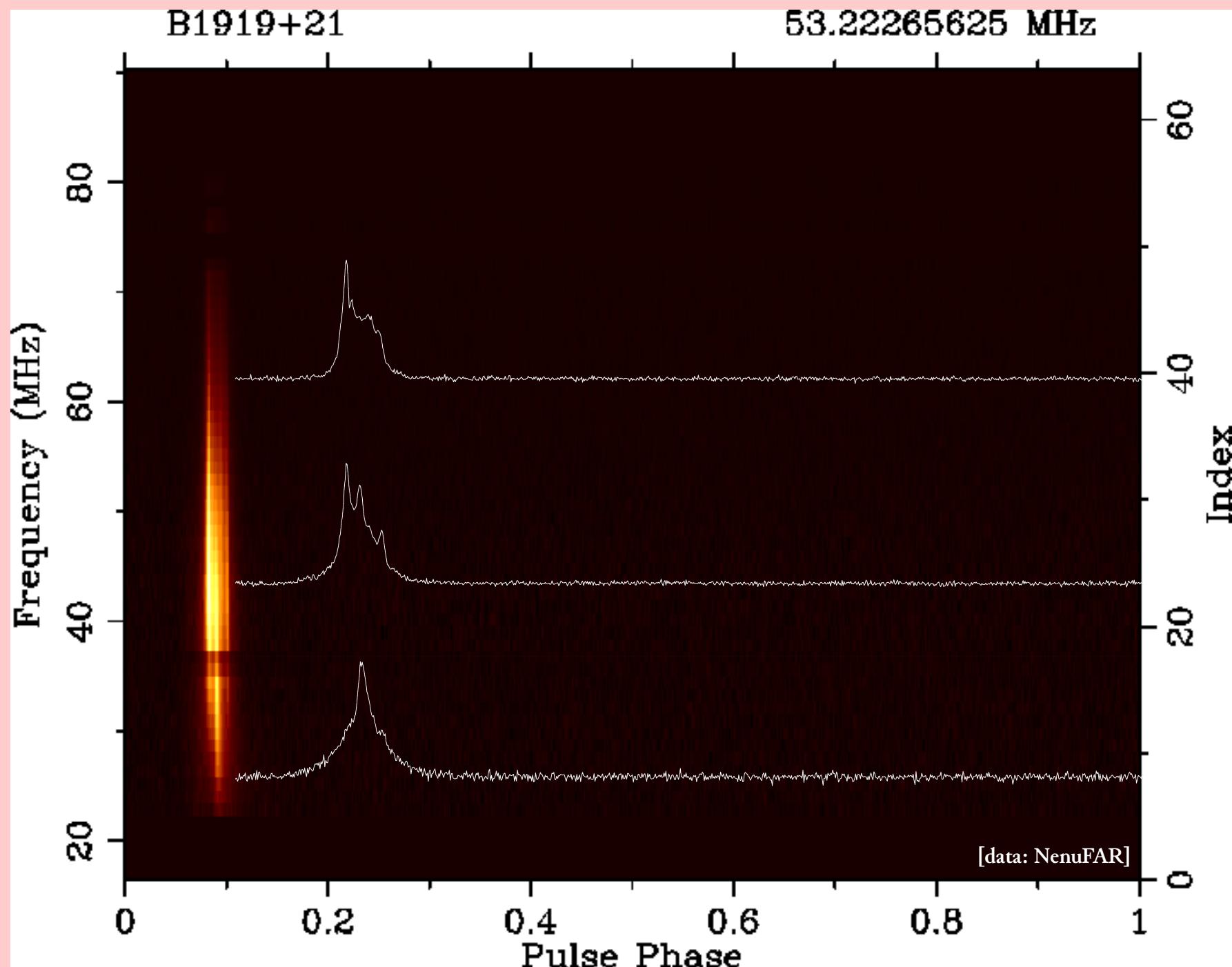
Bleeding-edge example:

→ B1919+21, the *original* pulsar (“LGM 1”) observed near its original detection frequency, from NenuFAR



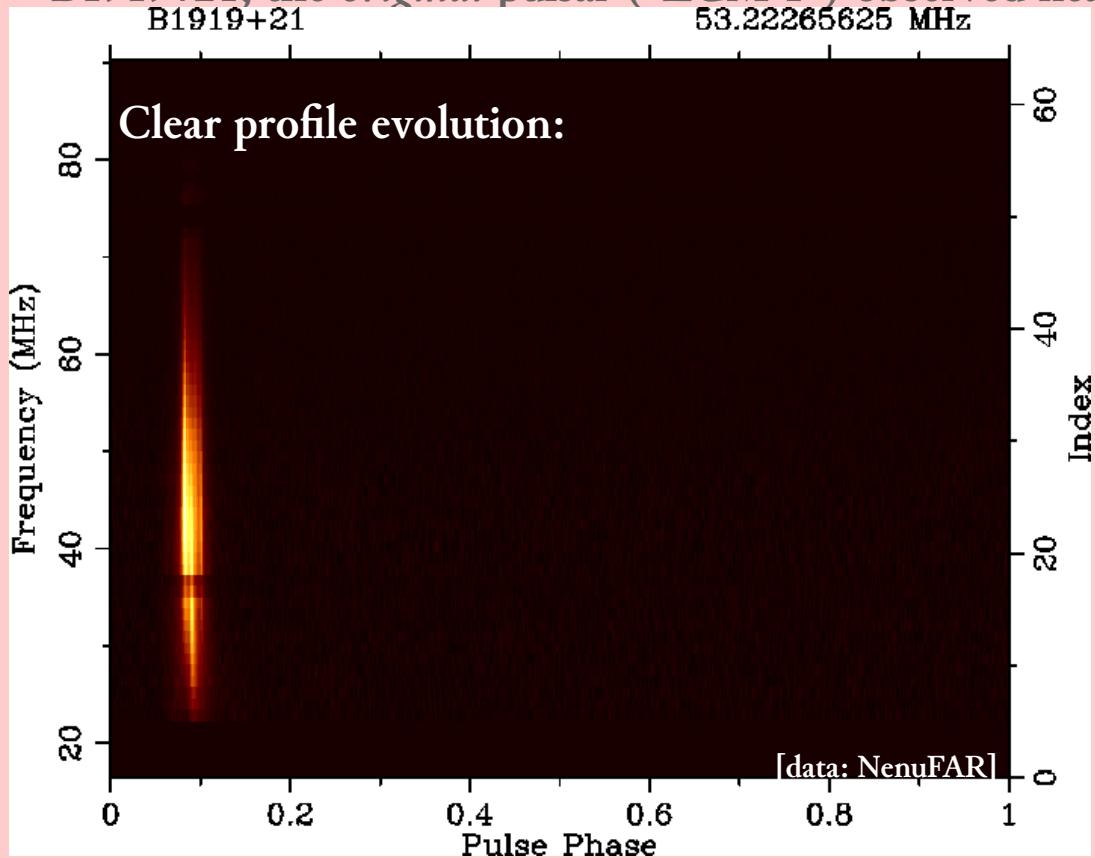
Bleeding-edge example:

→ B1919+21, the *original* pulsar (“LGM 1”) observed near its original detection frequency, from NenuFAR

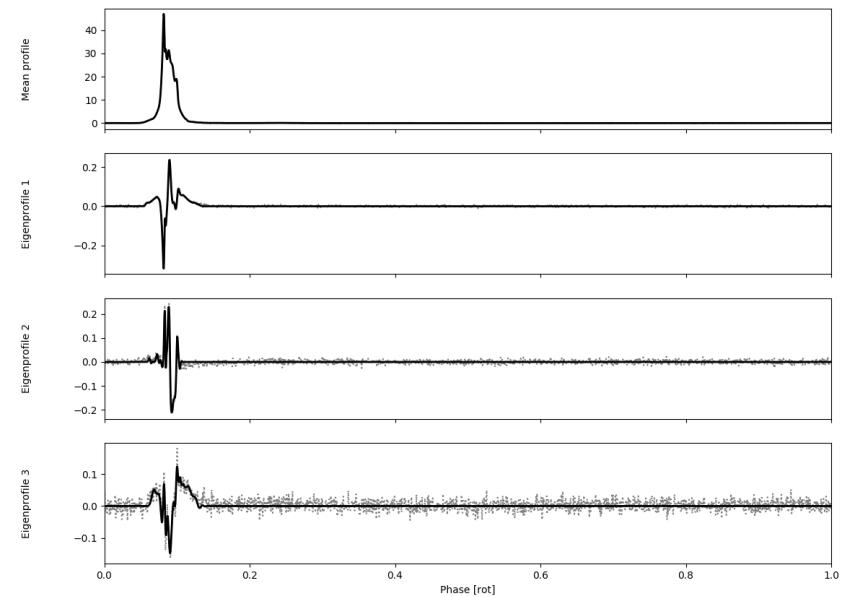


Bleeding-edge example:

→ B1919+21, the *original* pulsar (“LGM 1”) observed near its original detection frequency, from NenuFAR



PCA decomposition:

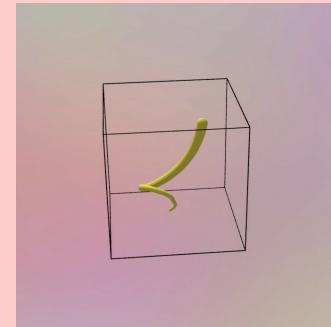


Movie of model:

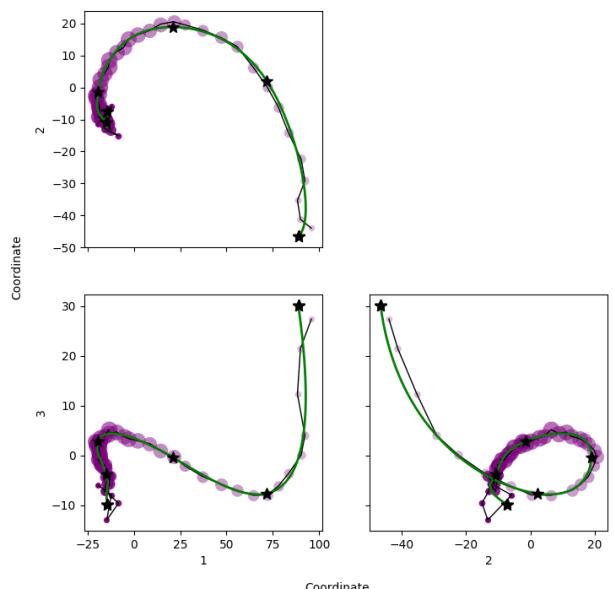
→ Method recently accepted for publication in *ApJ*, Pennucci (2019)

→ Bottom line: high-fidelity templates give better timing results

[movie: A. Bilous]



Spline interpolation model:



Profile-Domain Timing

Wide-band Profile Domain Pulsar Timing Analysis

L. Lentati^{1*}, M. Kerr², S. Dai², M. P. Hobson¹, R. M. Shannon^{2,3}, G. Hobbs², M. Bailes⁴, N. D. Ramesh Bhat⁵, S. Burke-Spolaor⁵, W. Coles⁶, J. Dempsey⁷, P. D. Lasky⁸, Y. Levin⁸, R. N. Manchester², S. Oslowski^{4,9,10}, V. Ravi¹¹, D. J. Reardon⁸, P. A. Rosado⁴, R. Spiewak^{4,12}, W. van Straten⁴, L. Toomey², J. Wang¹³, L. Wen¹⁴, X. You¹⁵, X. Zhu¹⁴

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¹³ Xinjiang Astronomical Observatory, Chinese Academy of Sciences, 150 Science 1-Street, Urumqi, Xinjiang 830011, China

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¹⁵ School of Physical Science and Technology, Southwest University, Chongqing, 400715, China

24 December 2016

ABSTRACT

We extend profile domain pulsar timing to incorporate wide-band effects such as frequency-dependent profile evolution and broadband shape variation in the pulse profile. We also incorporate models for temporal variations in both pulse width and in the separation in phase of the main pulse and interpulse. We perform the analysis with both nested sampling and Hamiltonian Monte Carlo methods. In the latter case we introduce a new parameterisation of the posterior that is extremely efficient in the low signal-to-noise regime and can be readily applied to a wide range of scientific problems. We apply this methodology to a series of simulations, and to between seven and nine yr of observations for PSRs J1713+0747, J1744–1134, and J1909–3744 with frequency coverage that spans 700–3600 MHz. We use a smooth model for profile evolution across the full frequency range, and compare smooth and piecewise models for temporal variations in DM. We find the profile domain framework consistently results in a precision compared to the standard analysis paradigm by as much as 40% for a given smoothness in the DM variations into the model further than we attribute to variation intrinsic to the pulses. Not accounting for this shape change of magnitude

Profile-Domain Timing

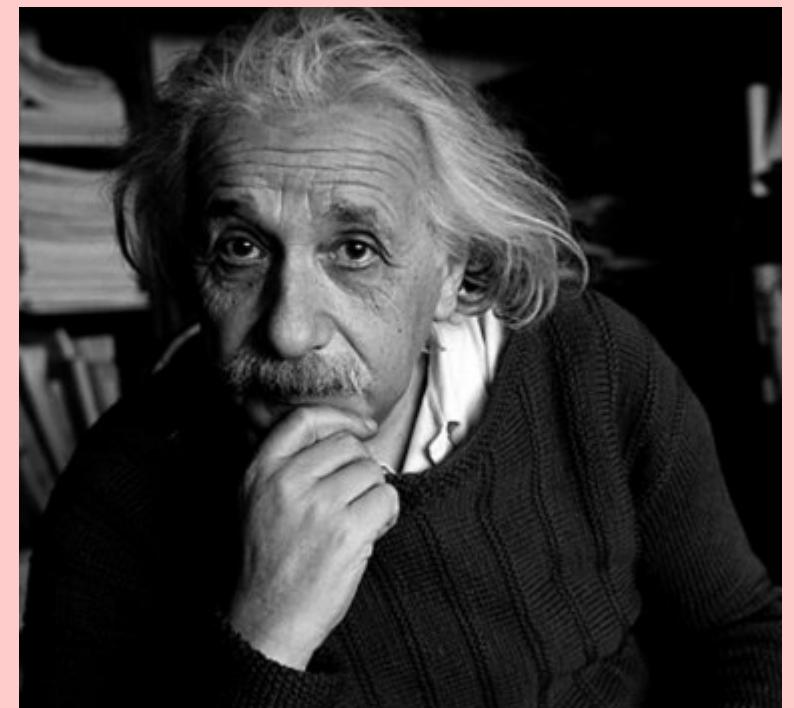
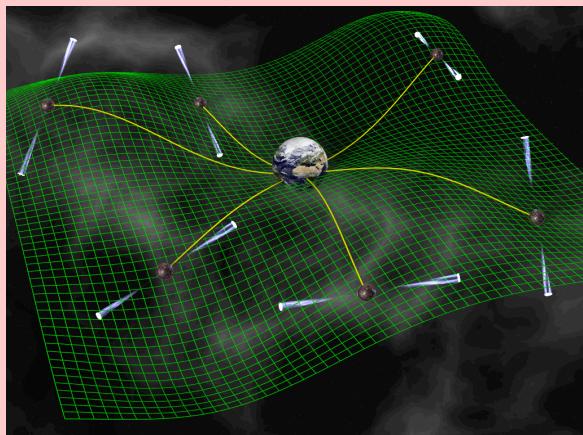
→ Several papers by Lentati et al., but other work by van Haasteren, too.

→ Take-home message is that clever people are looking for ways to obviate the need to measure TOAs as an intermediate step between obtaining the pulse profiles (which have time stamps) and pulsar timing models, noise analyses, and gravitational wave detection

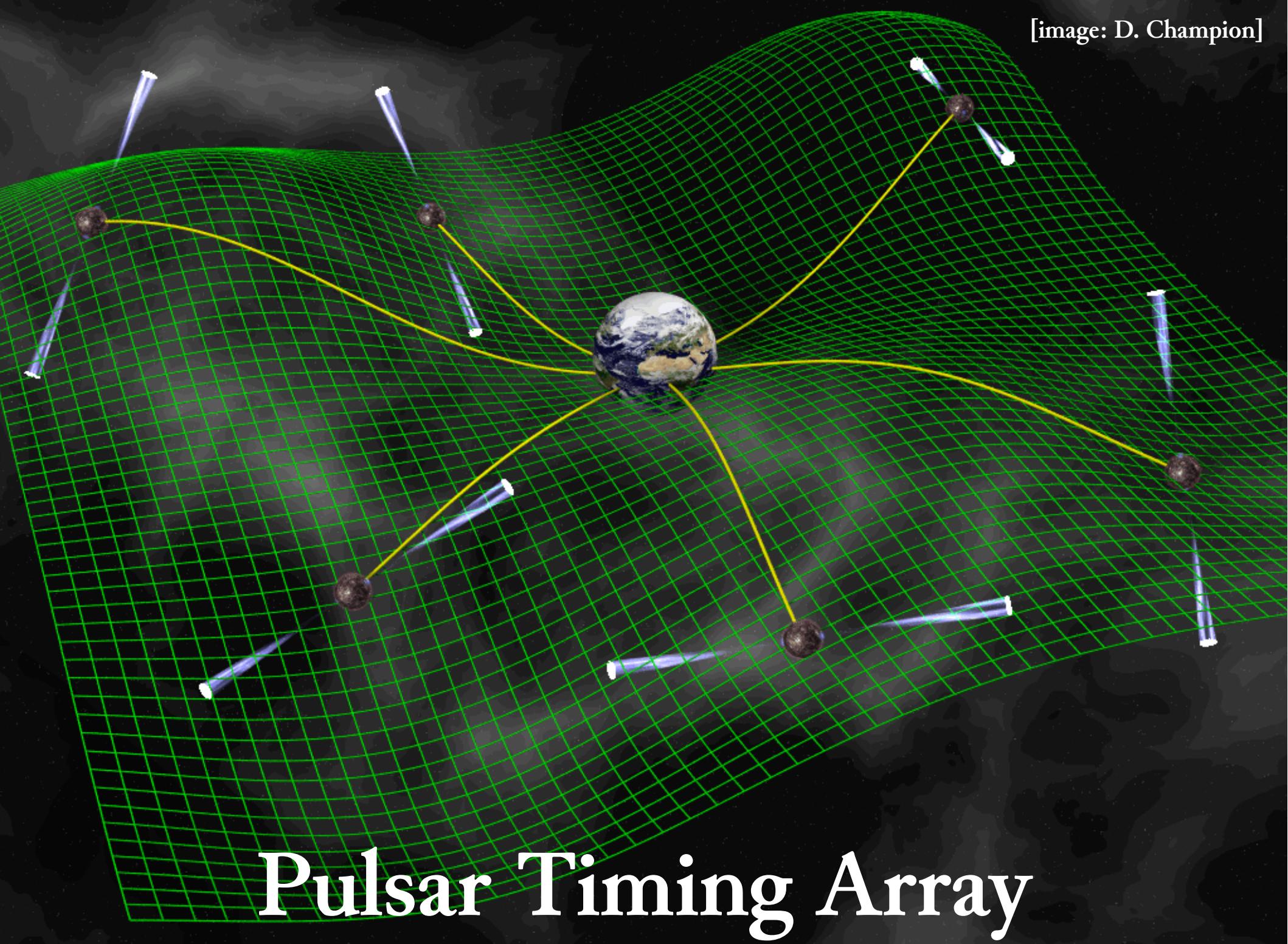
→ This works in a giant Bayesian likelihood framework which requires lots of computation

→ Fabulous idea, but it looks like practical TOAs will be around for some time more...

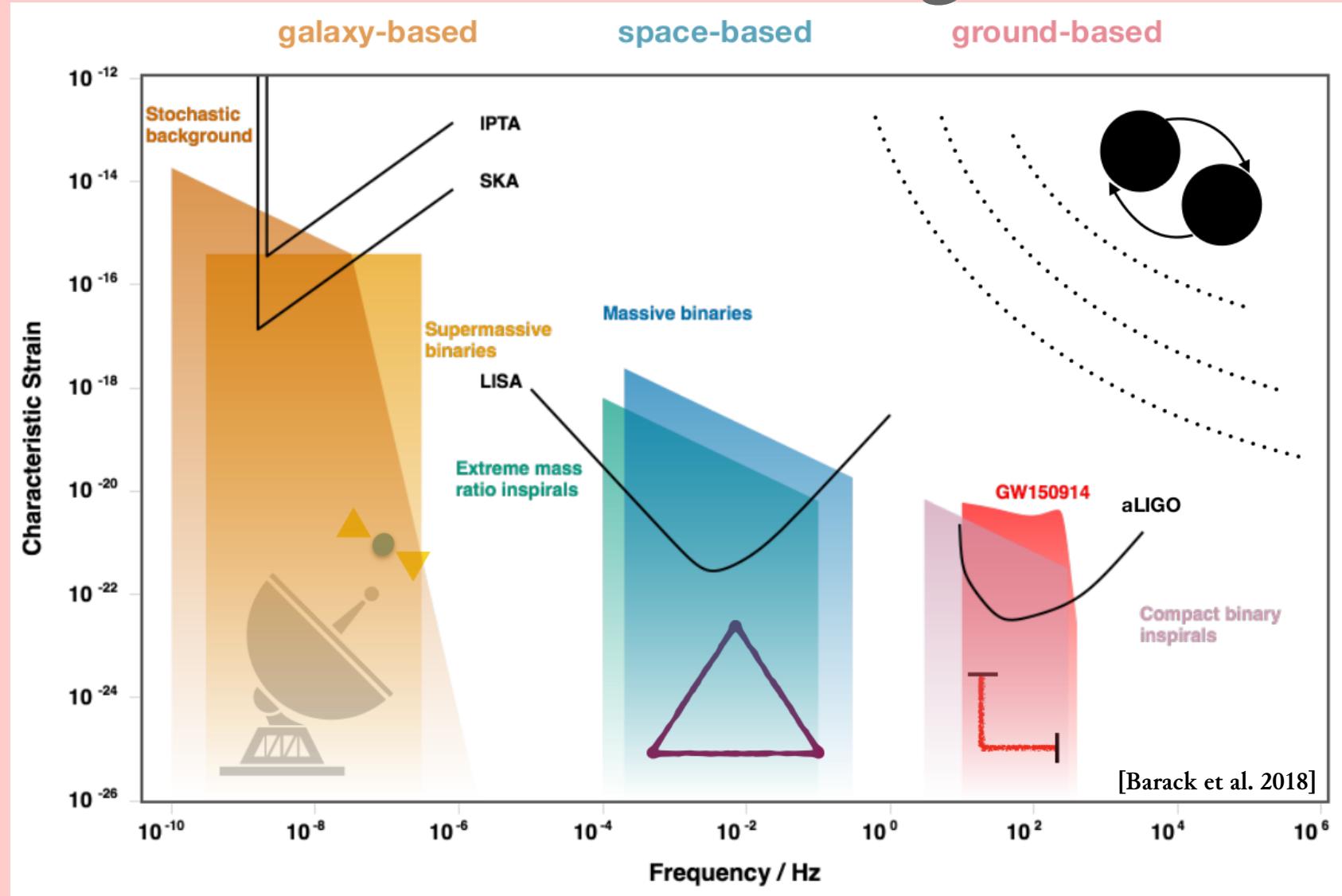
The PTA: A beautiful experiment...



[image: D. Champion]

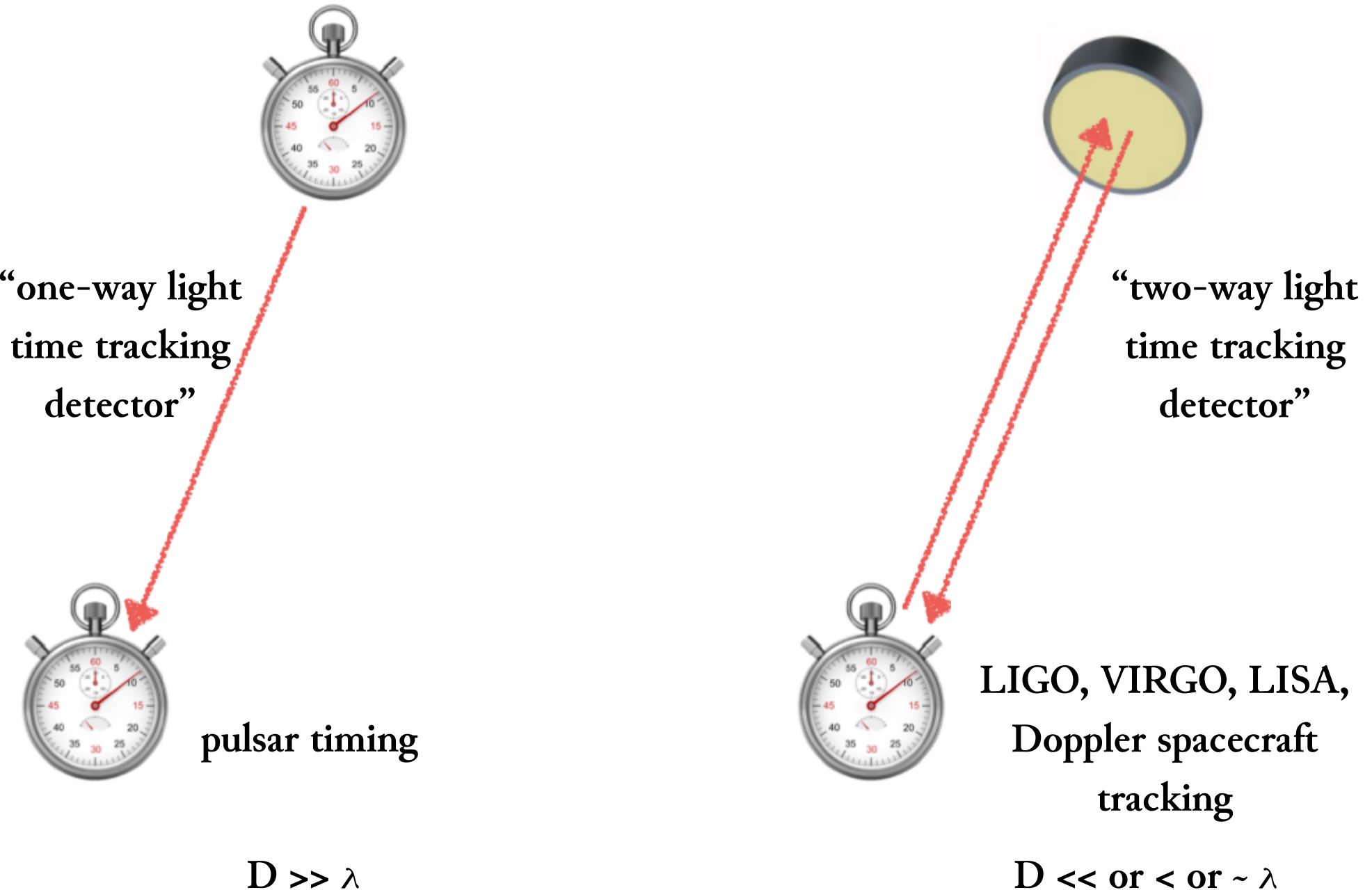


GW detection at a glance



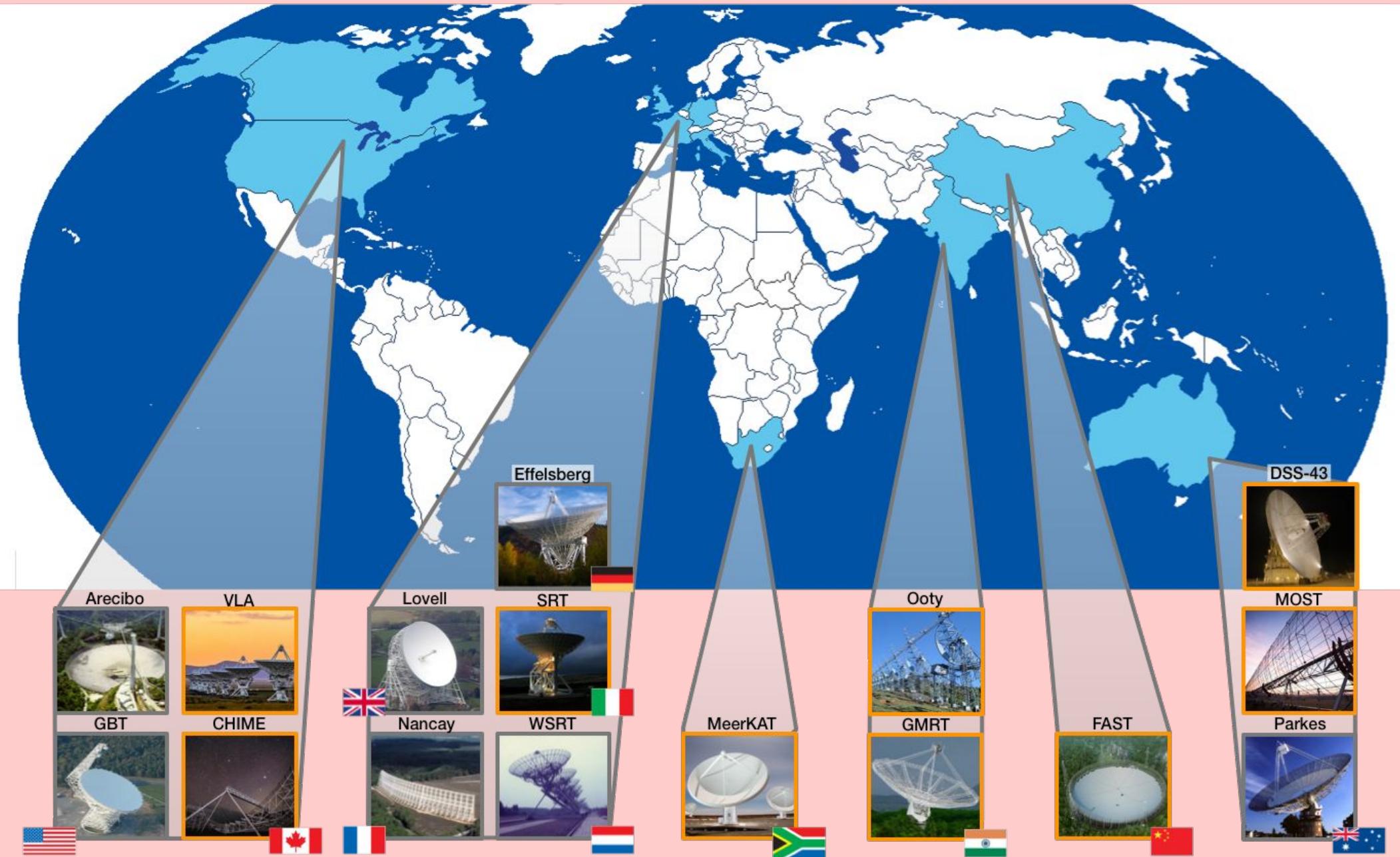
- Strain sensitivity determined by ~ timing stability of pulsar / experiment duration
- Frequency range bounded by ~ 1/experiment duration and ~ 1/sampling time

Quick comparison:



International Pulsar Timing Array (IPTA)

(www.ipta4gw.org)

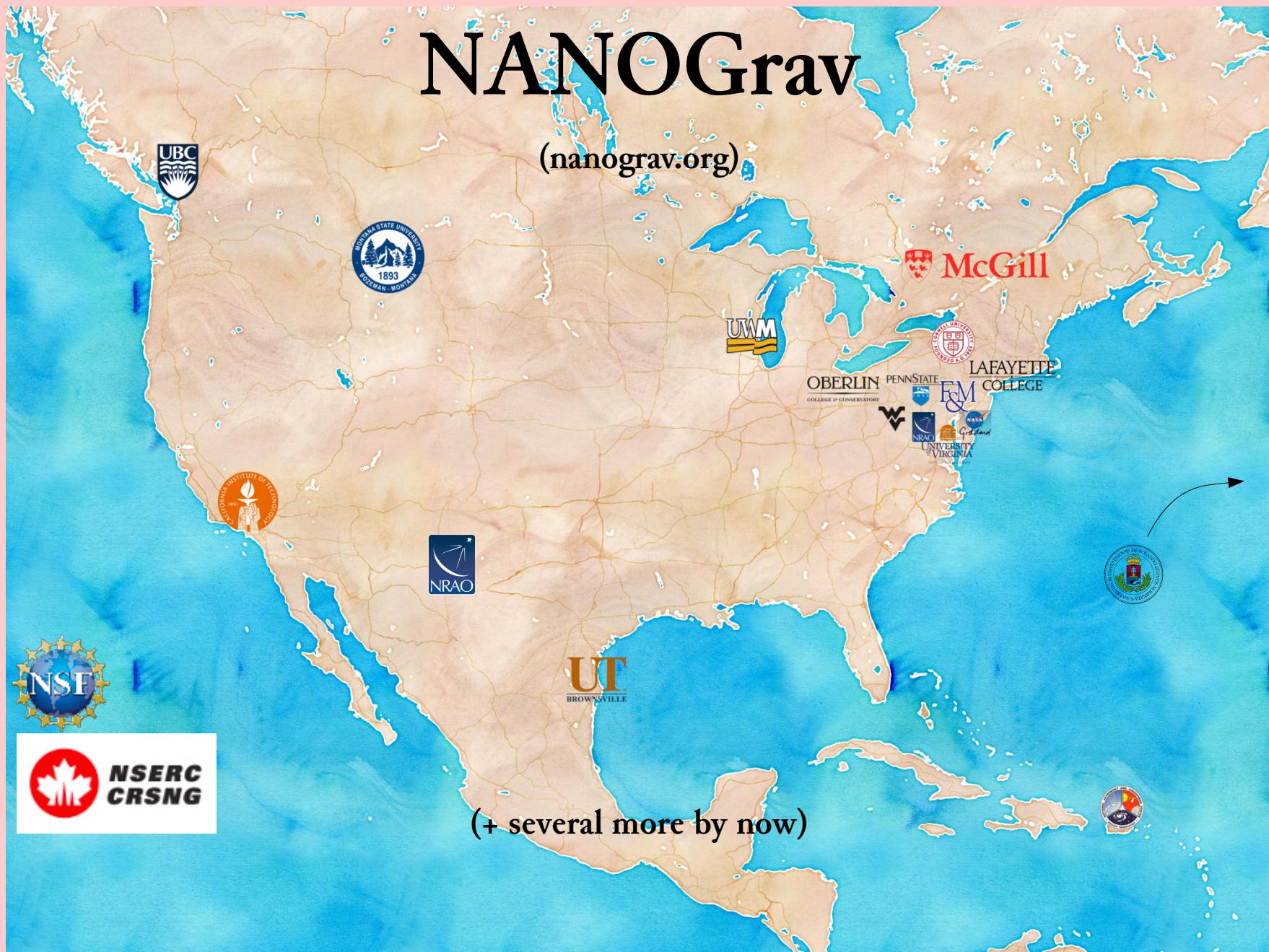




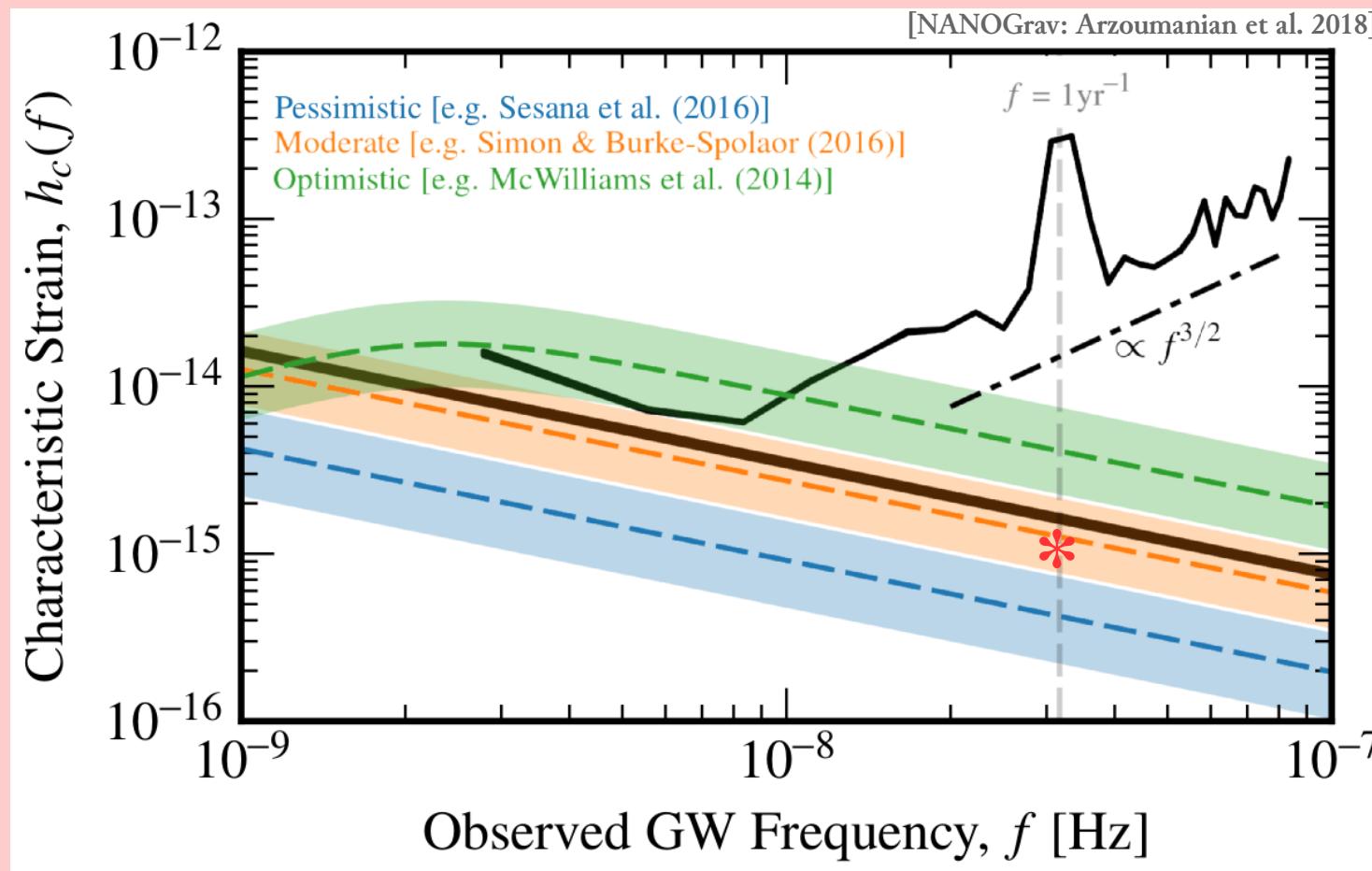
NORTH AMERICAN NANOHERTZ OBSERVATORY FOR GRAVITATIONAL WAVES

NANOGrav

(nanograv.org)

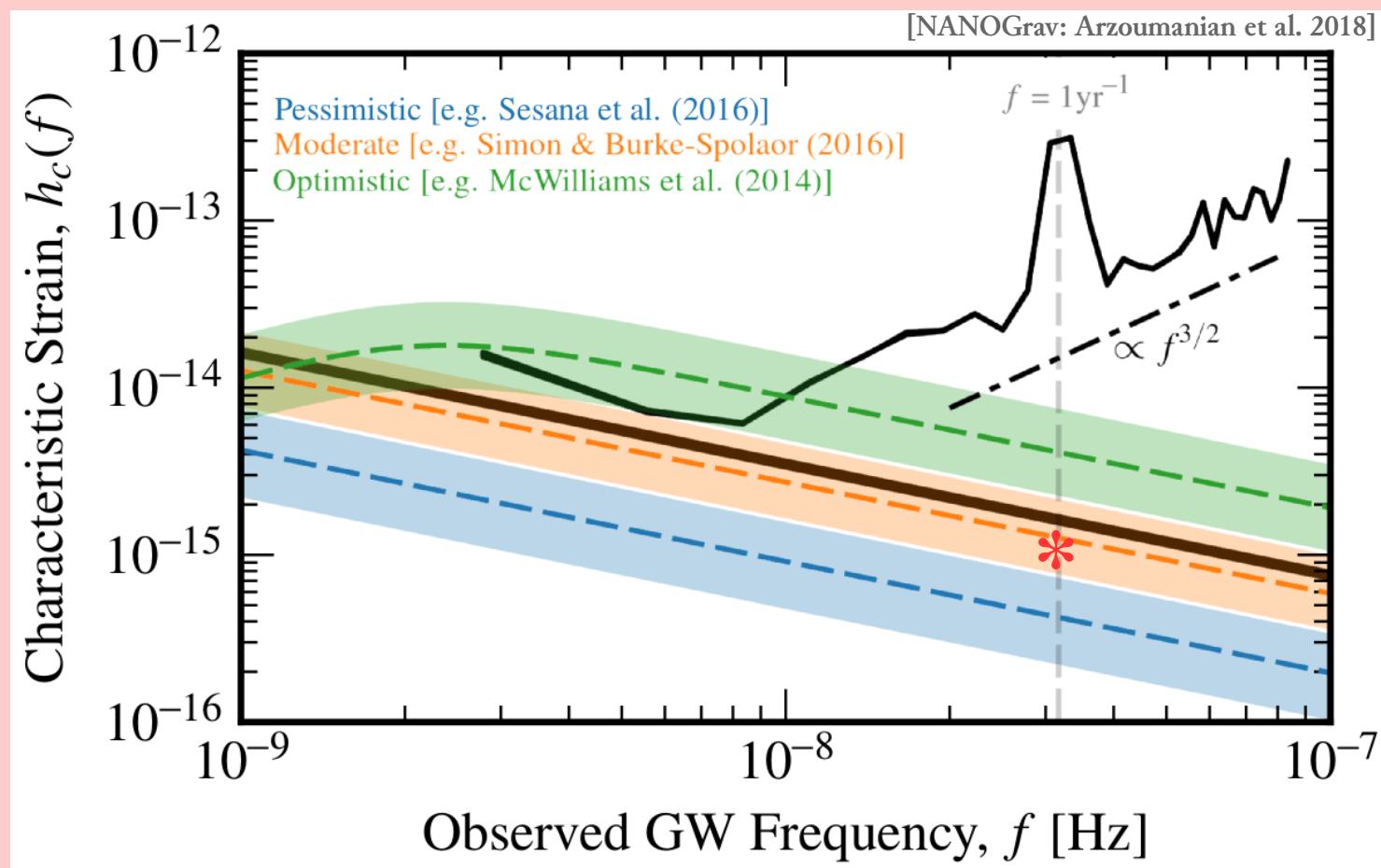


A recent PTA upper-limit plot



→ After accounting for Solar System ephemeris uncertainties, we find $A_{\text{GWB}} < 1.45 \times 10^{-15}$ @ frequency of 1 yr^{-1}

POP QUIZ 5



Why is there no sensitivity at a frequency of 1 yr^{-1} ?
[Hint: something to do with the timing model...]

Other Pulsar Timing Use-Cases:

- More about Pulsar Timing Arrays and nanohertz gravitational wave detection & astrophysics (that's why we're here!)
→ *see Andrea Lommen's talk Wednesday 14:30 – 15:30!*
- Probing/investigating/testing/studying the ISM, interiors of neutron stars, plasma physics, binary evolution, gravity & gravitational waves, SMBHB environments, planetary systems, time standards, supernova explosions, solar physics, stellar populations, interstellar navigation, etc, etc, etc.
→ *see Maura McLaughlin's talk Friday 09:30 – 11:00!*