Instituto de Optica (CSIC)

Image Processing and Vision Modeling Group

CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Log Gabor filtering

1.Abstract

Firstly proposed by Dennis Gabor in 1946 [1], the canonical coherent states of the Gabor filters are different versions of a Gaussian-shaped window shifted in time/space and frequency variables [2] [3], Gabor's work synthesizes the studies of Nyquist in Communication Theory in 1924 [4] and Heisenberg in Quantum Mechanics in 1927, by which he proposed the Gaussian shape as an optimal envelope for time-frequency representation turning the uncertainly principle from inequality into equality.

Some important characteristics of Gabor wavelets are [5]:

- Construction by a linear combination.
- Energy preservation in transform domain (Parseval's theorem).
- Non-orthogonality but an unconditional basis, a frame [6].
- Symmetry of the Fourier domain.
- Time/space and frequency shift-invariance.
- Localization: monomodal and isotropic.
- Regularity: smooth and infinitely derivable.
- Directionality: filters can be rotated to discriminate spectral features in multiples directions (orthogonal wavelets have well-known difficulties to discriminate more than three orientations).
- Complex modulation (odd/even phases): effective for analyzing different phased features like abrupt ridges or edges.

2.Self-invertible log-Gabor filters

One step further, the recent filter design proposed by Sylvain Fischer [22] come to solve some of the traditional disadvantages that have complicated the functionality of Gabor filters. Log-Gabor filters basically consist in a logarithmic transformation of the Gabor domain [23] which eliminates the annoying DC-component allocated in medium and high-pass filters.

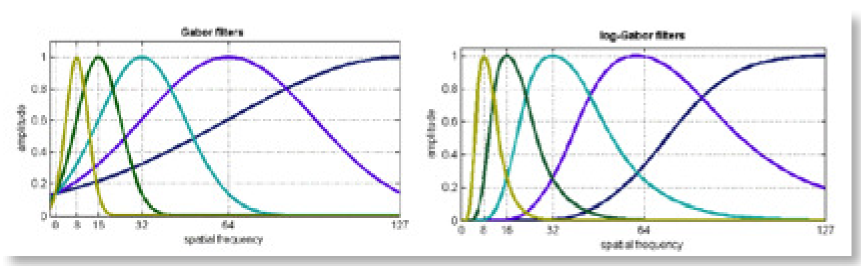

Fig. 1: Profiles of the frequency response of Gabor filters (left) and log-Gabor filters (right). Note that DC-component is minimized by the introduction of logarithms and the bands become more disjoined

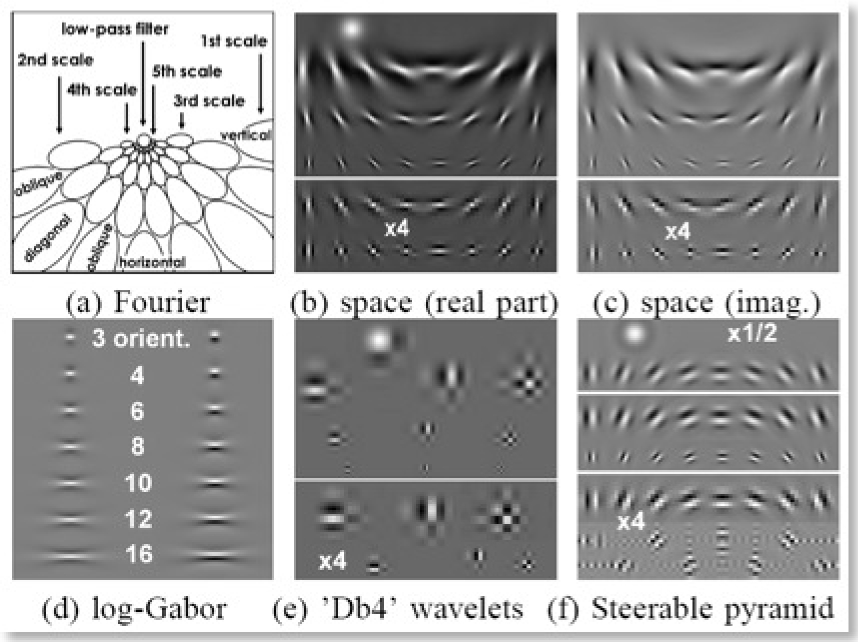

Fig. 2: Multiresolution schemes. **a.** Schematic contours of the log-Gabor filters in the Fourier domain with 5 scales and 8 orientations **b.** Real part in the spatial domain. **c.** Imaginary part in the spatial domain. **d.** In the proposed scheme the elongation of log-Gabor wavelets increases with the number of orientations (real parts in the left column and imaginary parts in the right column). **e. O**rthogonal wavelet filters 'Db4' for comparison. **f.** Steerable pyramid filters for comparison.

$$G_{pk} = G(\rho,\theta,p,k) = \exp\left(-\frac{1}{2}\left(\frac{\rho-\rho_k}{\sigma_\rho}\right)^2\right)\exp\left(-\frac{1}{2}\left(\frac{\theta-\theta_{pk}}{\sigma_\theta}\right)^2\right)$$

$$\text{with}\quad \begin{cases} \rho_k = \log_2(n) - k \\ \theta_{pk} = \begin{cases} \frac{\pi}{p}p & \text{if } k \text{ is odd} \\ \frac{\pi}{p}(p+\frac{1}{2}) & \text{if } k \text{ is even} \end{cases} \\ (\sigma_\rho, \sigma_\theta) = 0.996(\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{2}}\frac{\pi}{p}) \end{cases}$$

in which (ρ,θ) are the log-polar coordinates (octave scales), k indexes the scale and p is the orientation, the pair (ρ k; θ pk) corresponds to the frequency center of the filters, and (σρ; σθ) the angular and radial bandwidths. The main particularity of this novel scheme is the construction of the low-pass and high-pass filters [22]. Such a complete scheme approximates flat frequency response and therefore exact image reconstruction which is obviously beneficial for applications in which inverse transform is demanded, such as texture synthesis, image restoration, image fusion or image compression.

DOWNLOAD

Download Matlab programs to compute Log-Gabor filters. Please cite our papers [1-2] if you find this software useful.

- Log-Gabors filters package

REFERENCES

[1] Fischer, S., Cristóbal G., and Redondo, R., "Sparse overcomplete Gabor Wavelet representation based on local competitions", IEEE Trans. on Image Processing, 15, pp. 265-272, 2006

[2] Fischer, S., Sroubek, F., Perrinet, L., Redondo, R. and Cristóbal, G., "Self invertible Gabor wavelets", International Journal of Computer Vision, 75, pp. 231-246, 2007

[3]Fischer, S., Redondo, R. and Cristobal, G. "How to construct Log-Gabor filters", Open Access Digital CSIC Document, 2009

COPYRIGHT NOTICE
