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CLIMATE CHANGE AROUND THE WORLD

Per Krusell
Anthony A. Smith Jr.

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ABSTRACT

The economic effects of climate change vary across both time and space. To study these effects, this paper builds a global economy-climate model featuring a high degree of geographic resolution. Carbon emissions from the use of energy in production increase the Earth's (average) temperature and local, or regional, temperatures respond more or less sensitively to this increase. Each of the approximately 19,000 regions makes optimal consumption-savings and energy-use decisions as its climate (or regional temperature) and, consequently, its productivity change over time. The relationship between regional temperature and regional productivity has an inverted U-shape, calibrated so that the high-resolution model replicates estimates of aggregate global damages from global warming. At the global level, then, the high-resolution model nests standard one-region economy-climate models, while at the same time it features realistic spatial variation in climate and economic activity. The central result is that the effects of climate change vary dramatically across space---with many regions gaining while others lose---and the global average effects, while negative, are dwarfed quantitatively by the differences across space. A tax on carbon increases average (global) welfare, but there is a large disparity of views on it across regions, with both winners and losers. Climate change also leads to large increases in global inequality, across both regions and countries. These findings vary little as capital markets range from closed (autarky) to open (free capital mobility).

Per Krusell
Institute for International Economic Studies
Stockholm University
106 91 STOCKHOLM
SWEDEN
and CEPR
and also NBER
per.krusell@iies.su.se

Anthony A. Smith Jr.
Department of Economics
Yale University
P.O. Box 208268
New Haven, CT 06520
and NBER
tony.smith@yale.edu

[C]limate change is going to affect different nations to different degrees and in different ways. Unfashionable though these terms may be, there will be “winners” as well as “losers.”¹

1 Introduction

That human greenhouse gas emissions cause global warming is no longer questioned in the science community. Still, much uncertainty remains regarding how much warming there will be under different emission scenarios; the IPCC reports have, since their first version, narrowed their intervals of estimated warming but not by very significant amounts. In addition, how a given amount of warming will affect human welfare is also subject to much—if anything, to more—uncertainty. For these reasons, views differ substantially among commentators on the extent to which emissions need to be curbed. However, an arguably more important reason why they, and policymakers around the world, display quite dispersed attitudes toward climate action is that different parts of the world are expected to be affected very differently by global warming, as well as by the proposed measures to mitigate it. Here, we believe that economic research can be of great, if not indispensable, help.

The present paper is motivated by a need to understand better the heterogeneous costs and benefits involved in combating climate change. With a framework that allows a systematic account of heterogeneous effects, different policy options can then be assessed, including region/country-specific compensation schemes. We view our framework as a natural next modeling step in Nordhaus’s agenda: the core is his neoclassical growth model augmented with carbon-cycle and climate blocks but, in addition, now featuring rich regional heterogeneity both in economic and climate outcomes. Another important difference with Nordhaus’s work is that we study market settings explicitly: for the population, preferences, and technology given, we define a dynamic general equilibrium and can therefore expose our global economy to a range of policy interventions, such as taxes on carbon emissions.² Another point of reference is the macroeconomic literature—notably Aiyagari (1994)’s framework—focusing on consumer inequality, which materializes here across regions. In emphasizing dynamics of inequality, as climate change by nature is a transitional phenomenon, our setting is also related to our own previous work in Krusell & Smith (1998) and Krusell & Smith (1997).³

¹See p. 6 in Pumphrey (2008).

²If one studies planning problems, it is straightforward to derive what an optimal carbon tax would look like, but it is not possible to study various forms of sub-optimal policy.

³Our model here does not feature stochastic fluctuations in regional or aggregate temperatures, but it is feasible to include them by adapting the methods developed in Krusell & Smith (1998), as outlined in Technical Appendix E; we have implemented these methods successfully in earlier versions of this work.

A starting point for our work is that the physical and economic effects of climate change are nowhere near uniform across the globe. As the Earth warms, some regions of the globe, such as those in northern latitudes, actually warm more rapidly than others. At the same time, some regions, particularly cold ones, become more productive for economic activity as they warm, while hot regions, becoming even hotter, become less so. These spatially heterogeneous changes in productivity, in turn, induce economic resources to move across space. To study these effects, we thus build a dynamic, general-equilibrium, global model of economy-climate interactions featuring a high degree of geographic resolution. At the global level, the model nests standard one-region economy-climate models, such as the canonical DICE (Dynamic Integrated Climate Economy) model.⁴ At the regional level, by contrast, it features realistic spatial variation in climate and economic activity. The model therefore permits the quantitative evaluation of how climate change and policies designed to combat it affect different regions in the world in different ways. Put differently, the model serves as a laboratory in which we can quantify not just the aggregate (or average) effects of climate change and climate policy, but also their distributional effects across space. As the globe warms, for example, which regions gain and which lose, and by how much? If one set of regions imposes a tax on carbon emissions, do some regions gain while others lose?

The fundamental unit of analysis in the model is a $1^\circ \times 1^\circ$ cell, or region, containing land, with regions straddling more than one country subdivided to preserve national boundaries for a total of approximately 19,000 regions. Output (i.e., gross domestic product, or GDP) in each region is produced using physical capital, labor, and energy. In each period each region decides how much to consume and how much to save, either locally or abroad, as well as how much energy to use. Labor supply is exogenous but its productivity varies over time as a region's climate changes. Energy use emits carbon into the atmosphere, which warms the globe but not in a uniform fashion: some regions warm faster than others. The sensitivity of each region's temperature to the global temperature is calibrated using output from geophysical models of the Earth's climate.

The productivity of labor in each region is the product of two components. The first component does not depend on a region's climate (or average temperature): its initial value in each region is calibrated to replicate the global distribution of regional output in 1990 in the G-Econ (Geographically based Economic data) database and thereafter it grows at a constant annual rate. The second component, by contrast, varies with regional temperature according to an inverse U -shape normalized to have a maximum of one and a minimum of zero. This U -shape is calibrated so that the high-resolution economy-climate model constructed in this paper generates aggregate (global) damages from global warming that match those in standard representative-consumer economy-climate models. The predictions of the high-resolution climate-economy model for global aggregates, then, align with those delivered by the DICE model or its modern macroeconomic

⁴For an exposition of this model, see, for example, Nordhaus (2007).

counterpart in Golosov et al. (2014).

At the regional level, the optimal annual average temperature (at which the calibrated inverse U -shape governing how labor productivity varies with temperature reaches its peak) is approximately 12 degrees Celsius ($^{\circ}C$); an increase of regional temperature from 10 $^{\circ}C$ to 12 $^{\circ}C$ increases a region’s total factor productivity (TFP) by about 1%, while a further increase in annual average temperature from 12 $^{\circ}C$ to 14 $^{\circ}C$ reduces its TFP by about 2%.⁵

The main quantitative finding is that climate change affects regions very differently: output (GDP) grows dramatically in some regions (relative to trend), while it shrinks sharply in others; these regional variations in GDP growth, moreover, are much, much larger (in absolute value) than the changes in growth of global output wrought by climate change. Consequently, from a regional perspective, there are large disagreements about the welfare effects of carbon taxes: when a uniform carbon tax is imposed across all regions, with revenues redistributed locally as a lump sum so that there are no interregional transfers, some regions gain and others lose, often by large amounts that swamp the globally-averaged benefits of carbon taxes.

In the framework built here labor is assumed to be immobile and therefore does not migrate across regions in response to climate change. Nonetheless, there is significant adaptation to climate change. This adaptation occurs both across time—through intertemporal smoothing of consumption streams within a region—and across space—through movements of capital across regions. The framework therefore also permits leakage—movements of resources across space in response to differences in carbon taxes across regions. A key methodological finding is that shutting down capital markets has little effect on the quantitative results: in autarky, optimal saving behavior within regions comes close to equalizing the marginal product of capital (net of taxes) across regions, as would obtain under free capital mobility. The case of free capital mobility, moreover, lends itself relatively easily to numerical characterization thanks to an aggregation result that we show obtains in our framework.

Our main contribution is to offer a quantitative framework with regional heterogeneity at a high level of resolution. Since the first version of our work in 2014, a number of studies have developed settings that also feature regional heterogeneity. The most notable of these is arguably a set of economic geography papers: Desmet & Rossi-Hansberg (2015), Conte et al. (2021) Cruz & Rossi-Hansberg (2021), Desmet et al. (2021), Rudik et al. (2021), and Cruz & Rossi-Hansberg (2022). These frameworks (Cruz & Rossi-Hansberg (2021) in particular is a very rich model) focus precisely on the mobility of people; we document and quantify migration pressures in our framework (below, we document how much is gained from moving, on purely economic grounds, with and without climate change), but do not allow migration. Some of these alternative settings also have other

⁵The non-linearity in the estimated relationship between temperature and TFP is significant; an increase of 4 $^{\circ}C$ from the peak reduces TFP by about 6%, while an increase of 4 $^{\circ}C$ to the peak increases it by about 5%.

features that we do not incorporate here: endogenous technology and (endogenous) population growth.⁶ On the other hand, these frameworks do not have neoclassical growth underpinnings (in particular, there is no physical capital accumulation as in DICE) and the decision-making of agents is, *de facto*, static.⁷ In this sense, this alternative line of work is most of all complementary to the approach we follow here, which is a more straightforward continuation of Nordhaus’s research. Another feature we do not include here is the distinction between agricultural goods and other goods, a distinction that is of first-order importance for developing countries; Nath (2020) is an interesting recent example of research in this direction. Clearly, a fully adequate model would merge the key features of these settings. In the conclusions of the paper, we discuss possible ways forward in this regard.

The literature on climate change and economics has also expanded significantly in ways that do not focus on regional heterogeneity. One active area has been to relax the assumption of exogenous technology, e.g., Popp (2004), Acemoglu et al. (2012), Acemoglu et al. (2016), Acemoglu et al. (2019), Hassler et al. (2021a), Fried (2019), and Hassler et al. (2022); looking at endogenous technology through the lens of our regional model would be a valuable extension. Another line of work explores the performance of less than first-best tax incentives, both with an explicit second-best approach, as in Barrage (2019), or by comparison of policies that in different ways depart from first-best, as in Hassler et al. (2021b). The present paper most closely follows the latter approach; we believe this approach to be most useful in practice as real-world policy outcomes are very far from what is suggested by the climate-economy literature, despite uncertainty about the sensitivity of climate to greenhouse gases and the extent of economic damages from climate change: while there are disagreements on the exact size of a tax on carbon there is little dispute that we should apply a significant tax, but on average, across the world, the average carbon tax is, in fact, currently negative.⁸ Thus, working out the details of winners and losers from raising taxes “above the ground” is an eminently important, and nontrivial, task. One-region estimates of the benefits of even small increases in the tax suggest large, positive effects for the world as a whole.⁹

Another strand of the literature has considered the roles of uncertainty, risk, and sharp nonlinearities (also referred to as tipping points) and how to manage them; none of these are present here.¹⁰ As stated at the outset here, there is indeed much unknown, both about the climate’s development and the role of greenhouse gases for it and about how human welfare is affected

⁶Another early paper on mobility, using different techniques, is Brock et al. (2014). See also Benveniste et al. (2020).

⁷Households and firms do solve dynamic problems, but these reduce to static problems under specific maintained assumptions on technology and markets.

⁸Carbon use is subsidized by many governments, more than offsetting the positive taxes used in some countries and the emission trading system used in the European Union.

⁹See Hassler et al. (2021b).

¹⁰Important contributions include Weitzman (2012), Gollier (2013), Jensen & Traeger (2014), Lemoine & Traeger (2014), Heal & Millner (2014), Barnett et al. (2021), Cai & Lontzek (2019), and Gillingham et al. (2018).

(and how we can protect ourselves in the event of significant climate change), and it is also very difficult to elicit probabilities. The management of uncertainty is then key, not just how policymakers should design their climate policy but also how market participants already act when faced with such uncertainty. Deterministic models such as the benchmark here are still, we think, of practical value in assessing the role of uncertainty. As a short-cut to incorporating robust control or ambiguity explicitly, for example, one can, in particular, use a deterministic model calibrated with “outlier parameter values”, say, at the extreme ends of the intervals given in the IPCC reports, and solve for competitive equilibrium outcomes, both with and without (optimal or sub-optimal) policy interventions.

Risks associated with climate/weather outcomes and associated economic damages in a broad sense are relevant both at the local and aggregate levels and several papers have explicitly included different aspects of these risks in their analyses. Markets are potentially useful for managing risk, though arguably it is not easy to insure fully against climate risk across countries, even when risk is idiosyncratic. Along these lines, as part of the present project (as described in Technical Appendix E), we have solved models with idiosyncratic and aggregate risks, in particular, shocks to regional temperatures which, in turn, give rise to stochastic fluctuations in the global temperature too, despite the large number of regions. These models allow only precautionary saving—risk-free international borrowing and lending—as a means to insure; we leave further exploration of these risks to future research.

Finally, tipping points can also, in principle, be both local and global. Many specific highly non-linear mechanisms are known at the local level, and some—such as the melting of the Arctic ice cap—are global, but when all of them are considered in conjunction, they appear to lead to very limited global non-linearities, as discussed in detail in the IPCC reports. One embodiment of this statement is the relatively recent consensus that there is an approximately constant so-called Transient Climate Response to Cumulative Carbon Emissions (TCRE): the global temperature increase (since the pre-industrial era) at a point in time is roughly proportional to the sum of all carbon emissions up until that date.¹¹ The methods we use here are designed to deal with non-linearities, but since we use up-to-date descriptions of the climate and carbon cycle systems, our global climate system is approximately linear. For simplicity, our local projections of climate change are also linear, but the approach permits the incorporation of specific local tipping points.

We begin the analysis in Section 2 by detailing the model we use, including how we solve it numerically. Section 3 contains the calibration of the model’s functional forms and parameter values; this section contains our discussion of damage estimates. Section 4 contains all the results and Section 5 concludes. Technical Appendices (A–E) detailing some of the theoretical derivations

¹¹In other words, any given amount of further emissions, regardless of their distribution over time, will increase the future temperature by a constant times the given amount. See Allan et al. (2021).

and computational methods are available online.

2 The Global Economy-Climate Model

2.1 Overview

The model's central feature is the same feedback loop that lies at the heart of all economy-climate models: economic activity emits carbon into the atmosphere, raising the Earth's temperature and influencing economic activity in turn. The main innovation of the model is to study this feedback at a high degree of geographic resolution. The globe is divided into approximately 19,000 regions, each of which makes its own consumption-savings and energy-use decisions. These regions interact through the global climate system and through global financial markets. The aggregate carbon emissions of the different regions cause the globe to warm which, in turn, alters the productivity of different regions differently. The model, which can accommodate taxes on carbon emissions that vary across space and time, generates equilibrium time paths not only for global output and the global temperature but also for output and the temperature in every region.

2.2 Time and space

The globe is divided into M regions containing land, indexed by i , each of which corresponds to a $1^\circ \times 1^\circ$ cell defined by the lines of latitude and longitude at one-degree intervals. Regions containing more than one country are subdivided along country boundaries. Time is discrete, lasts forever, and begins in period 0. There is no uncertainty.

2.3 Consumers

Each region i contains a large number, N_i , of identical, price-taking consumers.¹² Consumers in different regions have identical preferences: they value streams of consumption, $\{c_{it}\}_{t=0}^\infty$, according to:

$$\sum_{t=0}^{\infty} \beta^t U(c_{it}),$$

where c_{it} is consumption in region i in period t .¹³ The felicity function takes the form

$$U(c) = \frac{c^{1-\nu} - 1}{1 - \nu},$$

¹²For simplicity, we abstract from population growth.

¹³Regional variables denoting quantities are expressed in per capita terms.

where $\nu^{-1} > 0$ is the elasticity of intertemporal substitution.¹⁴ The discount factor, β , is between zero and one. Consumers do not value leisure.

Consumers in region i are endowed with \bar{k}_i units of physical capital in period 0 and one unit of time in each period. The productivity of their labor varies over time; each unit of time delivers ℓ_{it} effective units of labor in region i in period t . Consumers take the sequence $\{\ell_{it}\}_{t=0}^{\infty}$ as given when making decisions, but its evolution depends in part on the (endogenous) evolution of their regional temperature, as described in Section 2.6.

Labor is immobile: consumers live and work only in the region in which they are located. Consumers can save either by investing in physical capital in their own region or by participating in a frictionless interregional, or global, bond market.¹⁵ Letting the period- t consumption good serve as the numéraire, consumers in region i earn a wage w_{it} in period t per effective unit of labor. The rental rate of capital in region i in period t is r_{it} and the (global) price of a (one-period) bond in period t is q_t . Capital depreciates at rate δ per period.

Each region contains a government that taxes carbon emissions into the atmosphere and rebates the proceeds as a lump-sum transfer, R_{it} , to the region's consumers. Consumers in region i , then, face the following sequence of budget constraints for $t \geq 0$:

$$c_{it} + k_{i,t+1} + q_t b_{i,t+1} = (1 + r_{it} - \delta)k_{it} + w_{it}\ell_{it} + b_{it} + R_{it}, \quad (1)$$

where k_{it} is a typical consumer's holdings of capital in region i in period t and b_{it} is a typical consumer's bond holdings in region i in period t .¹⁶ Consumers do not hold bonds in period 0. Given a sequence $\{\ell_{it}, r_{it}, w_{it}, R_{it}, q_t\}_{t=0}^{\infty}$, consumers in region i choose a sequence $\{c_t, k_{i,t+1}, b_{i,t+1}\}_{t=0}^{\infty}$ to maximize their lifetime utility of consumption subject to the sequence of budget constraints.

2.4 Technology

Effective units of labor per unit of time in region i in period t , ℓ_{it} , are the product of two time-varying components: $\ell_{it} = A_{it}D_{it}$. The first component, A_{it} , captures permanent differences in labor productivity across regions (i.e., those unrelated to changes in climate). It grows growing deterministically at a rate g common across regions:

$$A_{it} \equiv (1 + g)^t \bar{A}_i, \quad (2)$$

¹⁴When $\nu = 1$, $U(c) = \log(c)$.

¹⁵Section 2.10 develops a version of the model in which the global bond market is shut down.

¹⁶Consumers also own the region's firms, to be described in detail in Section 2.5, but because they make zero profits in each period these can be omitted from the budget constraint.

where \bar{A}_i is a region-specific parameter capturing permanent differences in productivity across regions at time 0.

The second component, D_{it} , captures variations in labor productivity stemming from changes in region i 's temperature. Thus, we follow Nordhaus in summarizing the effects of climate change on humans by including them, in effect, in total factor productivity (TFP).¹⁷ Specifically, $D_{it} \equiv D(T_{it})$, where T_{it} is region i 's temperature in period t , measured in degrees Celsius ($^{\circ}\text{C}$). The nonnegative continuous function D has an inverted U -shape with a unique maximum at a temperature of T^* and its maximum is normalized to one: $D(T^*) = 1$. Total labor supply in region i in period t , measured in effective units, is then $L_{it} \equiv N_i \ell_{it}$.

There are two sectors in each region, one producing final goods and one producing energy. Let the superscript c denote the final-goods sector and the superscript e denote the energy sector. Energy is an intermediate good used in both sectors. The final good can be used either for consumption or investment in the capital stock. The technology for producing final goods, y , uses three inputs, namely, capital, k^c ; labor, ℓ^c , measured in effective units; and energy, e^c , measured in gigatons of oil equivalent (Gtoes): $y = F(k^c, \ell^c, e^c)$, where F is twice continuously differentiable, exhibits constant returns to scale, and satisfies standard concavity properties.¹⁸ Capital and labor can move freely between the two sectors within a region.

The technology for producing energy uses the same three inputs: $e = \zeta^{-1} F(k^e, \ell^e, e^e)$, where ζ^{-1} is a parameter that measures the productivity of the energy sector relative to that of the final-goods sector.¹⁹ Energy, then, is not an exhaustible resource in this model; instead it corresponds most closely to coal, whose marginal extraction cost is positive and whose stock is sufficiently large that it is not scarce (in the sense that its Hotelling rent is zero). This treatment is close in spirit to that in Golosov et al. (2014) and is motivated by two facts: (i) the stock of fossil fuel with significant Hotelling rents is very limited and would, if fully used, only contribute to warming by a fraction of a degree Celsius; and (ii) the remaining fossil fuel is enormous and would contribute to so much warming that it is inconceivable for us to use it all up. An attractive, alternative assumption is made in Cruz & Rossi-Hansberg (2021), which models a marginal cost structure that rises with total accumulated extraction.²⁰

¹⁷The calibration of the TFP damages, in our work as well as in Nordhaus's, relies on damages of all kinds, including loss of life, damages to the capital stock, loss of biodiversity, etc., all valued relative to production.

¹⁸One Gtoe is 3.97×10^{16} BTUs (British thermal units).

¹⁹Notice that we are assuming identical isoquants in the two sectors, up to a relative productivity shifter ζ ; this slightly simplifies the structure but is not essential.

²⁰In general, when the marginal cost depends on past extraction the production of fossil fuel is a dynamic choice; this case is straightforward to analyze but would add computational burden, which is why we abstract from it here. In Cruz & Rossi-Hansberg (2021), the dynamic dependence is an externality, thus making market-made decisions static.

2.5 Firms and emissions

Each region contains a large number of identical, price-taking firms in each sector. In every period a typical firm in the final-goods sector maximizes profits: it solves

$$\max_{k_{it}^c, \ell_{it}^c, e_{it}^c} F(k_{it}^c, \ell_{it}^c, e_{it}^c) - r_{it}k_{it}^c - w_{it}\ell_{it}^c - (p + \tau_{it}\chi_t\psi) e_{it}^c,$$

where k_{it}^c is physical capital, ℓ_{it}^c is the demand for labor, and e_{it}^c is the demand for energy for a typical firm in the final-goods sector in region i in period t .²¹ In addition, p is the price (expressed in units of the final good) of one Gtoe and $\tau_{it}\chi_t\psi$ is the tax paid per Gtoe: each Gtoe generates ψ gigatons of carbon emissions, $\chi_t \in [0, 1]$ is the fraction of these carbon emissions that is captured and stored before entering the atmosphere in period t (so that χ_t measures the “dirtiness” of a unit of carbon emissions in period t), and τ_{it} is the tax rate per gigaton of carbon emissions into the atmosphere. The exogenous sequence $\{\chi_t\}_{t=0}^\infty$ decreases monotonically as energy becomes cleaner over time, and equals zero for $t \geq S$. Thus, we do not endogenize green technology—policy, for example, will not change its importance—but we can set the path for χ_t so as to mimic different assumptions about how it might evolve over time.²²

Similarly, a typical firm in the energy sector solves

$$\max_{k_{it}^e, \ell_{it}^e, e_{it}^e} p\zeta^{-1} F(k_{it}^e, \ell_{it}^e, e_{it}^e) - r_{it}k_{it}^e - w_{it}\ell_{it}^e - (p + \tau_{it}\chi_t\psi) e_{it}^e$$

in every period, where k_{it}^e is the demand for physical capital, ℓ_{it}^e is the demand for labor, and e_{it}^e is the demand for energy for a typical firm in the energy-goods sector in region i in period t .

Consumers own the firms in their region, but under constant returns to scale firms make zero profits so they do enter into consumers’ budget constraints.

2.6 Temperature

The global temperature in period t , T_t , expressed as a deviation from the pre-industrial global temperature, is determined by the stock of carbon in the atmosphere, S_t , measured in gigatons, in period t :

$$T_t = T(S_t) \equiv \lambda \frac{\log(S_t/\bar{S})}{\log(2)}, \quad (3)$$

where \bar{S} is the pre-industrial stock of carbon. The parameter λ measures the sensitivity of the global temperature to changes in the stock of atmospheric carbon: were this stock to double the

²¹Firms make zero profits because they are price-takers and F exhibits constant returns to scale.

²²An alternative physical interpretation of $1 - \chi$ is the fraction of a given unit of emission that is captured and sequestered.

global temperature would increase by λ °C.

Using a “pattern scaling” (or “statistical downscaling”) approach borrowed from climate science, the global temperature serves as a sufficient statistic for regional temperature:

$$T_{it} = \bar{T}_i + \gamma_i \Delta T_t \quad (4)$$

where \bar{T}_i is the temperature in region i in period 0 and $\Delta T_t \equiv T_t - T_0$, i.e., ΔT_t is the change in the global temperature relative to the temperature at time 0. The region-specific coefficients $\{\gamma_i\}_{i=1}^M$ measure the responsiveness, or sensitivity, of the temperature in region i to a change in the global temperature.

2.7 The carbon cycle

The final component of the model describes the evolution of the stock of carbon in the atmosphere. This stock interacts with other stocks of carbon in the biosphere and in the oceans (especially the lower oceans, which are by far the largest repository of carbon on Earth) in complex ways over long periods of time in a dynamic process known as the carbon cycle. Following Golosov et al. (2014), a parsimonious way to describe the evolution of the atmospheric carbon stock is to view it as the sum of two components: $S_t = S_{1t} + S_{2t}$. The first component, S_{1t} , lasts forever (at least on the time scales relevant to this model) in the sense that additions to it are permanent. The second component, S_{2t} , by contrast, depreciates to zero (absent further additions to it) as it leaks into the biosphere and oceans.²³

Let $e_{it} \equiv e_{it}^c + e_{it}^e$ denote Gtoes used in region i in period t . Then global emissions of carbon into the atmosphere in period t equal $\chi_t \psi E_t$, where $E_t \equiv \sum_{i=1}^M N_i e_{it}$ is global energy use in period t .²⁴ Fraction ϕ_1 of these emissions enters the permanent component of the stock of carbon in period t , whose law of motion is then:

$$S_{1t} = \phi_1 \chi_t \psi E_t + S_{1,t-1}, \quad (5)$$

where $\phi_2 \in (0, 1)$. Fraction $(1 - \phi_1)\phi_2$ of these emissions enters the depreciating component of the stock of carbon, where $\phi_2 \in (0, 1)$. These emissions later dissipate into the biosphere and oceans at a rate determined by the coefficient $\phi_3 \in (0, 1)$ in the law of motion for the depreciating stock:

$$S_{2t} = (1 - \phi_1)\phi_2 \chi_t \psi E_t + \phi_3 S_{2,t-1}. \quad (6)$$

²³The division into S_1 and S_2 does not allow a physical interpretation and is merely a convenient recursive way of summarizing the carbon cycle. With an appropriate calibration of the associated parameters, as described below, it delivers an approximately constant TCRC (Transient Climate Response to Cumulative Carbon Emissions).

²⁴Recall from Section 2.5 that χ_t is the fraction of carbon emissions that is captured and stored before entering the atmosphere in period t and ψ is gigatons of carbon emissions per Gtoe.

Finally, the remaining fraction, $(1 - \phi_1)(1 - \phi_2)$, of global atmospheric emissions in period t immediately dissipates into the biosphere and oceans. The initial stocks of atmospheric carbon, $S_{1,-1}$ and $S_{2,-1}$, are predetermined.

2.8 Equilibrium

This section defines a perfect-foresight equilibrium for the global economy-climate model under free capital mobility. The key equilibrium objects, common to all regions, are the path for the global temperature, $\{T_t\}_{t=0}^\infty$, and the path for the bond price, $\{q_t\}_{t=0}^\infty$. Given a region's pre-industrial temperature, \bar{T}_i , and the sensitivity, γ_i , of its regional temperature to changes in the global temperature, the global temperature path determines the path for that region's temperature. This path, together with the permanent component of a region's productivity, \bar{A}_{it} , in turn pins down the path for the regions's productivity (i.e., effective units of labor). Consumers in each region optimize, taking as given paths for regional productivity, regional factor prices, regional lump-sum transfers, and the bond price. Firms in each region also optimize, taking as given regional factor prices and tax rates. In equilibrium, regional factor markets clear in every period; the path for aggregate carbon emissions implied by optimal energy use in each region replicates the global temperature path taken as given; the global bond market clears in every period; and in every period each region's revenue from taxing carbon emissions equals the lump-sum transfer taken as given.

More formally, a perfect-foresight equilibrium is a collection of sequences, one for each region:

$$\{\ell_{it}, \ell_{it}^c, \ell_{it}^e, A_{it}, T_{it}, c_{it}, e_{it}, k_{it}, k_{it}^c, k_{it}^e, b_{i,t+1}, R_{it}, r_{it}, w_{it}\}_{t=0}^\infty, \quad i = 1, \dots, M,$$

an aggregate sequence, $\{E_t, S_{1t}, S_{2t}, T_t, q_t\}_{t=0}^\infty$, and a price p such that:

- (i) the sequence $\{c_{it}, k_{i,t+1}, b_{i,t+1}\}_{t=0}^\infty$ solves the consumer's problem in region i ;
- (ii) for $j = c, e$, the sequence $\{k_{it}^j, \ell_{it}^j, e_{it}^j\}_{t=0}^\infty$ solves the problem of a typical firm in sector j in region i ;
- (iii) factor markets clear in region i in every period: for all $t \geq 0$, $k_{it}^c + k_{it}^e = k_{it}$, $\ell_{it}^c + \ell_{it}^e = \ell_{it}$, and $e_{it}^c + e_{it}^e = \zeta^{-1} F(k_{it}^e, \ell_{it}^e, e_{it}^e)$;
- (iv) the final goods market in region i clears in every period: for all $t \geq 0$,

$$c_{it} + k_{i,t+1} - (1 - \delta)k_{it} + qb_{it} - b_{i,t+1} = F(k_{it}^c, \ell_{it}^c, e_{it}^c);$$

- (v) for all $t \geq 0$, region i 's temperature, T_{it} , is determined by the global temperature, T_t , according to equation (4);
- (vi) for all $t \geq 0$, effective units of labor (per unit of time) in region i are determined by $\ell_{it} = A_{it}D(T_{it})$, where A_{it} evolves according to equation (2);
- (vii) each region's government balances its budget in each period: for all $t \geq 0$, $R_{it} = \tau_{it}\chi_t\psi e_{it}$;
- (viii) the global bond market clears in each period: for all $t \geq 0$, $B_t \equiv \sum_{i=1}^M N_i b_{it} = 0$;
- (ix) the sequence $\{E_t, S_{1t}, S_{2t}\}_{t=0}^\infty$ satisfies equations (5) and (6) governing the carbon cycle; and
- (x) for all $t \geq 0$, the global temperature is determined by equation (3), with $S_t = S_{1t} + S_{2t}$.

2.9 A recursive formulation

This section recasts the model in an equivalent but more compact form in which each region, rather than consisting of consumers and firms with separate objectives, consists instead of a large number of identical consumer-entrepreneurs with the same preferences over streams of consumption as the consumers described in Section 2.3.²⁵ The consumption-savings problem faced by each entrepreneur can be expressed recursively in the form of a dynamic program that facilitates the computation of the equilibrium defined in Section 2.8.

Each entrepreneur is endowed with one unit of time in each period and operates a (common) backyard technology that uses capital, energy, and the entrepreneur's labor, measured in effective units, to produce final goods.²⁶ Specifically, in period t an entrepreneur with k units of capital and ℓ effective units of labor chooses energy, e , to maximize output of final goods less taxes on energy:

$$g_t^e(k, \ell, \tau_{it}) \equiv \arg \max_e (G(k, \ell, e) - \tau_{it}\chi_t\psi e), \quad (7)$$

where $G(k, \ell, e) \equiv F(k, \ell, e) - \zeta e$ is output of final goods (given the inputs, k , ℓ , and e) and $\tau_{it}\chi_t\psi e$ is taxes paid by an entrepreneur in region i who uses e Gtoes in period t .

A typical entrepreneur in region i then solves the following dynamic program (in which x' denotes next period's value of the variable x), taking as given paths for the two stocks of atmospheric carbon, the global bond price, the regional carbon tax, and regional lump-sum transfers:

$$v_t(\omega, A; \bar{T}_i, \gamma_i) = \max_{c, k', b'} [U(c) + \beta v_{t+1}(\omega', A'; \bar{T}_i, \gamma_i)] \quad (8)$$

²⁵Technical Appendix A demonstrates this equivalence formally.

²⁶Each entrepreneur in region i is also endowed with \bar{k}_i units of physical capital in period 0.

subject to:

$$\begin{aligned}
\omega &= c + k' + q_t b' \\
A' &= (1 + g)A \\
T'_i &= \bar{T}_i + \gamma_i(T_{t+1} - T_0) \\
\ell' &= A'D(T'_i) \\
e' &= g_{t+1}^e(k', \ell', \tau_{i,t+1}) \\
\omega' &= G(k', \ell', e') - \tau_{i,t+1}\chi_{t+1}\psi e' + (1 - \delta)k' + b' + R_{i,t+1}.
\end{aligned}$$

The period- t value function, v_t , of a typical entrepreneur in region i depends on two state variables: total resources after production of final goods (“wealth”), ω , and the permanent component of productivity, A . It is indexed by two time-invariant parameters: the region’s pre-industrial temperature, \bar{T}_i , and the sensitivity, γ_i , of its temperature to changes in the global temperature.

The first constraint in the entrepreneur’s problem states that current wealth is split between consumption, investment in the (regional) capital stock, and purchases/sales of interregional bonds. The second constraint states that the permanent component of regional productivity (i.e., the component unaffected by regional temperature) grows at rate g . The third constraint connects the regional temperature to the global temperature. The fourth constraint pins down effective units of labor. The fifth constraint states that the entrepreneur chooses energy optimally, given capital and labor. Finally, the sixth and last constraint states that next period’s wealth is the sum of four components: output of final goods less taxes on energy; the depreciated value of the capital stock; bond holdings; and the regional lump-sum transfer.

The solution to this problem is a pair of time-varying rules for choosing capital and bond holdings, one for each region: $k' = g_{it}^k(\omega, A)$ and $b' = g_{it}^b(\omega, A)$. Each of these rules depends on the region, i , not only because pre-industrial temperature, \bar{T}_i , and sensitivity to changes in the global temperature, γ_i , vary by region but also because the path for carbon taxes can vary arbitrarily across regions.

The period-0 wealth of a typical entrepreneur in region i , ω_{i0} , is given by:

$$\omega_{i0} = F(\bar{k}_i, \ell_{i0}, e_{i0}) + (1 - \delta)\bar{k}_i - \zeta e_{i0},$$

where $\ell_{i0} = \bar{A}_i D(T_{i0})$, $e_{i0} = g_0^e(\bar{k}_i, \ell_{i0}, \tau_{i0})$, and T_{i0} satisfies equations (3)–(6).²⁷ Denote by Γ_0 the initial distribution of regions across the state vector (ω, A) ; i.e., $\Gamma_0 \equiv \{(\omega_{i0}, \bar{A}_i)\}_{i=1}^M$. Given Γ_0 and a collection of paths, $\{\{\tau_{it}, R_{it}\}_{t=0}^\infty\}_{i=1}^M$, for regional taxes and transfers, the collection of decision rules, $\{\{g_{it}^k, g_{it}^b\}_{t=0}^\infty\}_{i=1}^M$, in conjunction with the laws of motion for the climate system, generates

²⁷This expression for ω_{i0} imposes government budget balance in period 0.

a path for the global temperature, $\{T_t\}_{t=1}^\infty$, and a path for global (or aggregate) bond holdings, $\{B_t\}_{t=1}^\infty$.

A perfect-foresight equilibrium is then a collection of decision rules; a path for the global temperature; a path for the global bond price, $\{q_t\}_{t=0}^\infty$; and a collection of paths for regional taxes and transfers such that:

- (i) for each region i , the decision rules $\{g_{it}^k, g_{it}^b\}_{t=0}^\infty$ are optimal given $\{T_t, q_t\}_{t=0}^\infty$ and $\{\tau_{it}, R_{it}\}_{t=0}^\infty$;
- (ii) given Γ_0 and $\{\{\tau_{it}, R_{it}\}_{t=0}^\infty\}_{i=1}^M$, the collection of decision rules generates $\{T_t\}_{t=0}^\infty$;
- (iii) $B_t = 0$ for all $t \geq 0$; and
- (iv) each regional government balances its budget in every period.

2.10 Autarky

Under autarky the interregional bond market is shut down so that regions interact only through the climate system which determines the global temperature (and, consequently, regional temperature). Formally, in the recursive formulation of the model, under autarky the first constraint in the dynamic program (25) faced by a typical entrepreneur reads: $\omega = c + k'$, so that the entrepreneur can save only by investing in the regional capital stock. Likewise, the last constraint determining the evolution of wealth then reads:

$$\omega' = G(k', \ell', e') - \tau_{i,t+1} \chi_{t+1} \psi e' + (1 - \delta)k' + R_{i,t+1}.$$

Finally, in the definition of equilibrium the bond price drops out along with the requirement that the global bond market (were it to exist) clear.

2.11 Theoretical characterization: aggregation

Under an additional restriction on the production function, the model aggregates exactly, provided that the interregional bond market is allowed to operate; that is, the behavior of the global temperature and the global macroeconomic aggregates does not depend on the distribution of capital across regions when capital can flow freely between regions. This feature of the model simplifies its computation.

To that end, assume that $F(k, \ell, e) = H(k^\alpha \ell^{1-\alpha}, e)$, where $\alpha \in (0, 1)$ and H exhibits constant returns to scale in its two arguments. Define $h(\cdot) \equiv H(1, \cdot)$. In this case, $G(k, \ell, e) = \Phi(x) k^\alpha \ell^{1-\alpha}$, where $\Phi(x) \equiv h(x) - \zeta x$ and $x \equiv e/k^\alpha \ell^{1-\alpha}$ is a measure of energy intensity. Given k , ℓ , and τ_{it} ,

the optimal choice for energy for an entrepreneur in region i in period t , e_{it}^* , is then proportional to $k^\alpha \ell^{1-\alpha}$:

$$e_{it}^* \equiv g_t^e(k, l, \tau_{it}) = x_{it}^* k^\alpha \ell^{1-\alpha}, \quad (9)$$

where the optimal energy intensity, x_{it}^* , satisfies the first-order condition

$$\Phi'(x_{it}^*) = \tau_{it} \chi_t \psi, \quad (10)$$

so that $x_{it}^* = h'^{-1}(\zeta + \tau_{it} \chi_t \psi)$. Given this choice for energy, output of final goods less taxes on energy is equal to $\Pi_{it} k^\alpha \ell^{1-\alpha}$, where

$$\Pi_{it} \equiv \Phi(x_{it}^*) - \tau_{it} \chi_t \psi x_{it}^*.$$

Finally, the marginal net return to capital in region i in period t is then $\alpha \Pi_{it} (k/\ell)^{\alpha-1} - \delta$.

With free capital mobility, in every period each entrepreneur equates the marginal return from investing in region-specific capital to the (common) net return on the risk-free bond. Marginal returns to capital are therefore equalized across regions in equilibrium. Consequently, in equilibrium the capital stock in region i in period t can be expressed as a fraction of the global capital stock, $K_t \equiv \sum_{i=1}^M N_i k_{it}$, in period t :

$$N_i k_{it} = \frac{N_i \theta_{it}}{\theta_t} K_t,$$

where $\theta_{it} \equiv \ell_{it} \Pi_{it}^{\frac{1}{1-\alpha}}$ and $\theta_t \equiv \sum_{i=1}^M N_i \theta_{it}$. From this result it follows that, with free capital mobility, global output of final goods, net of energy taxes, depends only on the global capital stock and not on its distribution across regions:

$$\sum_{i=1}^M N_i y_{it} = \sum_{i=1}^M N_i \Pi_{it} k_{it}^\alpha \ell_{it}^{1-\alpha} = \Theta_t K_t^\alpha,$$

where the coefficient $\Theta_t \equiv \theta_t^{1-\alpha}$; this coefficient, depends on the global temperature in period t , the coefficient, χ_t , measuring the dirtiness of emissions in period t , and the set of regional taxes, $\{\tau_{it}\}_{i=1}^M$, in period t . Thus, it is a function of exogenous variables only.

As shown in Technical Appendix B, with free capital mobility the path for global capital solves the dynamic program of a stand-in global consumer whose utility depends only on global consumption, C , and not on its distribution across regions:

$$V_t(\Omega) = \max_{K'} [U(C) + \beta V_{t+1}(\Omega')] \quad (11)$$

subject to: $\Omega = C + K'$ and $\Omega' = \Theta_{t+1} (K')^\alpha + (1 - \delta) K' + \sum_{i=1}^M N_i R_{i,t+1}$, where V_t is the period- t

value function of the stand-in consumer and Ω is global wealth at the beginning of the period. The stand-in consumer takes as given a path for the global temperature, which in turn pins down the sequence $\{\Theta_t\}_{t=0}^\infty$, and a path for global (or aggregate) transfers (i.e., $\sum_{i=1}^M N_i R_{it}$, $t = 0, \dots, \infty$). The solution to the stand-in consumer's problem is a time-varying rule for choosing (global) capital: $K' = g_t(\Omega)$.

The competitive-equilibrium paths for the global temperature, global capital, and the collection of regional transfers then satisfy three conditions: one, given paths for the global temperature and global transfers, the sequence of decision rules, $\{g_t\}_{t=0}^\infty$, solves the problem of the stand-in consumer; two, given initial global wealth, $\Omega_0 \equiv \sum_{i=1}^M N_i \omega_{i0}$, and the path for regional transfers, the sequence of decision rules generates the path for global temperature; and, three, each region's government balances its budget in every period, i.e., $R_{it} = \tau_{it} \chi_t \psi e_{it}^*$ for all i and t .

The marginal product of capital on its equilibrium path pins down the (global) gross interest rate between periods t and $t+1$: $q_t^{-1} = \alpha \Theta_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta$. Measured in units of the period-0 consumption good, the "lifetime wealth" of a typical consumer in region i is then the sum of period-0 wealth and the present value of future labor income and transfers:

$$\omega_i \equiv \omega_{i0} + \sum_{t=0}^{\infty} q_t (w_{i,t+1} \ell_{i,t+1} + R_{i,t+1}),$$

where the wage per effective unit of labor, w_{it} , equals the marginal product of labor in region i in period t , i.e., $w_{it} = (1 - \alpha) \Pi_{it} (k_{it} / \ell_{it})^\alpha$. Letting $C_t \equiv \sum_{i=1}^M N_i c_{it}$ denote global consumption in period t , with free capital mobility consumption in region i in period t is then, in equilibrium, a time-invariant fraction of global consumption:

$$N_i c_{it} = \frac{N_i \omega_i}{\sum_{i=1}^M N_i \omega_i} C_t,$$

where the denominator in the fraction is global lifetime wealth.

2.12 Optimal emissions taxes

By construction, the problem of the stand-in consumer-entrepreneur studied in Section 2.11, a device for studying competitive equilibrium in the global model when capital is freely mobile, does not internalize the (global) externality caused by atmospheric carbon emissions. Instead, the stand-in consumer takes as given the path for global temperature when solving his problem; in a perfect-foresight equilibrium the optimal behavior of the stand-in consumer replicates this path. This section returns to that problem, viewed now as that of a social planner who seeks to maximize the welfare of the stand-in consumer (the planner therefore puts no weight on how

consumption is distributed across regions), taking into account how his actions influence the global temperature. By internalizing the carbon externality, the planning problem delivers an optimal path for carbon taxes, one that can be used to implement the socially optimal outcome in a (decentralized) competitive equilibrium. This paper later compares the competitive equilibrium allocation in which all regional governments implement this path for carbon taxes to the laissez-faire competitive equilibrium in which there are no such taxes.

In every period the planner allocates the global capital stock efficiently across regions so that marginal returns to capital are equalized. Efficient allocation of capital implies that, in every period, the capital stock in each region is proportional to global capital:

$$N_i k_{it} = \frac{N_i \ell_{it}}{L_t} K_t,$$

where $L_t \equiv \sum_{i=1}^M N_i \ell_{it}$. In addition, in every period, the planner chooses the same energy intensity, x_t , in every region. Global output of final goods then equals $\Phi(x_t) \Lambda_t K_t^\alpha$, where $\Lambda_t \equiv L_t^{1-\alpha}$, and global energy use equals $x_t \Lambda_t K_t^\alpha$. The coefficient Λ_t depends solely on the global temperature in period t ; the global temperature, in turn, depends on the stock of atmospheric carbon, S_t , in period t , according to equation (3). To emphasize this dependence, let $\Lambda_t = \Lambda(S_t)$.

Given initial conditions $S_{1,-1}$, $S_{2,-1}$, and K_0 , the planner chooses a sequence $\{C_t, K_{t+1}, x_t, S_{1t}, S_{2t}\}_{t=0}^\infty$ to maximize $\sum_{t=0}^\infty \beta^t U(C_t)$ subject to, for all $t \geq 0$, the global resource constraint, i.e.,

$$C_t + K_{t+1} - (1 - \delta)K_t = \Phi(x_t) \Lambda(S_{1t} + S_{2t}) K_t^\alpha, \quad (12)$$

and the laws of motion, embodied in equations (5) and (6), for the two components of the stock of atmospheric carbon, with $E_t = x_t \Lambda(S_{1t} + S_{2t}) K_t^\alpha$. Technical Appendix C derives the first-order conditions for this problem and uses them to characterize the socially optimal path for taxes on carbon emissions. It also shows that when all regions impose this path for taxes, the competitive-equilibrium allocation defined in Section 2.11 coincides with the socially optimal allocation as defined here.

2.13 Computation

Computing the competitive equilibrium, under either free capital mobility or autarky, requires finding a fixed point in two sequences: the path for global temperature and the path for the set of regional transfers. Under free capital mobility, given such sequences, iterate backwards on a stationary version (i.e., one in which the trends in permanent productivity have been removed) of the stand-in consumer's Bellman equation (11) to obtain a sequence of value functions (and corresponding decision rules). Because $\chi_t = 0$ for $t \geq S$, atmospheric carbon emissions equal zero

after period S , so that the global temperature eventually converges to a steady state once the depreciating stock of atmospheric carbon converges to zero. The backward iterations can therefore proceed from some time period greater than S such that the steady state has been (nearly) reached. Armed with these (time-varying) decision rules, run the global economy forwards in time to obtain new sequences for the global temperature and the set of regional transfers (setting them equal to regional tax revenues). Continue iterating on them until they converge. Technical Appendix D gives complete details.

Under autarky the computation of the competitive equilibrium follows the same steps, but rather than iterate on the (single) Bellman equation of a stand-in consumer it is necessary to solve the dynamic program (25) of each region separately. These dynamic programs are indexed by each region's period-0 temperature, \bar{T}_i , and sensitivity, γ_i , to changes in the global temperature. Given a path for the global temperature, each pair (\bar{T}_i, γ_i) generates a unique path for the temperature in region i and consequently a unique path for that region's productivity (as measured by efficiency units of labor in that region). Rather than solve directly approximately 17,000 dynamic programs, one for each such path, Technical Appendix D shows how by interpolating across values of (\bar{T}_i, γ_i) the number of dynamic programs to be solved can be reduced to approximately 700. To speed up the computations even further, these dynamic programs are split into groups of approximately 35 and solved in parallel (given a common path for the global temperature and region-specific paths for transfers) on a cluster with 20 CPUs. Stationary versions of each of the 700 dynamic programs are again solved backwards from the eventual steady state.

The solution to the social planning problem described in Section 2.12 works instead directly with the first-order conditions to that problem described in Technical Appendix C. Given a path for the one-period-ahead interest rate (i.e., the intertemporal marginal rate of substitution) and paths for the two (scaled) Lagrangian multipliers associated with the two equations governing the carbon cycle, use the first-order conditions for capital and energy intensity to generate paths for aggregate (global) capital, aggregate energy use, and the two stocks of carbon. These paths imply a path for aggregate consumption which, in turn, implies a new path for the one-period-ahead interest rate. Next, generate new paths for the two multipliers by iterating backwards, starting from the steady-state values of these multipliers, using equation (21) in Technical Appendix C. This equation uses the two first-order conditions for the two stocks of carbon to express the current-period multipliers in terms of next period's multipliers. Continue iterating on the paths for the one-period-ahead interest rate and the two multipliers until they converge. Once again, Technical Appendix D gives all the details.

3 Calibration

This section specifies the functional forms used in the quantitative model, chooses values for its many parameters, and specifies its initial conditions.

3.1 Time

One time period corresponds to one year. The initial time period, $t = 0$, corresponds to the year 1990.

3.2 Regional GDP and population

The G-Econ database (specifically, GEcon 2.11) tabulates gross domestic product (GDP) and population for every $1^\circ \times 1^\circ$ cell containing land for the model's base year, 1990.²⁸ The cell boundaries correspond to the 360 degrees of longitude and 180 degrees of latitude, for a total of $360 \times 180 = 64,000$ such cells covering all of the Earth's surface (both land and water). To preserve country boundaries, GEcon 2.11 subdivides cells containing the land of more than one country, for a grand total of 27,451 cell-countries. GEcon 2.11 contains GDP and population for 19,235 of these cell-countries in 1990; these are the regions that comprise the basic unit of analysis in the global economy-climate model. These regions cover all (or part, if there is a coastline) of 16,859 distinct $1^\circ \times 1^\circ$ cells. [Figure 1](#) displays the (natural) logarithm of GDP in 1990 for each of these 16,859 cells.²⁹

3.3 Technology

As in Section 2.11, the production function takes the form $F(k, \ell, e) = H(k^\alpha \ell^{1-\alpha}, e)$, where k is capital, ℓ is effective units of labor, and e is energy use. The function H is assumed to have a constant elasticity of substitution, $(1 - \rho)^{-1}$, between its two arguments:

$$H(k^\alpha \ell^{1-\alpha}, e) = (\sigma(k^\alpha \ell^{1-\alpha})^\rho + (1 - \sigma)e^\rho)^{1/\rho},$$

where $\rho \geq 0$ and the share parameter, σ , is between zero and one. The use of this particular functional form is motivated by Hassler et al. (2021a), who find it to fit U.S. data quite well, in the short run with a very low elasticity of substitution and in the medium run with an elasticity

²⁸The GEcon 2.11 Excel spreadsheet is no longer available on the G-Econ [website](#) but can be obtained from the authors upon request. GEcon 2.11 also contains data for 1995, 2000, and 2005.

²⁹In all of the maps, the color bar on the right breaks observations into nine groups with equal numbers of regions: for example, one-ninth of the observations on the log of GDP in 1990 are between 0.9 and 6.6, another one-ninth are between -0.1 and 0.9 , etc.

close to one ($\rho = 0$); in the baseline experiments, ρ is set to zero, so that F is Cobb-Douglas in its three arguments.

The growth rate, g , of the permanent component of productivity, A_{it} , in each region is set to 0.01 (or 1% per year). The (annual) rate of depreciation of the capital stock, δ , is set to 0.06.

In equilibrium, the price, p , of one Gtoe (in units of the consumption good) equals ζ , the parameter governing the productivity of the final-goods sector relative to that of the energy sector. Assuming that there are no carbon taxes, each region chooses optimal energy intensity $x^* = h'^{-1}(\zeta)$.³⁰

To choose a value for p (and hence ζ), first define the ratio $s_{it} \equiv pe_{it}/y_{it}$, i.e., the ratio of energy production (expressed in units of the consumption good) in region i in period t to the output of final goods in that region in that period. This ratio is the same across regions and time:

$$s_{it} = \frac{px^*k_{it}^\alpha\ell_{it}^{1-\alpha}}{\Phi(x^*)k_{it}^\alpha\ell_{it}^{1-\alpha}} = \frac{px^*}{\Phi(x^*)} \equiv s.$$

The ratio s is, therefore, also equal to the ratio of global energy production to global output of final goods. Golosov et al. (2014) reports that in 2008 global energy use consisted of 3.315 Gtoes of coal, 4.058 Gtoes of oil, 2.302 Gtoes of green energy, and 2.596 Gtoes of natural gas. It also reports that there are 1.58 tons of coal per Gtoe of coal and that the price of coal in 2008 was \$74 per ton, so the global value of coal production was \$387.6 ($= 3.315 \times 1.58 \times 74$) billion in 2008. Next, it reports that a ton of oil generates 0.846 tons of carbon and that the price of a ton of oil is \$606.5 per ton of carbon generated by its use, implying that the price of oil is \$513.1 ($= 606.5 \times 0.846$) per ton. The global value of oil production in 2008 was therefore \$2082.2 ($= 4.058 \times 513.1$) billion. Valuing green energy production using the same price per Gtoe as oil as in Golosov et al. (2014), the global value of green energy in 2008 was \$1181.2 ($= 2.302 \times 513.1$) billion. Finally, the price for natural gas in 2008 was about $\$6.8 \times 10^{-6}$ per BTU.³¹ One BTU equals 1.055 kilojoules (kJ), one kJ equals 2.4×10^{-5} kgoes (kilograms of oil equivalent), and one kgoe equals 10^{-3} tons of oil equivalent, implying that the price in dollars of one Gtoe of natural gas is:

$$\frac{6.8 \times 10^{-6}}{1.055 \times (2.4 \times 10^{-5}) \times 10^{-3}} = 268.6.$$

The global value of natural gas production in 2008 was then \$697.3 ($= 2.596 \times 268.6$). Summing across the four sources of energy, the global value of global energy production in 2008 was \$4.35 trillion, or about 6.2% of global GDP of \$70 trillion. The ratio of energy production to production

³⁰When $\rho > 0$, $h(x) = (\sigma + (1 - \sigma)x^\rho)^{1/\rho}$ and $h'(x) = (1 - \sigma)(\sigma x^{-\rho} + 1 - \sigma)^{\frac{1-\rho}{\rho}}$. When $\rho = 0$, $h(x) = x^{1-\sigma}$ and $h'(x) = (1 - \sigma)x^{-\sigma}$.

³¹This is the spot price at the Henry Hub terminal in Louisiana; see this [graph](#).

of final goods, s , is therefore set to 0.062.

Summing GDP across the 19,235 regions, global GDP in the model's base year, 1990, was \$30.55 trillion. Summing again across the four sources of energy, global energy production in 2008 was 12.272 ($= 3.315 + 4.059 + 2.302 + 2.596$) Gtoes. In the model, apart from transitional dynamics (and in the absence of taxes), aggregate quantities grow at the rate g (i.e., 1% per year). To target global energy production in 2008, global energy production in 1990 is $12.272/1.01^{18} = 10.260$ Gtoes. By definition of the ratio s , the price, p , of one Gtoe in the model equals sY_0/E_0 , where Y_0 is global GDP in 1990 and E_0 is global energy production in 1990, i.e., p is set to $0.062 \times 30.55/10.26 = 0.185$.

Define

$$\hat{s} \equiv \frac{pE_0}{Y_0 + pE_0} = \frac{s}{1 + s}.$$

Then, using the first-order condition $h'(x^*) = p$ and the fact that

$$\frac{Y_0 + pE_0}{E_0} = \frac{h(x^*)}{x^*},$$

it is straightforward to show that

$$\sigma = 1 - \hat{s} \left(\frac{p}{\hat{s}} \right)^\rho.$$

When $\rho = 0$, as in the baseline experiments, $\sigma = 1 - \hat{s} = 0.942$.

Capital's share of income, α , is set to 0.36.

As noted above, one Gtoe of oil generates 0.846 tons of carbon. Golosov et al. (2014) reports that one Gtoe of coal generates 0.716 tons of carbon, so one Gtoe of coal generates 1.131 ($= 1.58 \times 0.716$) tons of carbon. Green energy is assumed to generate zero carbon emissions. Finally, one Gtoe of natural gas generates approximately 0.6 tons of carbon.³² The parameter ψ is the ratio of the amount of carbon emissions, measured in gigatons of carbon (GtCs), to the amount of energy used, measured in Gtoes. In 2008 global emissions of carbon were 8.741 ($= 3.315 \times 1.131 + 4.059 \times 0.846 + 2.302 \times 0 + 2.596 \times 0.6$) GtCs, so that ψ is set to $8.741/12.272 = 0.712$.

3.4 Preferences

The elasticity of intertemporal substitution, ν , is set to one, so that the felicity function is the (natural) logarithm. On the economy's eventual balanced-growth path, after atmospheric carbon emissions have shrunk to zero and the global temperature has stabilized, the consumer's Euler

³²This [website](#) from the U.S. Energy Information Administration states that natural gas generates 117.0 pounds of carbon dioxide per million BTUs while diesel fuel and heating oil generate 161.3 pounds of carbon dioxide per million BTUs. One Gtoe of oil generates 0.846 tons of carbon, so one Gtoe of natural gas generates $(117.0/161.3) \times 0.846 = 0.614$ tons of carbon.

equation implies that the steady-state interest rate is: $\beta^{-1}(1+g)^\nu - 1$. The discount factor, β , is set to 0.985, so that the steady-state interest rate is 2.53% per year.

3.5 Clean energy

The time-varying parameter $\chi_t \in [0, 1]$ is the fraction of carbon emissions that is captured and stored before entering the atmosphere in period t ; it is assumed to decline monotonically to zero as energy becomes cleaner over time, reaching zero in period $S = 300$ and remaining there forever after, so that all energy is, in effect, green starting in period S . Before then, χ_t evolves according to a logistic function of time:

$$\chi_t = 1 - \left(1 + \exp \left(\log \left(\frac{0.01}{0.99} \right) \frac{t - n_{0.5}}{n_{0.01} - n_{0.5}} \right) \right)^{-1},$$

where $n_{0.01}$ is the time period in which $\chi_t = 0.01$ and $n_{0.5}$ is the time period in which $\chi_t = 0.5$. The two parameters, $n_{0.01}$ and $n_{0.5}$, are set to 10 and 125, respectively. By 2125, then, half of carbon emissions are abated via carbon capture and storage.³³ Figure 2 graphs this function from 1990 to 2290 (i.e., from $t = 0$ to $t = 300$).

3.6 Carbon cycle

Following Golosov et al. (2014), 25% of a freshly-emitted unit of carbon into the atmosphere remains there indefinitely, so that $\phi_1 = 0.25$. In addition, the half-life of a freshly-emitted unit of carbon is 30 years and the half-life of a fresh addition to the depreciating stock of carbon is 300 years. These last two restrictions imply that $\phi_3 = (0.5)^{1/300} = 0.998$ and

$$\phi_2 = \frac{0.5 - \phi_1}{(1 - \phi_1)\phi_3^{30}} = 0.36.$$

Initial values, $S_{1,-1}$ and $S_{2,-1}$, for the two stocks of carbon are set so that the two carbon stocks equal (approximately) 684 and 118, respectively, in the year 1999 (corresponding to $t = 10$).³⁴ Specifically, assume that global energy use in 1990, i.e., E_0 , is 10.260 Gtoes (as set above) and then grows at 1% per year thereafter (as it does in the model in the absence of taxes, apart from minor deviations arising from transitional dynamics). The implied path of global emissions from 1990 to 1999 is $\psi\chi_t(1+g)^t E_0$, $t = 0, \dots, 9$. Iterating the laws of motion (5) and (6) for the two

³³Alternatively, one can view the fraction χ_t as representing the period- t mix of green and dirty sources of energy; this mix shifts towards green energy over time and by 2290 all energy is green.

³⁴Golosov et al. (2014) set the carbon stocks to these values in the year 2000 instead; the resulting differences in global temperature are negligible.

carbon stocks from $t = 0$ to $t = 9$ then yields:

$$S_{1,9} = \phi_1 \psi E_0 \sum_{s=0}^9 \chi_s (1+g)^s + S_{1,-1}$$

$$S_{2,9} = (1 - \phi_1) \phi_2 \psi E_0 \sum_{s=0}^9 \phi_3^{9-s} \chi_s (1+g)^s + \phi_3^{10} S_{2,-1}.$$

The initial values $S_{1,-1}$ and $S_{2,-1}$ solve these equations when the targets $S_{1,9}$ and $S_{2,9}$ are set to 684 and 118, respectively.

3.7 Temperature

This section discusses choices for the parameters and initial conditions that determine time paths for the global and regional temperatures.

3.7.1 Climate sensitivity

As in Golosov et al. (2014), the sensitivity, λ , of the global temperature to changes in the stock of atmospheric carbon is set to 3: a doubling of this stock increases the global temperature by λ °C.

3.7.2 Pattern scaling

The coefficients, γ_i , governing the sensitivity of each region's temperature to the global temperature are set using the output of five coupled geophysical models of the Earth's climate, all of which are part of the Coupled Model Intercomparison Project Phase 5 (also known as CMIP5) and simulate climate and weather at a high degree of geographic resolution.³⁵ The five models are CCSM (Community Climate System Model), CESM (Community Earth System Model), CanESM2 (Canadian Earth System Model), HadGEM (Hadley Centre Global Environment Model), and MPI-ESM (Max Planck Institute Earth System Model). Each model is run using a path for global carbon emissions called RCP (Representative Concentration Pathway) 2.6; this is a “low” emissions path but the coefficients are largely insensitive to using other RCPs. The methodology for computing the coefficients is based on the idea of “pattern scaling” which uses changes in global variables to predict changes in their regional counterparts. Specifically, following Tebaldi & Arblaster (2014), for each model m calculate

$$\gamma_{im} = \frac{T_{im}^{2081-2100} - T_i^{1986-2005}}{T_m^{2081-2100} - T^{1986-2005}},$$

³⁵This [website](#) provides an overview of CMIP5.

where $T_m^{2081-2100}$ is the average global temperature in model m from 2081 to 2100, $T^{1986-2005}$ is the historical average global temperature from 1986 to 2005, $T_{im}^{2081-2100}$ is the average global temperature in region i in model m from 2081 to 2100, and $T_i^{1986-2005}$ is the historical average temperature in region i from 1986 to 2005. The coefficient γ_i is then set to the average of the γ_{im} s, i.e., $\gamma_i = 5^{-1} \sum_{m=1}^5 \gamma_{im}$. Tebaldi & Arblaster (2014) and Lopez et al. (2013) argue that the assumption of linearity implicit in these calculations, i.e., that changes in regional temperature scale linearly with changes in global temperature, is a reasonable one.

Figure 3 displays these coefficients for the 16,859 $1^\circ \times 1^\circ$ cells in the economy-climate model. They range from 0.4 in coastal regions to 5.2 in the far northern latitudes. The area-weighted average of these coefficients is 1.30: when the surface of the entire globe warms by 1° the average surface temperature over land increases by 1.30° .³⁶

3.7.3 Regional temperatures in the base year

Wilmott & Matsuura (2009) provide monthly average temperatures over land on a $0.5^\circ \times 0.5^\circ$ grid for the period 1900 to 2008.³⁷ Averaging across months yields annual average temperatures for each of these cells. Annual average temperatures for the 16,859 $1^\circ \times 1^\circ$ cells in the global economy-climate model are then equal to an area-weighted average of the four $0.5^\circ \times 0.5^\circ$ cells that comprise it. Figure 4 displays the average of these average annual temperatures over the period 1901 to 1920. Regional temperatures in period 0, i.e., \bar{T}_i , $i = 1, \dots, M$, are then set according to: $\bar{T}_i = T_i^{1901-1920} + \gamma_i \Delta T_{1910-1990}$ where $T_i^{1901-1920}$ is the average annual temperature in region i for the period 1901 to 1920, γ_i is the sensitivity of region i 's temperature to changes in global temperature, and $\Delta T_{1910-1990} = 0.91$ is the change in the global temperature from 1910 to 1990.³⁸

3.8 Regional permanent productivities and initial capital stocks

Efficiency units of labor (per capita) in region i in period 0 equal $\bar{a}_i \equiv D(\bar{T}_i)\bar{A}_i$, where \bar{A}_i is the permanent component of region i 's productivity in period 0, \bar{T}_i is temperature in region i in period 0, and the function D describes how productivity varies with regional temperature. To calibrate \bar{a}_i and the initial capital stock (per capita), \bar{k}_i , in each region, impose two sets of restrictions. First, in line with Caselli & Feyrer (2007), assume that marginal net returns to capital are equalized across regions (as they are in every period in the model with full capital mobility); second, require that GDP per capita in each region in period 0 match the data on GDP per capita in the G-Econ

³⁶By definition, the area-weighted average of the sensitivity coefficients for all 64,800 $1^\circ \times 1^\circ$ cells on the globe, including both land and water, is 1.

³⁷See, in particular, the file `air_temp2009.tar.gz` available at the URL provided in the references.

³⁸According to this graph prepared by Berkeley Earth, the global temperature in 1910 was about $0.3^\circ C$ higher than in the pre-industrial era. In addition, $S_0 = 768.8$ (absent taxes), so the global temperature in 1990, relative to the pre-industrial era, was $T(S_0) = 1.21$, implying in turn that $\Delta T_{1910-1990} = 1.21 - 0.3 = 0.91$.

database (GEcon 2.11) in 1990. To normalize the \bar{a}_i s, assume in addition that at time 0 the global economy is on a balanced-growth path so that the (common) marginal net return to capital equals the steady-state interest rate, i.e., $r_{ss} \equiv \beta^{-1}(1 + g)^\nu - 1$.

In the absence of taxes, all regions choose the same energy intensity, x^* , in every period. GDP per capita in region i in period 0 is then equal to $\Phi(x^*)\bar{k}_i^\alpha \bar{a}_i^{1-\alpha}$ and the marginal net return to capital in region i in period 0 is equal to: $\alpha\Phi(x^*)(\bar{k}_i/\bar{a}_i)^{\alpha-1} - \delta$. Letting y_i^{1990} be GDP per capita in 1990 in GEcon 2.11, set \bar{a}_i and \bar{k}_i to satisfy the two equations:

$$\begin{aligned}\Phi(x^*)\bar{k}_i^\alpha \bar{a}_i^{1-\alpha} &= y_i^{1990} \\ \alpha\Phi(x^*)(\bar{k}_i/\bar{a}_i)^{\alpha-1} - \delta &= r_{ss}.\end{aligned}$$

Section 3.9 calibrates the function D . Given D , permanent productivity in region i in period 0, \bar{A}_i , can be recovered by calculating the ratio $\bar{a}_i/D(\bar{T}_i)$.

3.9 Productivity and regional temperature

The function $D(T_i)$ measures how a region's efficiency units of labor vary with regional temperature, T_i . The last ten years of literature has contributed a number of empirical studies, using both aggregate and microeconomic data, on how human welfare is negatively affected by warming—through effects on agricultural production, aggregate income, various amenities, mortality, or the incidence of conflict.³⁹ Many of these studies document U-shaped relationships between temperature and damages. Our approach is consistent with this finding: to calibrate D , the essential idea is to choose it so that aggregate damages from global warming in the (high-resolution) global-economy climate model match those in representative-agent economy-climate models (i.e., those with a single global consumer), such as Nordhaus's pioneering DICE model, as laid out, for example, in Nordhaus (2007). The result of this procedure, in particular, is an inverted U-shape for TFP as a function of temperature.

The DICE model embeds aggregate damages from global warming as a reduction in global TFP; i.e., these damages can be expressed as a fraction of global GDP, holding factor inputs fixed, that varies with the global temperature. In Nordhaus (2007), in particular, that fraction, $\pi(T)$, is an increasing convex function of the global temperature, T (again expressed as a deviation from the pre-industrial global temperature):

$$\pi(T) = \frac{\varphi T^2}{1 + \varphi T^2},$$

³⁹See, among others, Deschênes & Greenstone (2007), Dell et al. (2014), Burke et al. (2015a), Burke et al. (2015b), Barreca et al. (2016), Harari & Ferrara (2018), and Cruz & Rossi-Hansberg (2021).

where the parameter φ is set to 0.0028388. [Figure 5](#) graphs this function for $T \in [0, 5]$. It ranges from no damages when $T = 0$ to damages equal to 6.6% of world GDP when $T = 5$. The damage function π implies that if the global temperature were suddenly to change from T to the global temperature in 1990, T_{1990} , then global GDP would change by a percentage equal to

$$\hat{\pi}_{DICE}(T) \equiv 100 \left(\frac{1 + \varphi T^2}{1 + \varphi T_{1990}^2} - 1 \right).$$

Given a function D , it is straightforward to derive the counterpart of $\hat{\pi}_{DICE}$ for the global economy-climate model. Under the assumptions on the function H made in Section 3.3, regional output $y_{it} = D(T_{it})^{1-\alpha} \Phi(x_{it}) k_{it}^\alpha \bar{A}_i^{1-\alpha}$, so that the function $\tilde{D}(\cdot) \equiv D(\cdot)^{1-\alpha}$ captures how variations in a region's temperature influence its total factor productivity (TFP). For that reason the calibration procedure targets \tilde{D} , the “climate component” of regional TFP, directly rather than D itself. For parsimony assume that \tilde{D} is indexed by four parameters:

$$\tilde{D}(T_i) = \begin{cases} (1-d) \exp(-\kappa^+(T_i - T^*)^2) + d & \text{if } T_i \geq T^* \\ (1-d) \exp(-\kappa^-(T_i - T^*)^2) + d & \text{if } T_i \leq T^* \end{cases}$$

The nonnegative parameter d is a lower bound on \tilde{D} (as T_i diverges from T^*) and is set to 0.02. The parameter T^* is the optimal regional temperature: $\tilde{D}(T_i)$ attains its maximum of one when $T_i = T^*$. The parameters κ^+ and κ^- determine how quickly $\tilde{D}(T_i)$ declines from its maximum as T_i deviates from T^* .⁴⁰

As shown in Section 2.11, with free capital mobility global GDP in period t equals $\Theta_t K_t^\alpha$, where K_t is the global capital stock and Θ_t is a coefficient that depends, among other things, on the set of regional labor productivities (i.e., efficiency units of labor in each region). When there are no taxes, every region chooses the same energy intensity $x^* = h'^{-1}(p)$, so that:

$$\begin{aligned} \Theta_t &= \Phi(x^*) \left(\sum_{i=1}^M N_i \ell_{it} \right)^{1-\alpha} \\ &= (1+g)^t \Phi(x^*) \left(\sum_{i=1}^M N_i \bar{a}_i \frac{D(T_{it})}{D(\bar{T}_i)} \right)^{1-\alpha} \\ &= (1+g)^t \Phi(x^*) \left(\sum_{i=1}^M N_i \bar{a}_i \frac{D(\bar{T}_i + \gamma_i \Delta T_t)}{D(\bar{T}_i)} \right)^{1-\alpha} \\ &\equiv (1+g)^t \Phi(x^*) \Theta(\Delta T_t; T^*, \kappa^+, \kappa^-), \end{aligned}$$

⁴⁰Because, by construction, $D(\cdot) = \tilde{D}(\cdot)^{\frac{1}{1-\alpha}}$, D , like \tilde{D} , has an inverted U -shape, attains its maximum at T^* , and is bounded between 0 and 1.

where, again, ΔT_t is the change in the global temperature relative to $t = 0$ (corresponding to 1990) and the dependence of the function Θ on the three free parameters characterizing the inverted U -shape \tilde{D} has been made explicit. In the global economy-climate model, then, holding global capital constant, a change in global temperature from $T_{1990} + \Delta T$ to T_{1990} would change global GDP by the percentage

$$\hat{\pi}(\Delta T; T^*, \kappa^+, \kappa^-) \equiv 100 \left(\frac{\Theta(0; T^*, \kappa^+, \kappa^-)}{\Theta(\Delta T; T^*, \kappa^+, \kappa^-)} - 1 \right).$$

To set the parameters T^* , κ^+ , and κ^- , then, require that damages from global climate change, as measured by the percentage change in global GDP that would be induced by a reversion of the global temperature to its level in 1990, be the same in both the DICE model and the global-economy climate model for three different values of the global temperature, labelled T^i , $i = 1, 2, 3$:

$$\hat{\pi}_{DICE}(T^i) = \hat{\pi}(T^i - T_{1990}; T^*, \kappa^+, \kappa^-), \quad i = 1, 2, 3.$$

Setting $T^1 = 1$, $T^2 = 2.5$, and $T^3 = 5$ and then solving these three (nonlinear) equations for the three unknown parameters yields: $T^* = 11.58$, $\kappa^+ = 0.00311$, and $\kappa^- = 0.00456$.

Figure 6 graphs \tilde{D} given the choices for its four parameters. \tilde{D} reaches its peak of one at 11.6°C ; it declines from this peak asymmetrically, with declines in a region's temperature leading to larger losses in TFP than equivalent increases: for example, a 5°C increase in a region's temperature from the optimal temperature lowers its TFP by 7% to 0.93 while a 5°C decrease lowers it by 11% to 0.89.

Figure 7 displays on a map the climate component of regional TFP, i.e., the value of $\tilde{D}(T_i^{1901-1920})$, for each region i , multiplied by 100 so that the scale varies from 2 to 100, where $T_i^{1901-1920}$ is the historical average temperature in region i from 1901 to 1920. Figure 8 graphs the share of global GDP in 1990 against global variations in the climate component of regional TFP, showing that the bulk of this output is produced in regions whose average annual temperatures are near the optimal temperature of 11.6°C . In particular, 22% of global GDP is produced in regions very near the peak of \tilde{D} , specifically, in regions for which the climate component of regional TFP is larger than 0.995. Finally, Figure 9 graphs the share of global population in 1990 against global variations in the climate component of regional TFP. As for the share of global GDP, there is a spike in the share of the population living in regions with high values of this component of regional TFP (i.e., regions whose average annual temperatures are close to the optimal temperature): 10.6% of the global population lives in regions for which $\tilde{D}(T_i^{1901-1920})$ is larger than 0.995. But there is also a large share of the global population living in regions for which the climate component of regional TFP is half as large, in the general vicinity of 0.55.

4 Results

This section presents the findings, first for aggregate variables and then for regional variables. One set of experiments maintains *laissez-faire* (no carbon taxes) and another set imposes a (common) path for carbon taxes on some or all regions. The central message is that the distributional effects of global warming are large: many regions gain while others lose. A harmonized global carbon tax (administered locally so that there are no transfers across regions) improves average welfare but this average gain is swamped by the large disparity of views across regions: there are both big winners and big losers from a carbon tax. This distribution of gains and losses also increases incentives to migrate; we report quantitative measures of these incentives in Section 4.2.1.

A secondary, methodological, finding is that the structure of capital markets affects the findings only to a small degree: the behavior of the aggregates is almost identical under either free capital mobility or autarky and regional outcomes, with some minor exceptions, are very close too under the two market structures.

4.1 Aggregate variables

The global economy-climate is calibrated, under free capital mobility, so that aggregate damages from global warming match those in standard representative-agent economy-climate models (such as DICE). By construction, then, the behavior of the aggregates in the global-economy climate model matches that generated by models such as DICE, provided that technology and preferences are otherwise identical. An important methodological finding is that under autarky the behavior of the aggregates is virtually unchanged, so that it suffices to report results under free capital mobility.

Figure 10 shows the time path of global carbon emissions from 1990 to 2390 with free capital mobility both under *laissez-faire* (the top line) and under optimal carbon taxes (the bottom line) when all regions impose the same path for such taxes. Figure 11 shows this tax path from 1990 to 2288 (just before all energy becomes green); as explained in Technical Appendix C, it is derived from the solution to the planning problem laid out in Section 2.12. The optimal tax is \$49.3 per ton of carbon emissions in 1990 and grows exponentially thereafter at a declining rate that asymptotes to 1% per year. Carbon emissions peak around 2090, both with and without taxes, and then decline to zero by 2290 as energy becomes entirely green by then, but the peak in emissions is much lower with taxes than without. Figure 12 shows the path of the stock of atmospheric carbon from 1990 to 2390, again both with (the top line) and without (the bottom line) taxes and Figure 13 shows the corresponding paths for the global temperature, which varies directly with the stock of atmospheric carbon according to equation (3). Both variables peak around 2190 (both with and without taxes), with the global temperature reaching a peak of about 4.4 °C

(relative to the pre-industrial temperature) without taxes and a peak of about 3.6°C with taxes. Eventually, after emissions drop to zero and the stock of atmospheric carbon asymptotes to a new higher level than in 1990 (because a fraction of emissions remains in the atmosphere “forever”), the global temperature converges to a new steady-state level somewhat lower than at its peak but substantially higher than in 1990.

Comparing to other studies, we obtain a tax intervention similar to the one, for example, in Golosov et al. (2014), but the paths for carbon emissions and temperature increase more modestly here because we assume that fossil-fuel emissions become increasingly green over time. To obtain sharper reductions in optimal emissions, and lower resulting optimal temperatures—say, in line with the 1.5- or 2-degree international targets—one would need to assume more potent warming, larger damages, or discount factors significantly closer to 1. Our benchmark calibration is in line with Nordhaus’s.

Turning to the economic variables, Figure 14 shows the path of global GDP under free capital mobility from 1990 to 2390, both with and without taxes. In this graph the long-run trend in global GDP has been removed, so that the lines then show deviations from a trend path of 1% starting at global GDP of \$30.55 trillion in 1990. From 1990 to about 2090, global GDP with taxes falls by more (relative to trend) than without taxes. Global GDP eventually asymptotes to a higher level (relative to trend) with taxes than without taxes because the global temperature asymptotes to a higher level without taxes than with taxes, though in either case there is a permanent loss of output because the eventual steady-state global temperature is higher than in 1990. Without taxes global GDP reaches its nadir (relative to trend) just after 2190, when it is about 7.3% below the trend that would have obtained starting in 1990 without further global warming. With taxes, global GDP reaches its nadir just before 2190, at about 5.5% below trend.

Figure 15 shows the path of global consumption under free capital mobility from 1990 to 2390, both with and without taxes, again with the trend removed. The behavior of global consumption, both with and without taxes, is similar to that of global GDP, though consumption is initially (and briefly) higher with taxes than without, later falling by more under taxes before asymptoting to a higher level with taxes than without (though, again, in both cases there is a permanent loss of consumption).

4.2 Regional variables

This section discusses the distributional effects of climate change and climate policy; i.e., it discusses the evolution of the regional variables over time. Section 4.2.1 considers the *laissez-faire* scenario with no carbon taxes and Section 4.2.2 discusses scenarios with carbon taxes.

4.2.1 Regional variables under laissez-faire

Each run of the model generates time paths for annual temperature, GDP, and productivity in each of the 19,235 regions, for a total of approximately 11.5 million data points over the course of, say, 200 years. The best way to apprehend such a large amount of data is to use visual representations of it. To begin, Figures 16a to 16u show the evolution of regional temperatures, in increments of a decade, from 2000 to 2200 under free capital mobility (and, again, no carbon taxes). The reader is invited to click through the maps in these figures to watch how temperature evolves over time at different points on the globe, and then return to the text with the (internal) links provided.⁴¹ Alternatively, this [movie](#) animates the spatial evolution of temperature one year at a time. Because the range of annual average temperatures across the globe is so large, from -65°C at one extreme to 31°C at the other, the changes in regional temperatures wrought by global warming can seem small in comparison (though their economic consequences can nonetheless be large). This [movie](#), instead, shows the change in regional temperature, with all regions starting at zero in 1990. Because regional sensitivities to a change in the global temperature vary widely too, from 0.4 to 5.2, different parts of the globe warm more than others as the global temperature rises, with locations in northern latitudes, in particular, warming the most.

Because the evolution of regional temperature (and other variables) differs only slightly under autarky, this section focuses mainly on the laissez-faire case. But this section does later provide evidence that under autarky marginal products of capital across regions are close to being equalized along the transition to the eventual balanced-growth path, as they are by construction with free capital mobility.⁴²

Next, Figures 17a to 17u show the evolution of regional TFP, in particular, $100 \times \tilde{D}(T_{it})$, around the globe in response to changing regional temperature (and see also this [movie](#)). Watch as the dark-red band of highest productivity across the mid-latitudes in the Northern Hemisphere gradually moves north over time: in North America, for example, this band runs initially across the center of the continental United States and then moves north into southern Canada, and in Western Europe it gradually moves into northern Germany and southern Scandinavia from France and the southern half of the United Kingdom.

Finally, Figures 18a to 18u show the evolution of regional GDP, expressed as a percentage deviation from a 1% trend beginning in 1990 (see also the accompanying [movie](#)). As discussed in Section 4.1, global GDP, at its nadir near 2190, falls by 7.3% (relative to trend) as the globe warms. By contrast, regional GDP displays a large dispersion of outcomes relative to the aggregate outcome: GDP in many regions rises, often by large amounts, relative to trend, while in others it

⁴¹It is most informative to click through the maps in full-screen mode, easily enabled by typing **Ctrl-L**.

⁴²In addition, Section 4.2.2 compares the regional welfare gains from carbon taxes under both free capital mobility and autarky.

falls, by equally large amounts. Regions that gain the most are those whose initial temperatures are well below the optimal temperature of 11.6°C , especially if, in addition, their sensitivities to increases in temperature are large. The most prominent examples are Russia, particularly its northern reaches, and the more northern parts of Canada. Regions that lose the most are those that are already far “too hot” relative to the optimal temperature: India, northern parts of Africa, and Brazil are the most prominent examples here.

An alternative way to “watch” the evolution of regional GDP is to track a sequence of relative-frequency histograms depicting the distribution of percentage changes in GDP (relative to trend) across regions, as is undertaken in Figures 19a to 19u and in the accompanying [movie](#). These histograms show only those regions with percentage changes in GDP (relative to trend) between -80% and 200% ; in 2190, these regions comprise about 85% of the total number of regions. As the globe warms, the distribution of percentage changes in GDP by region spreads out and eventually stabilizes as the global temperature reaches its peak and then begins declining toward its eventual steady-state level (albeit one substantially higher than in 1990). By 2190 GDP has fallen (relative to trend) in 57% of the 19,235 regions and risen in the rest; the median percentage decline across regions is -11% .

This [movie](#) animates a sequence of relative-frequency histograms showing how the distribution of marginal products of capital (MPKs) across regions evolves over time in autarky (with free capital mobility, of course, they are equalized at every point in time). Initially, in 1990, by construction these MPKs are equalized and eventually, as the global economy converges to its new balanced-growth path in which global warming has ceased, they become equalized again because each region eventually equates its MPK (net of depreciation) to the (common) rate of time preference (properly adjusted for steady-state growth). Along the transition, though, the distribution of MPKs, shown in yellow in the [movie](#), spreads out (before collapsing again), but the bulk of the distribution stays close to the (common) MPK along the transition path with free capital mobility (and this MPK, in turn, does not move much over time because the equilibrium interest rate does not much move either). Weighting the MPKs by regional GDP, the spread of MPKs in the histograms, shown instead in red in the [movie](#), is even smaller. Thus, in autarky, most regions, through their own (isolated) savings decisions, choose paths for their regional capital stocks that are close to those that obtain under free capital mobility. Although there are some differences, they are too small to dilute the main message that global warming leads to very different outcomes at different points on the globe, with many regions gaining and others losing, often by large amounts.⁴³

In the global economy-climate model constructed here, capital flows across space but there is

⁴³A model with even less reallocation of capital across space would keep the regional distribution capital fixed at its initial value over time, save for a common trend. The results are in [Figure 20](#), revealing large losses from misallocation.

no migration. Nonetheless, the model can be used to quantify the extent to which climate change influences incentives to migrate. To this end, [Figure 21](#) displays lifetime wealth (at time zero) for a typical consumer in each region after removing permanent differences in productivity. The resulting spatial variation in lifetime wealth in the figure thus stems solely from the effects of global warming on regional productivity (if there were no warming lifetime wealth, after removing regional differences in permanent productivity, would be the same across the globe). Once again, there are large disparities in how climate change affects people around the world, now in terms of their incentives to migrate. How the welfare of a resident of region i would change by moving to region j at time zero depends both on the numbers in the figure and on how starkly the initial positions differ (i.e., on how large is the difference between the permanent components of productivity in the two locations). The welfare gains from moving can be large. Moving to any part of Florida from any part of India, for example, would lead to a huge improvement in economic welfare even without global warming; with it, this change becomes much larger, as the figure illustrates. The gap in welfare between Florida and Kamchatka, by contrast, has shrunk because of warming (while remaining high and positive). One can also compute “border pressures”—increases or decreases in migration pressure between adjacent regions—as well; they are nontrivial, but we leave out the details here.

4.2.2 Regional variables with a carbon tax

This section discusses the distributional effects of taxes on carbon emissions. In the first set of experiments, all regions impose the same path for (locally-financed) carbon taxes, namely, the one discussed in Section 4.1 which implements, in a competitive equilibrium, the socially optimal allocation described in Section 2.12.⁴⁴ In the second set of experiments, only regions in the United States and China impose this path, with all other regions setting carbon taxes to zero.

With taxes on carbon, regional temperature, regional TFP, and regional GDP evolve in a way that is qualitatively similar to their evolution without carbon taxes, though because the global temperature rises by less the quantitative effects are somewhat muted in comparison. For the sake of brevity, a detailed discussion of the behavior of these variables in the presence of carbon taxes is omitted in favor of focusing on how such taxes affect welfare across regions. To wit, [Figure 22](#) displays, as of time zero, how much each region gains or loses in a competitive equilibrium with free capital mobility in which there are “harmonized” (i.e., common) carbon taxes across all regions. Once again, there is a large disparity of views: some regions lose more than the equivalent of 30% of consumption per year, while others gain as much as the equivalent of 10% of consumption per year. “Hot” regions (i.e., those whose initial annual average temperatures in 1990 are above

⁴⁴Recall that the planner in this case maximizes the welfare of a stand-in global consumer, putting no weight on inequality of consumption across regions.

the optimal temperature) generally gain from carbon taxes, while those in “cold” regions (whose annual average temperatures in 1990 are below the optimum) tend to lose. Global warming leads hot regions to become even more unproductive, and so those regions prefer to arrest global warming using carbon taxes. Cold regions, on the other hand, enjoy productivity gains when the global warms and so oppose putting brakes on global warming.

Figure 23 displays the gains and losses from the same carbon taxes in an autarkic competitive equilibrium. There are some differences, but, again, they are not large enough to dilute the overarching message that the distribution of gains and losses from carbon taxes is widely spread, with many regions gaining and many losing, often by large amounts.

Another useful way to look at the distribution of gains and losses arising from the imposition of carbon taxes is to construct a relative-frequency histogram using all 19,235 regions: Figure 24 does so with free capital mobility and Figure 25 does so in autarky. The two distributions are qualitatively and quantitatively similar, though the left tail of this distribution is a little thicker, and the mass to the right of a zero consumption equivalent a little smaller, under free capital movement than in autarky.

There are additional useful summary measures of the distribution of gains and losses induced by carbon taxes that can be derived from these histograms. With free capital mobility, 56% of regions gain. But population is not spread equally across regions: 84% of the world’s population gains from carbon taxes. Nor is global GDP: if each region gets a single vote weighted by its GDP then regions representing 68% of the world’s GDP in 1990 gain. In terms of consumption-equivalents (i.e., percentages of consumption per year), the average such gain across regions is actually negative, at -2.1% of consumption per year. But weighted by regional GDP, the average gain is positive, at 0.6% , and weighted instead by regional population it is again positive, at 1.7% .

Again, one key takeaway from these findings is that the distributional effects of carbon taxes swamp their average effect: the stand-in global consumer in the planning problem, for example, gains a consumption equivalent of only 0.4% per year, belying the large heterogeneity in views across regions.

A final way to characterize the gains and losses from carbon taxes is to aggregate regions into countries. One can, for example, calculate the fraction of a country’s population that would vote for carbon taxes (in other words, the fraction that would gain from the imposition of a harmonized global carbon tax, again administered locally region-by-region). Figure 26 displays these fractions for all the world’s countries. They range from close to zero in “cold” countries (such as Russia, Mongolia, Canada, the United Kingdom, and those in Scandinavia) to close to one in “hot” countries (such as Brazil, India, those in the Middle East, Australia, and the entirety of Africa). Few countries, in fact, lie in between these two extremes, but among them are the United States at about three-quarters in favor of carbon taxes, France at about two-thirds, and a

few countries stretching from parts of Central Europe to Afghanistan at close to one-half.

Alternatively, rather than asking countries to vote, instead calculate consumption-equivalent welfare gains for a stand-in consumer in each country, as was calculated above for the global stand-in consumer (and keep in mind, again, these consumption equivalents ignore distributional effects within countries). [Figure 27](#) displays these country-level consumption equivalents. Compared to the 0.4% gain for the global stand-in consumer, there is again a wide disparity of gains and losses across countries, with the stand-in Mongolia consumer losing the equivalent of close to 20% of consumption per year and stand-in consumers in some Saharan African countries gaining close to 5% per year.

In a second set of tax experiments, only regions in the United States and China impose a carbon tax (in particular, the same path imposed in the first set of experiments); all other regions in the world set carbon taxes to zero. In this case, there is “leakage”: capital flows not only to regions with higher regional TFP, but also to regions with lower taxes. Average welfare gains are much smaller in this case because the global temperature rises by more than it does when all regions impose carbon taxes (the global stand-in consumer, for example, gains the equivalent of only 0.1% of consumption per year rather than 0.4% when all regions impose carbon taxes). The total number of regions gaining, however, is the same in both sets of tax experiments: 56% of regions gain when only the United States and China impose a carbon tax. But it is not the same set of regions: in the second set of tax experiments, 27% of regions in the United States and 27% of regions in China gain, whereas in the first set of experiments these percentages are noticeably higher: 41% and 36%, respectively. The overall number of regions gaining (across the entire globe) remains unchanged because a larger fraction of regions in the rest of the world now gains: 60%, rather than 58% in the first set of experiments.

4.3 Global inequality

Under laissez-faire, climate change leads to large increases in inequality in GDP per capita, both across regions and across countries. As shown in [Figure 28](#), the 75–25 ratio (i.e., the ratio of the 75th percentile to the 25th percentile) for the distribution of GDP per capita across regions increases by a factor of 1.7 from 1990 to 2190 under laissez-faire. Similarly, as shown in [Figure 29](#), the 90–10 ratio for this distribution increases by a factor of 5.3 over the same period. Under the optimal carbon tax, these increases in inequality are somewhat smaller but remain quite pronounced.

[Figure 30](#) and [Figure 31](#) show that the 75–25 and 90–10 ratios for the distribution of GDP per capita across countries also increase substantially under laissez-faire, by a factor of 1.21 for the 75–25 ratio and by a factor 1.46 for the 90–10 ratio over the period from 1990 to 2190. Again, under the optimal carbon tax, these increases remain almost as large.

5 Conclusion and Next Steps

In this paper we offer a model of climate change with very high regional resolution. It is to be viewed as a conceptually straightforward extension of Nordhaus’s path-breaking work in its multi-regional approach. In addition, we offer a fully micro-founded dynamic general equilibrium model and, hence, an appropriate framework for conducting counterfactual experiments where policy can vary both over time and across space. We wish to emphasize that, like our earlier work emphasizing inequality across consumers (Krusell & Smith (1998)), we think of the present contribution mainly as a proof of concept. At the same time it is, of course, also quantitatively realistic—we calibrate as best we can given available data and at the global level we nest standard one-region climate-economy models. The resulting multi-regional framework, we believe, offers rich and important messages. Arguably, the most important of our findings is the extreme extent to which the effects of climate change, and hence the effects of mitigation policy such as carbon taxes, differ around the world: these changes dwarf any global average estimates reported in this literature. Hence, large disagreements are to be expected as to how to deal with climate change. Of course, the world’s citizens are not all selfishly thinking only about their own local welfare, but the disparities are so large that any serious policy evaluation ought to give them center stage.

Many further developments of our work here appear fruitful; indeed the literature has grown significantly in many fruitful directions since we started the present research. We have emphasized that our setting does not allow the migration of people (while of course capital can move) and that it would be desirable to include it; on the one hand, migration would be a form of adaptation, making the losses from climate change less significant, but, on the other, the literature has also found that migration leads to conflict, thus generating larger losses than those we estimate. Migration is very challenging to analyze in a region-rich framework: if any agent can move from anywhere to anywhere between t and $t + 1$, but at a cost, the economy’s state variable becomes significantly more complex, and a discrete choice across all regions becomes intractable. The approach taken in the economic geography literature offers tractability by adopting specific assumptions on costs and offers interesting insights.⁴⁵ A number of other features can be added to the present setting without losing numerical tractability; these features include uncertainty (early versions of the present research did consider idiosyncratic and aggregate uncertainty, jointly), the consideration of agriculture as a separate good (an extension we did not pursue but which would be feasible), free movement of people across regions but within countries (straightforward), population growth, and a richer energy sector, including endogenous, directed technical change. We look forward to

⁴⁵Dynamic decision making can be avoided if moving costs are multiplicative over time and reflexive: moving from Kamchatka to Florida may be very costly but moving back again makes the net cost of these moves zero. Also, one can employ idiosyncratic extreme-value shocks, say, to spatial preferences, for easier computation; see, e.g., Rudik et al. (2021). We hope that one of these assumptions can be merged with a neoclassical model featuring individual saving while preserving numerical tractability.

pursuing a number of these ourselves.

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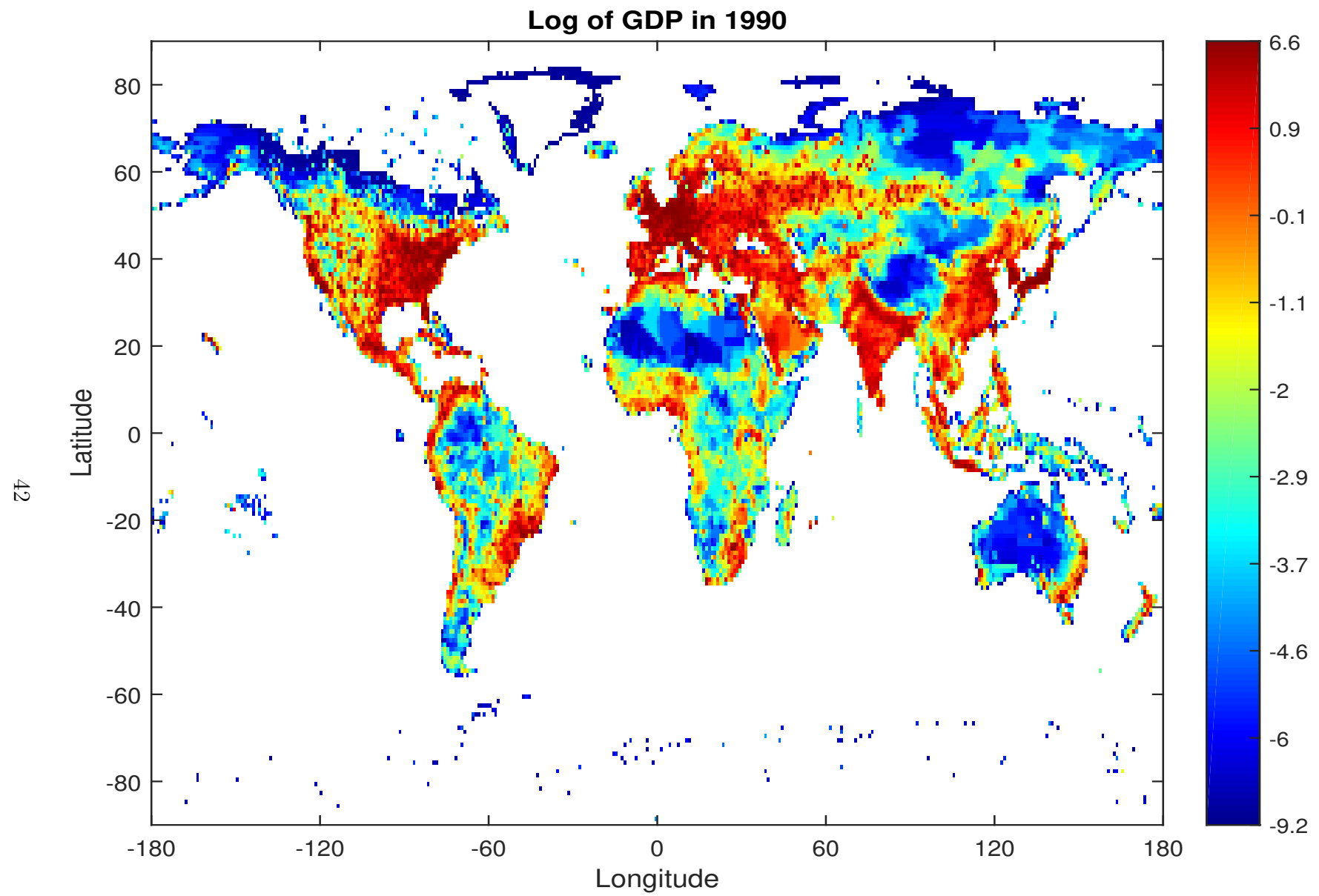


Figure 1: Log of GDP
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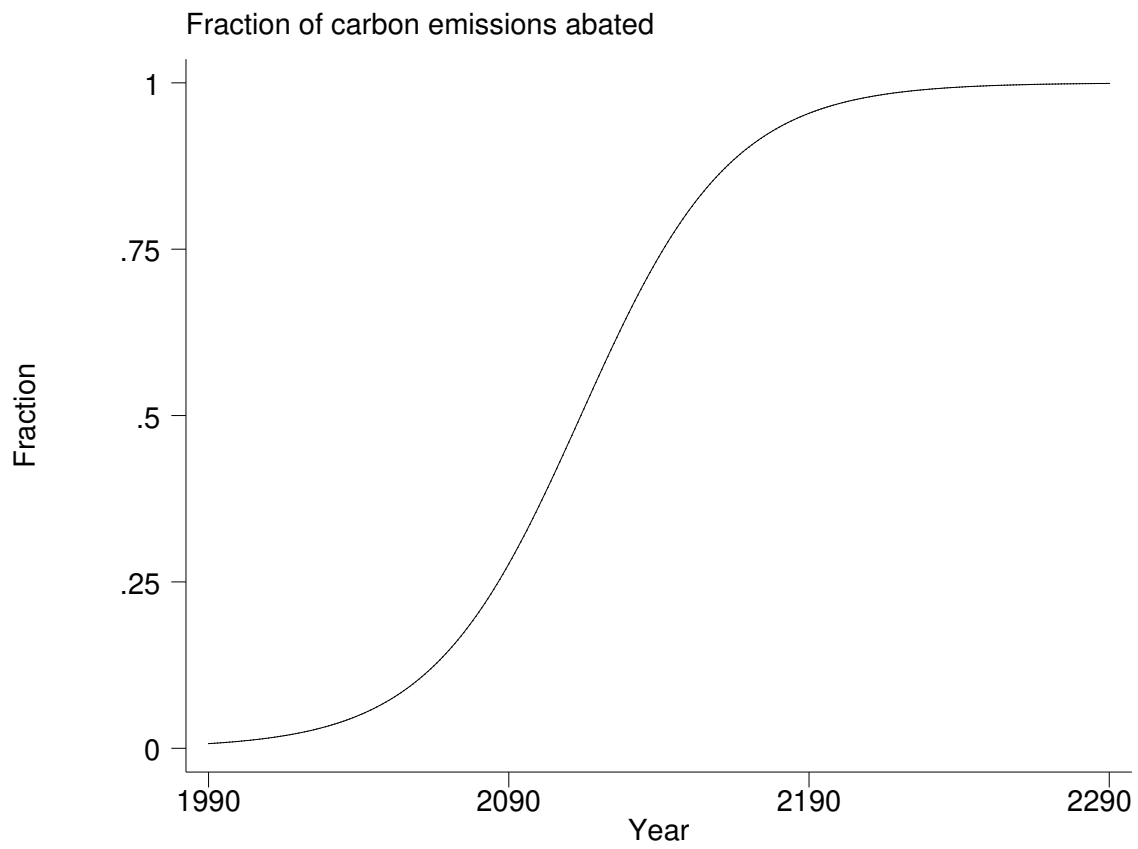


Figure 2: Fraction of Carbon Emissions Abated
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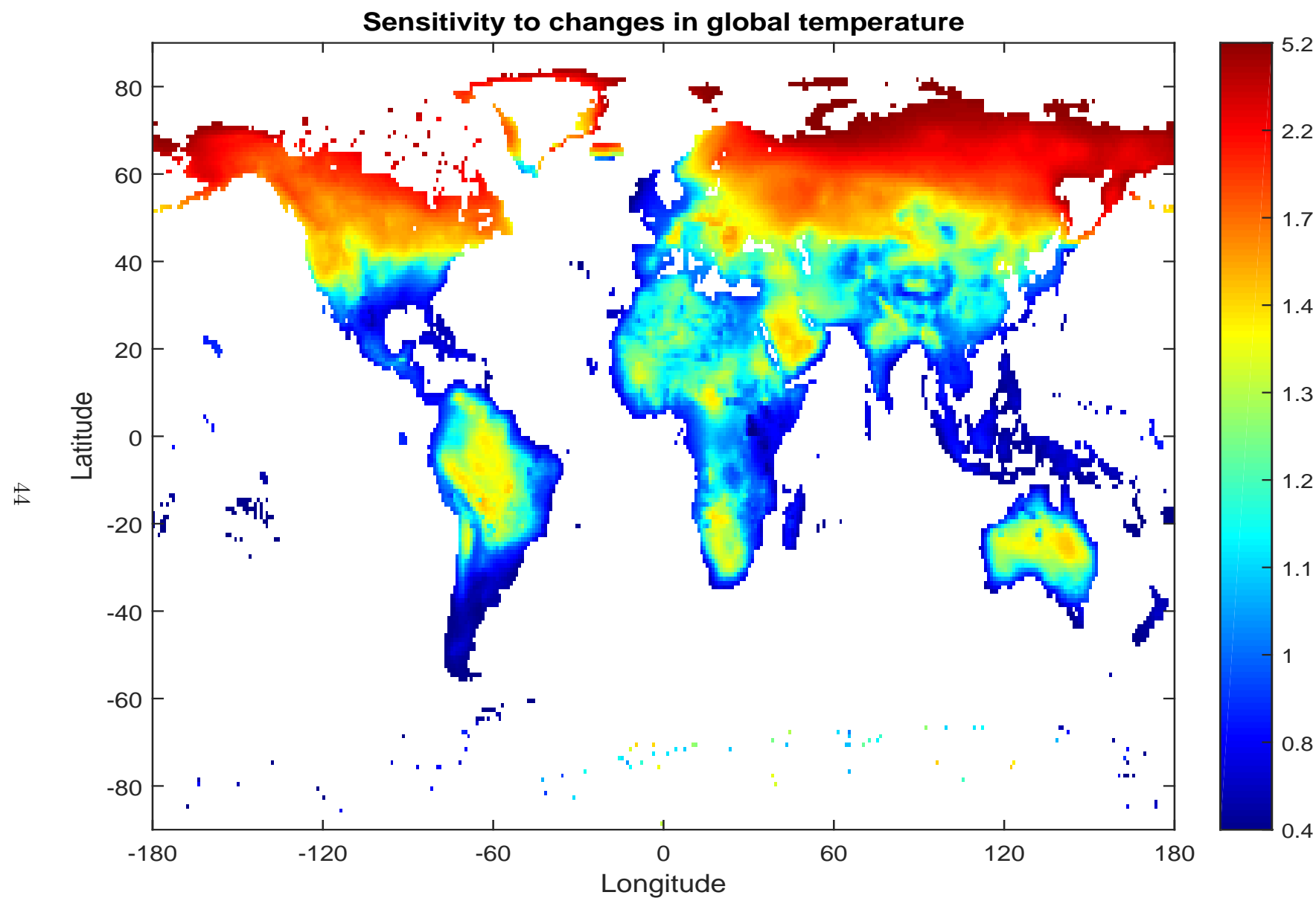


Figure 3: Change in Regional Temperature in Response to a 1° Change in Global Temperature
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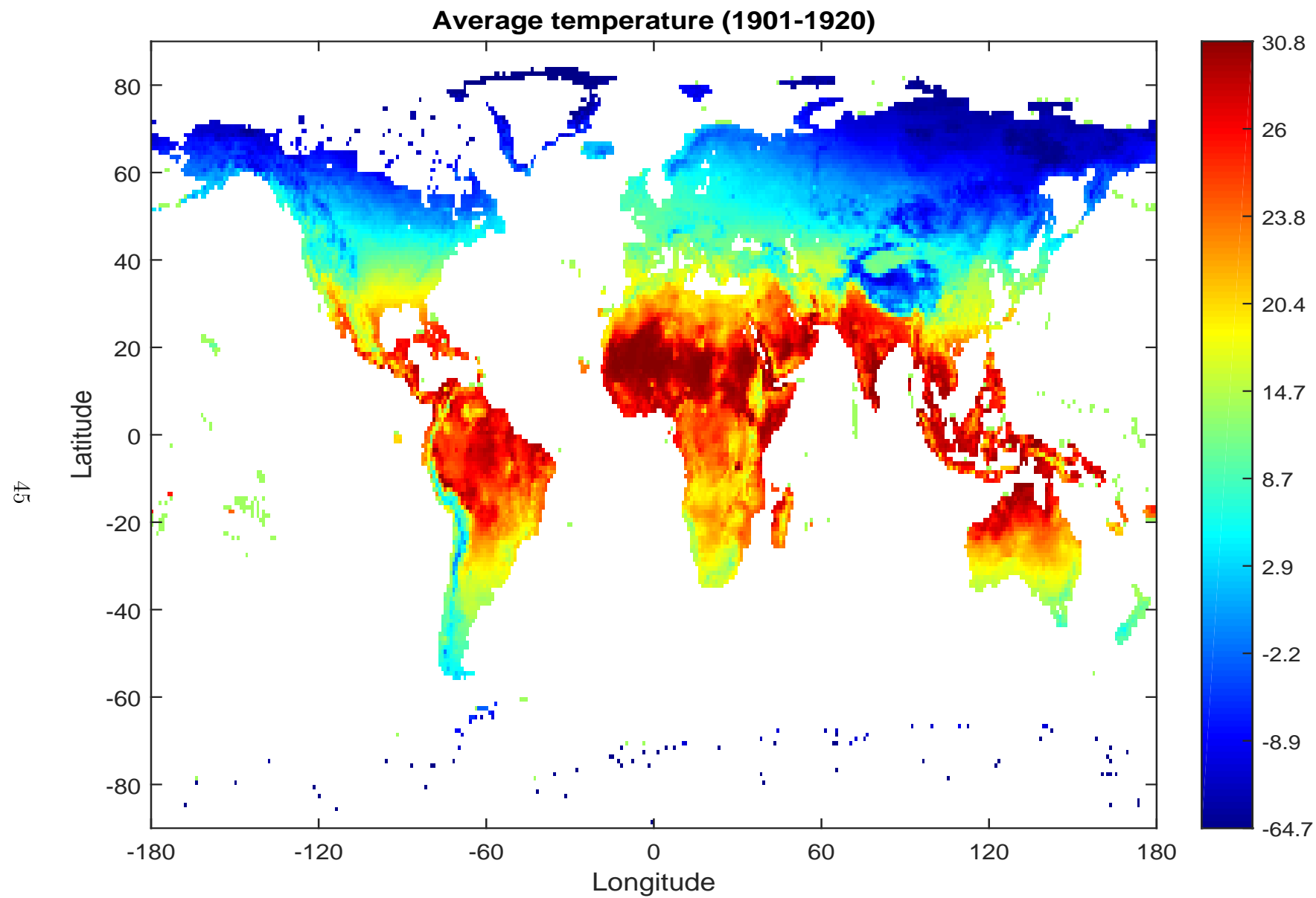


Figure 4: Average Annual Temperature in $^{\circ}C$, 1901 to 1920
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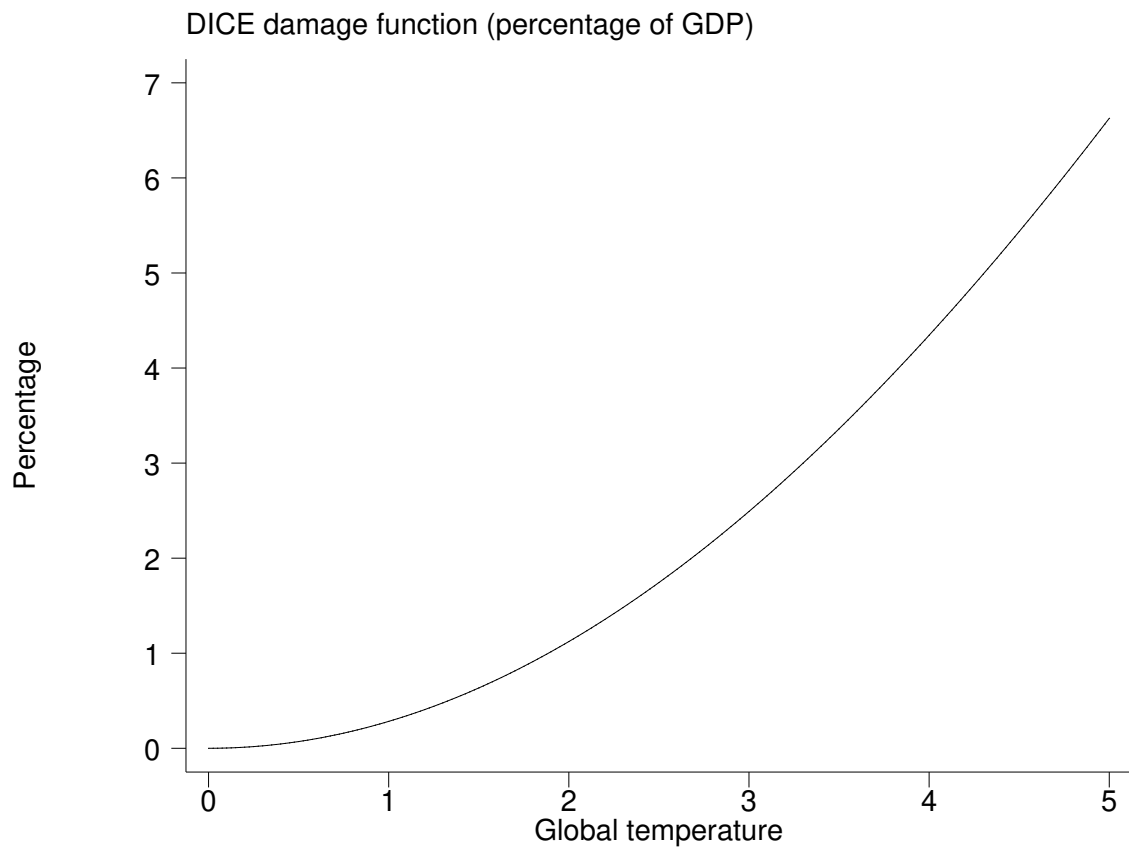


Figure 5: Aggregate Damages from Global Warming in the DICE Model
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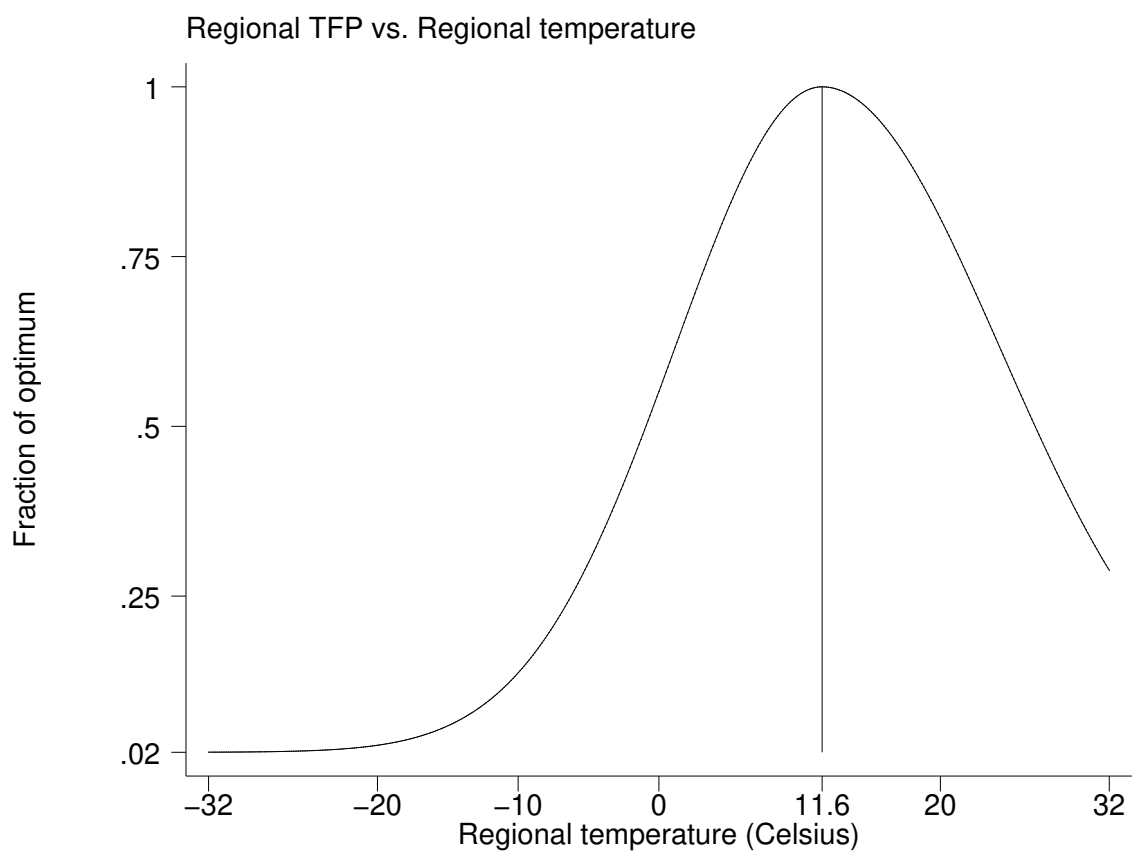


Figure 6: Regional TFP as a Function of Regional Temperature
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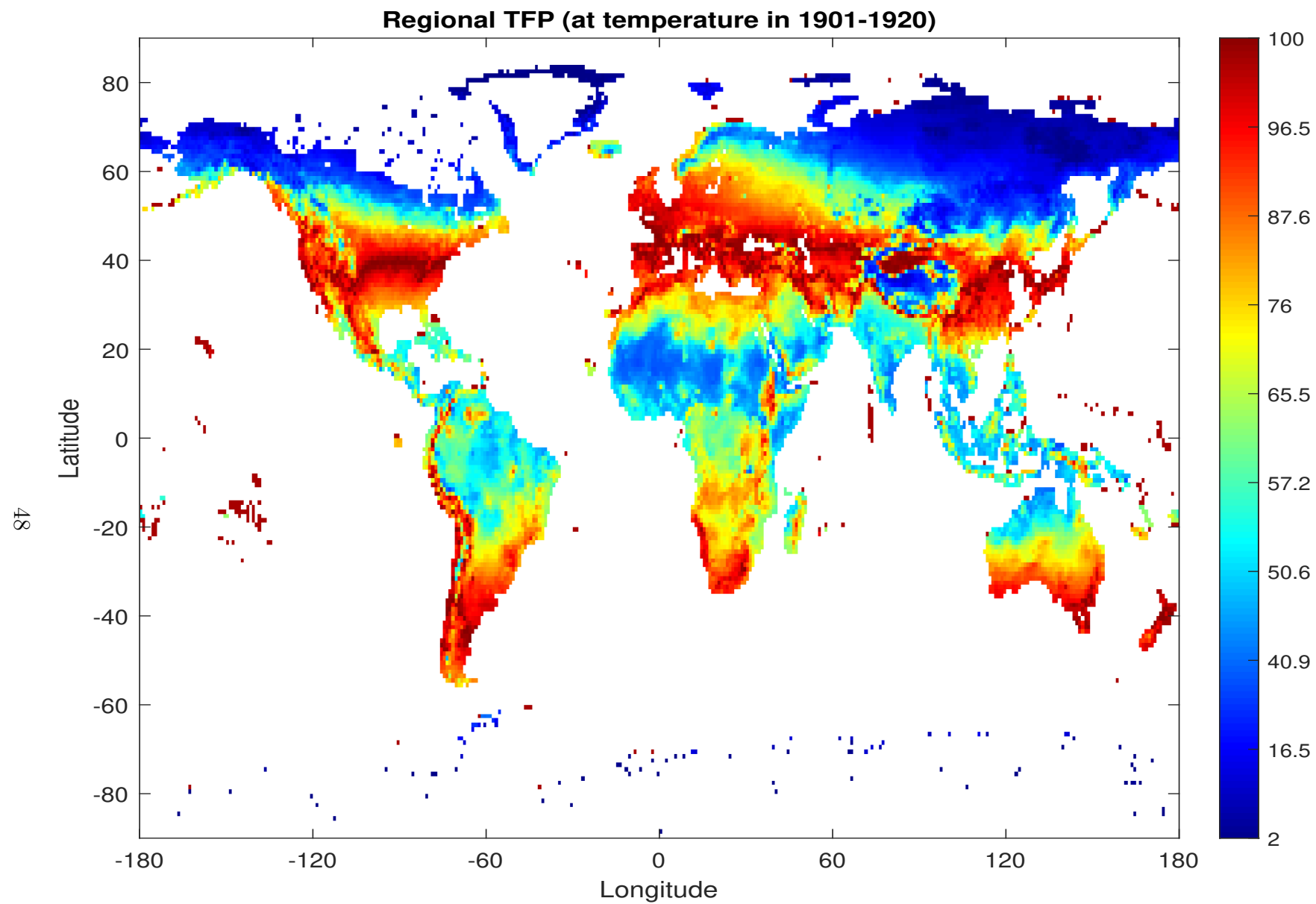


Figure 7: Regional TFP, or $100 \times \tilde{D}(T_i)$, at Average Annual Temperature from 1901 to 1920
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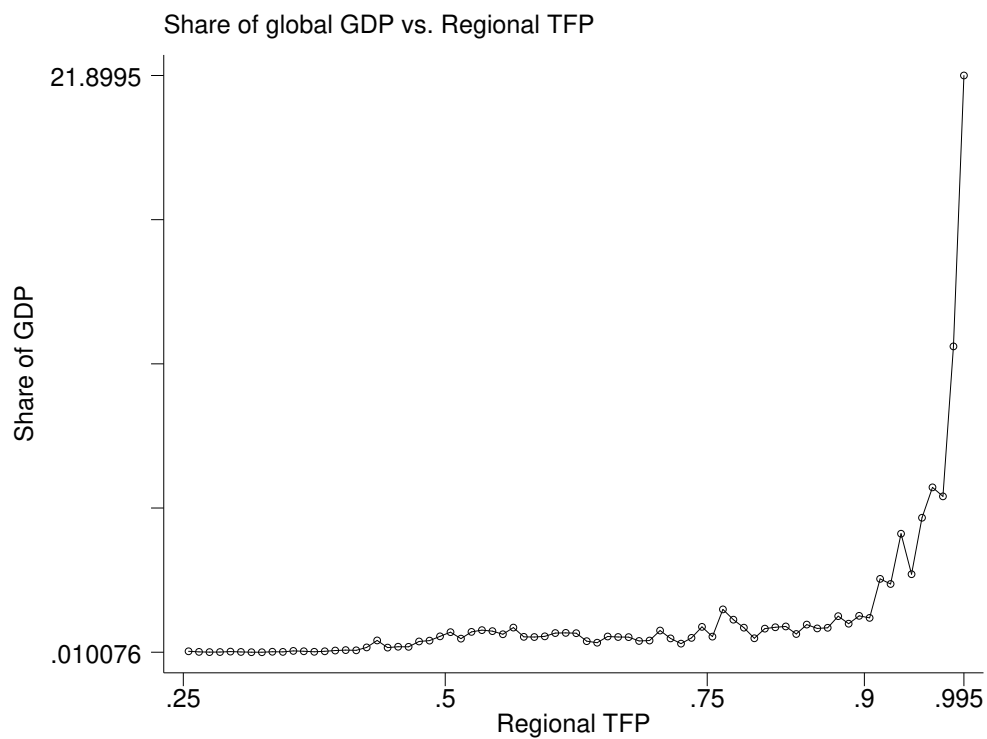


Figure 8: Share of global GDP vs. Regional TFP
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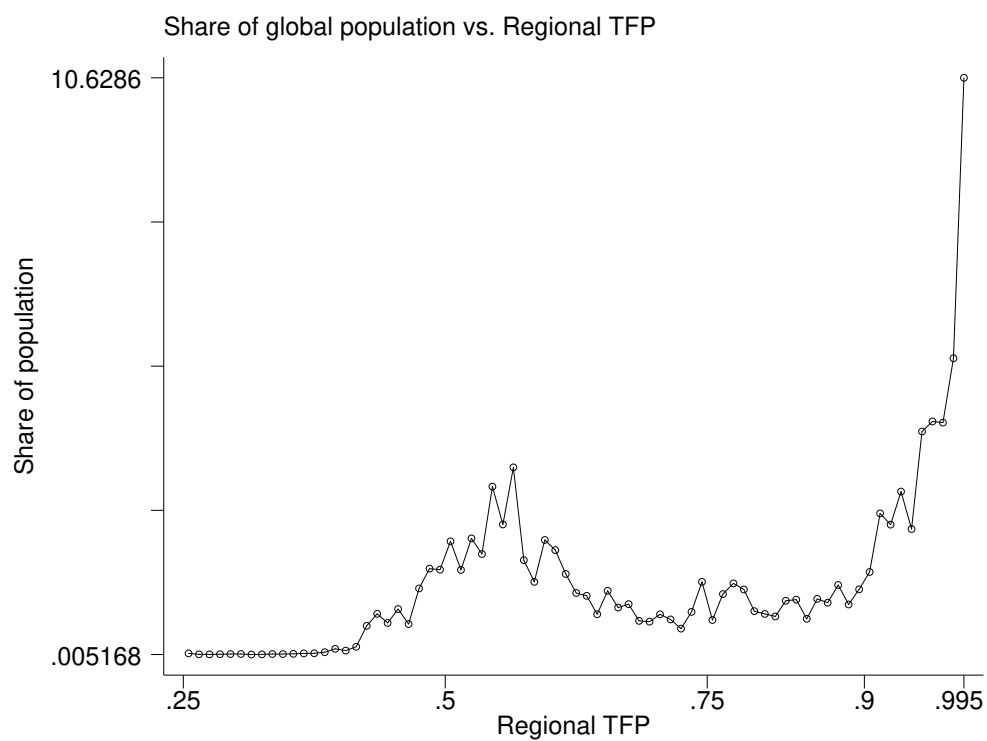


Figure 9: Share of global population vs. Regional TFP
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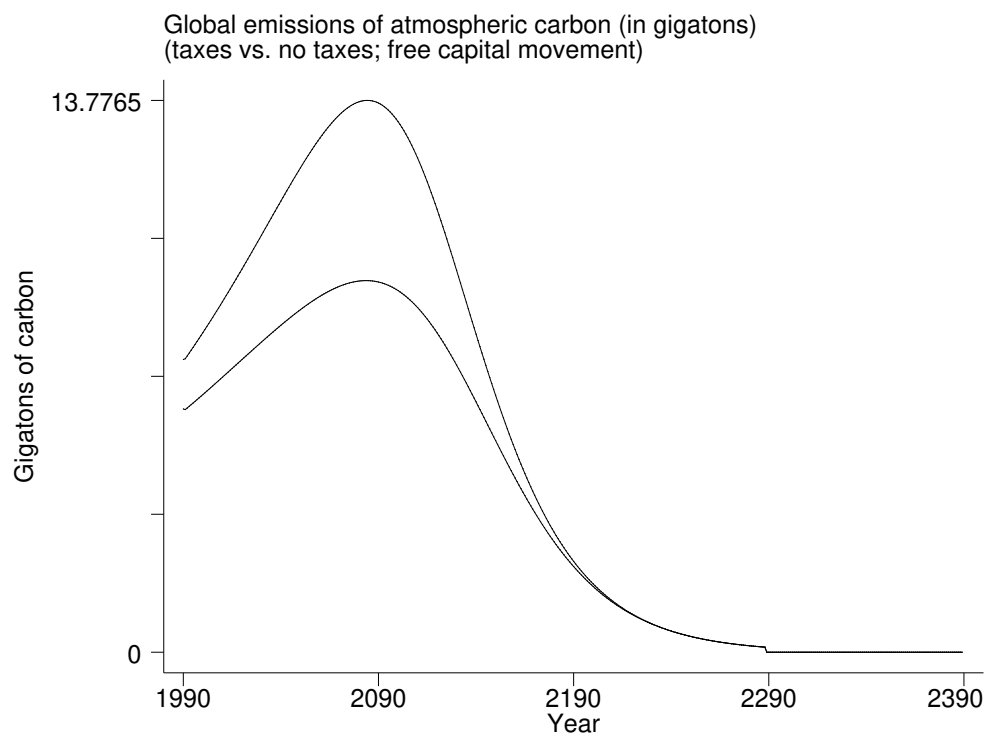


Figure 10: Global Emissions of Atmospheric Carbon
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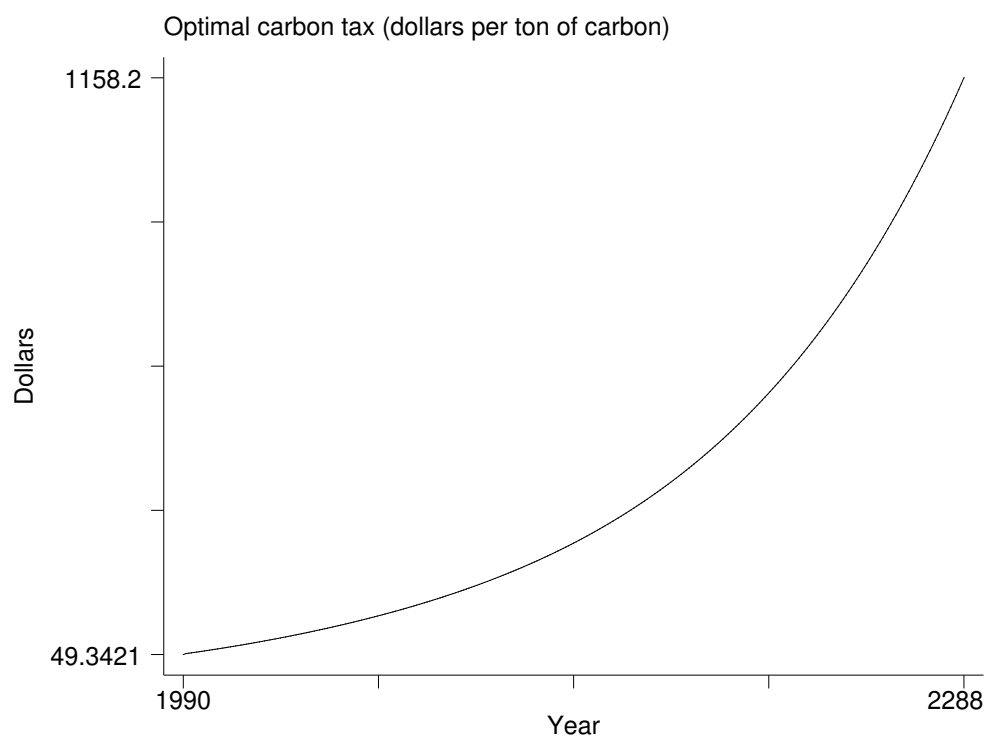


Figure 11: Optimal Path for Tax on Carbon Emissions
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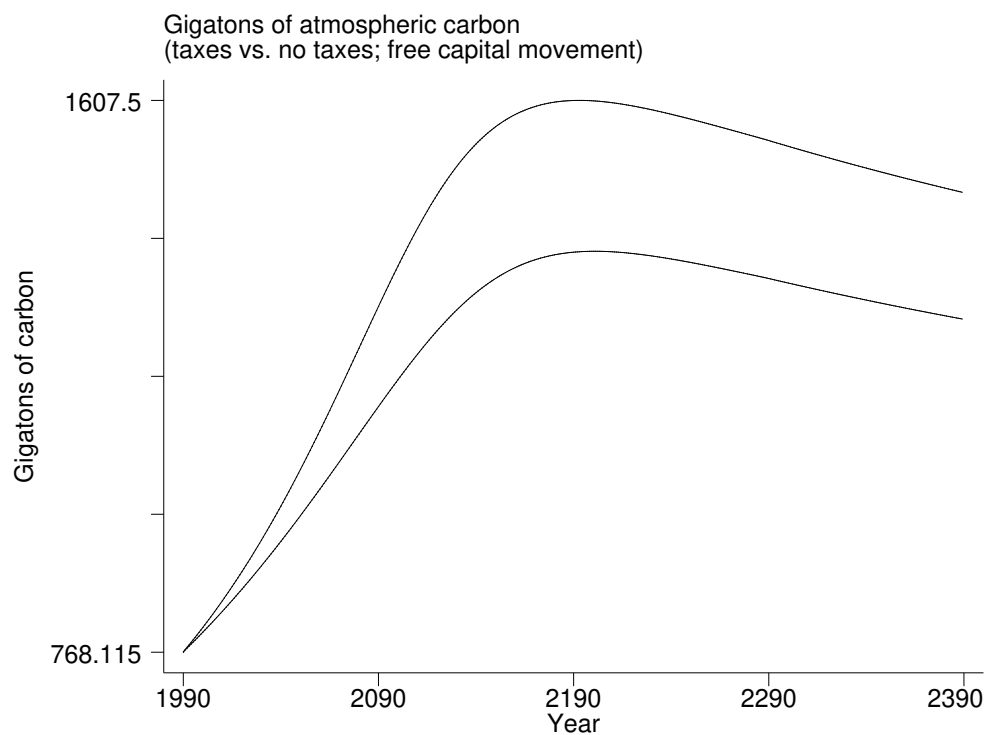


Figure 12: Stock of Atmospheric Carbon
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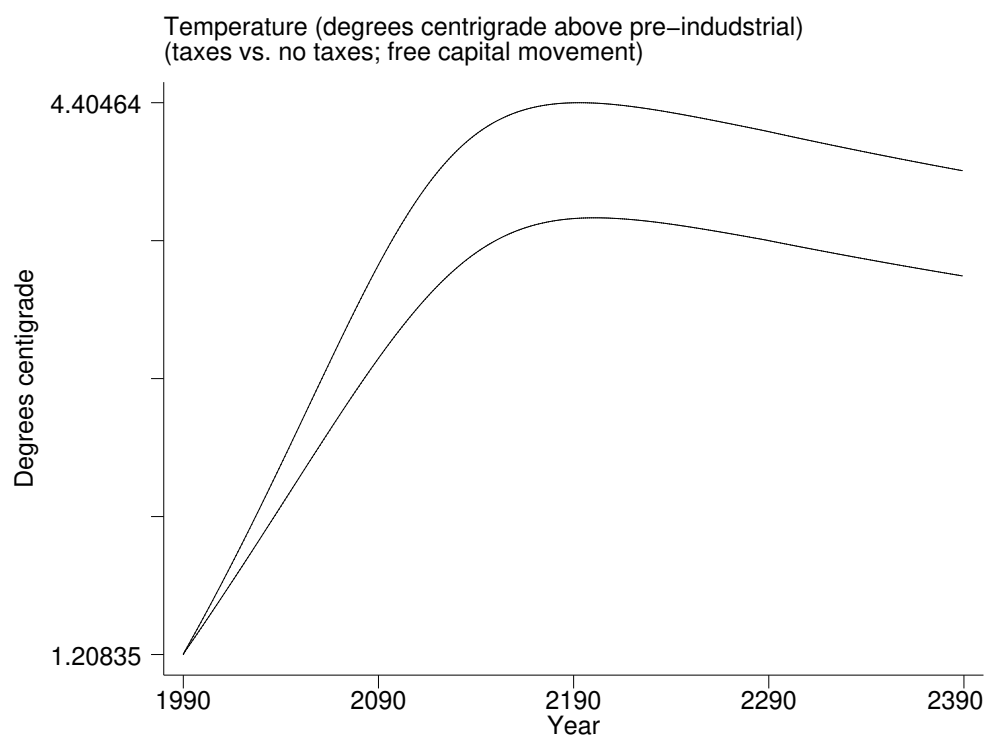


Figure 13: Global Temperature
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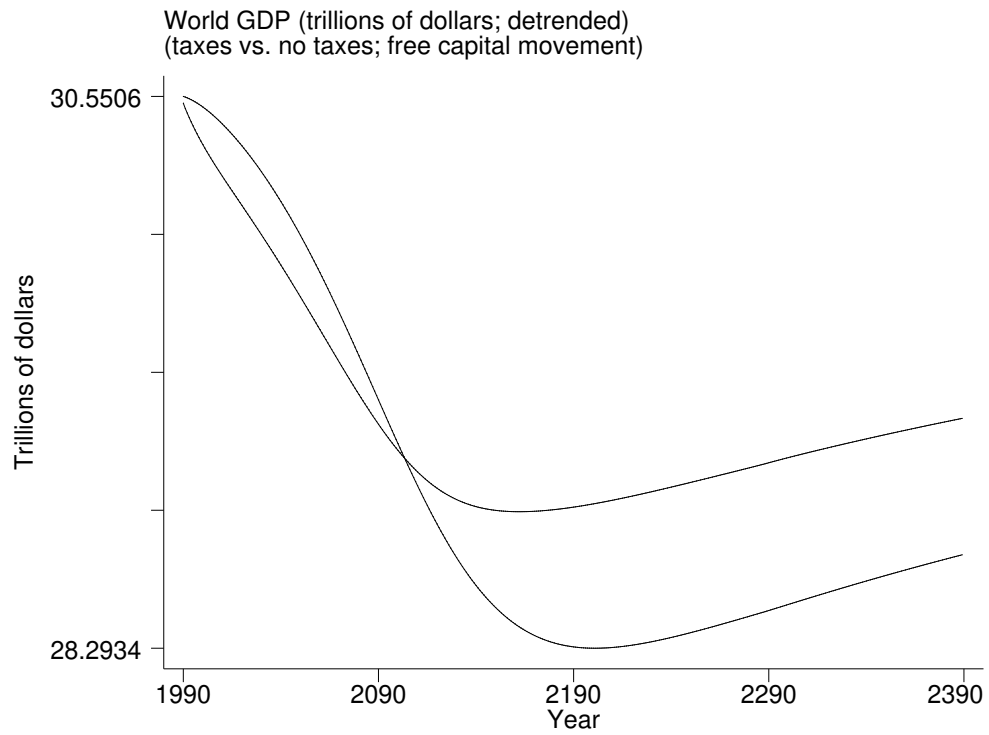


Figure 14: Global GDP (detrended)
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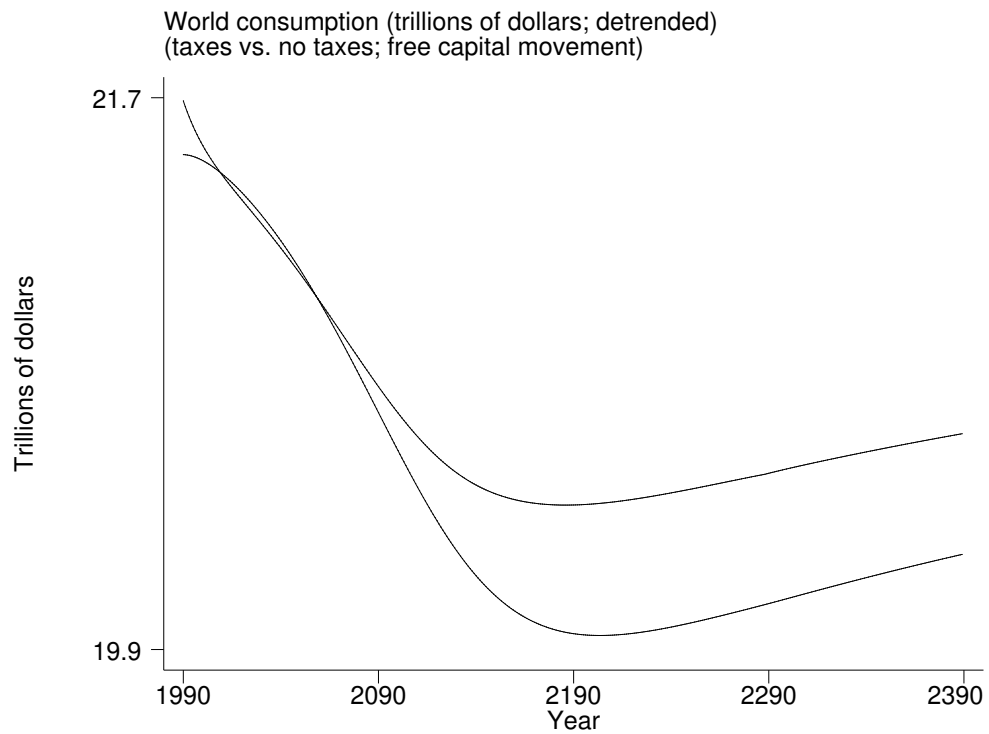


Figure 15: Global Consumption (detrended)
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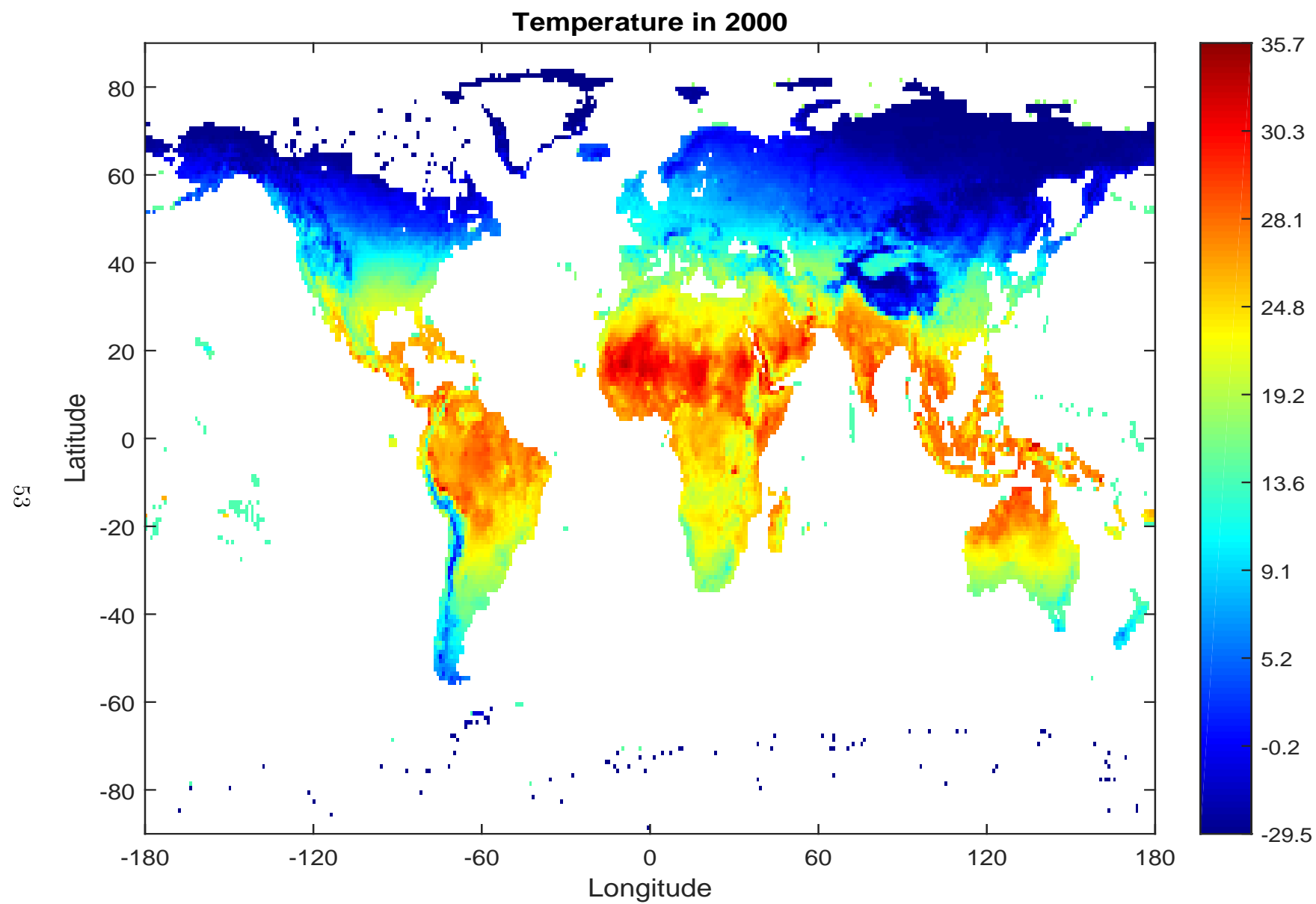


Figure 16a: Regional Temperature
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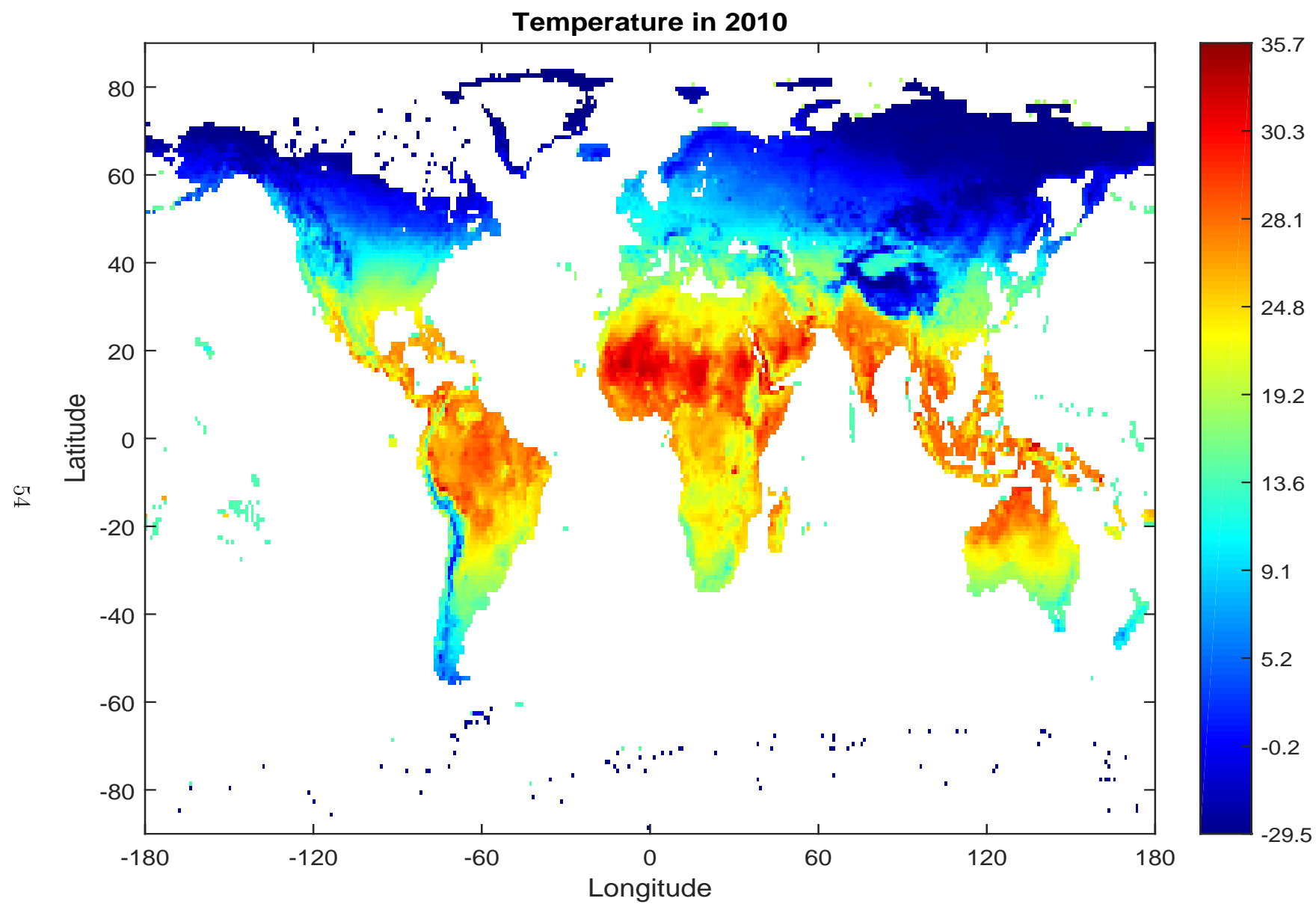


Figure 16b: Regional Temperature
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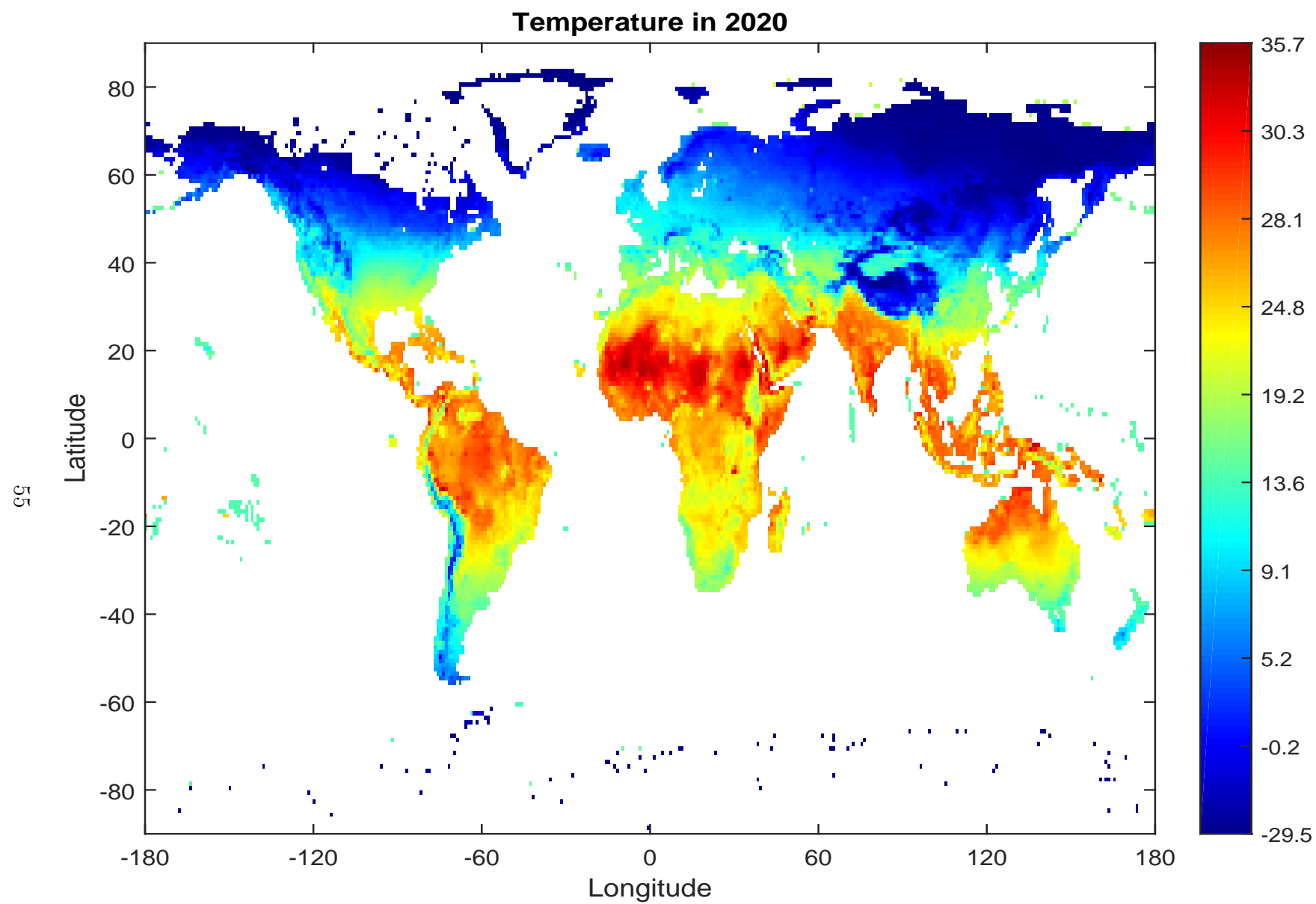


Figure 16c: Regional Temperature
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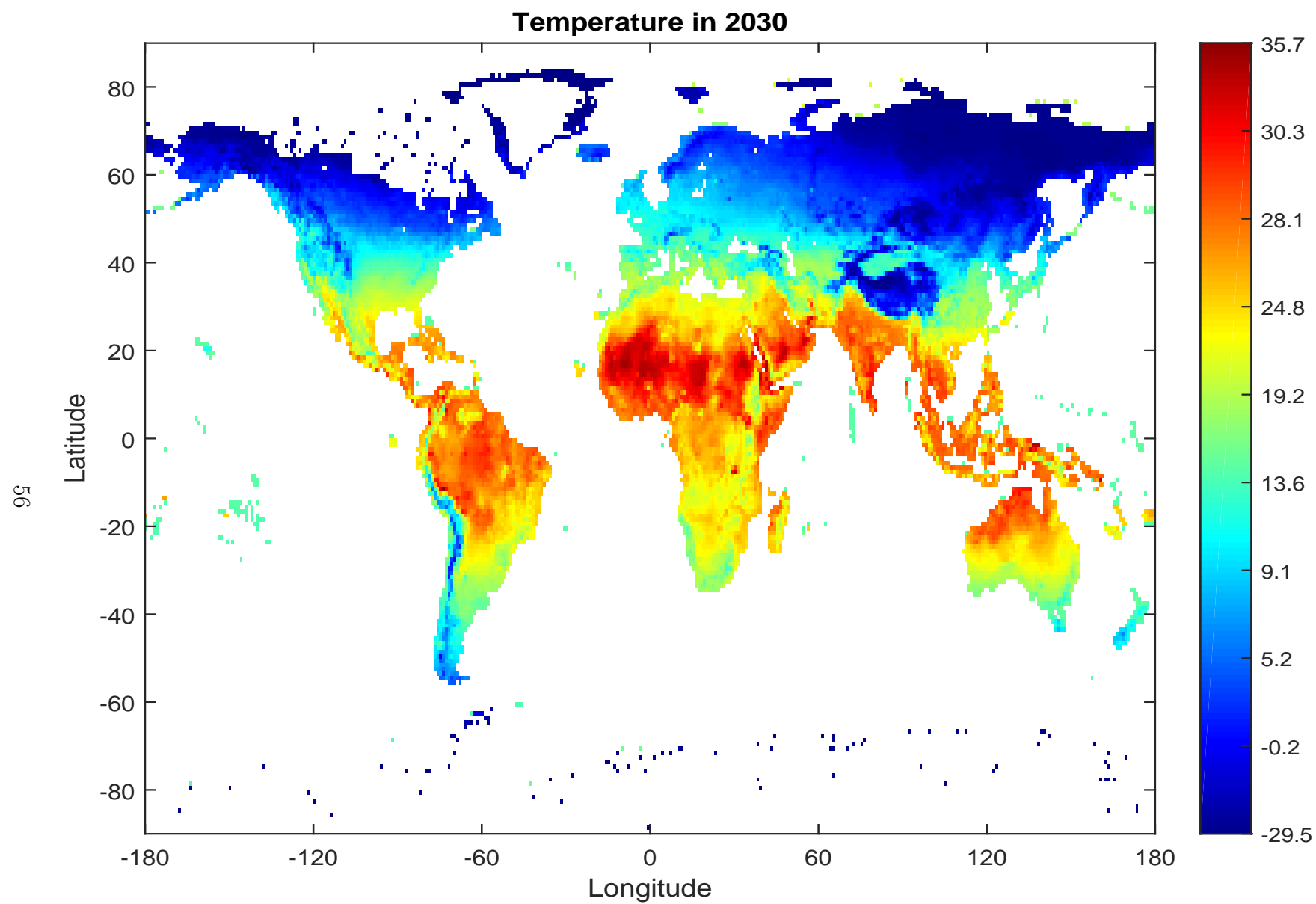


Figure 16d: Regional Temperature
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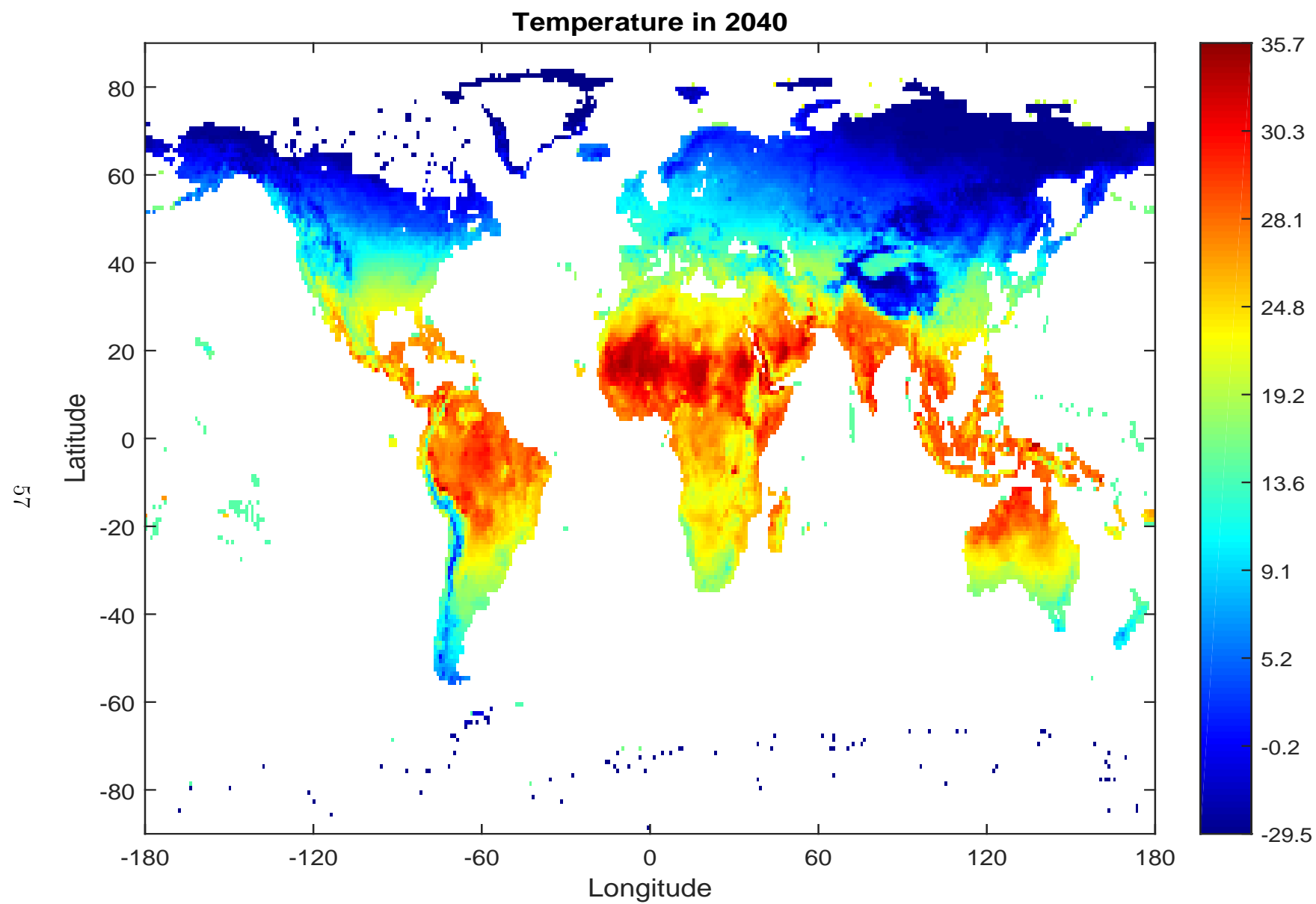


Figure 16e: Regional Temperature
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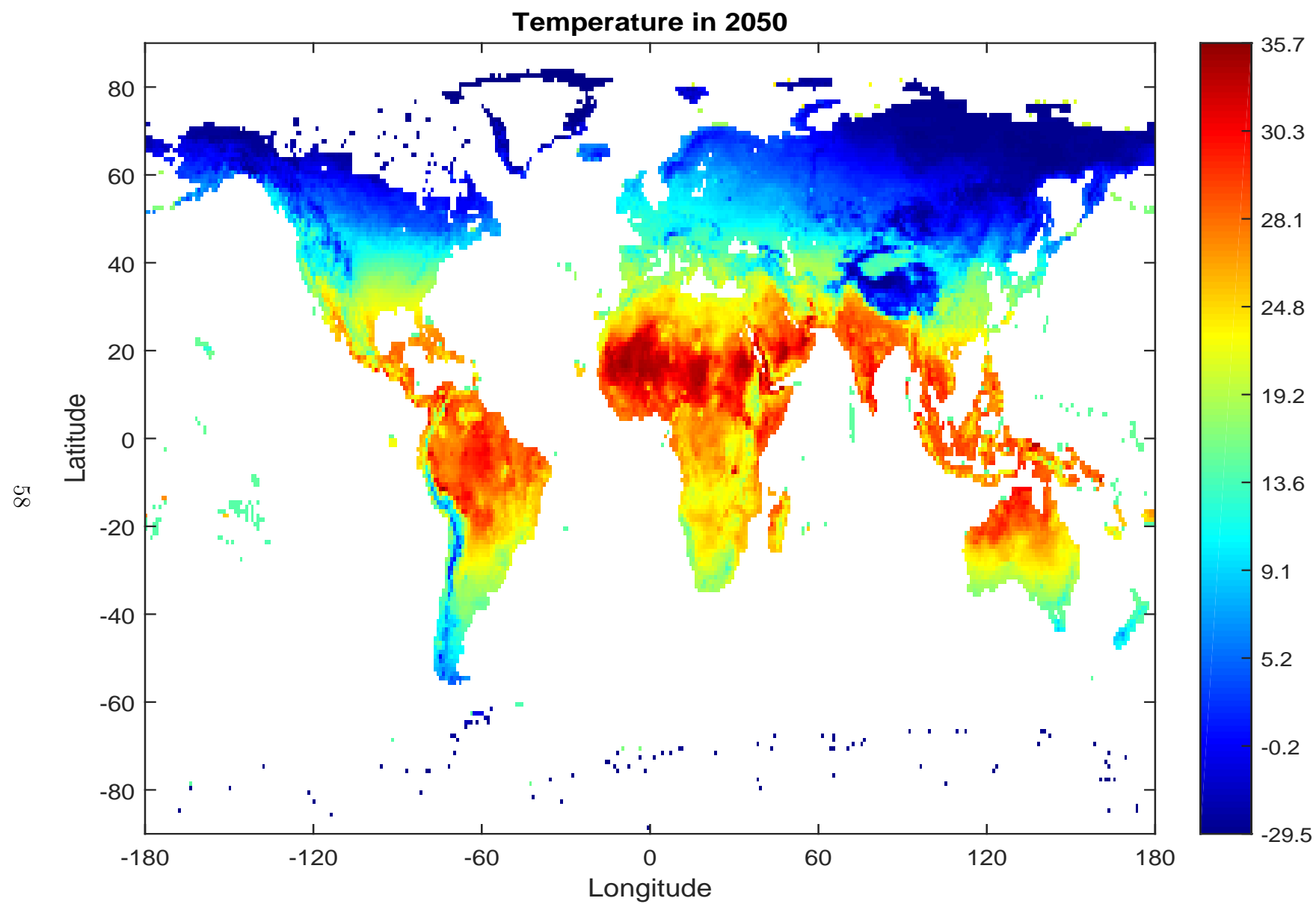


Figure 16f: Regional Temperature
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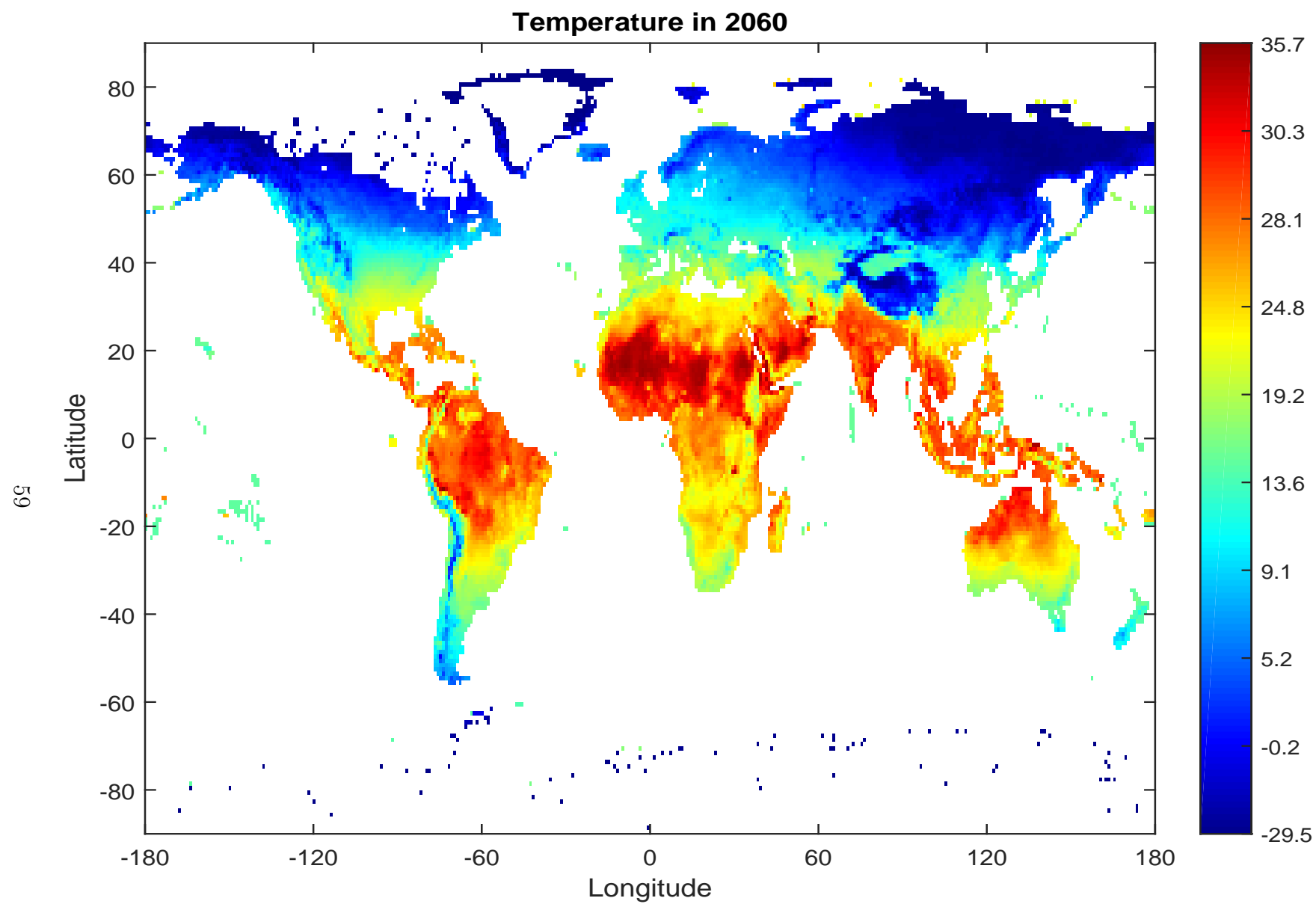


Figure 16g: Regional Temperature
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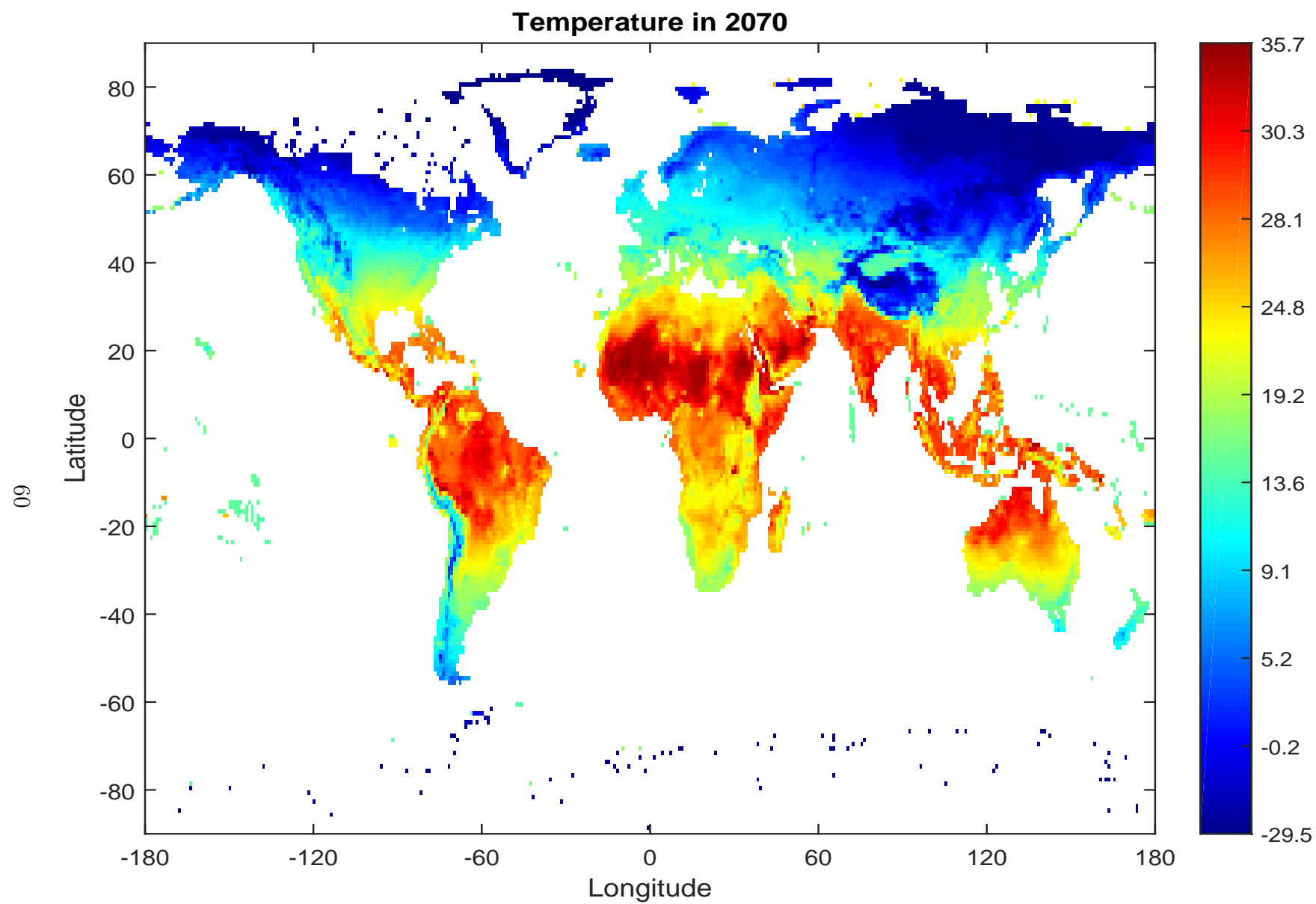


Figure 16h: Regional Temperature
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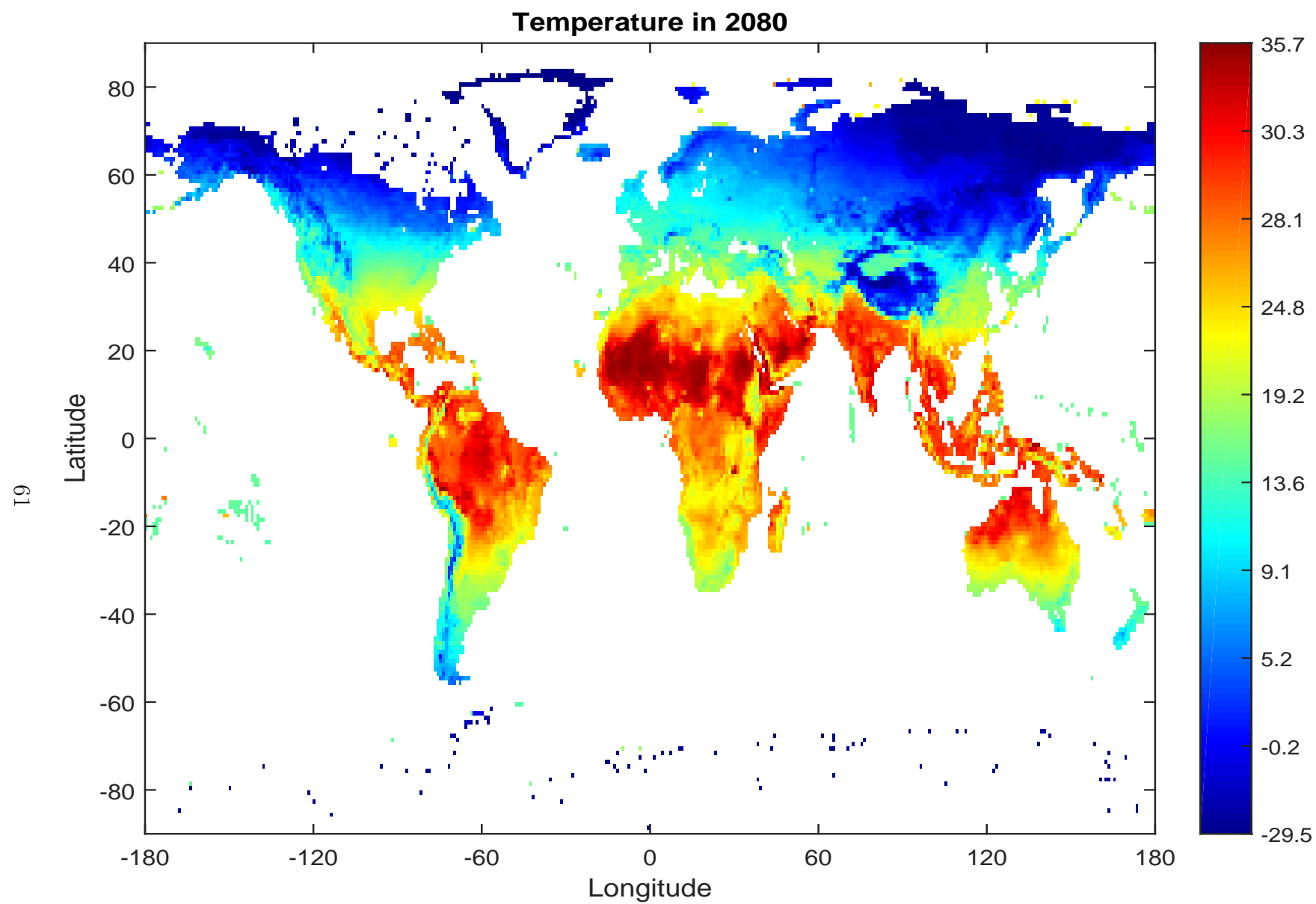


Figure 16i: Regional Temperature
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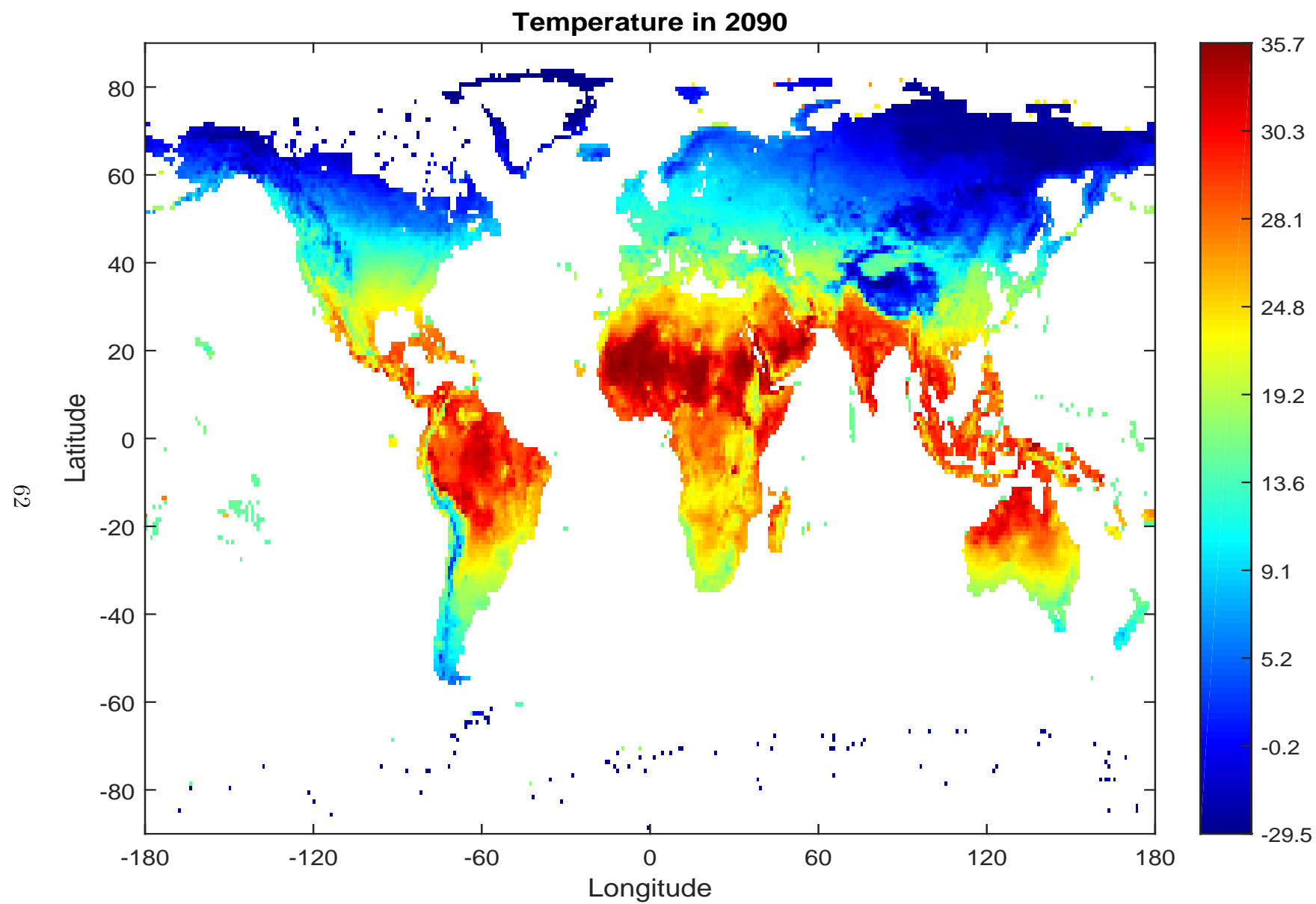


Figure 16j: Regional Temperature
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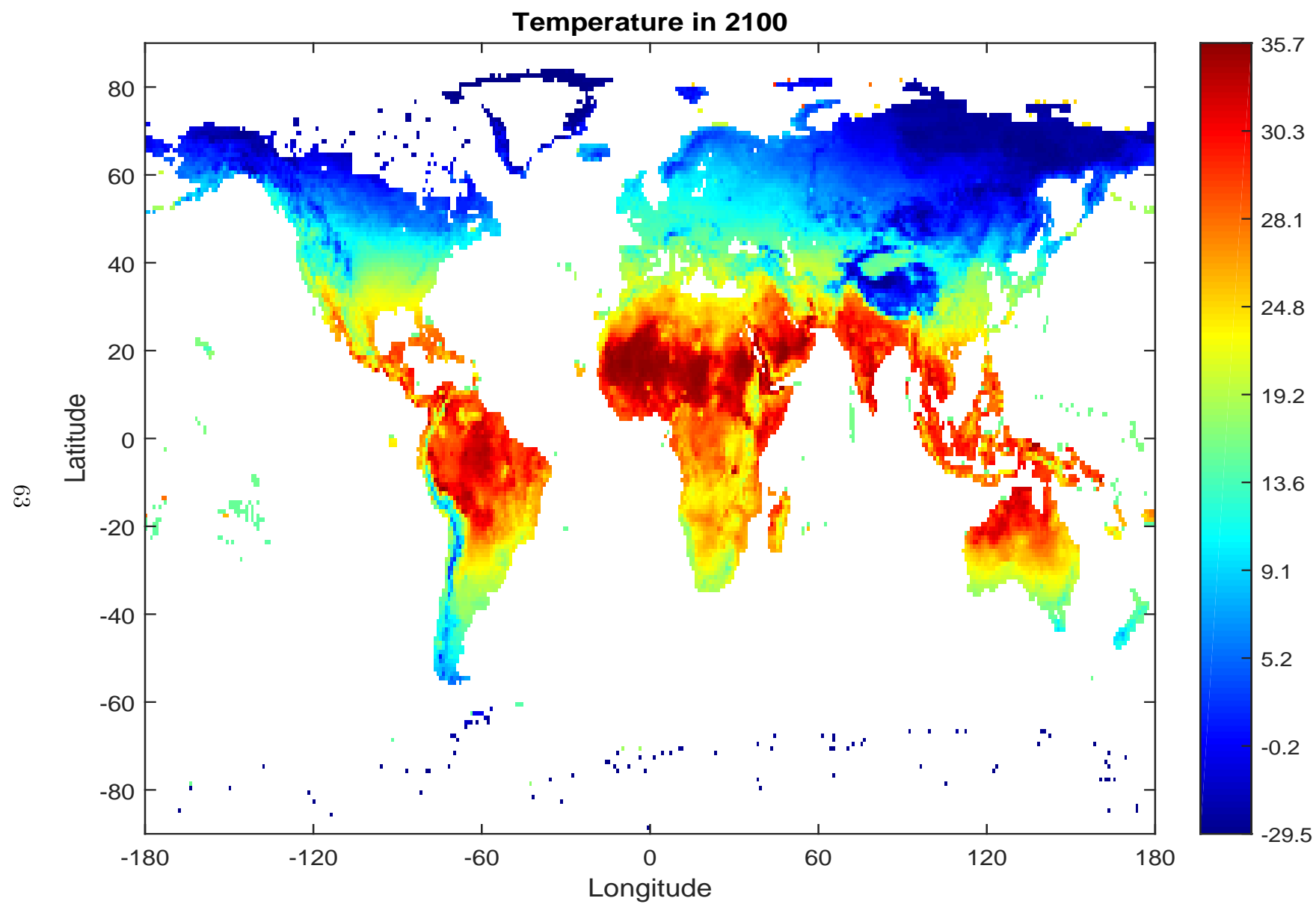


Figure 16k: Regional Temperature
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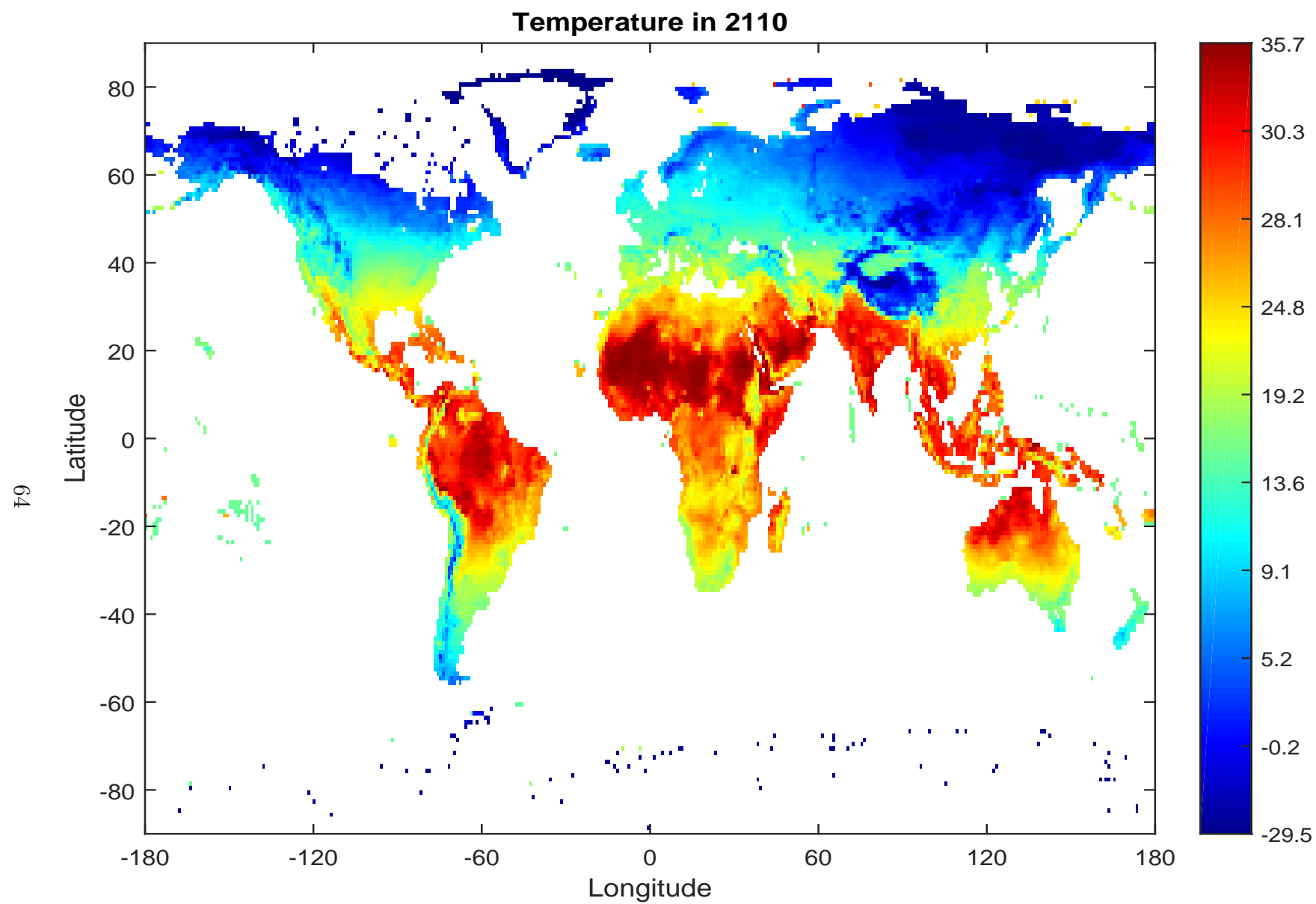


Figure 16l: Regional Temperature
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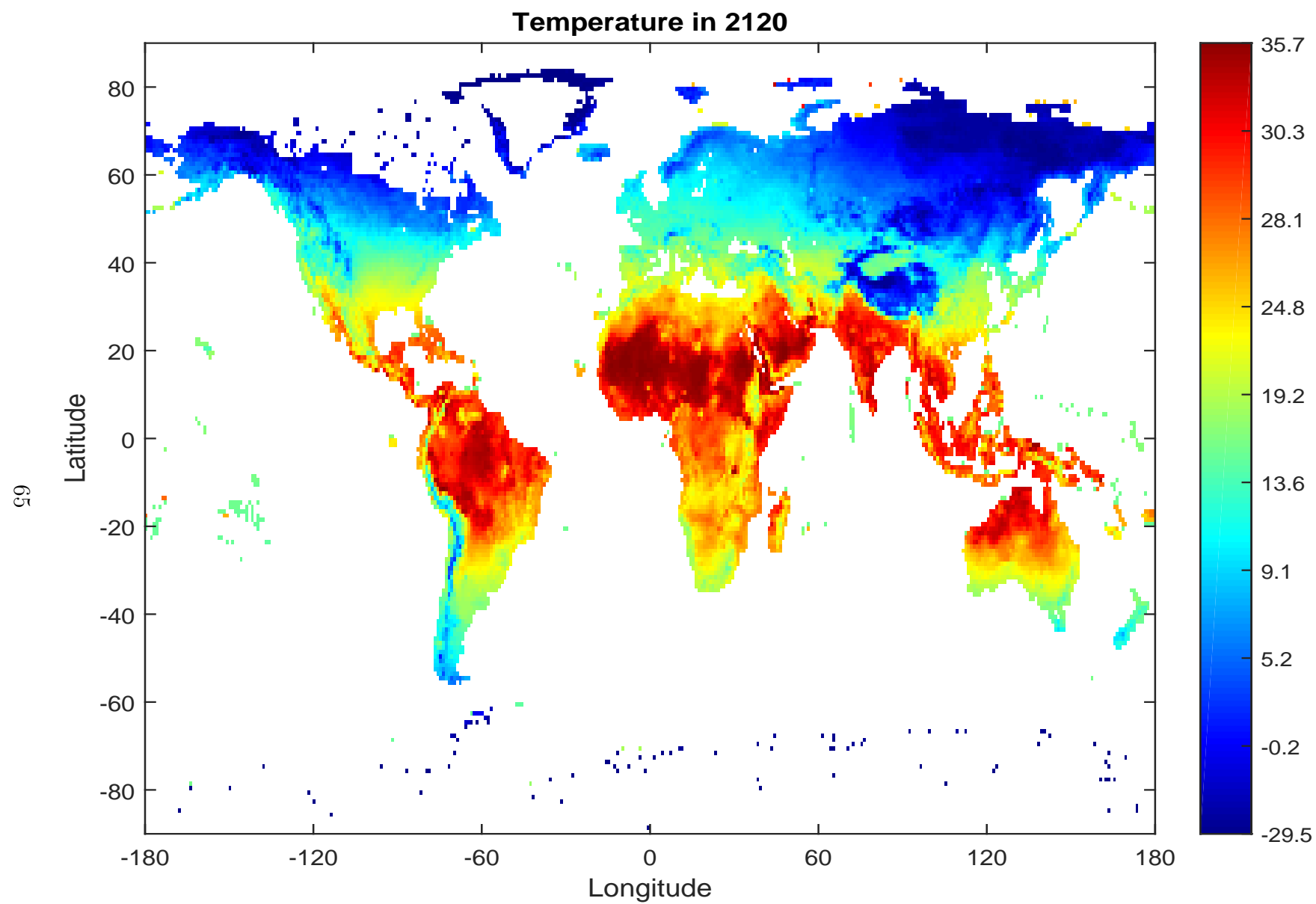


Figure 16m: Regional Temperature
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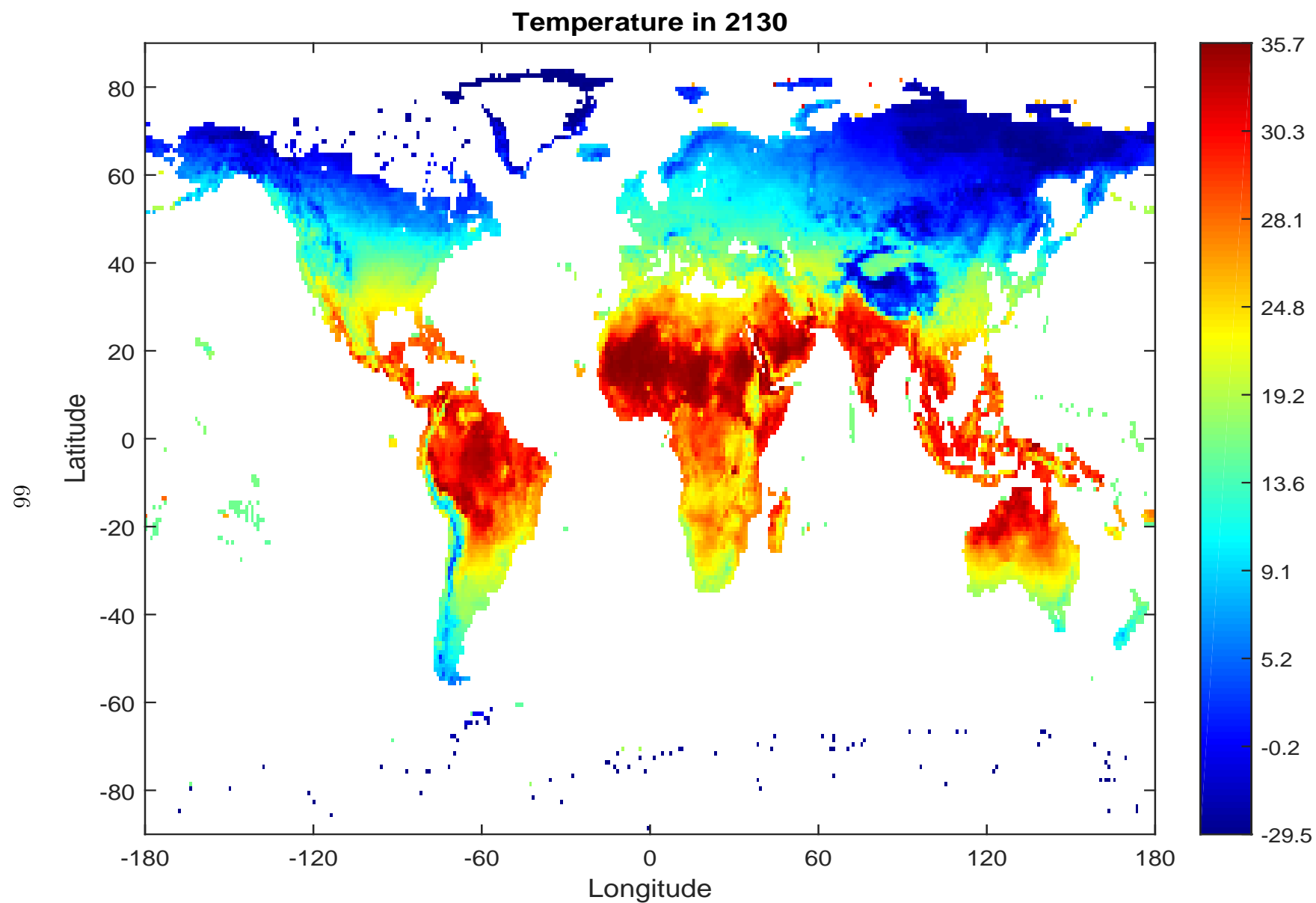


Figure 16n: Regional Temperature
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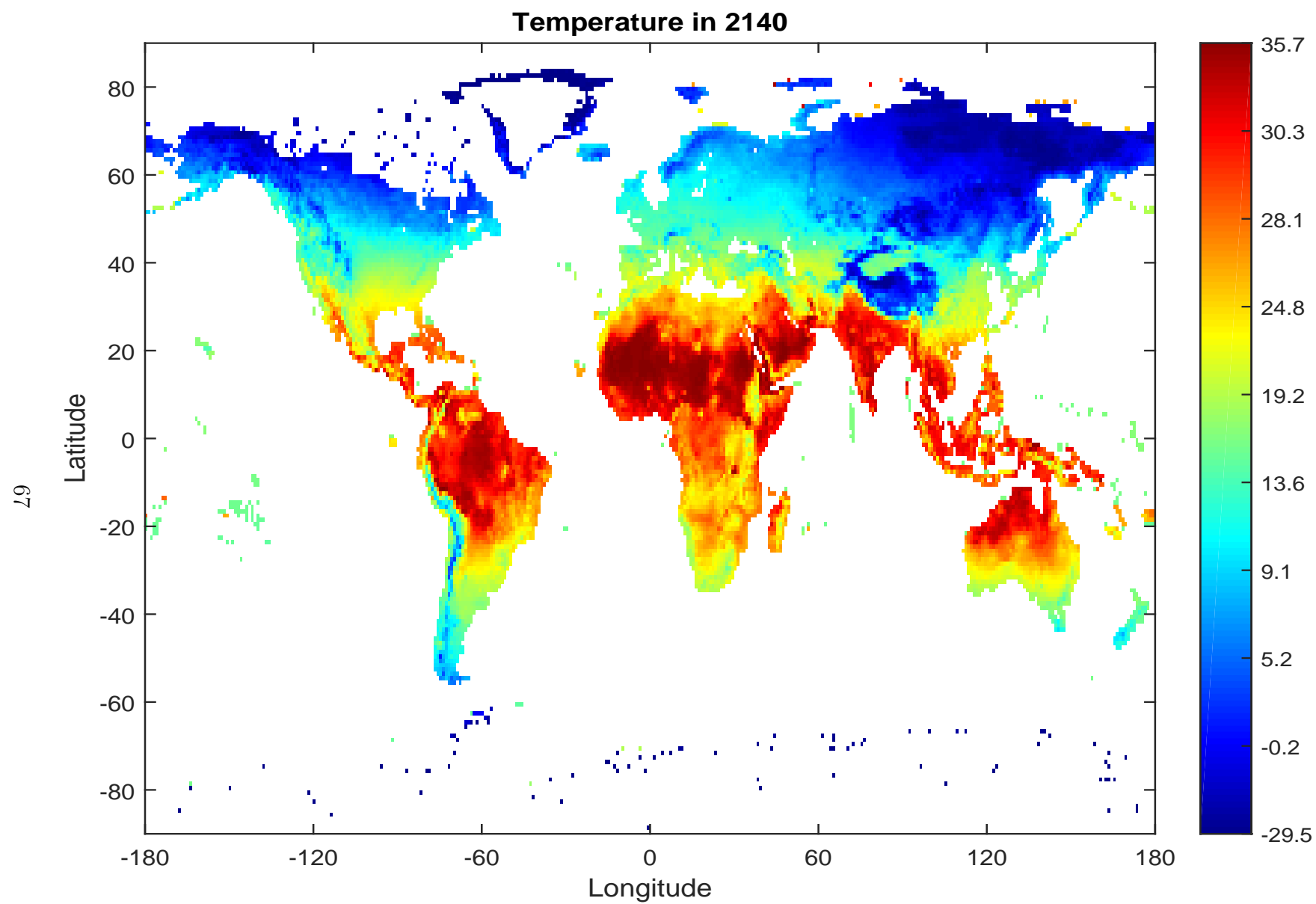


Figure 16o: Regional Temperature
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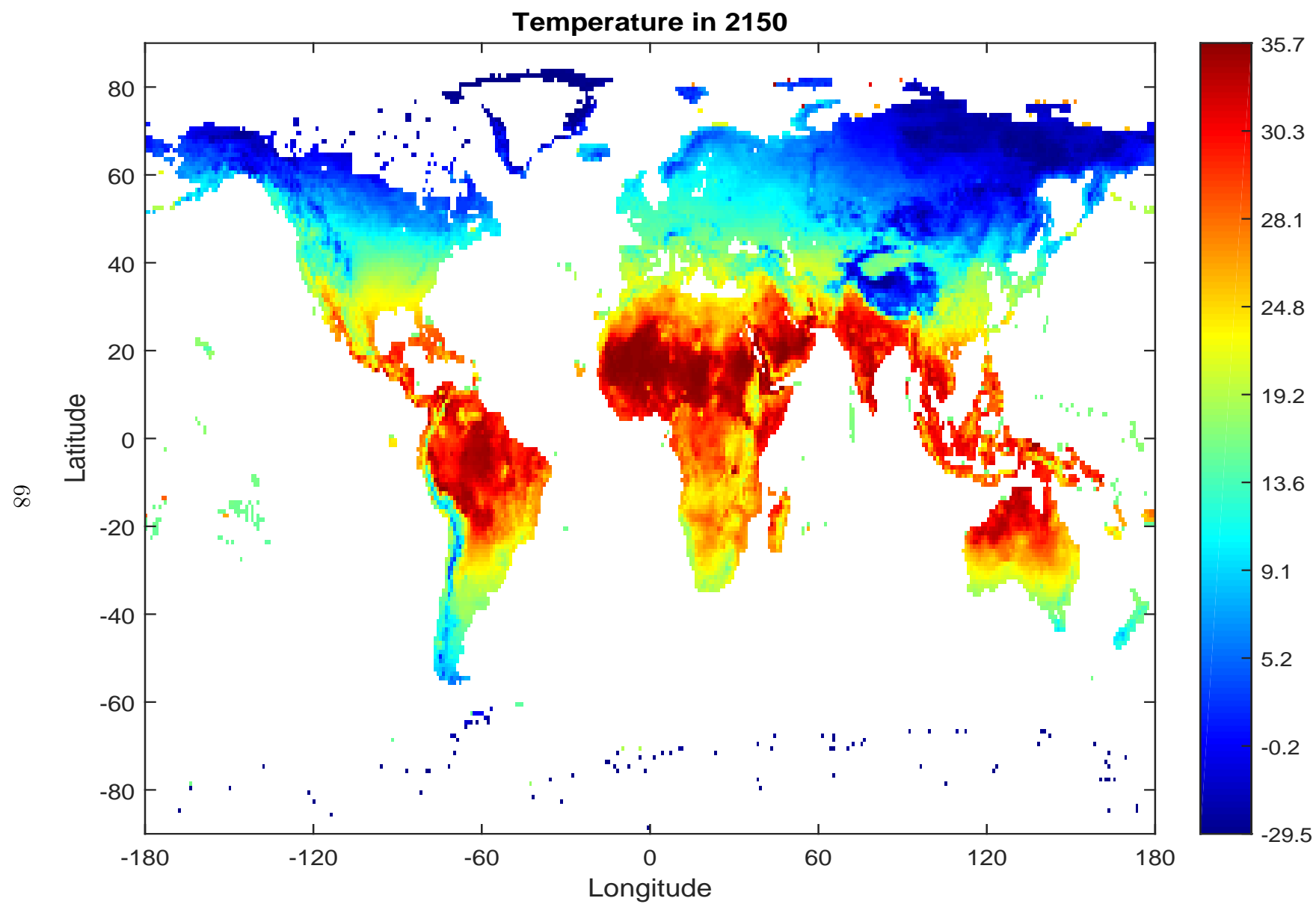


Figure 16p: Regional Temperature
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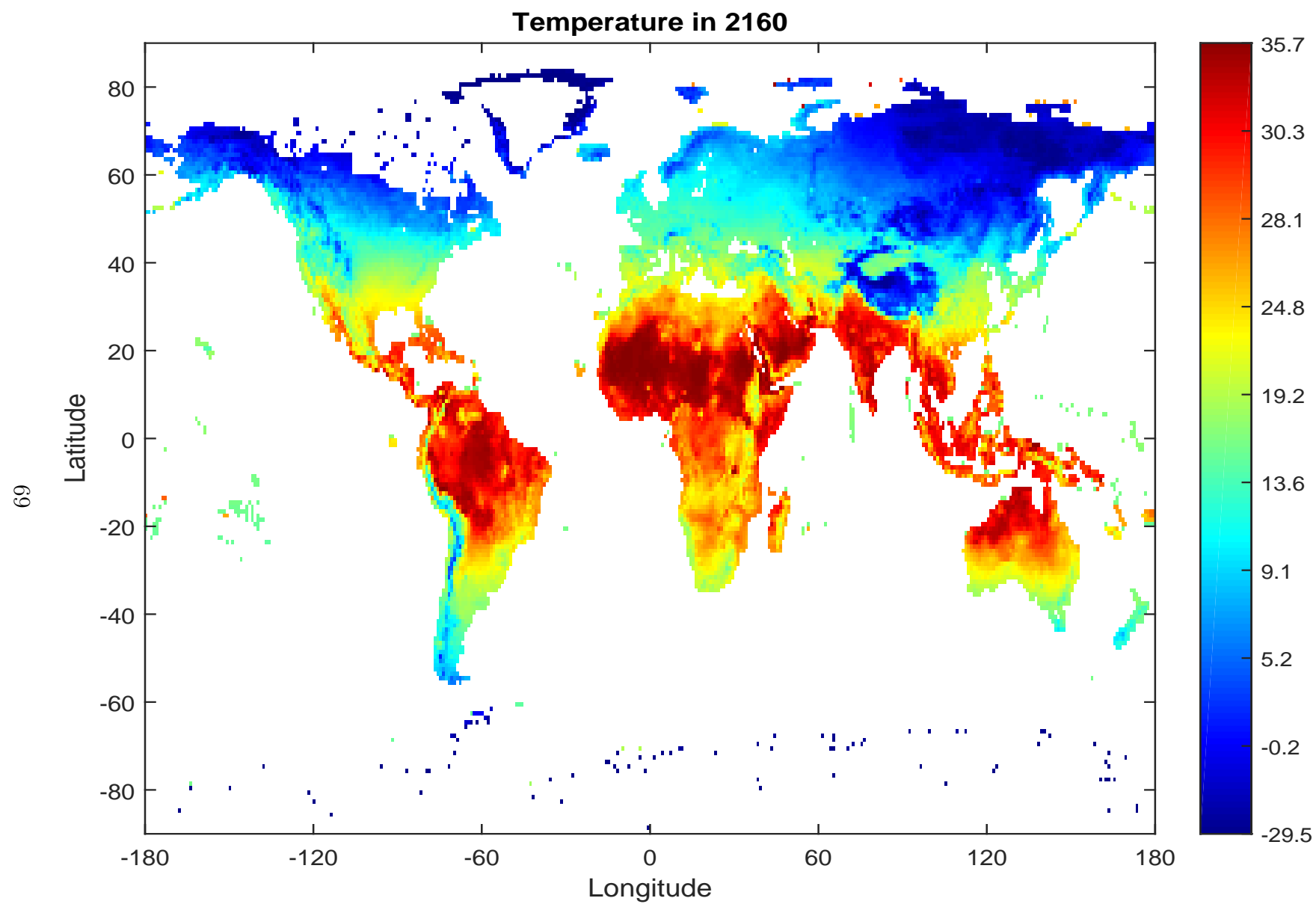


Figure 16q: Regional Temperature
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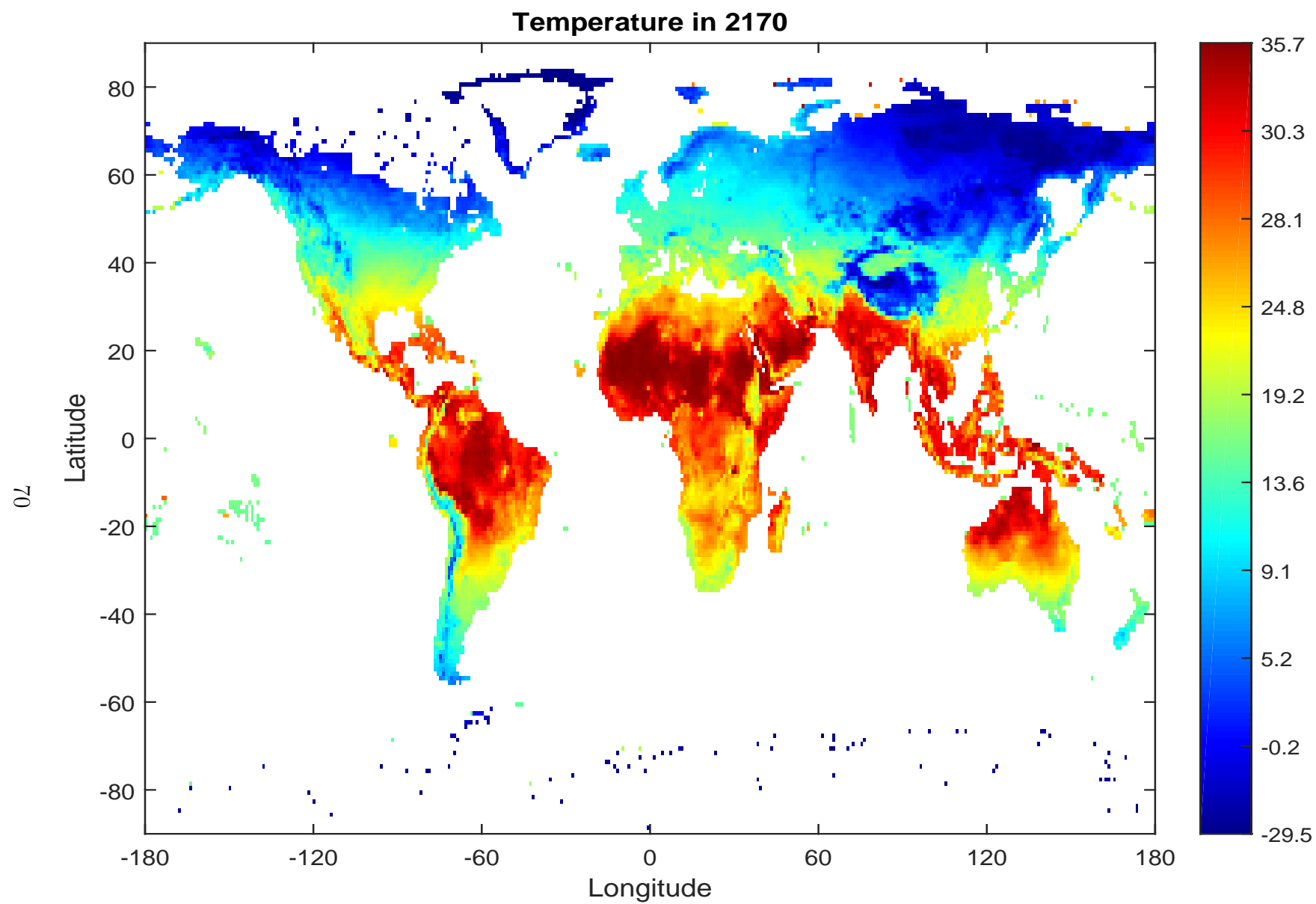


Figure 16r: Regional Temperature
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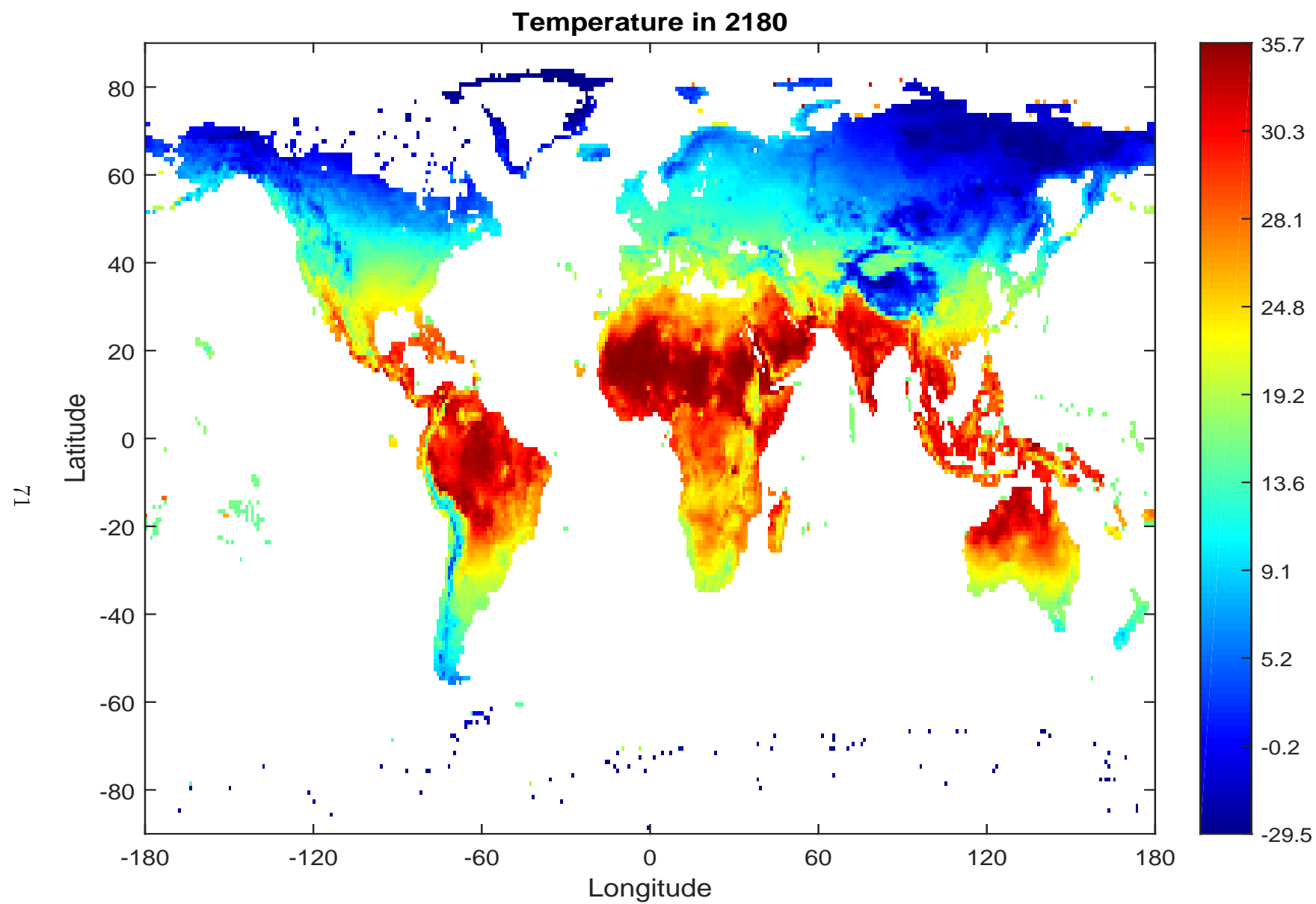


Figure 16s: Regional Temperature
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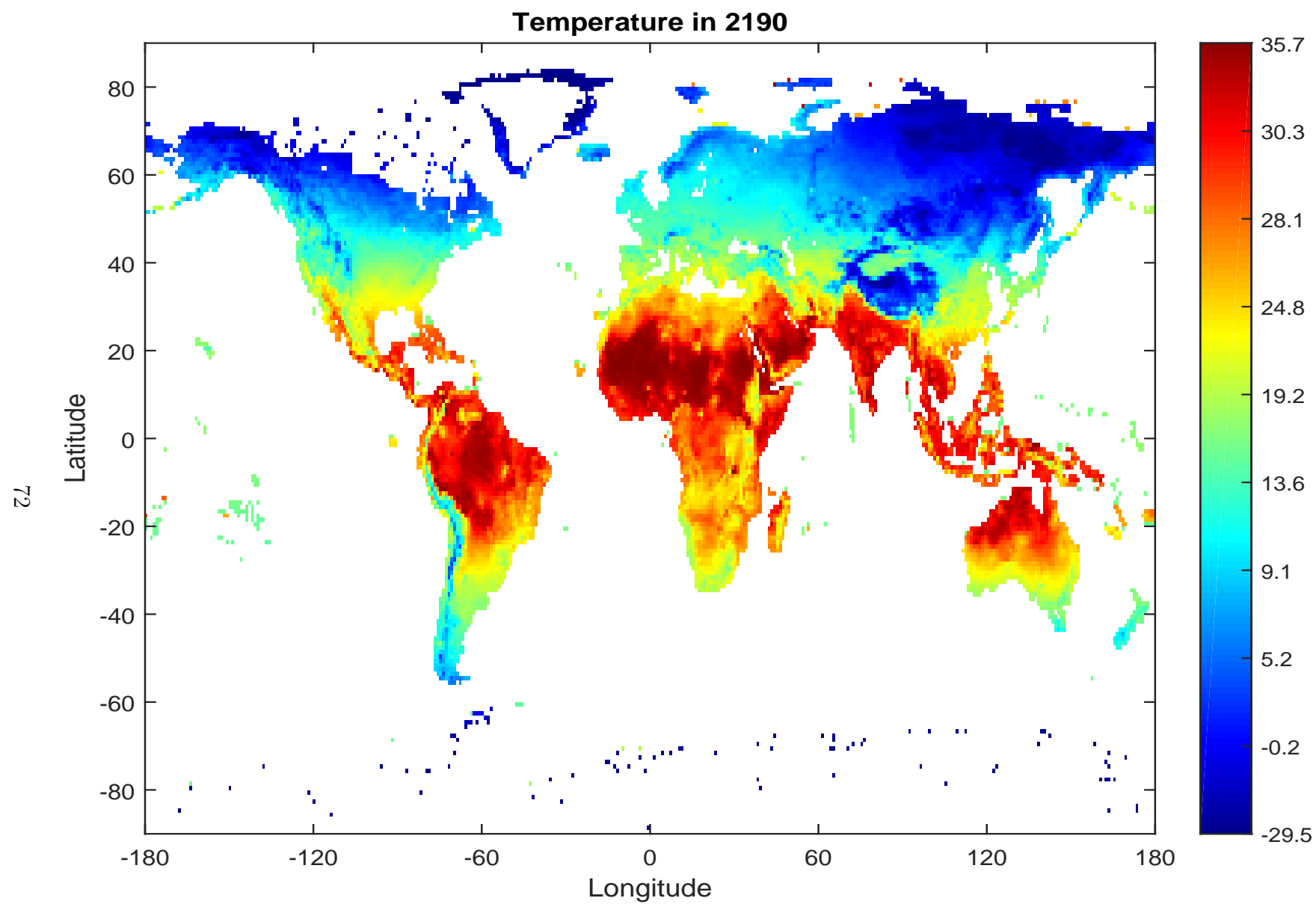


Figure 16t: Regional Temperature
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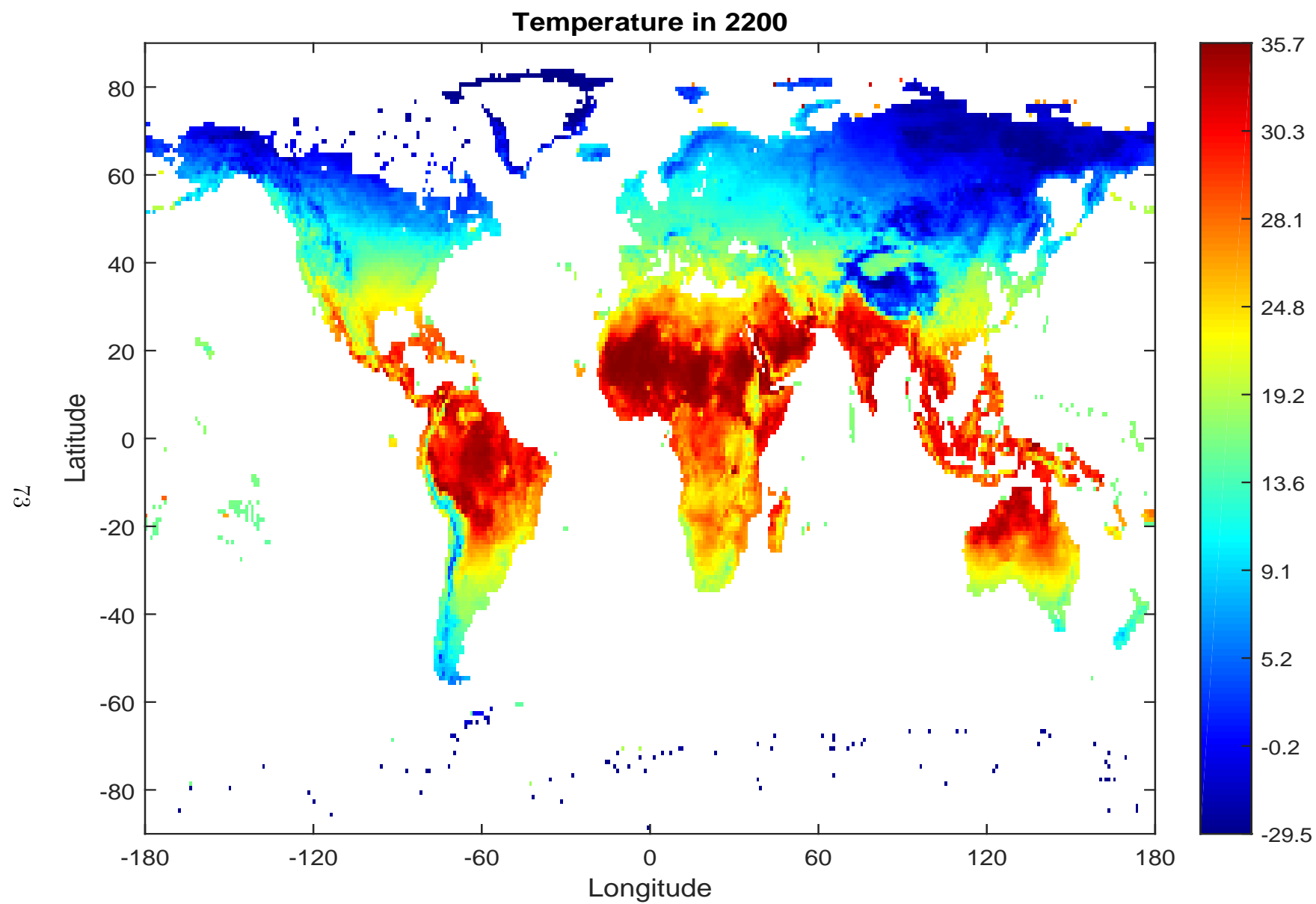


Figure 16u: Regional Temperature
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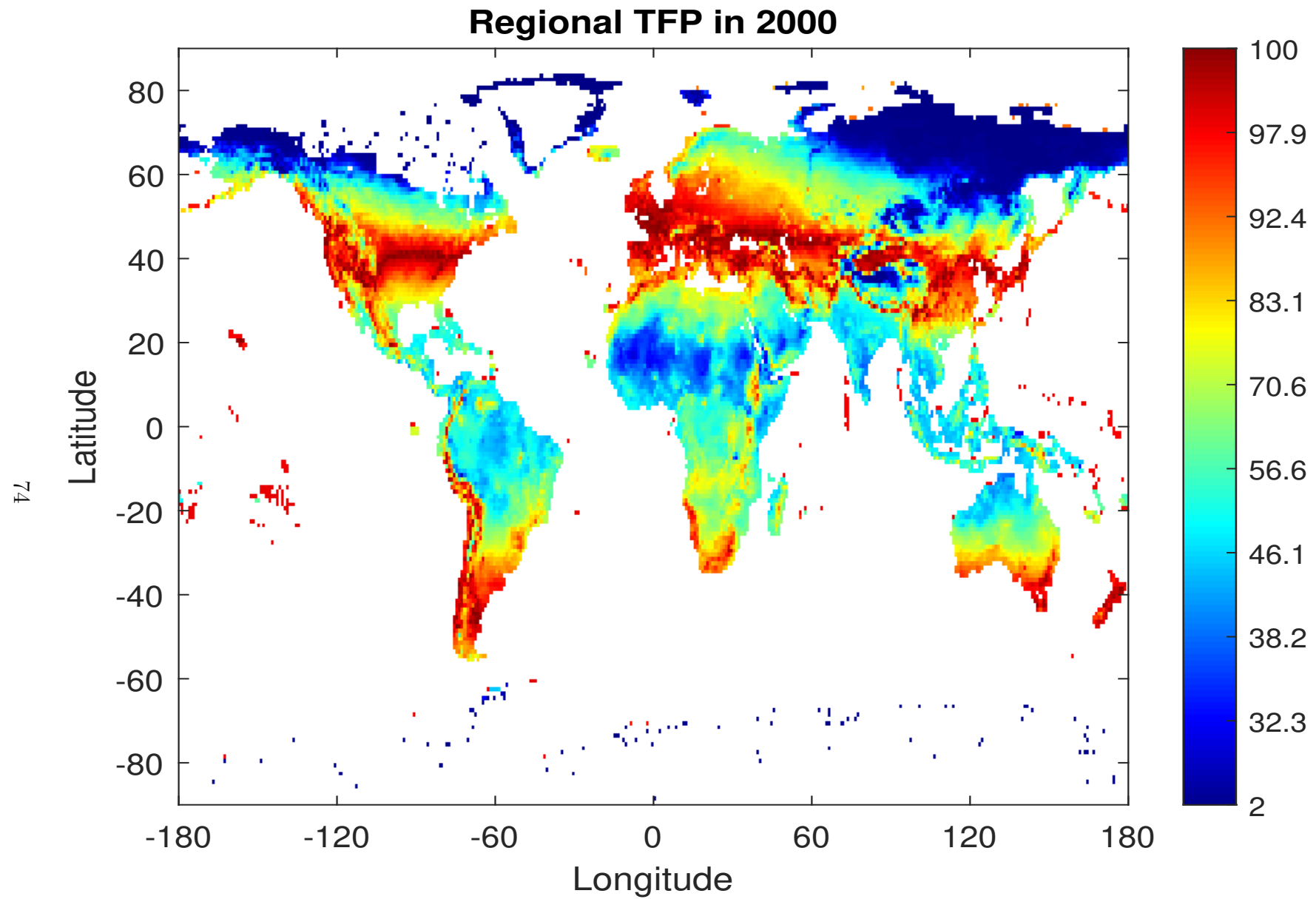


Figure 17a: Regional TFP, or $100 \times \tilde{D}(T_i)$
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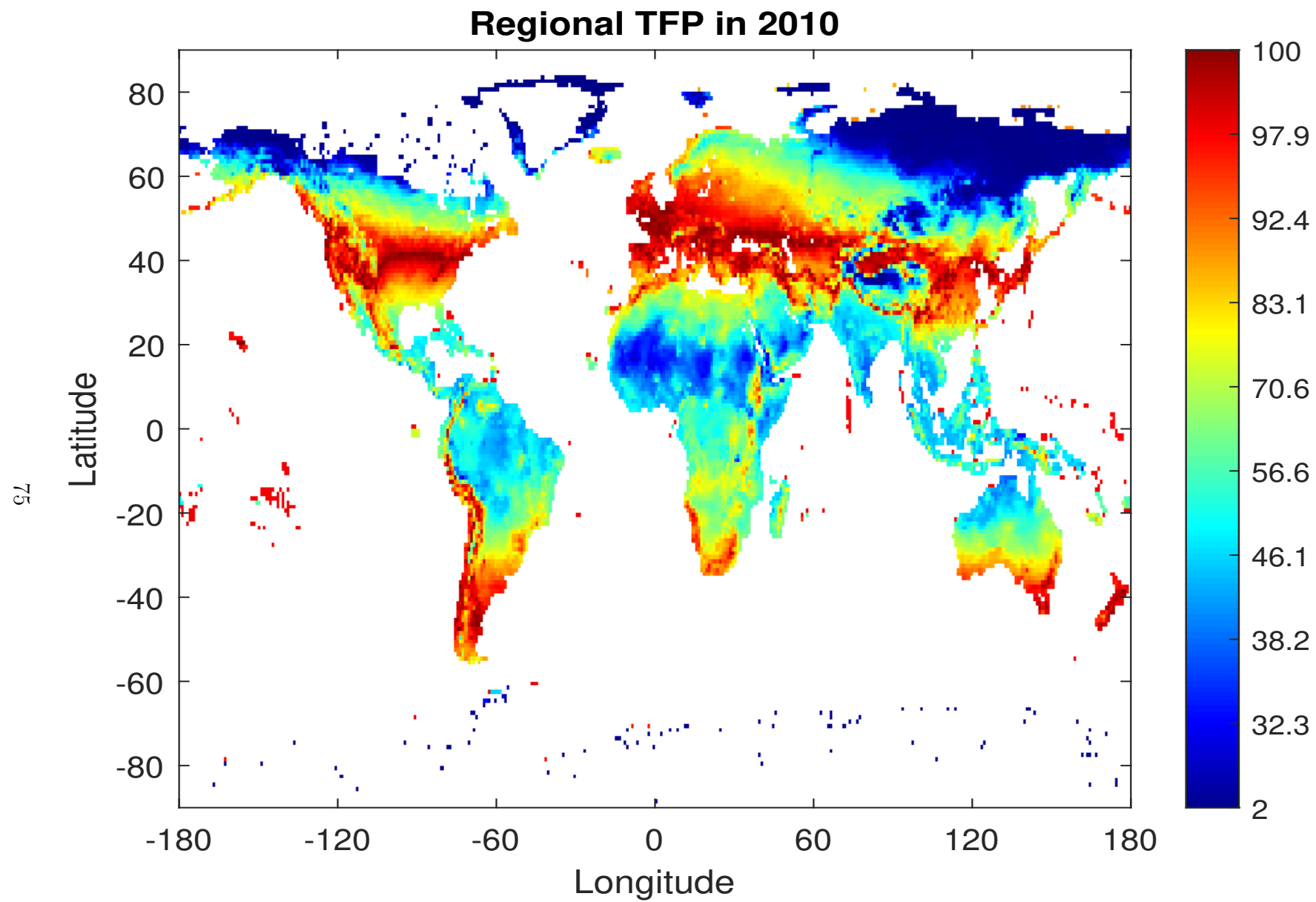


Figure 17b: Regional TFP, or $100 \times \tilde{D}(T_i)$
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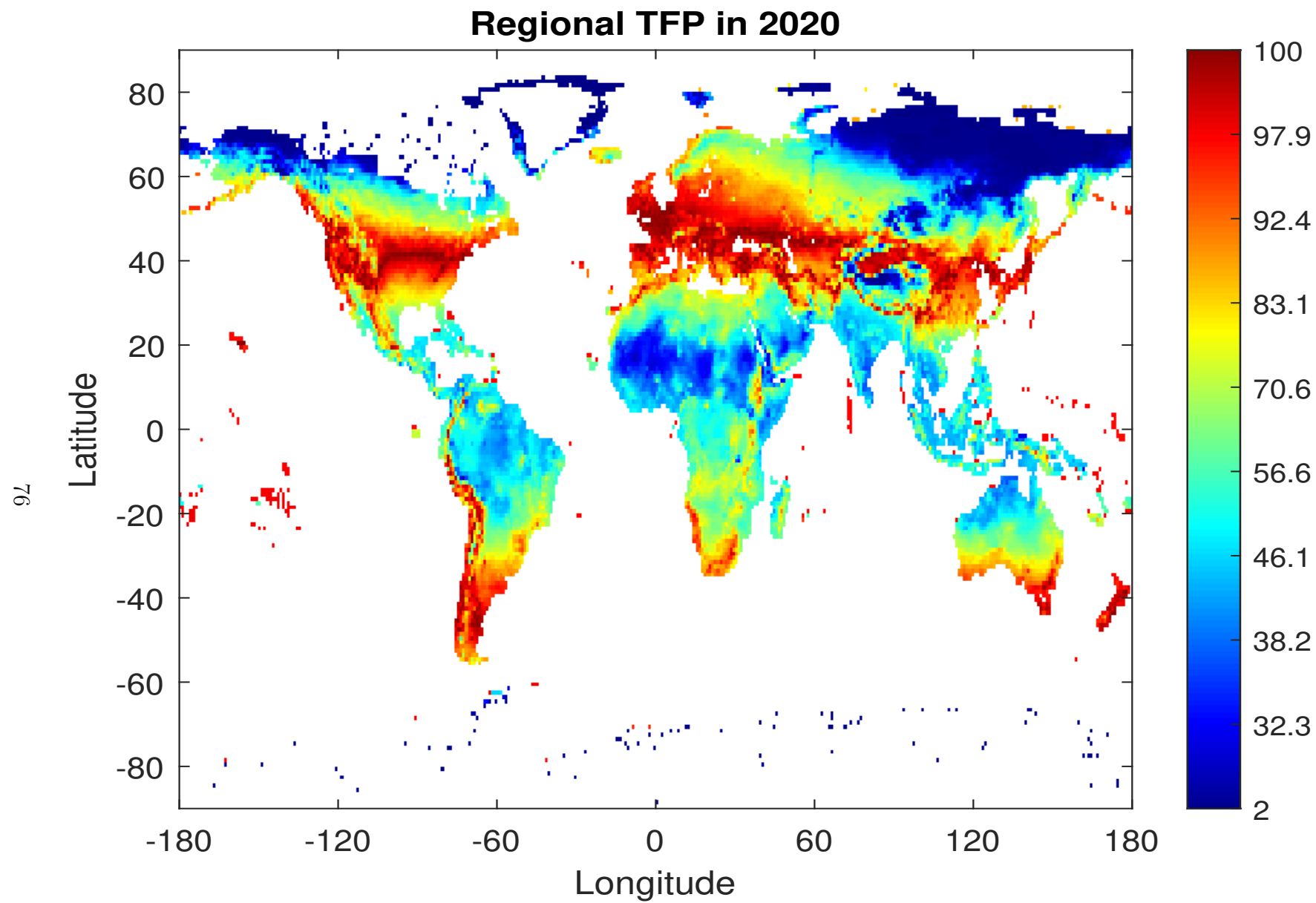


Figure 17c: Regional TFP, or $100 \times \tilde{D}(T_i)$
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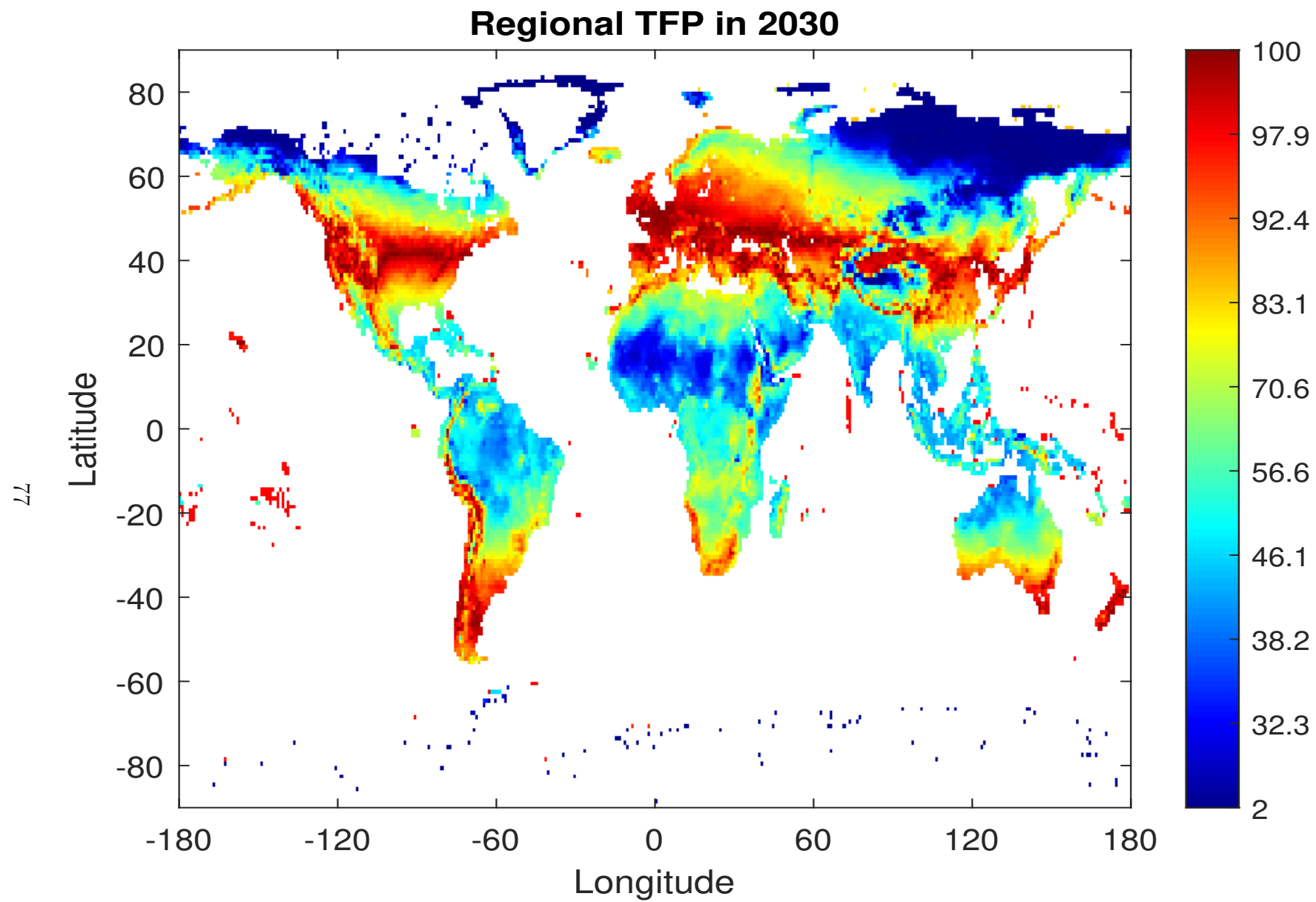


Figure 17d: Regional TFP, or $100 \times \tilde{D}(T_i)$
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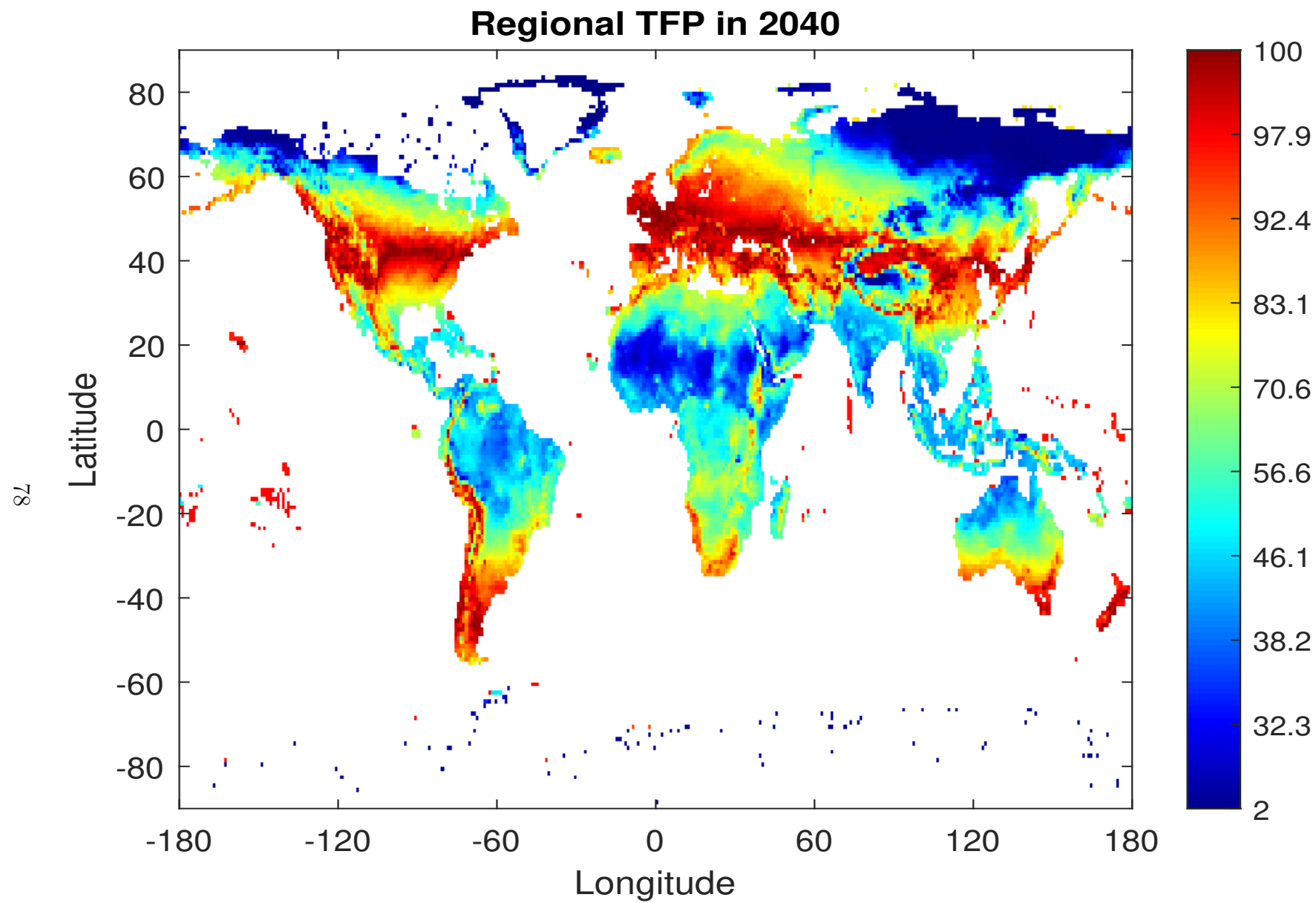


Figure 17e: Regional TFP, or $100 \times \tilde{D}(T_i)$
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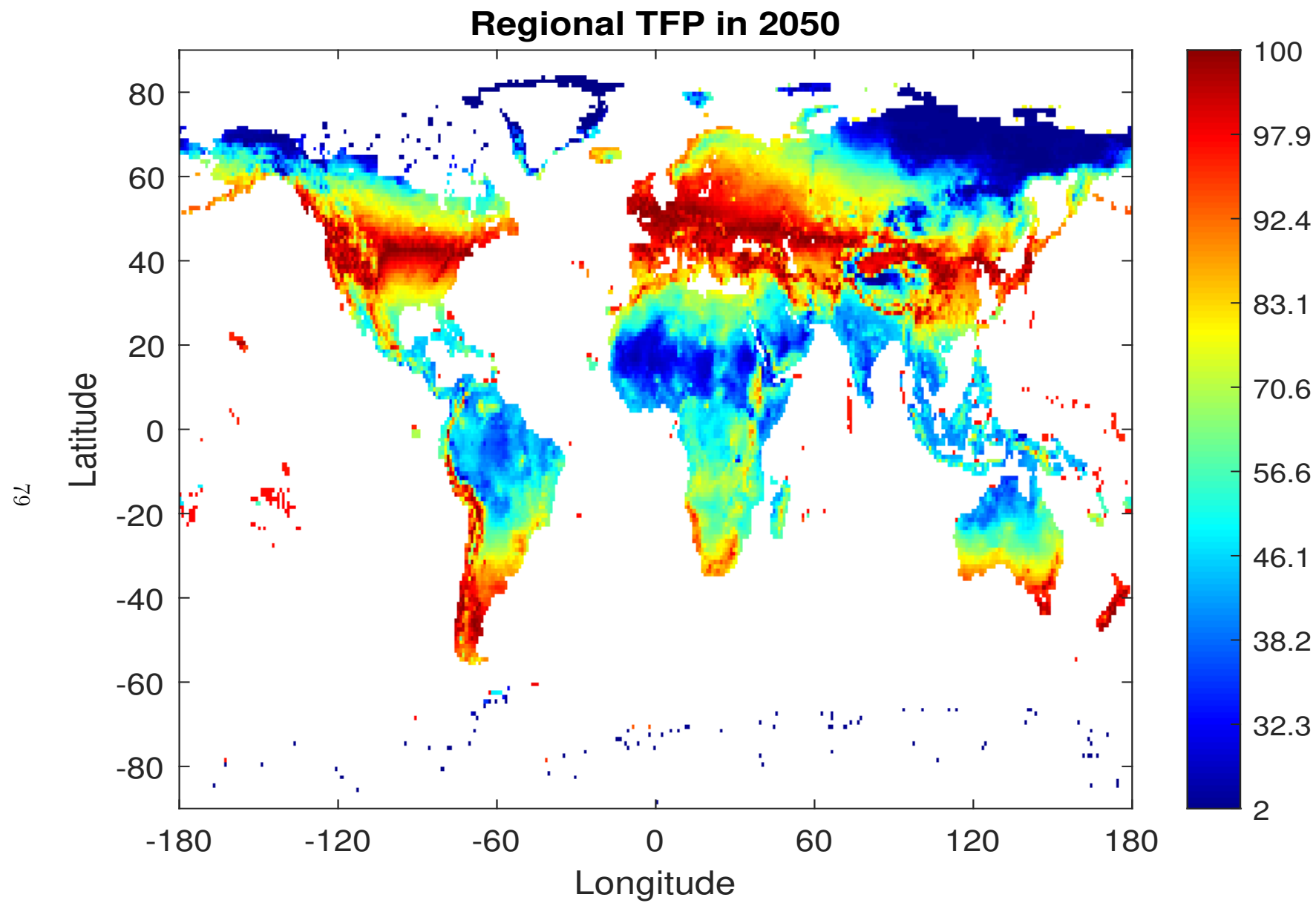


Figure 17f: Regional TFP, or $100 \times \tilde{D}(T_i)$
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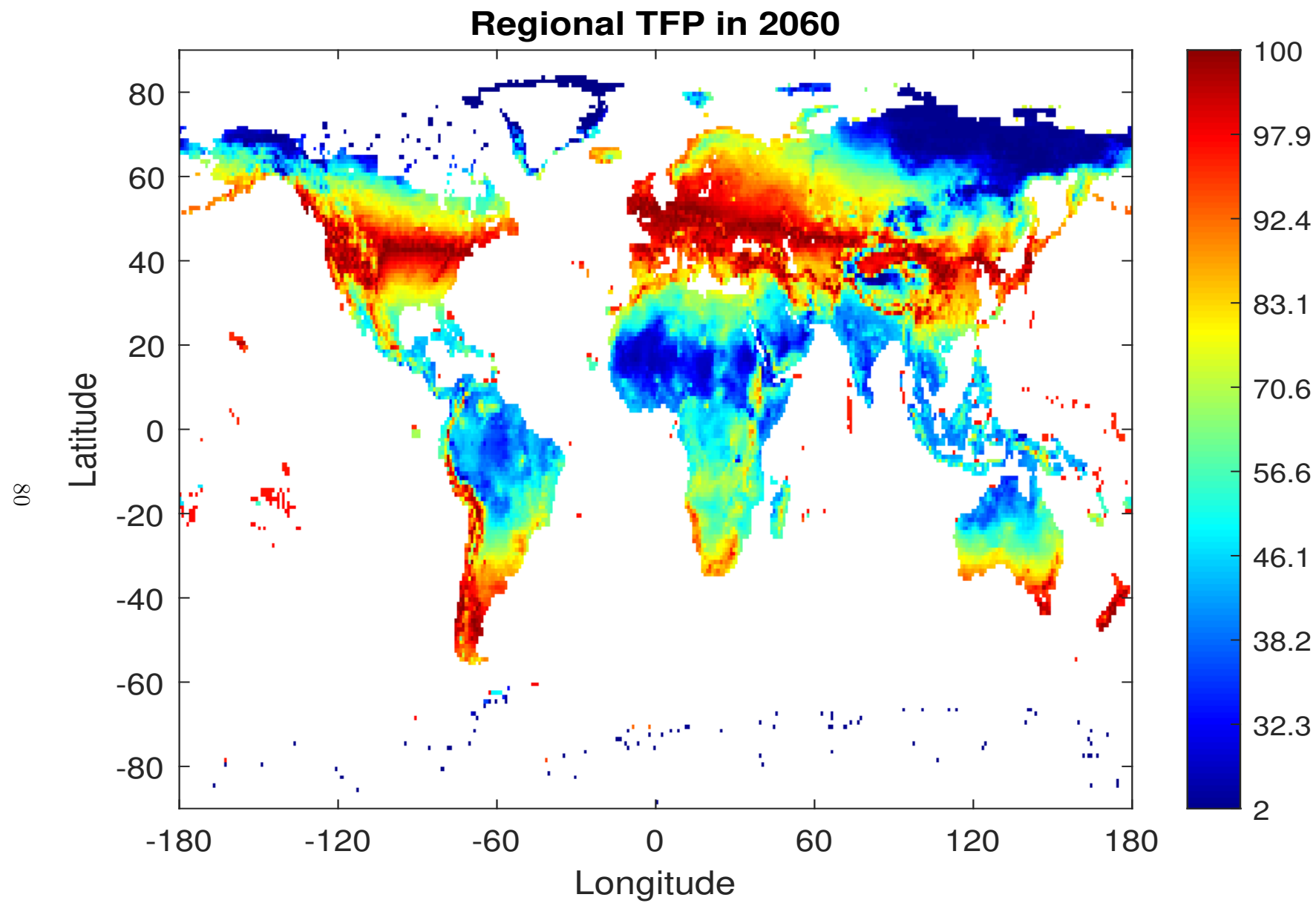


Figure 17g: Regional TFP, or $100 \times \tilde{D}(T_i)$
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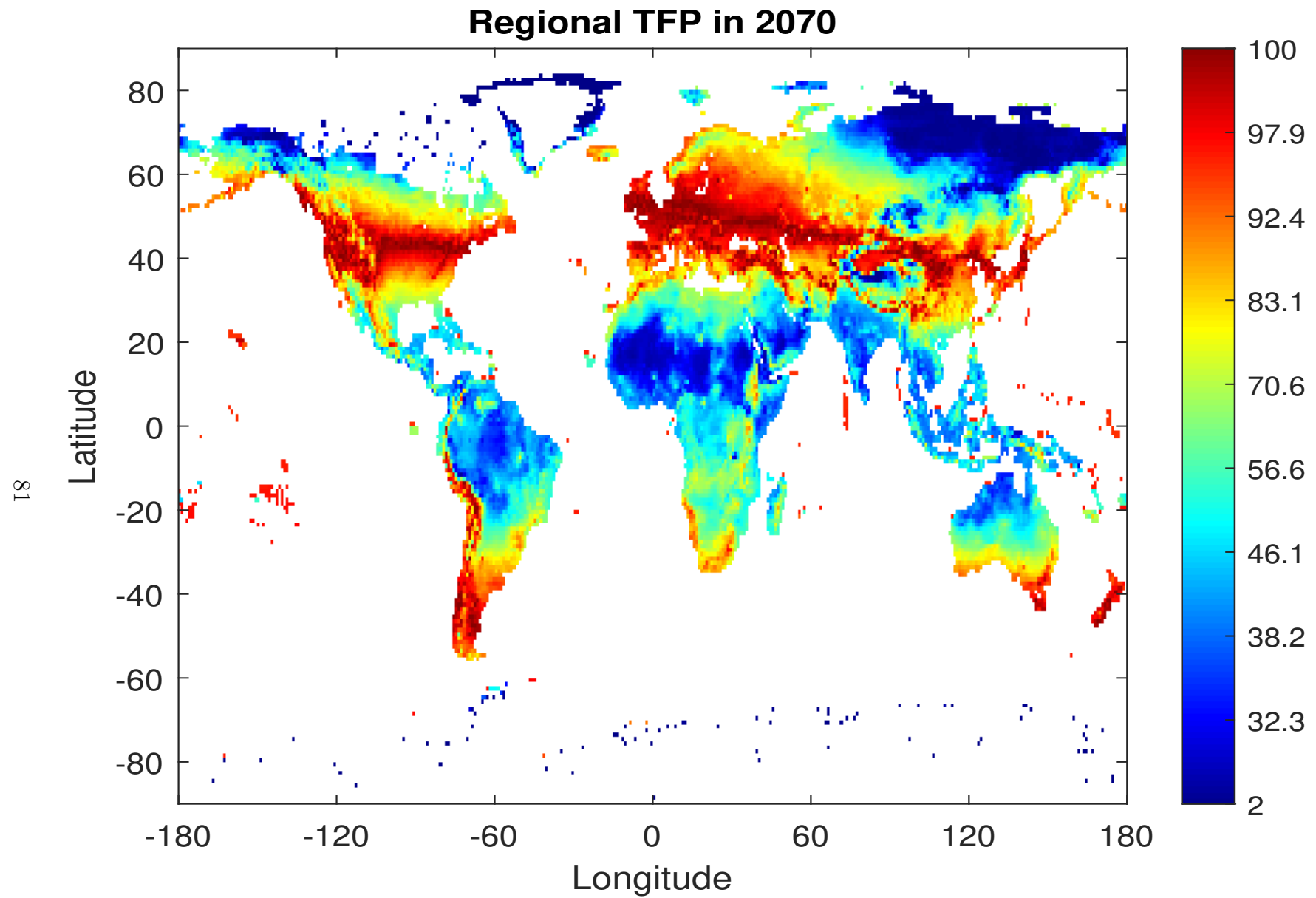


Figure 17h: Regional TFP, or $100 \times \tilde{D}(T_i)$
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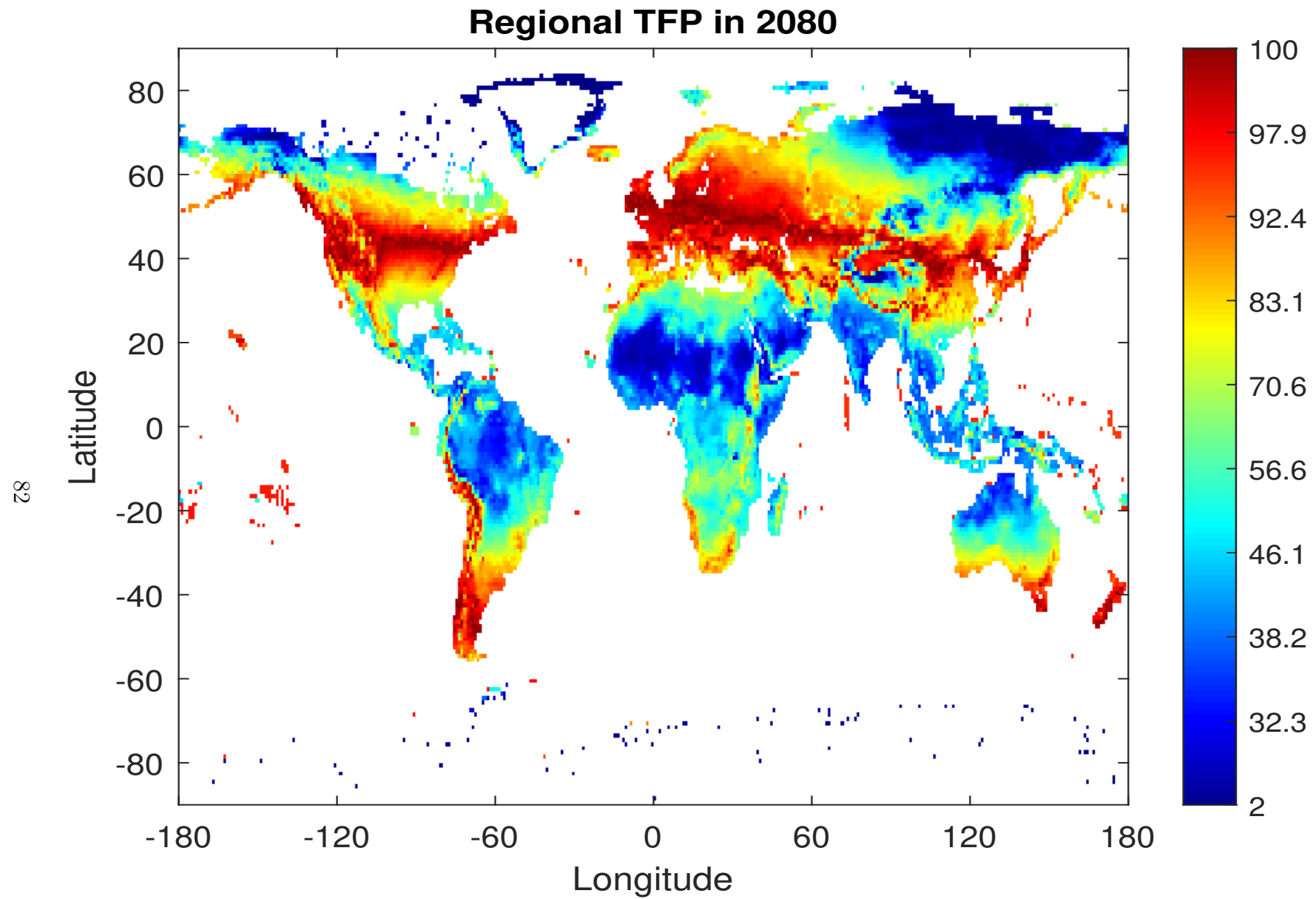


Figure 17i: Regional TFP, or $100 \times \tilde{D}(T_i)$
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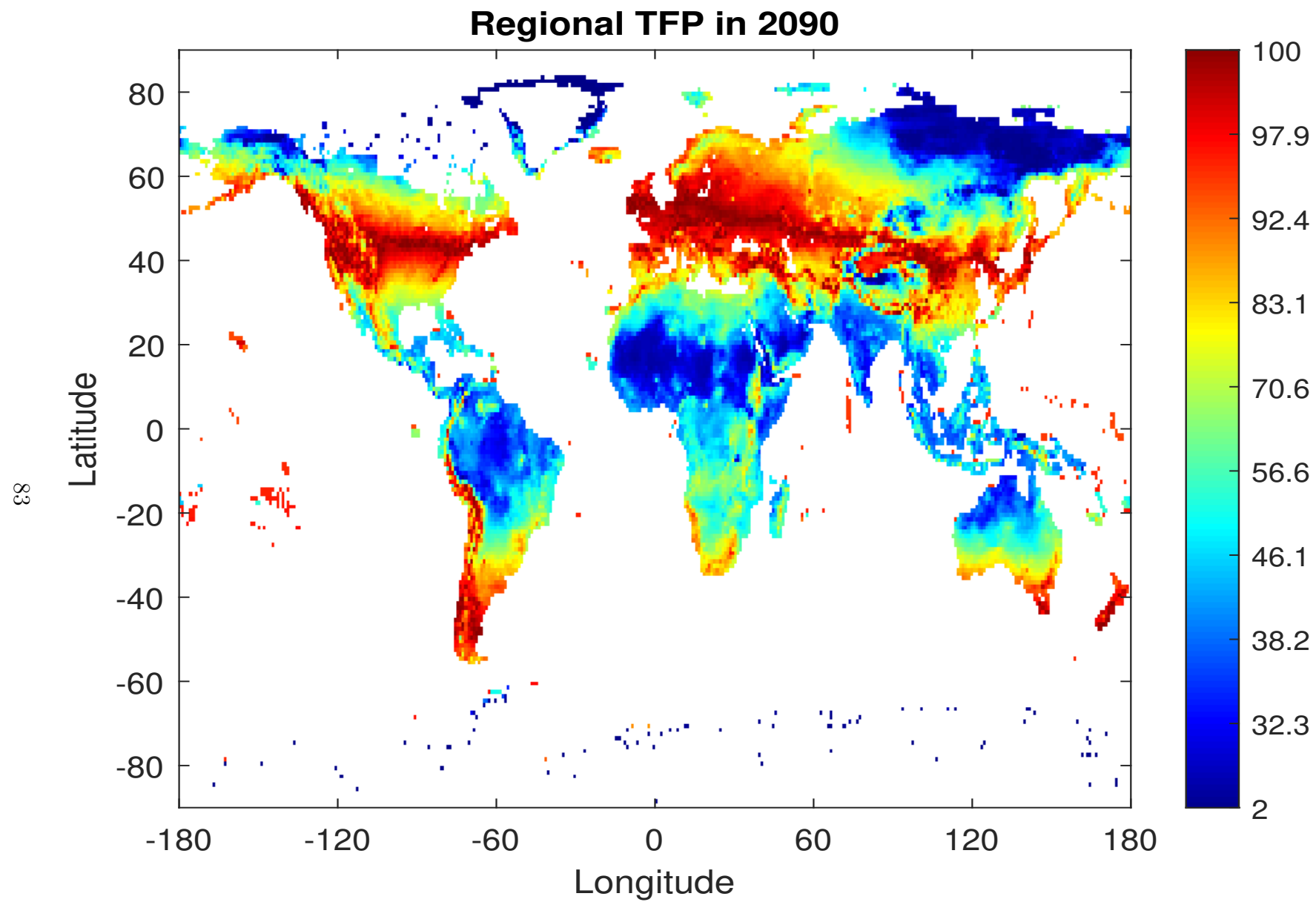


Figure 17j: Regional TFP, or $100 \times \tilde{D}(T_i)$
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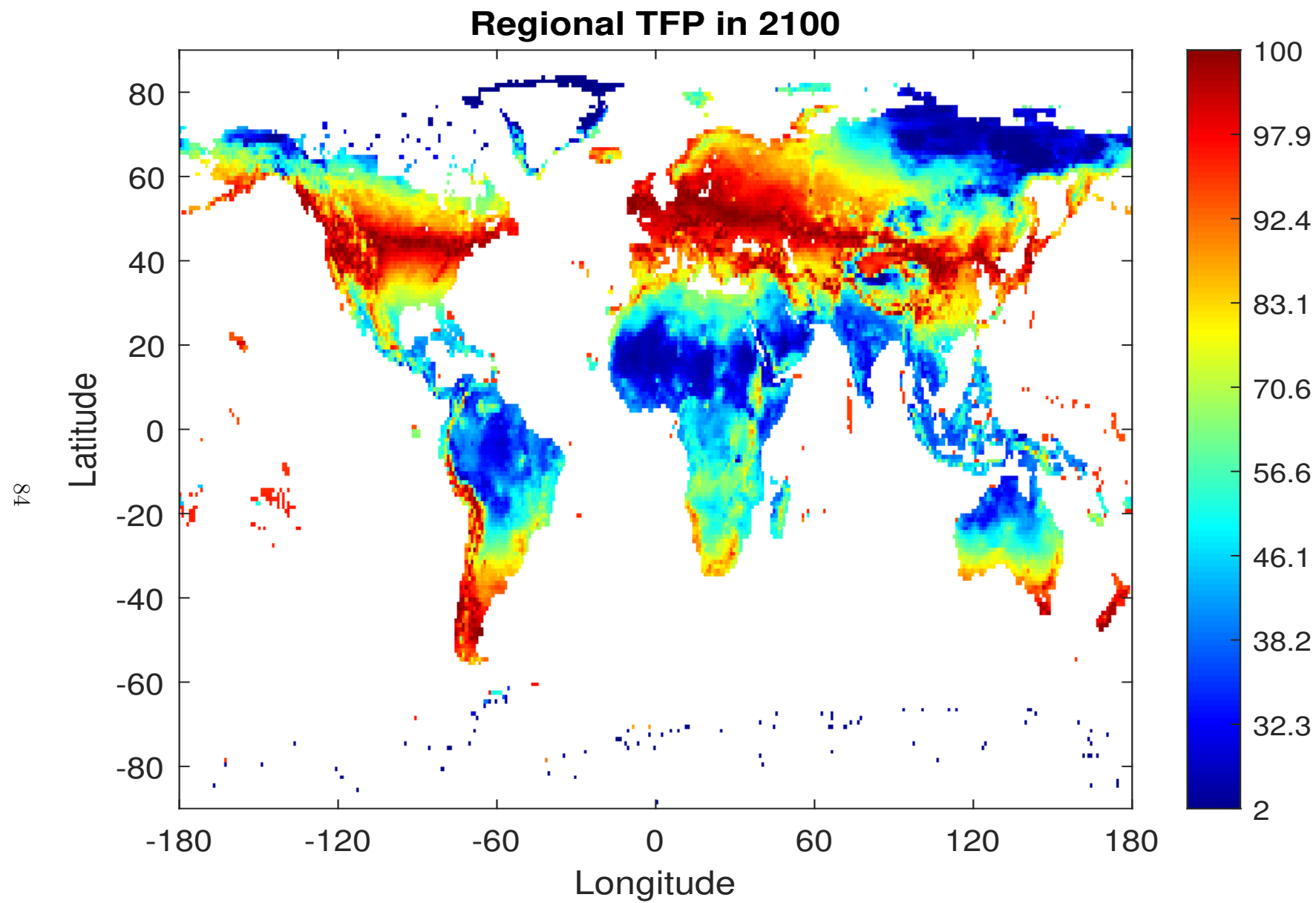


Figure 17k: Regional TFP, or $100 \times \tilde{D}(T_i)$
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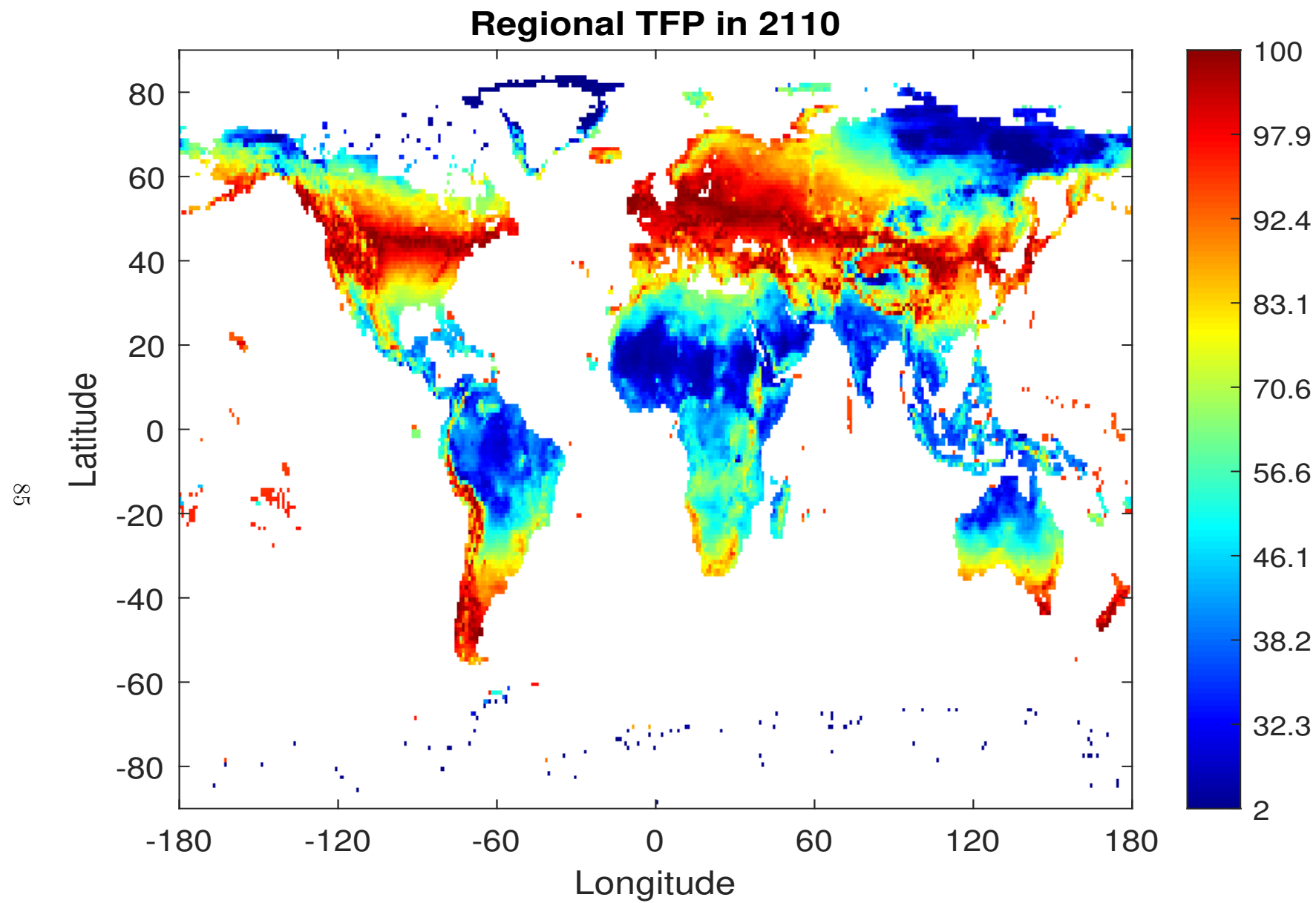


Figure 17l: Regional TFP, or $100 \times \tilde{D}(T_i)$
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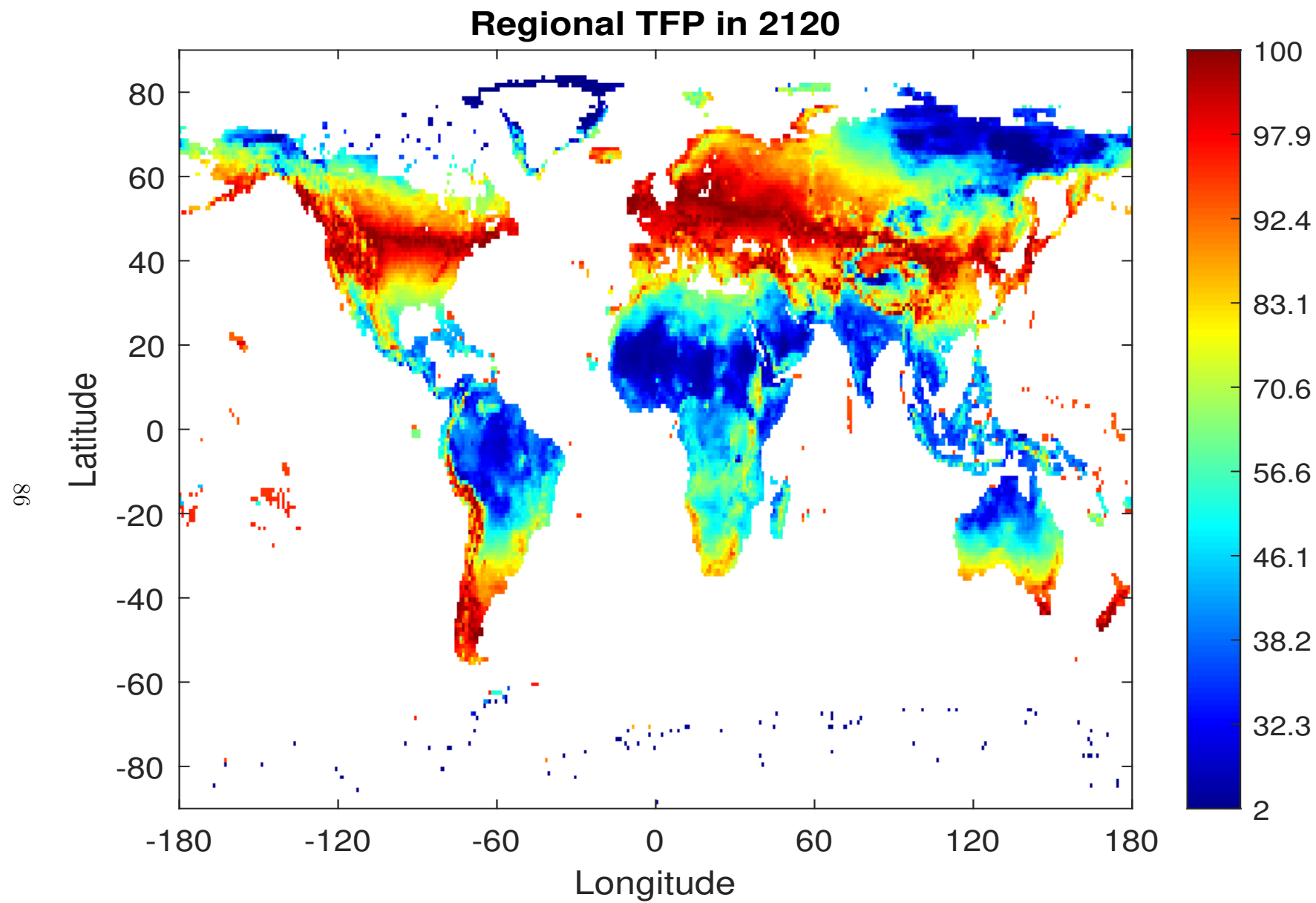


Figure 17m: Regional TFP, or $100 \times \tilde{D}(T_i)$
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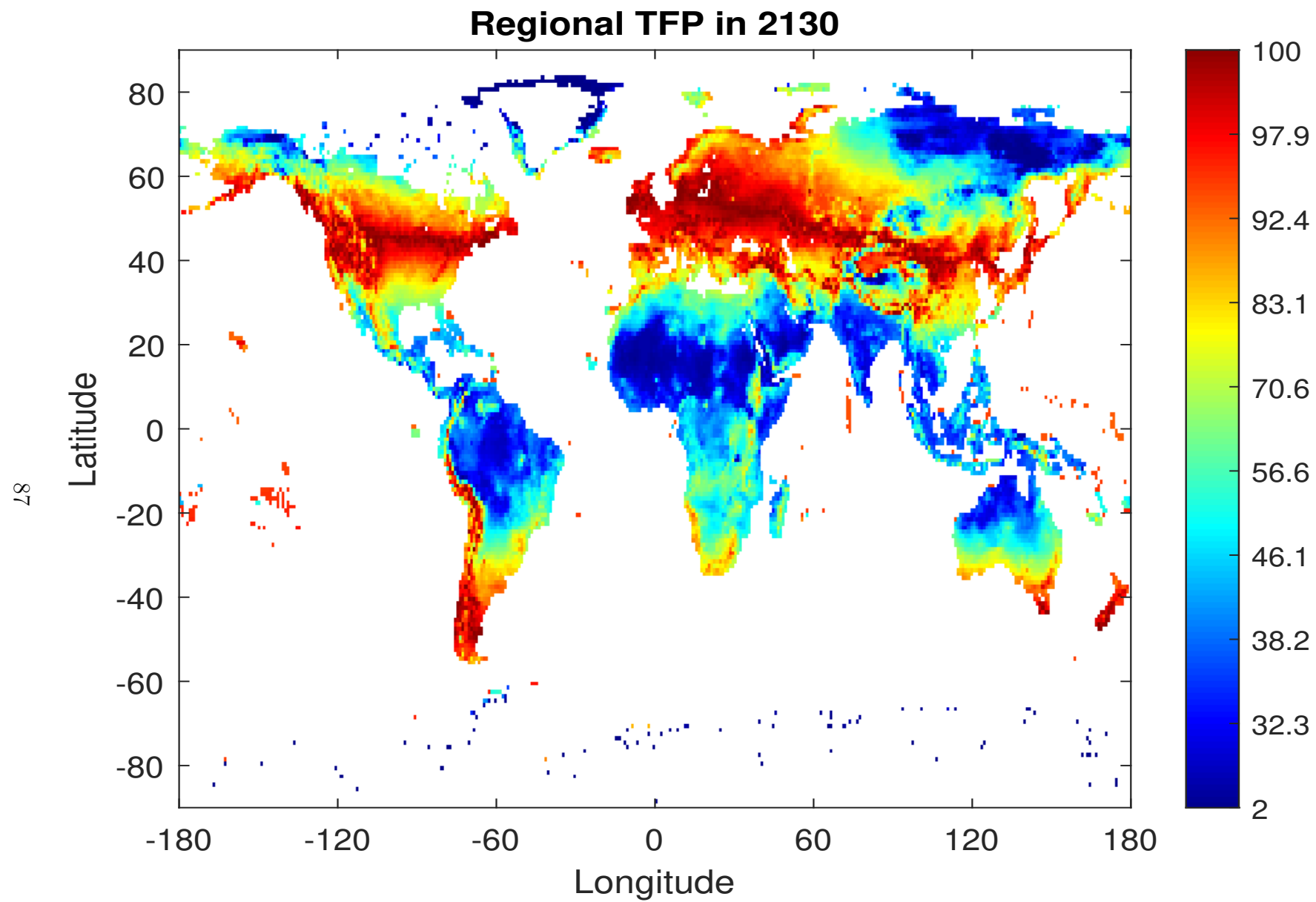


Figure 17n: Regional TFP, or $100 \times \tilde{D}(T_i)$
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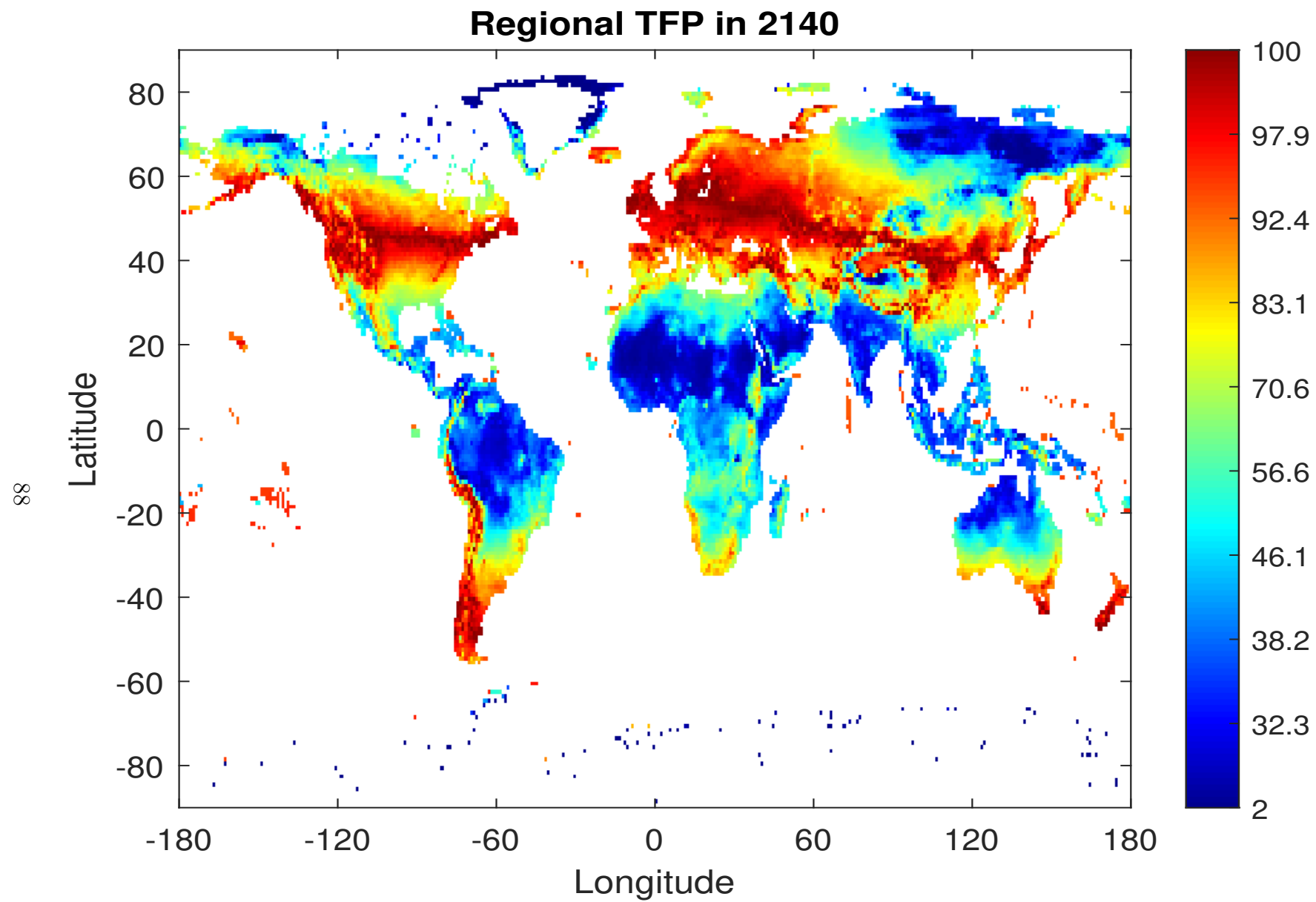


Figure 17o: Regional TFP, or $100 \times \tilde{D}(T_i)$
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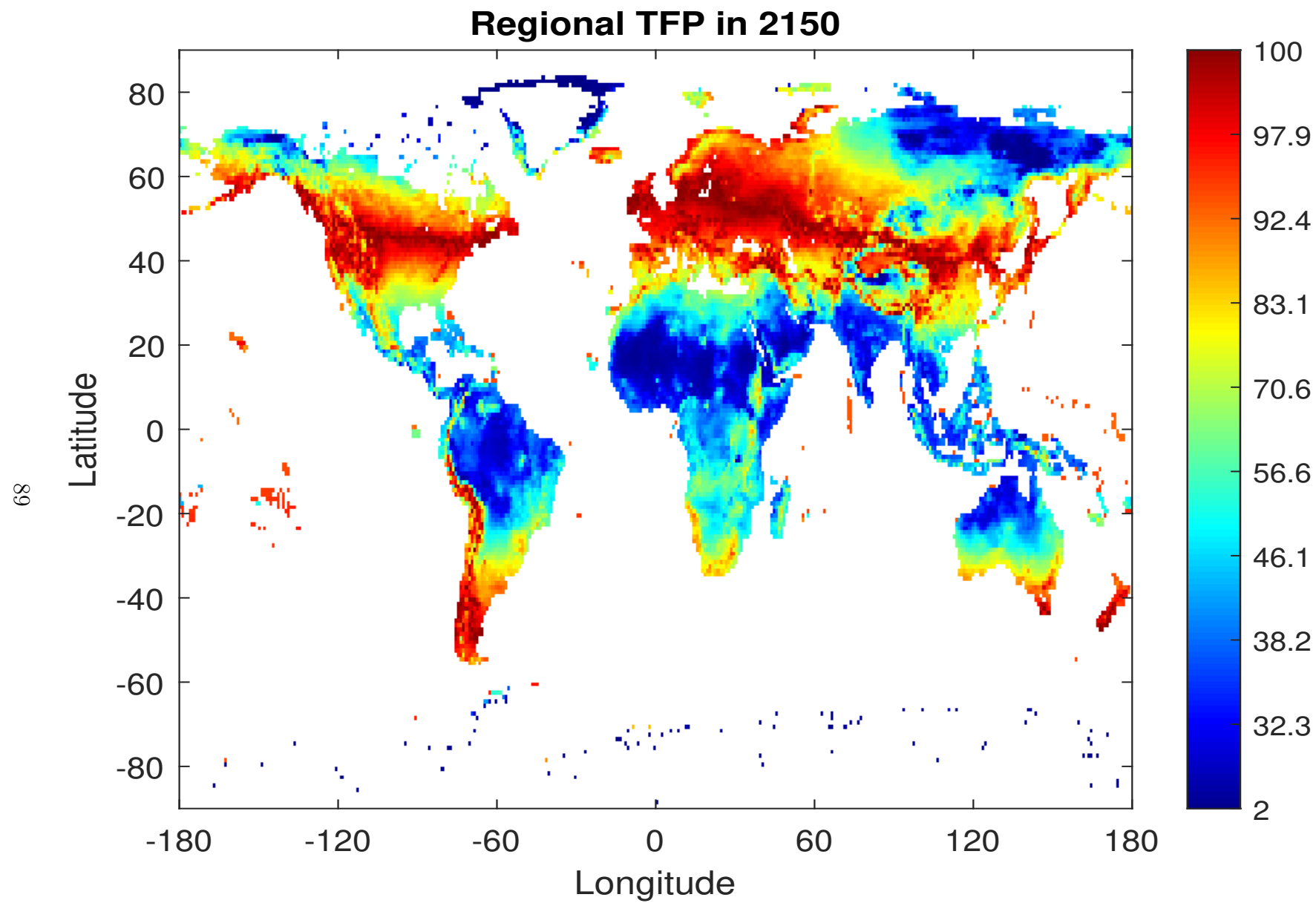


Figure 17p: Regional TFP, or $100 \times \tilde{D}(T_i)$
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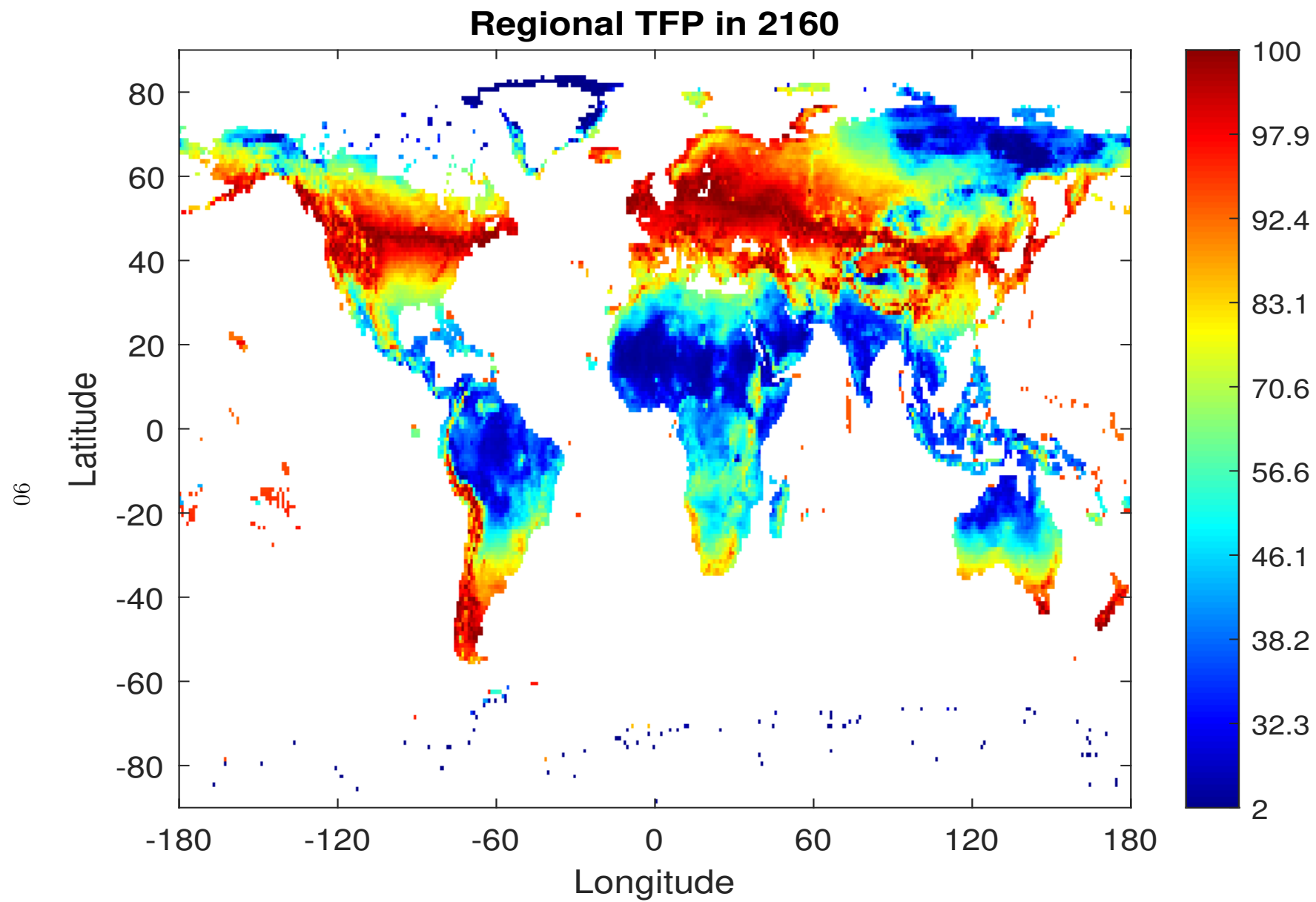


Figure 17q: Regional TFP, or $100 \times \tilde{D}(T_i)$
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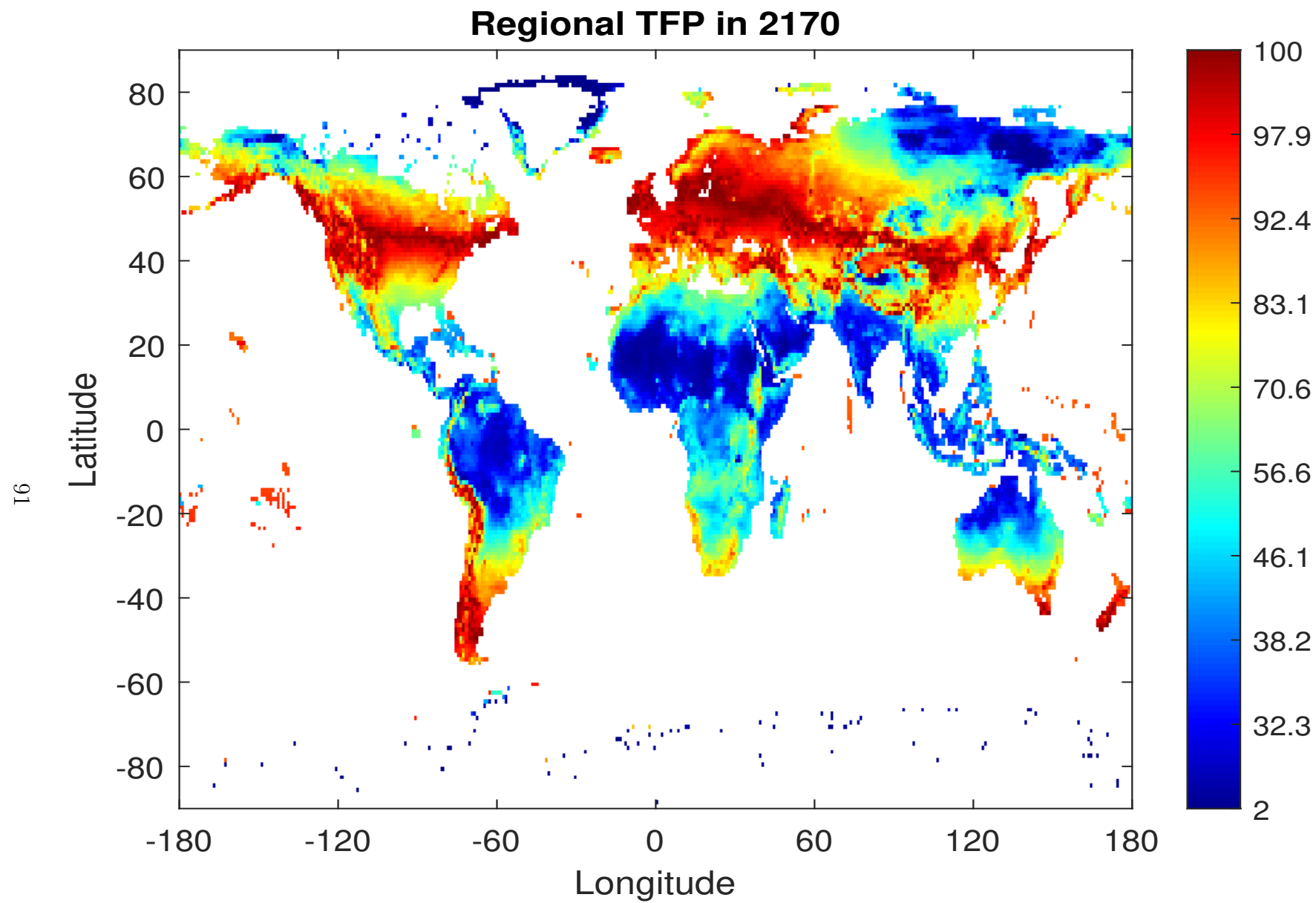


Figure 17r: Regional TFP, or $100 \times \tilde{D}(T_i)$
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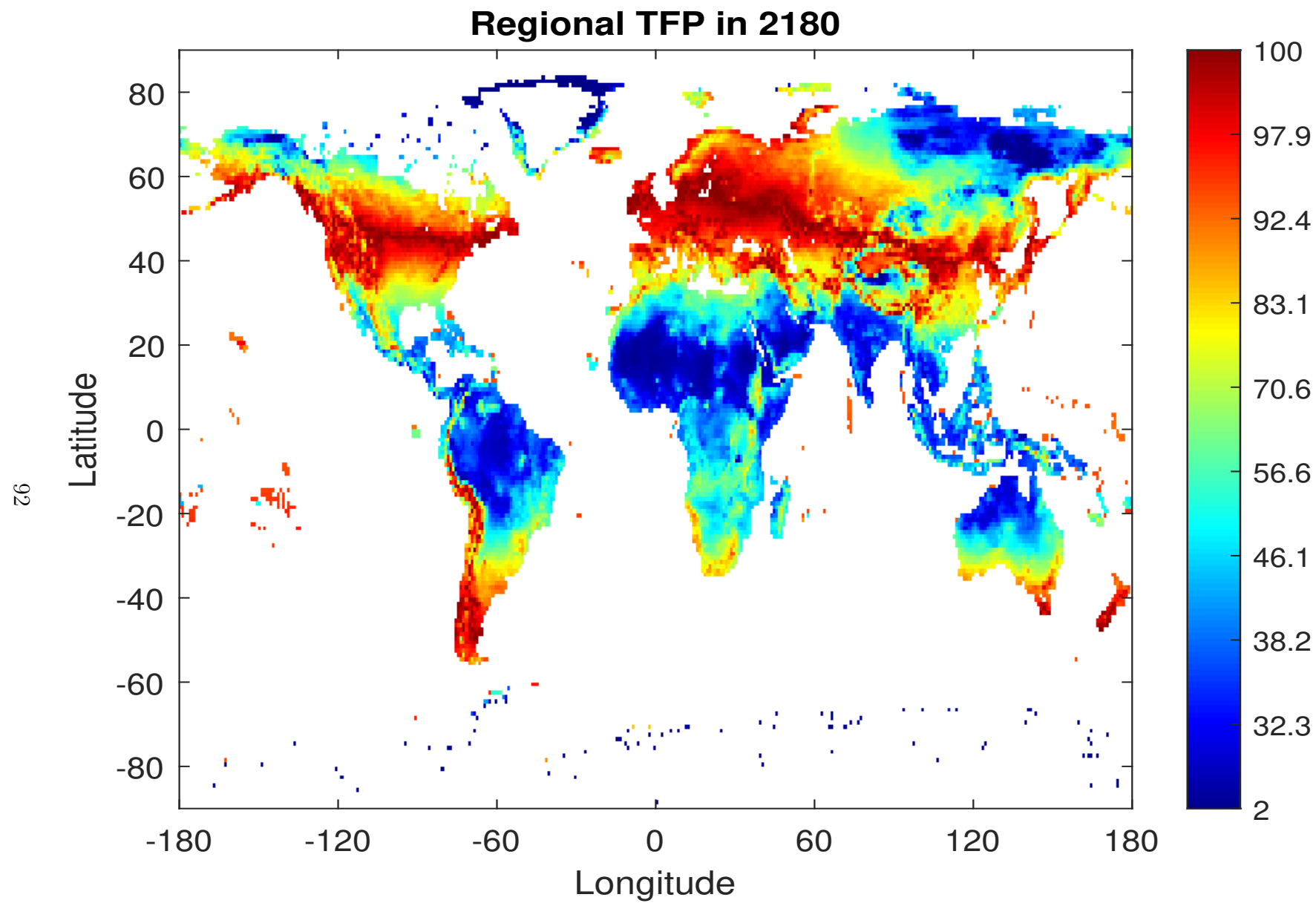


Figure 17s: Regional TFP, or $100 \times \tilde{D}(T_i)$
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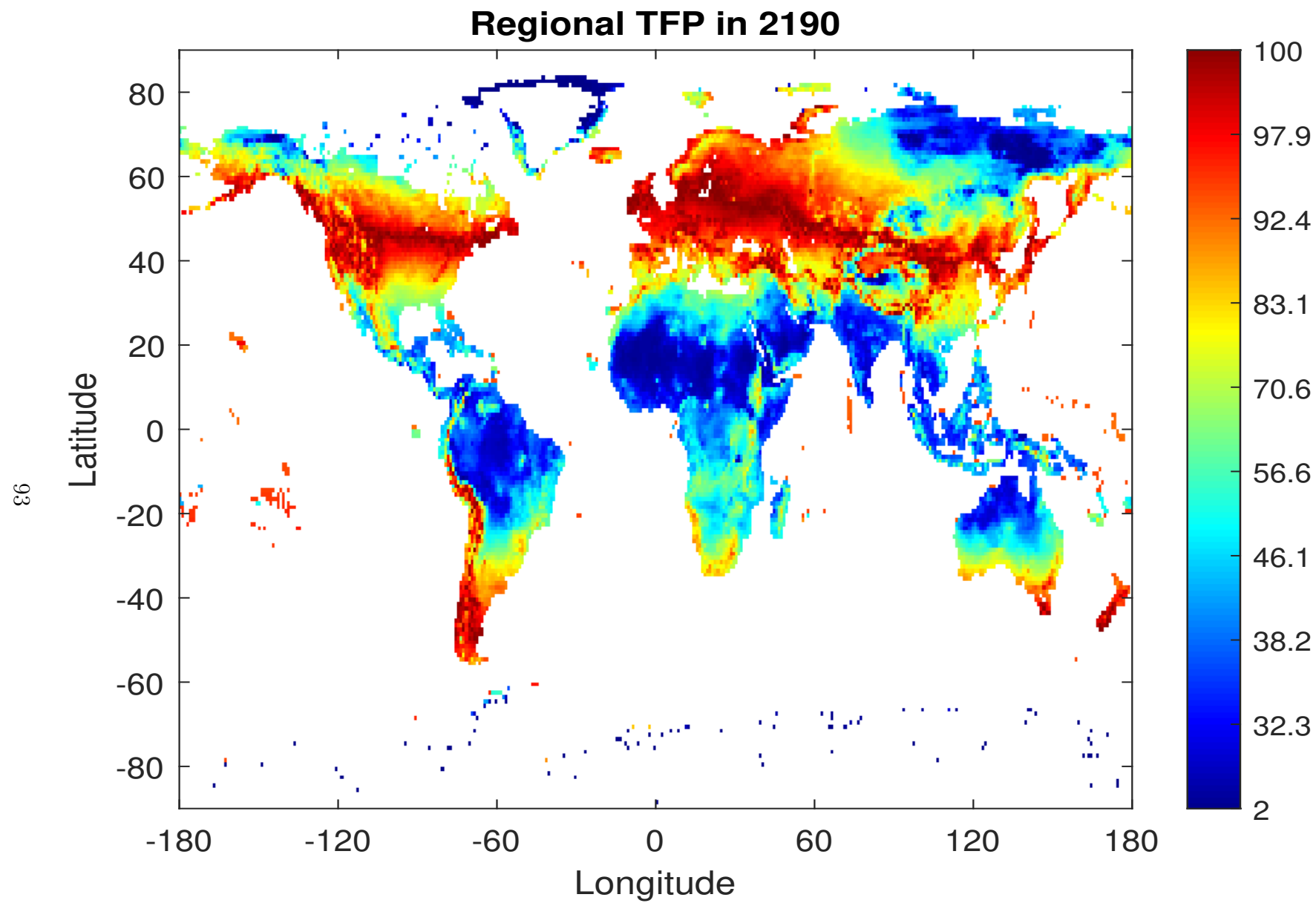


Figure 17t: Regional TFP, or $100 \times \tilde{D}(T_i)$
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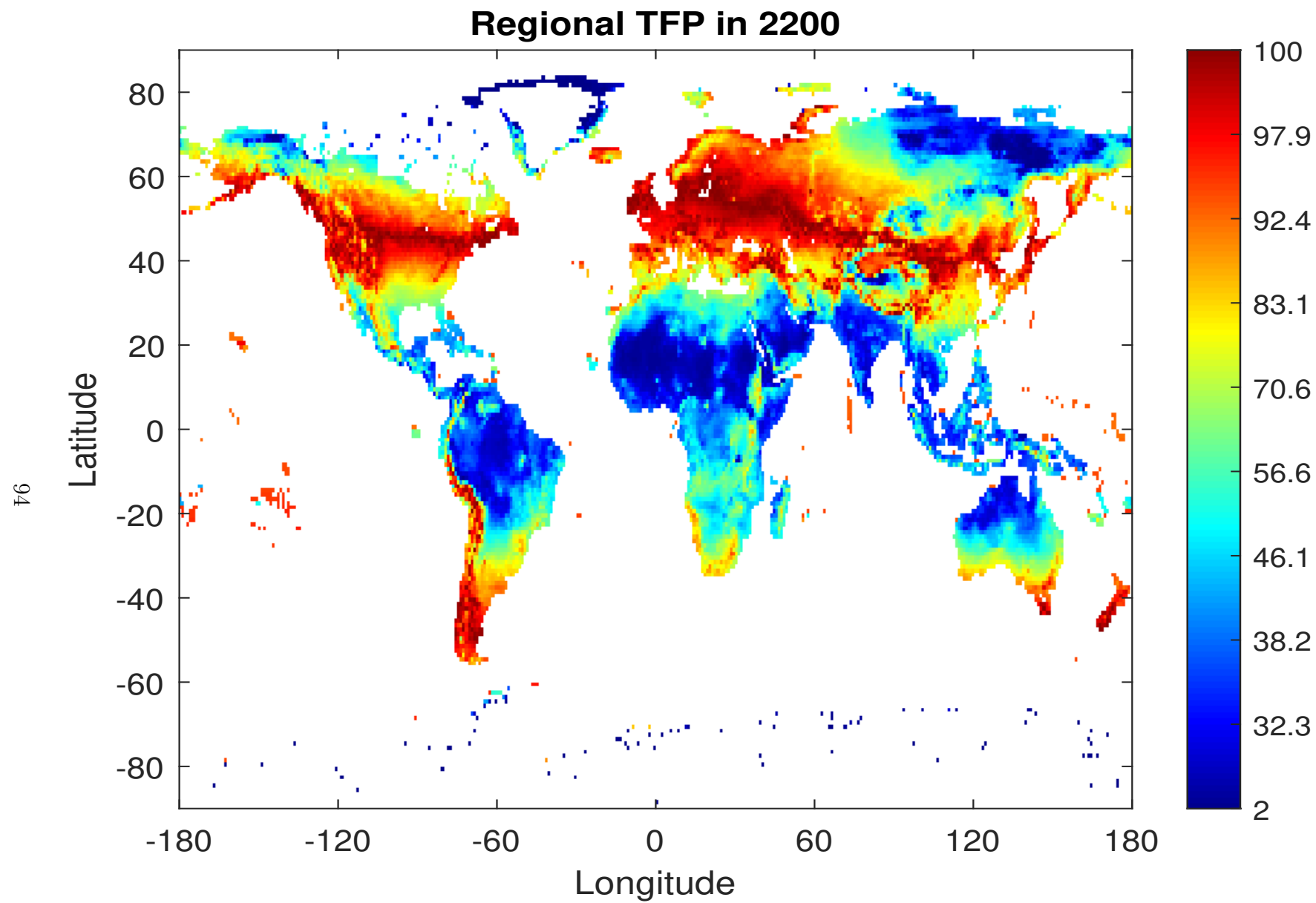


Figure 17u: Regional TFP, or $100 \times \tilde{D}(T_i)$
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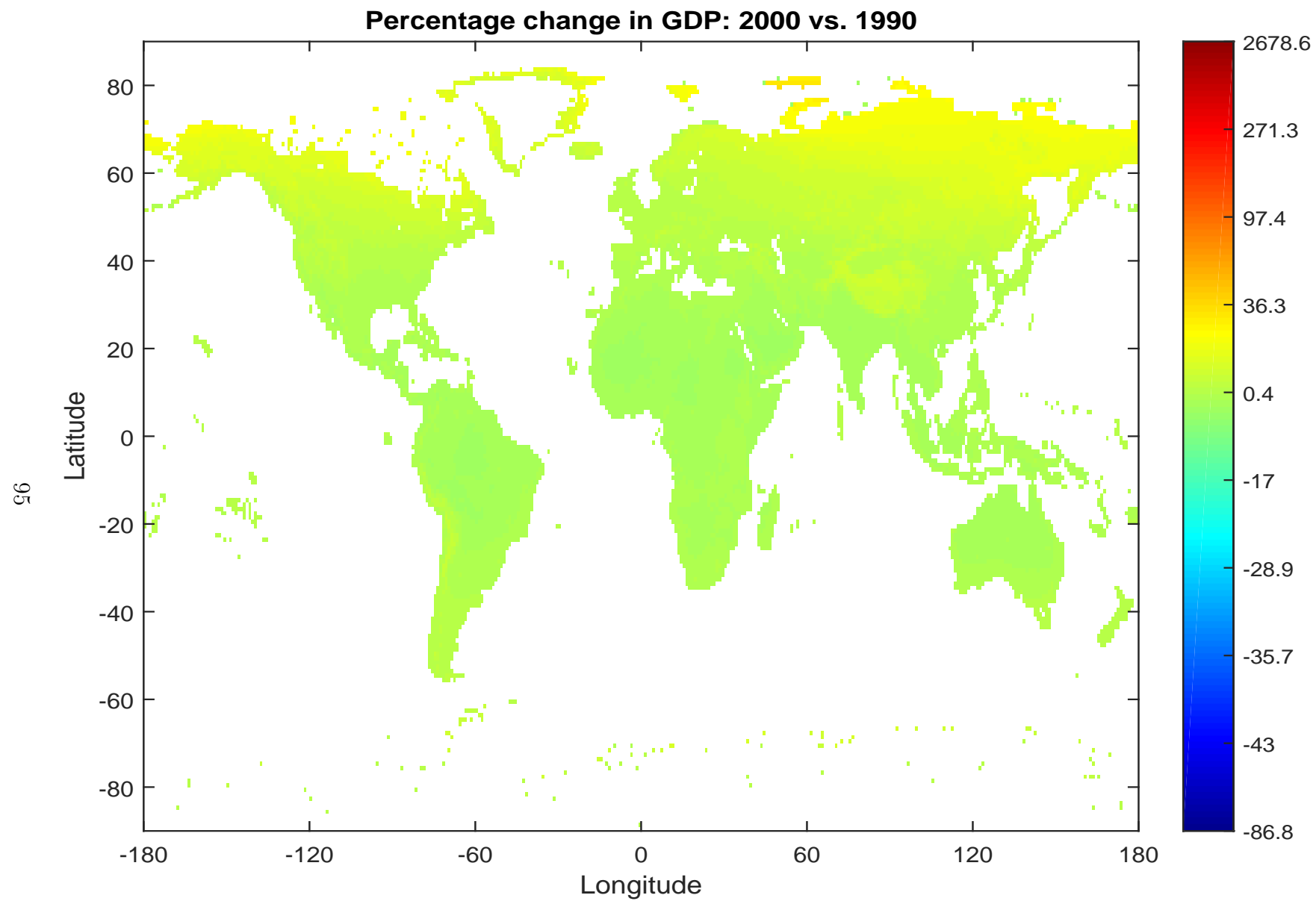


Figure 18a: Percentage Change in Regional GDP (relative to trend)
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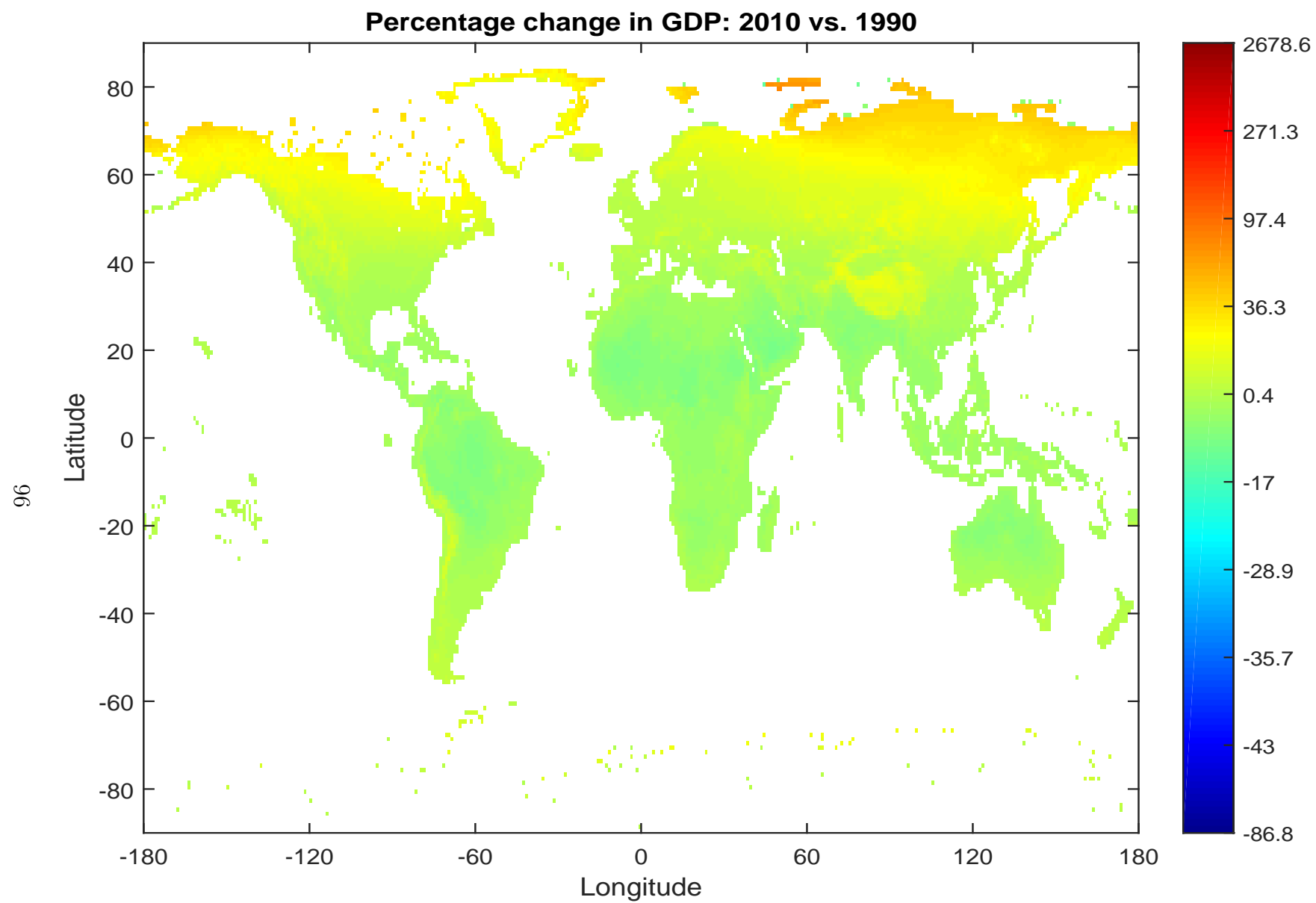


Figure 18b: Percentage Change in Regional GDP (relative to trend)
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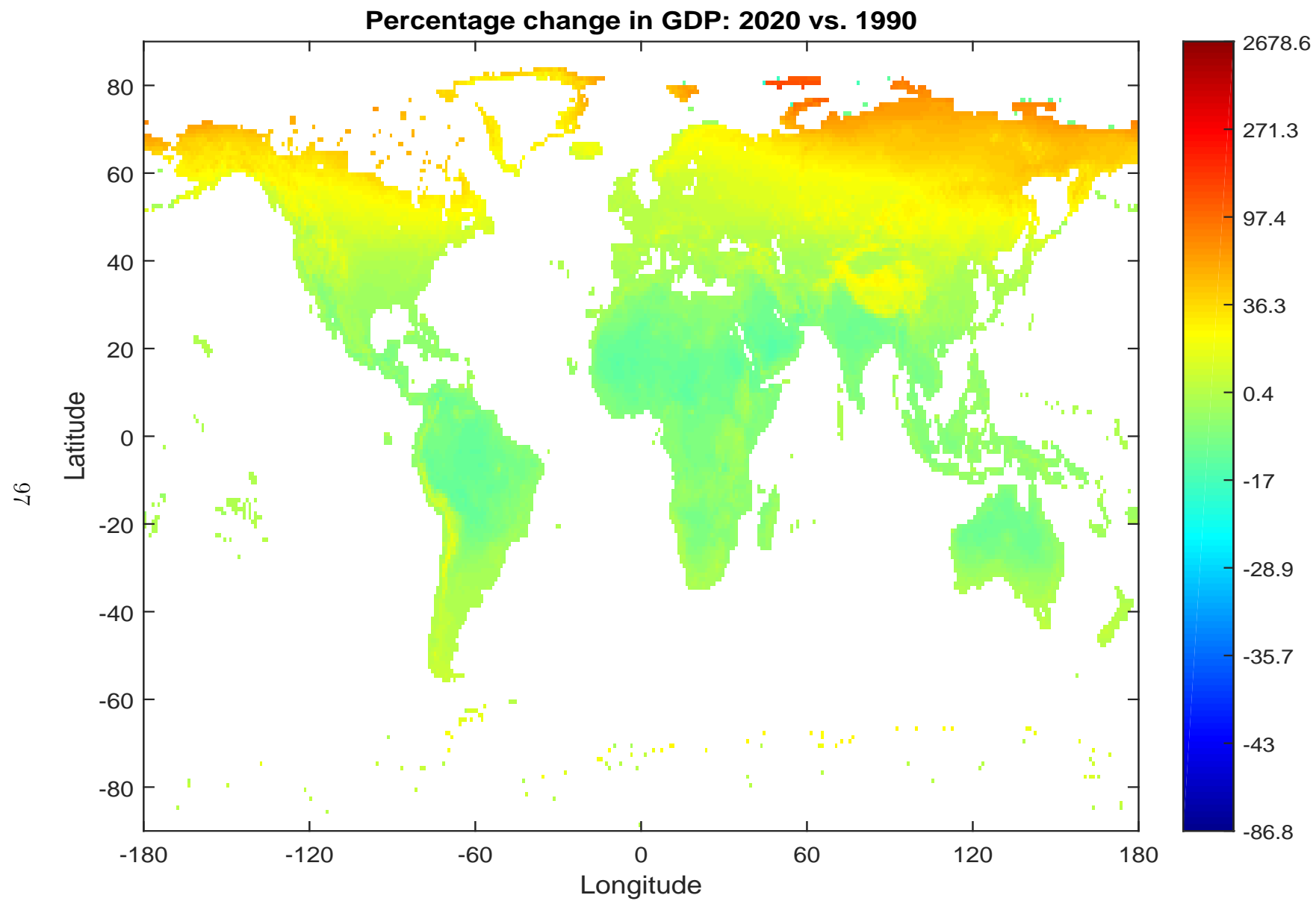


Figure 18c: Percentage Change in Regional GDP (relative to trend)
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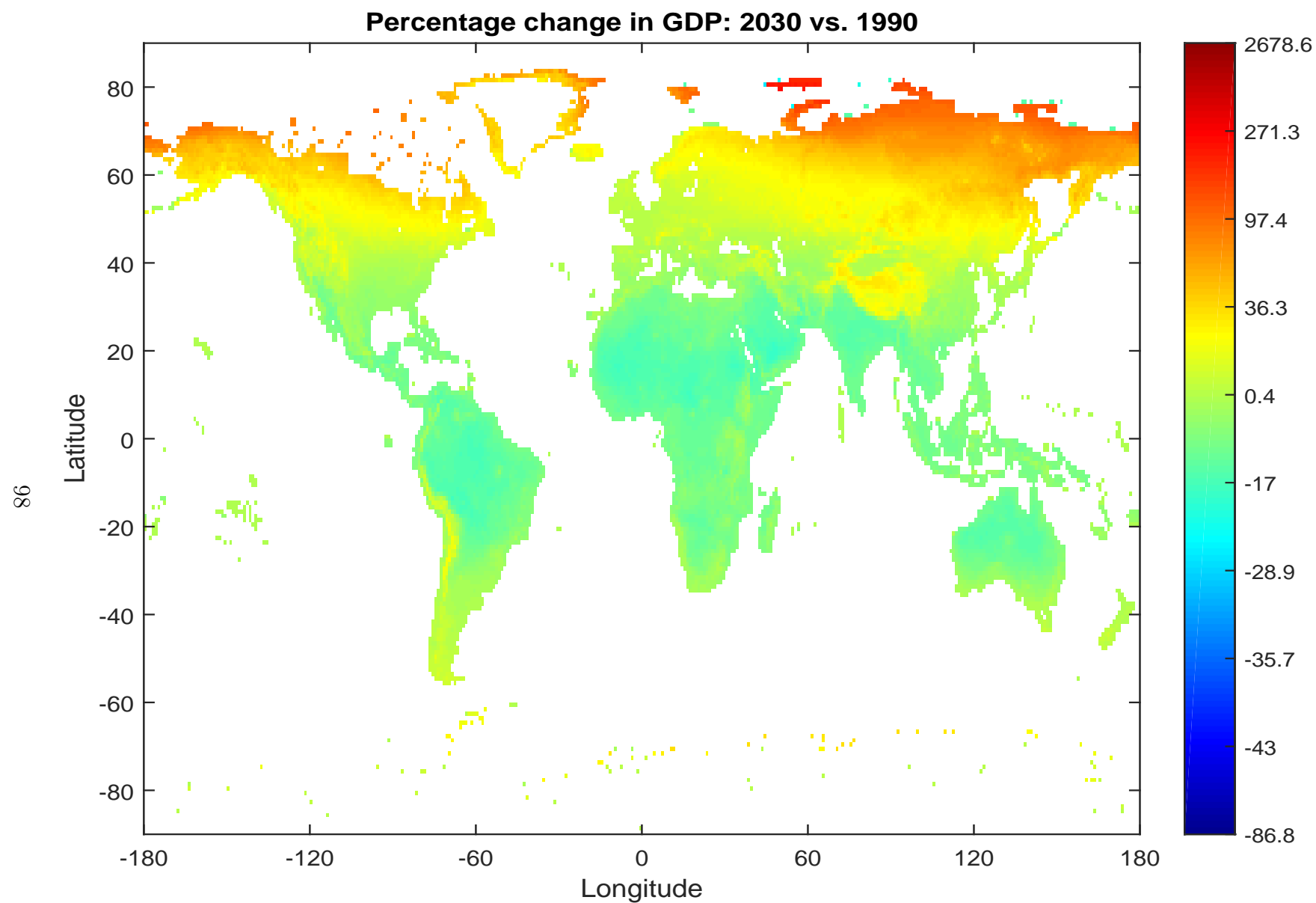


Figure 18d: Percentage Change in Regional GDP (relative to trend)
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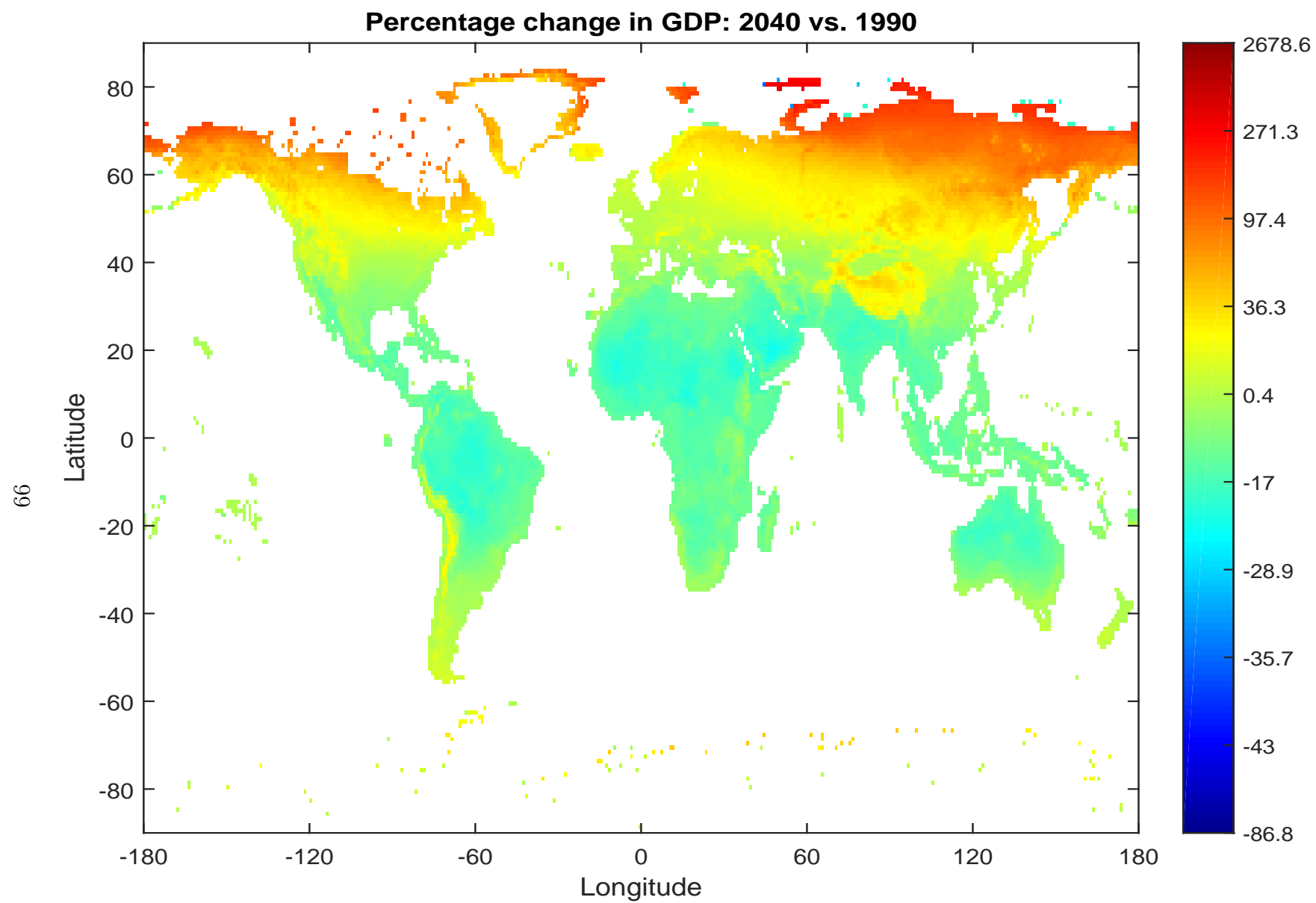


Figure 18e: Percentage Change in Regional GDP (relative to trend)
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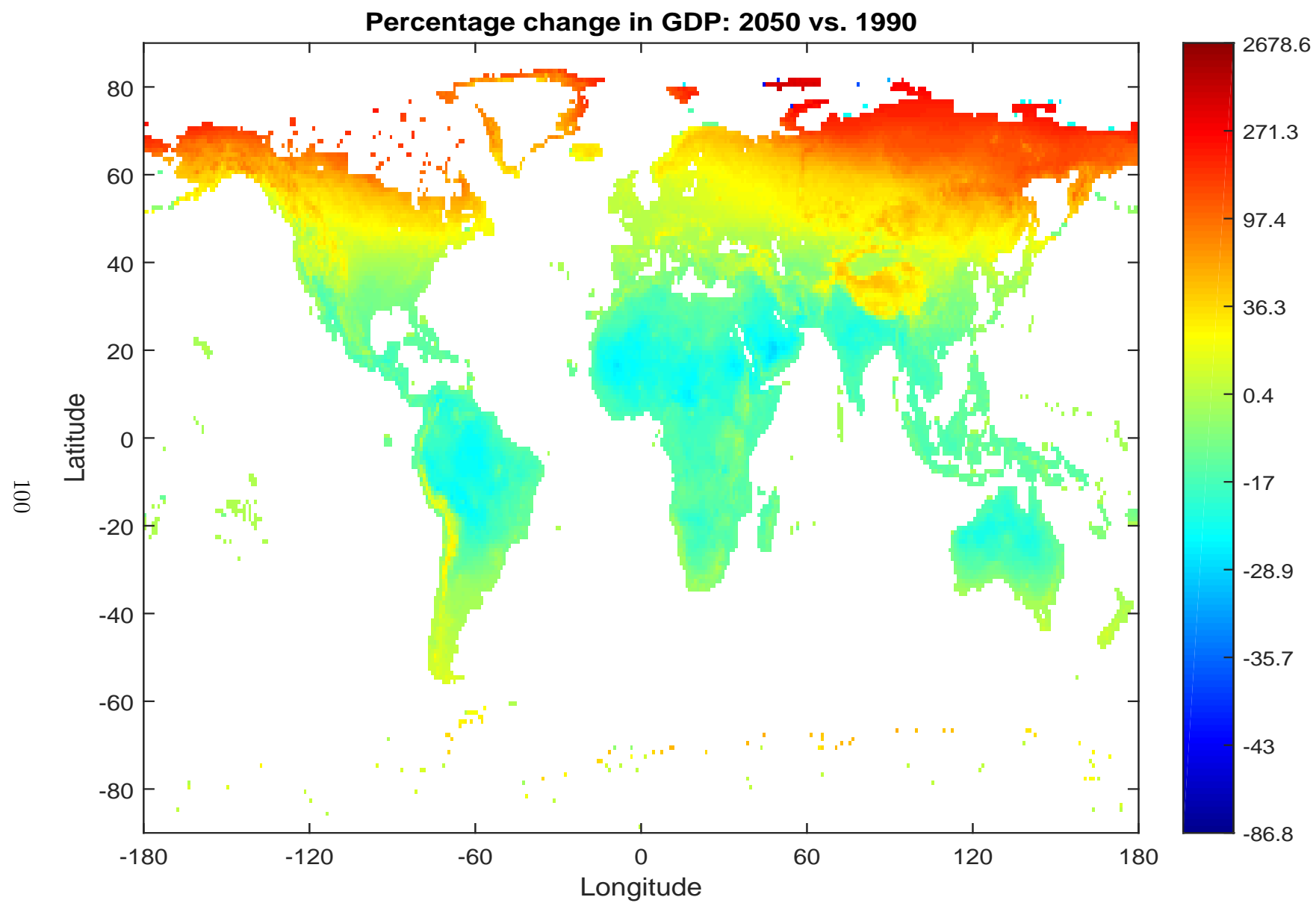


Figure 18f: Percentage Change in Regional GDP (relative to trend)
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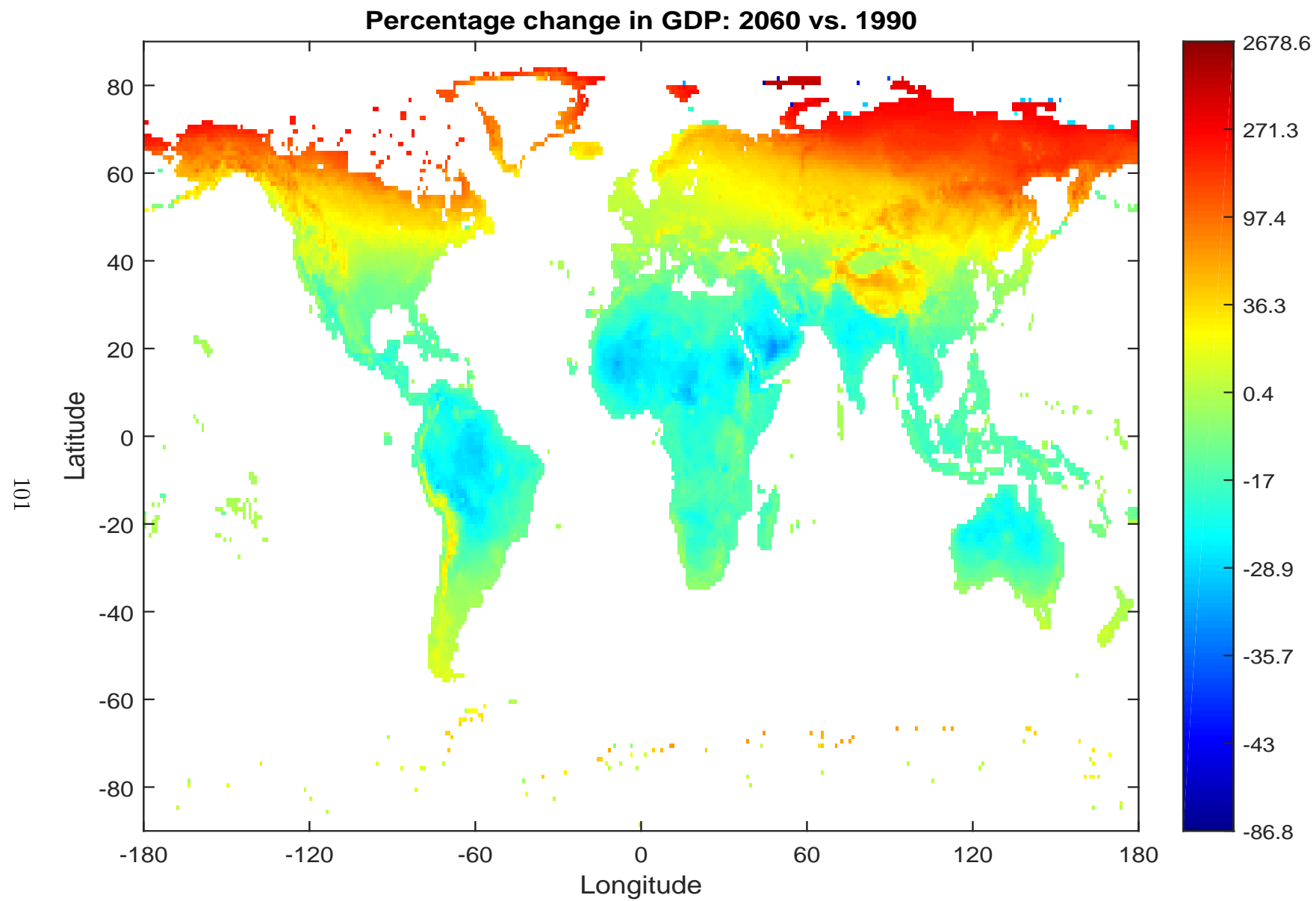


Figure 18g: Percentage Change in Regional GDP (relative to trend)
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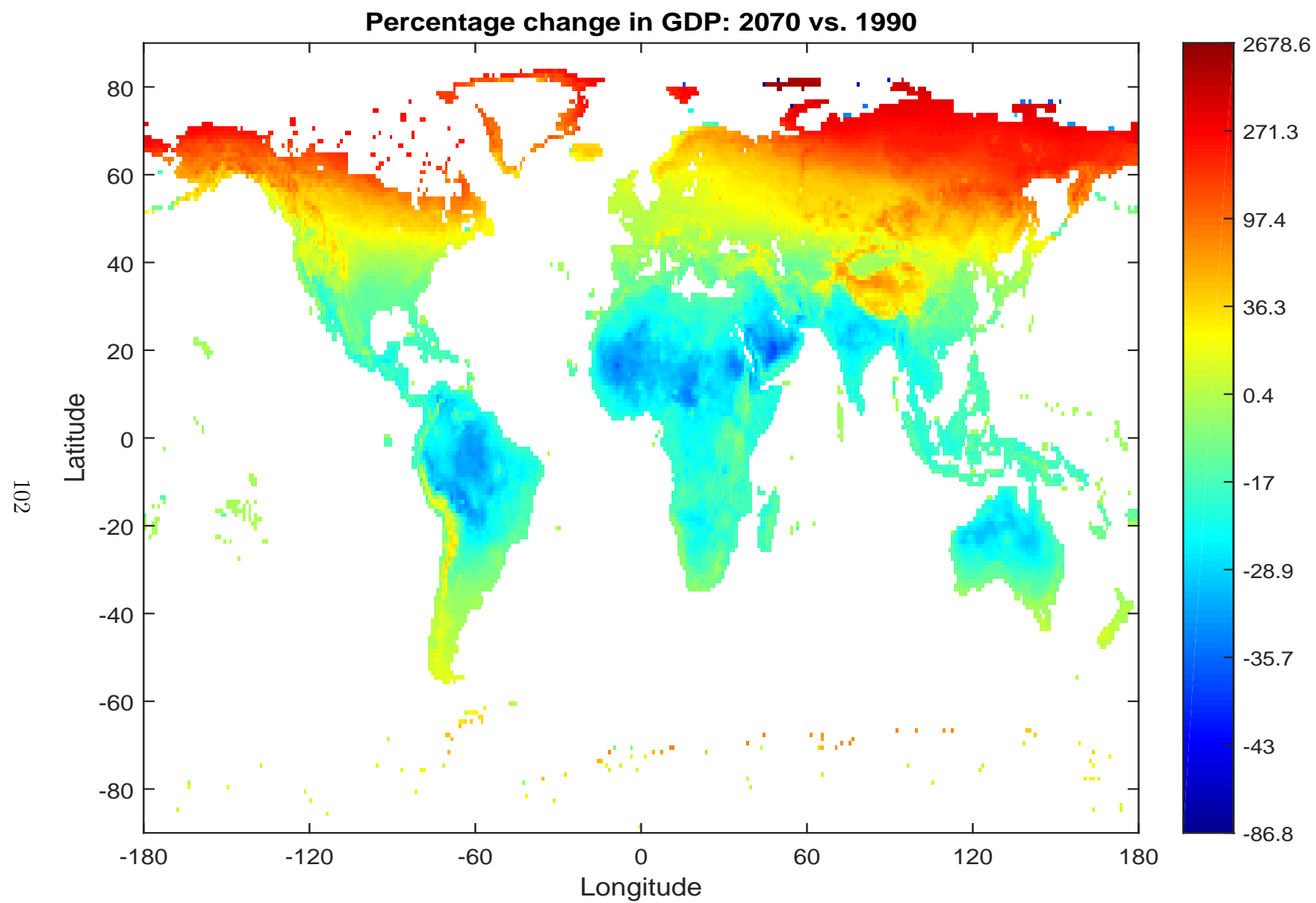


Figure 18h: Percentage Change in Regional GDP (relative to trend)
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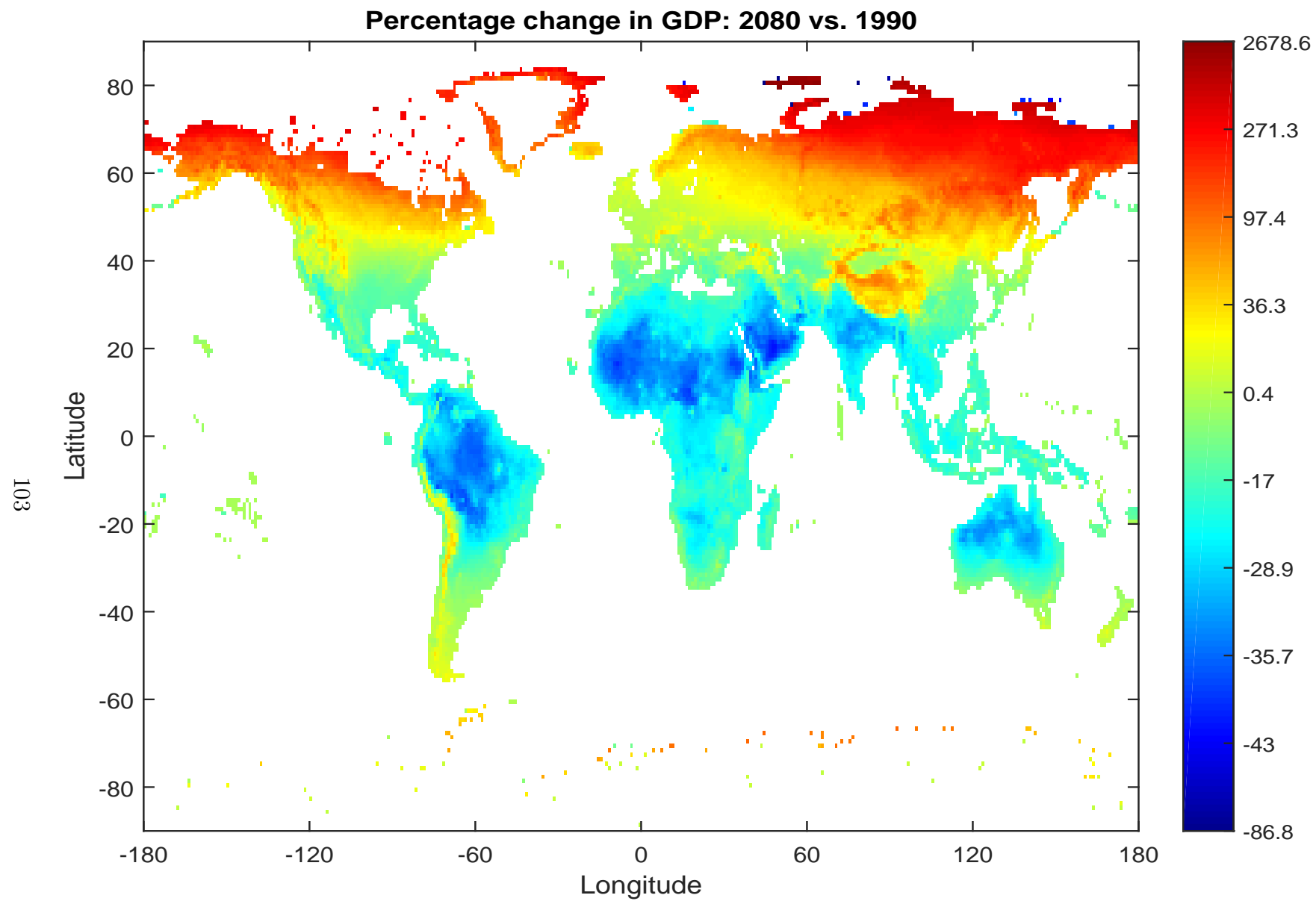


Figure 18i: Percentage Change in Regional GDP (relative to trend)
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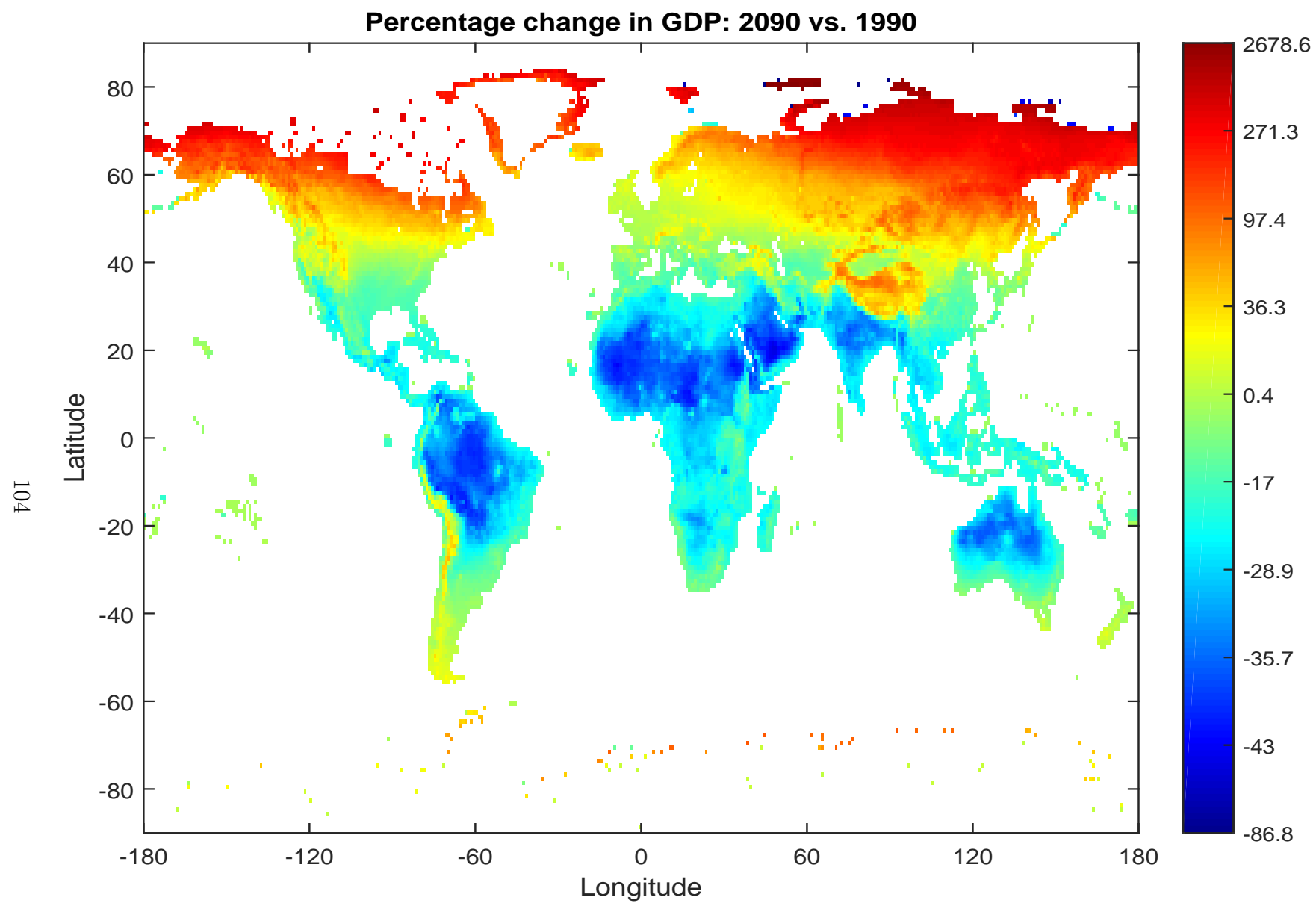


Figure 18j: Percentage Change in Regional GDP (relative to trend)
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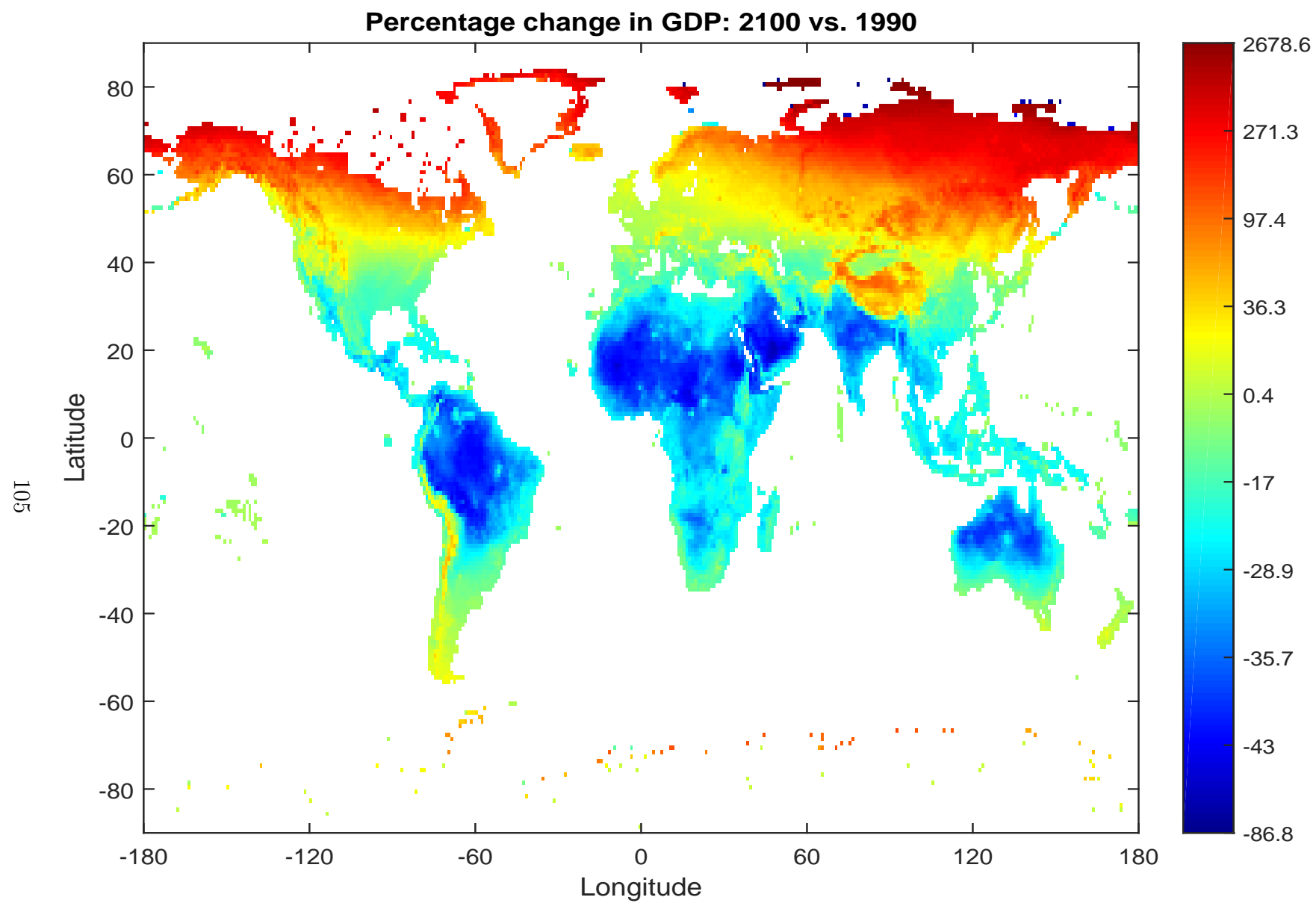


Figure 18k: Percentage Change in Regional GDP (relative to trend)
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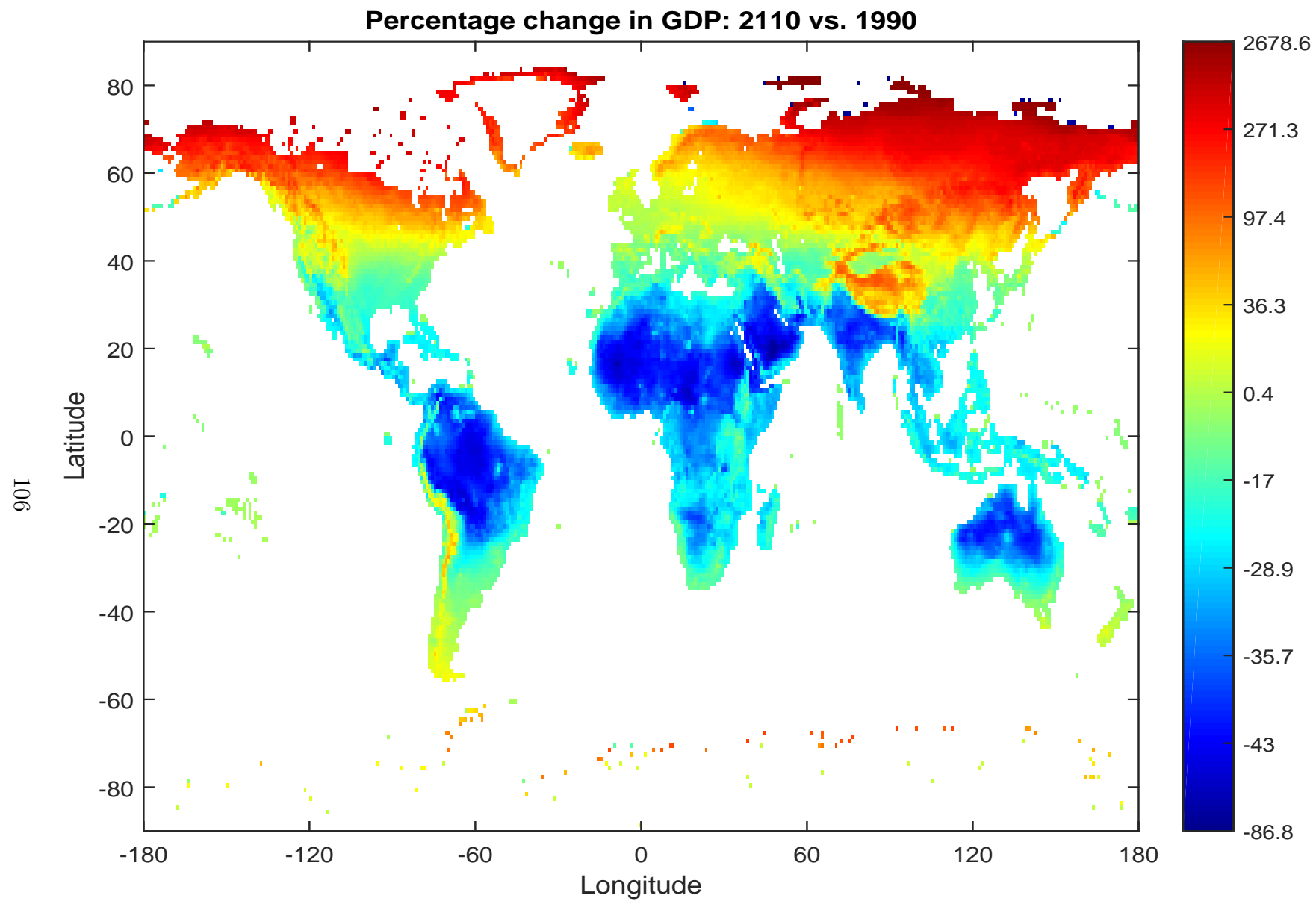


Figure 18l: Percentage Change in Regional GDP (relative to trend)
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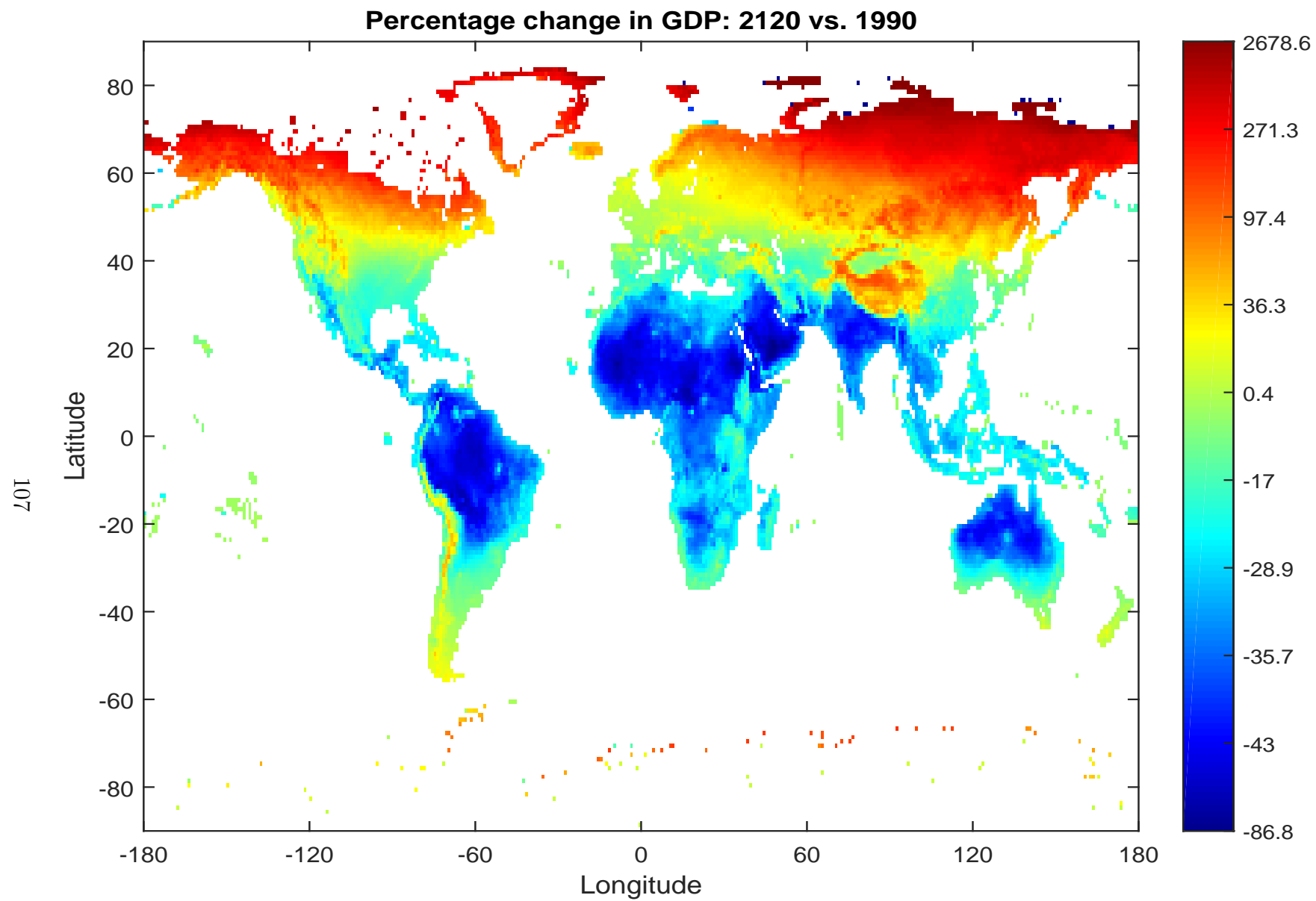


Figure 18m: Percentage Change in Regional GDP (relative to trend)
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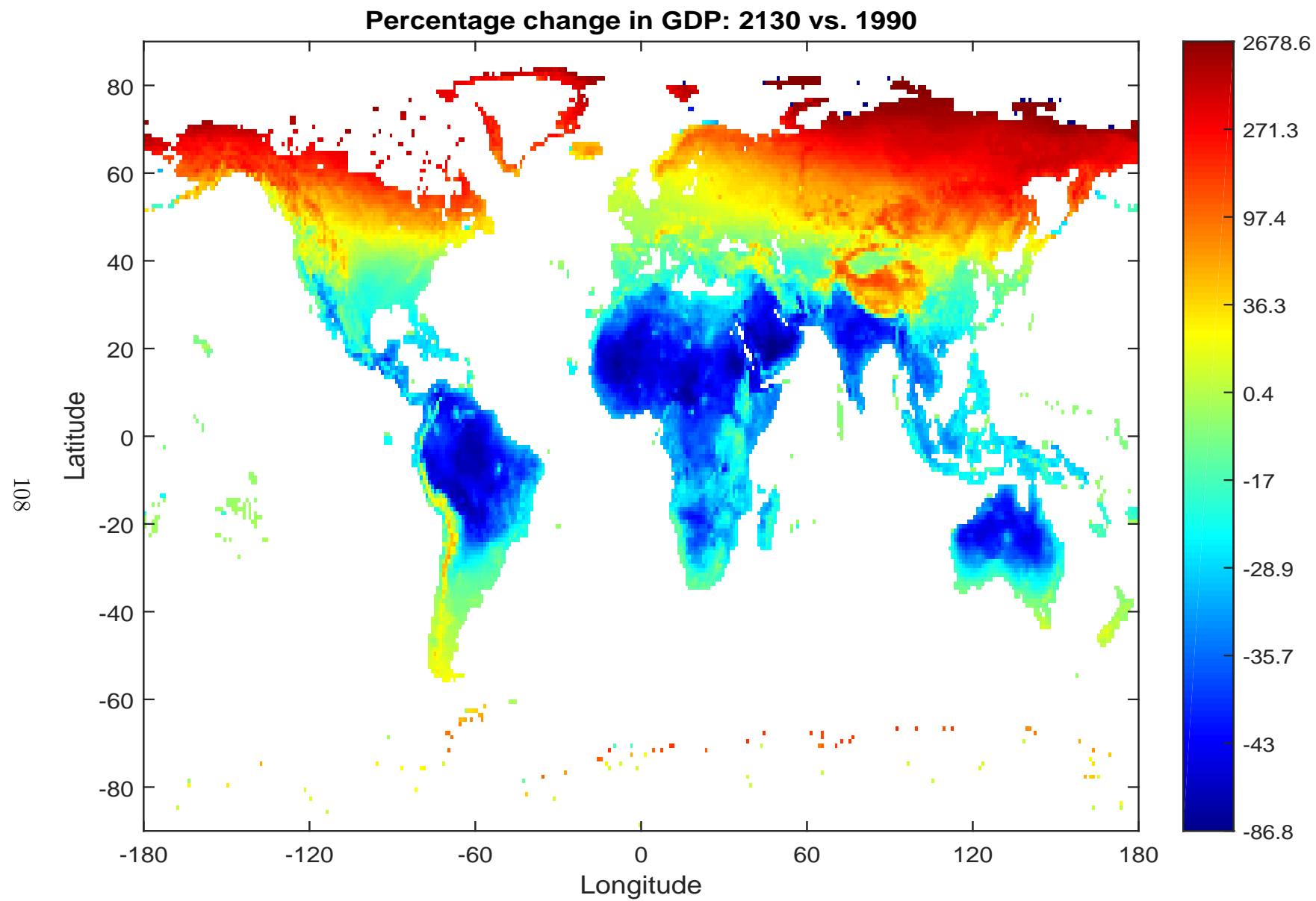


Figure 18n: Percentage Change in Regional GDP (relative to trend)
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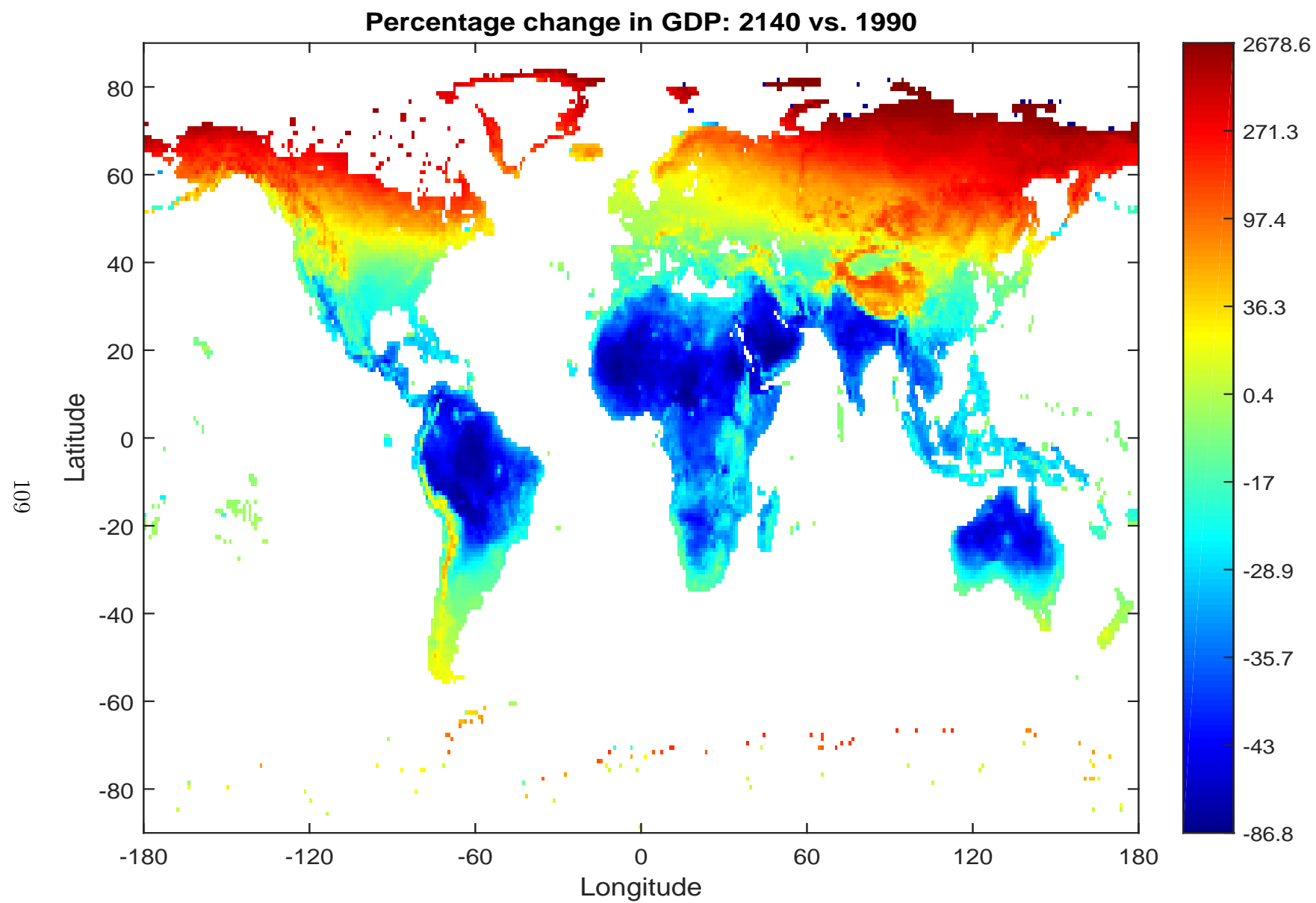


Figure 18o: Percentage Change in Regional GDP (relative to trend)
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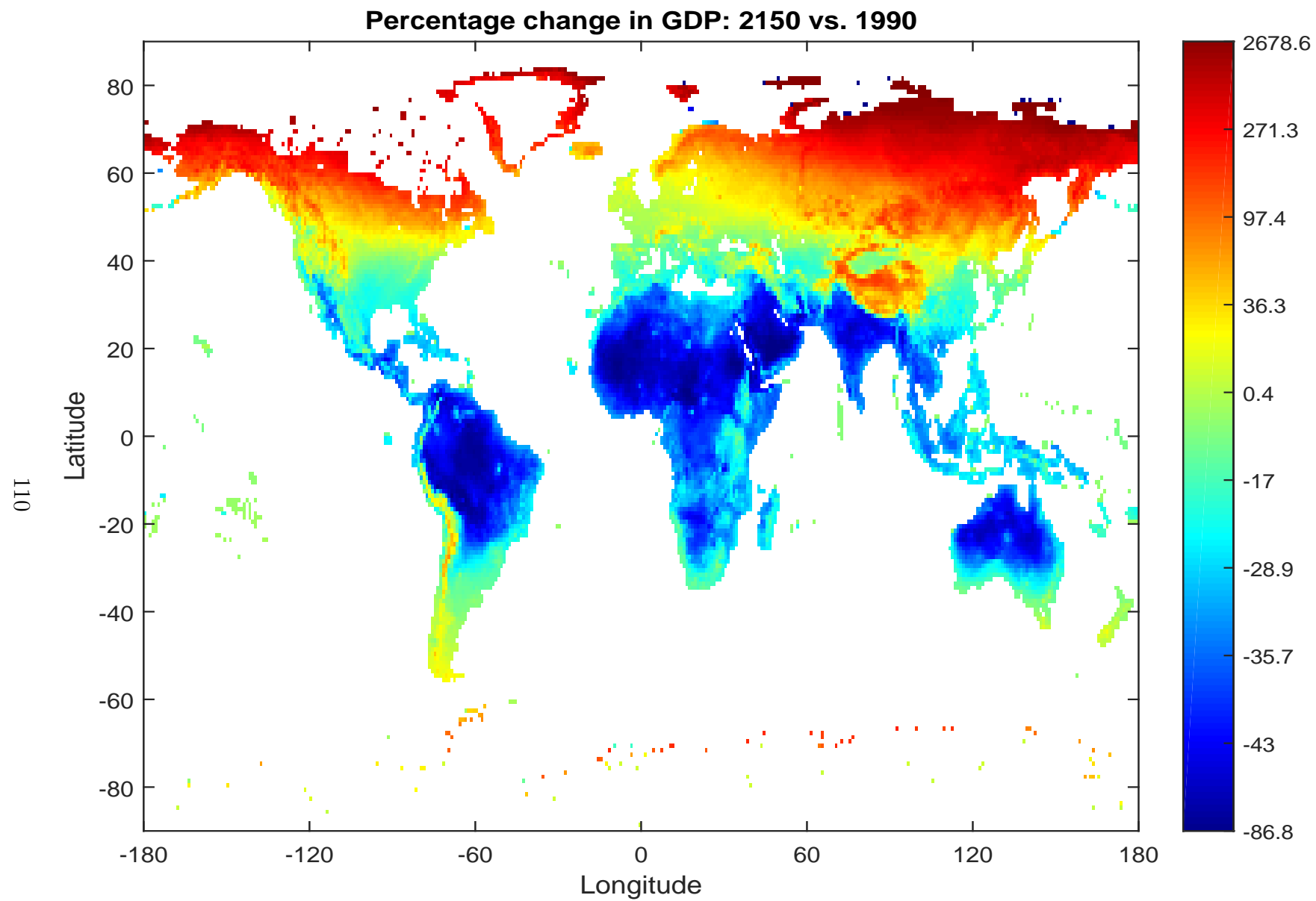


Figure 18p: Percentage Change in Regional GDP (relative to trend)
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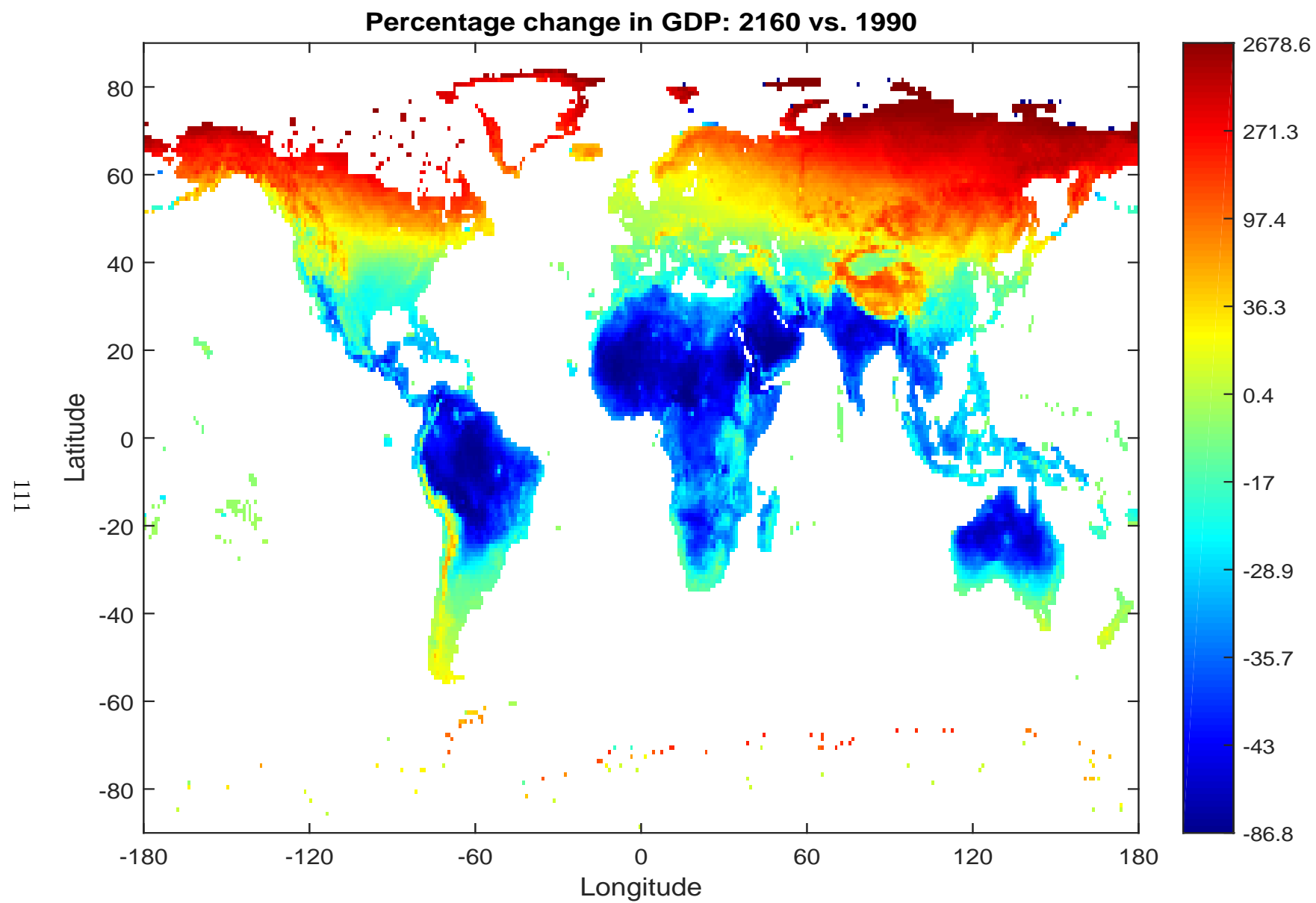


Figure 18q: Percentage Change in Regional GDP (relative to trend)
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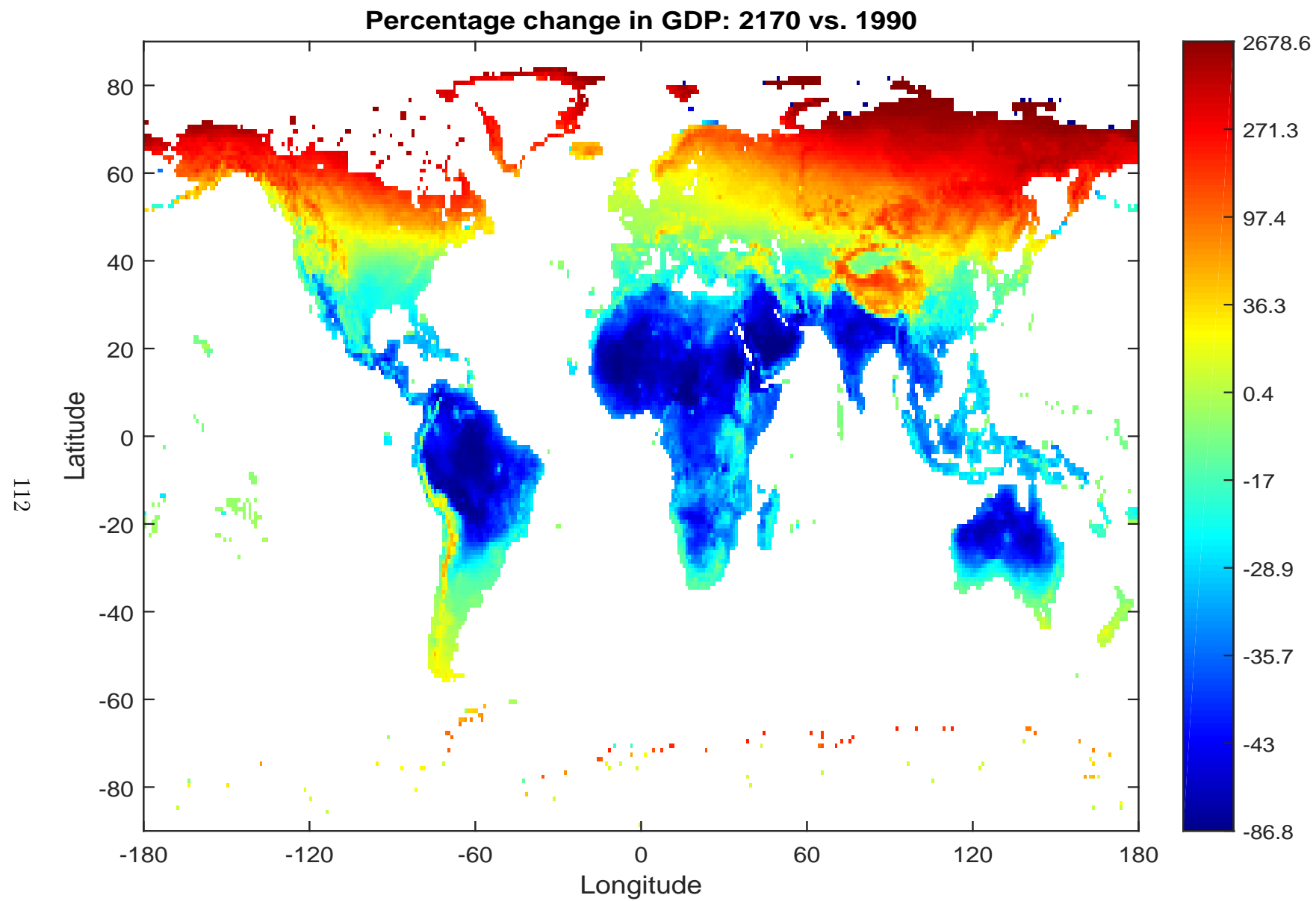


Figure 18r: Percentage Change in Regional GDP (relative to trend)
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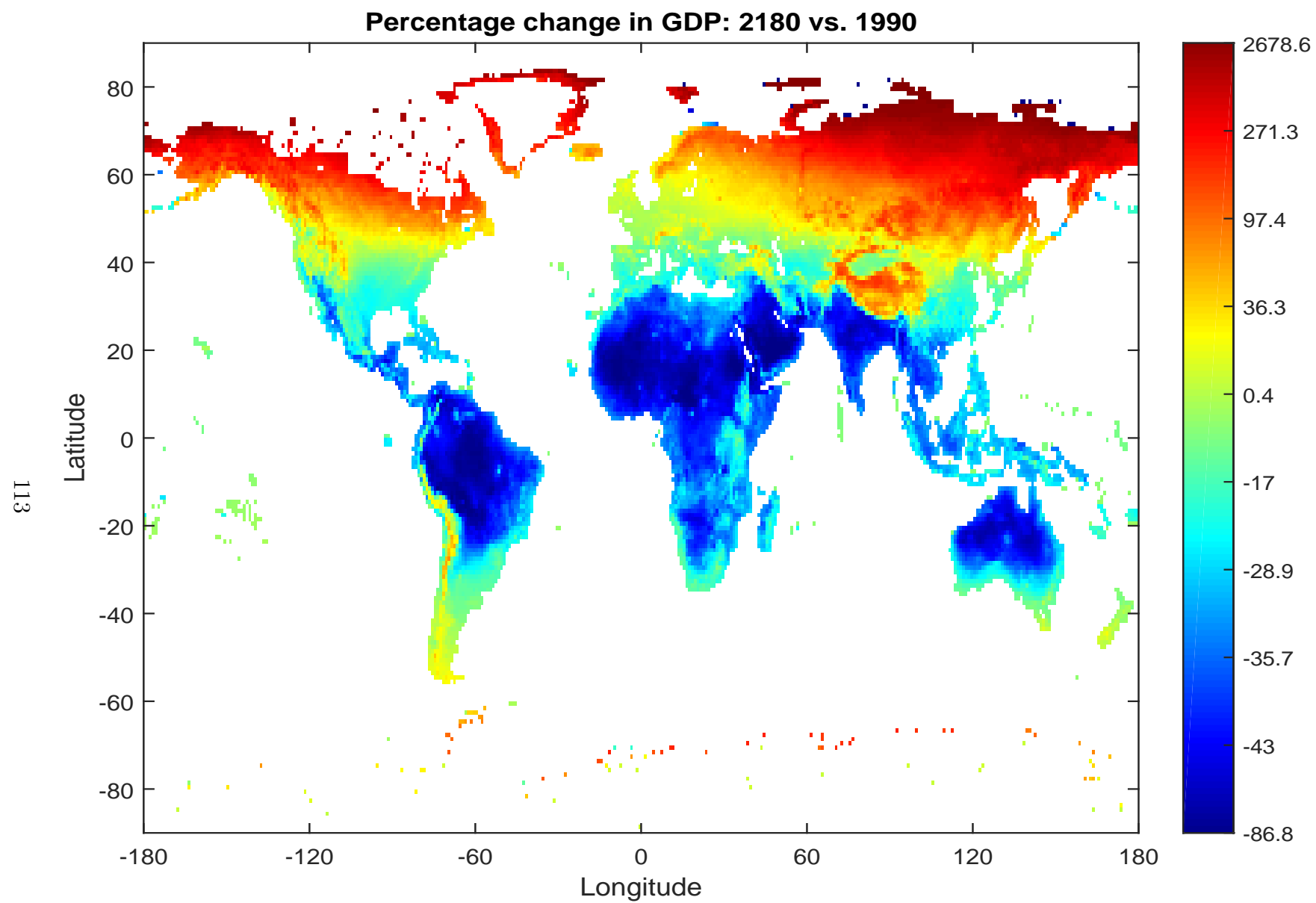


Figure 18s: Percentage Change in Regional GDP (relative to trend)
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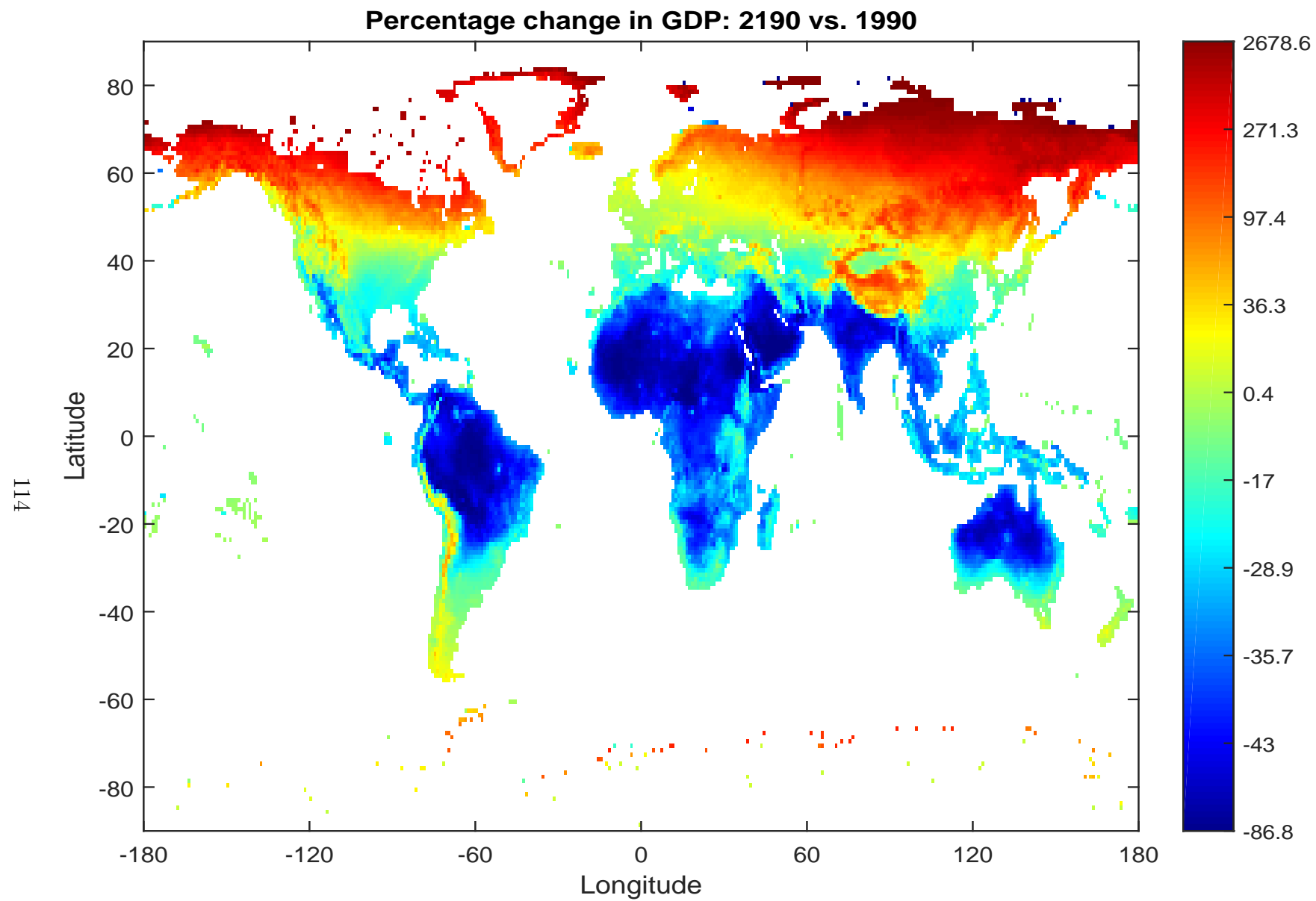


Figure 18t: Percentage Change in Regional GDP (relative to trend)
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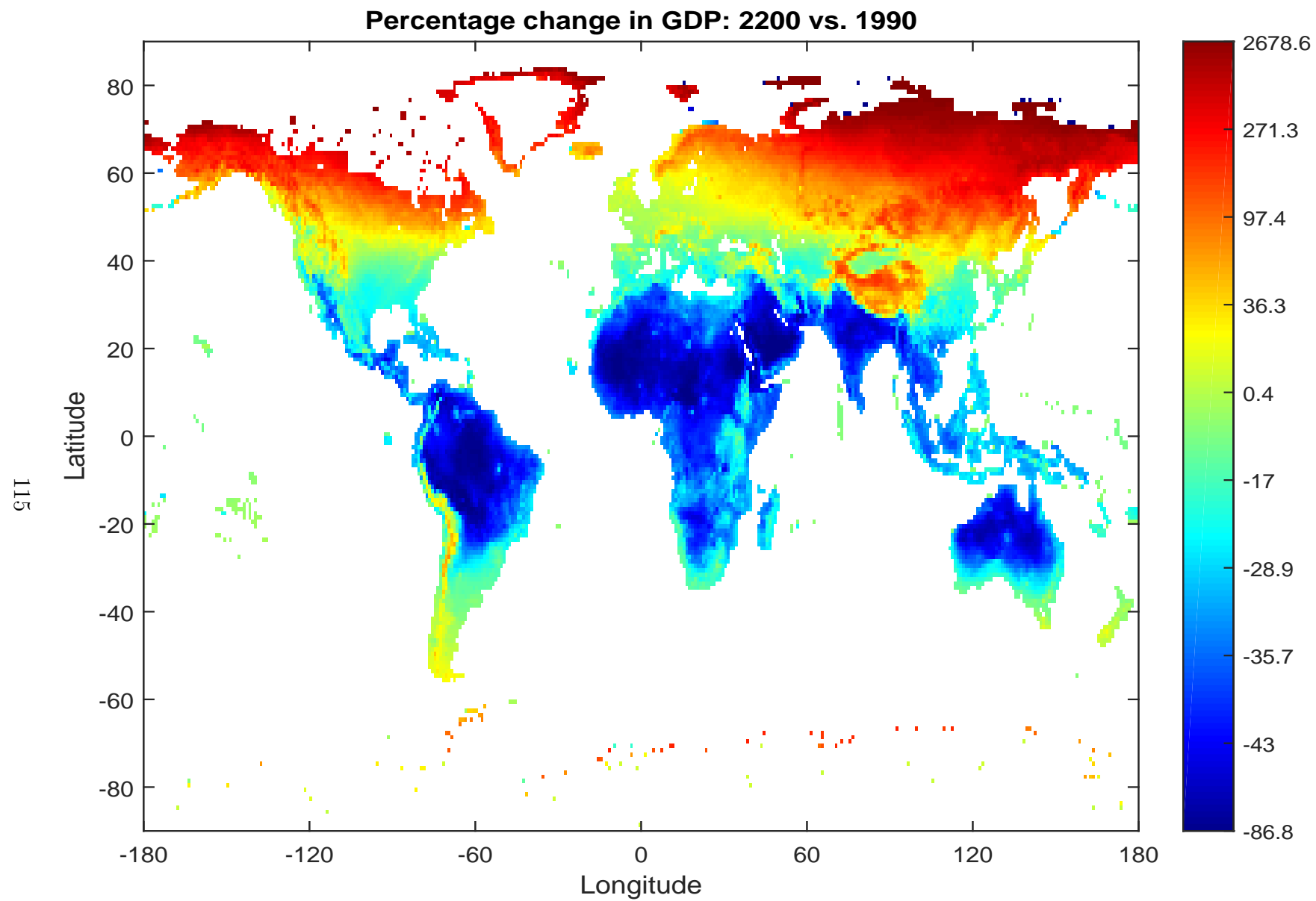


Figure 18u: Percentage Change in Regional GDP (relative to trend)
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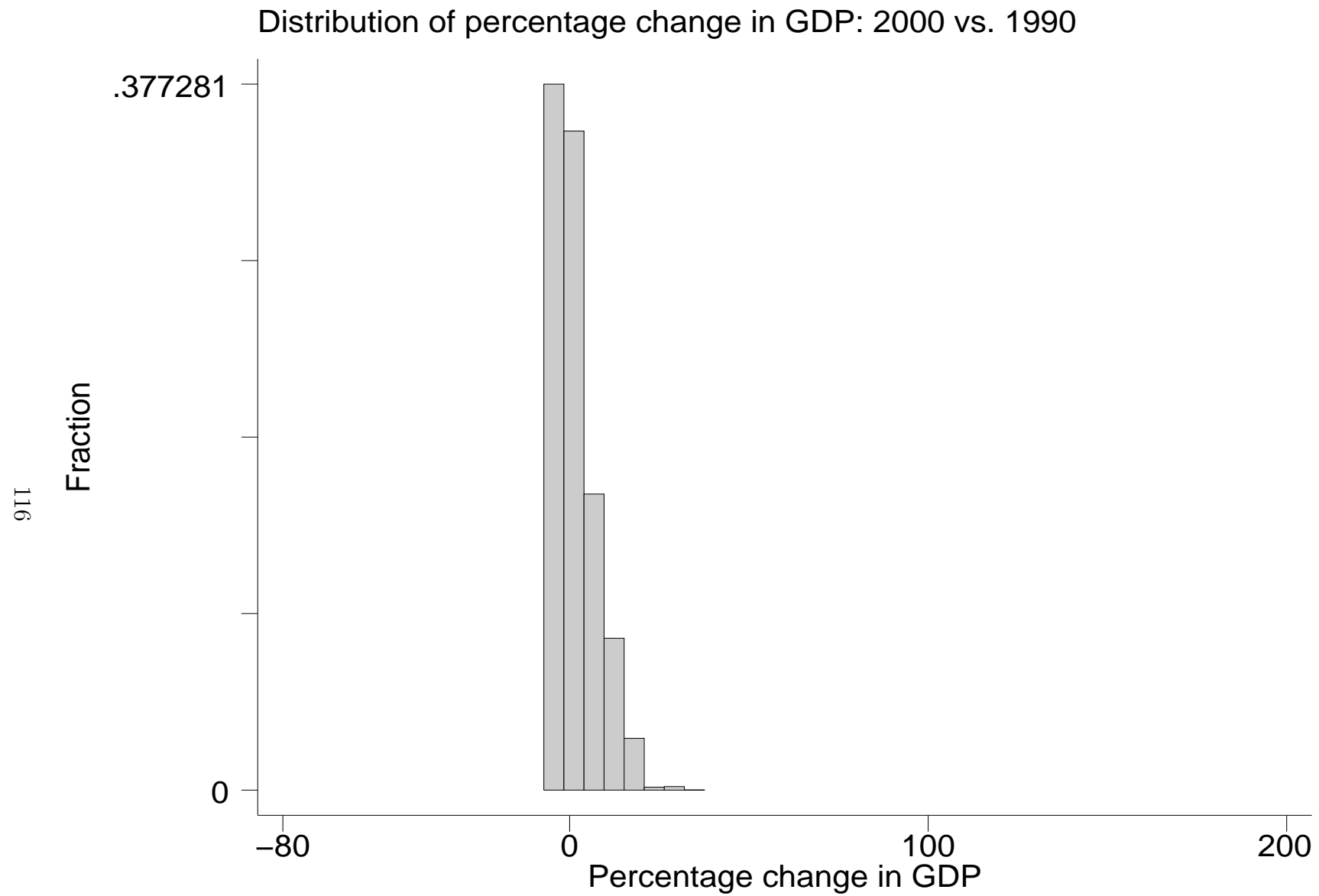


Figure 19a: Distribution of Percentage Changes in Regional GDP (relative to trend)
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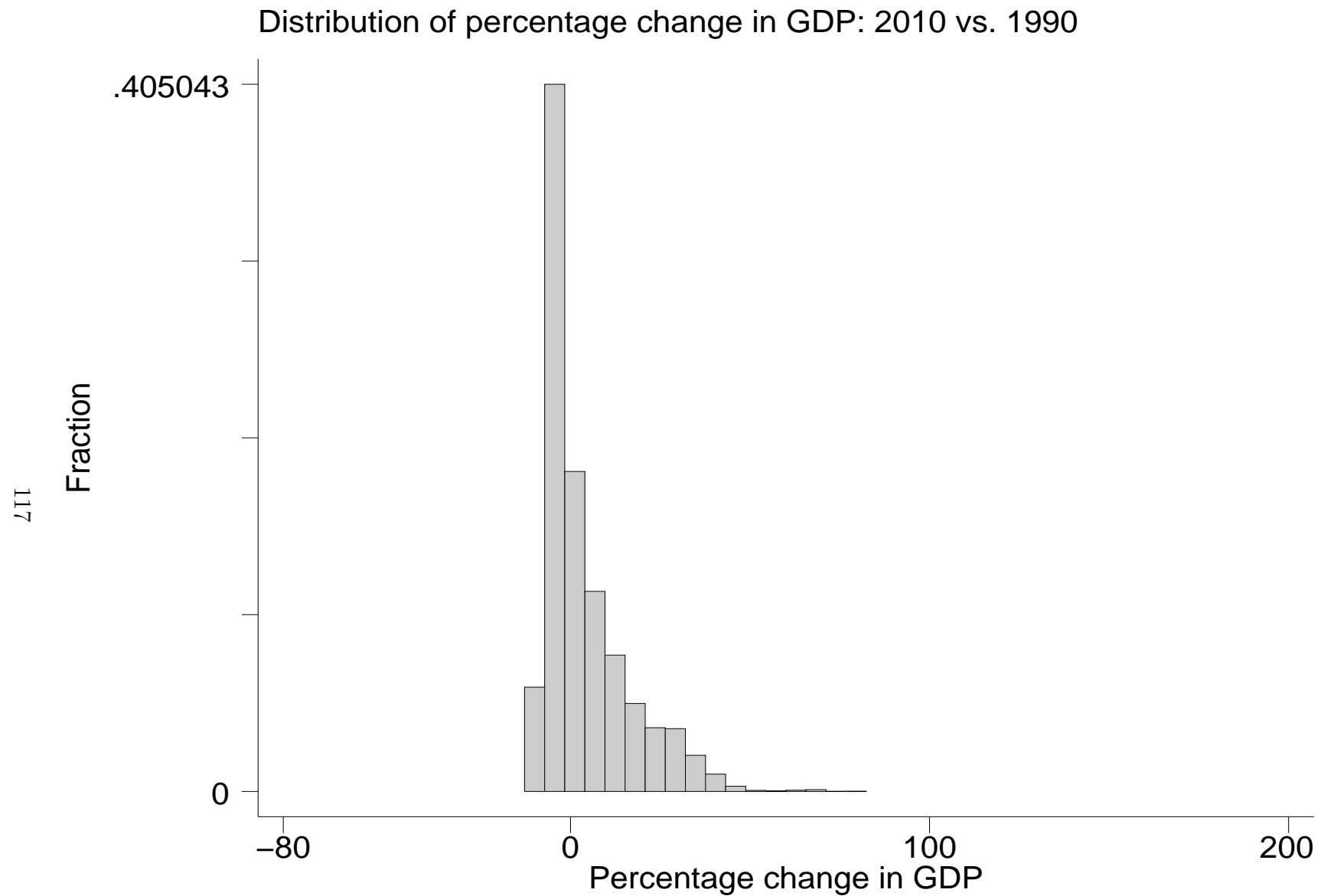


Figure 19b: Distribution of Percentage Changes in Regional GDP (relative to trend)
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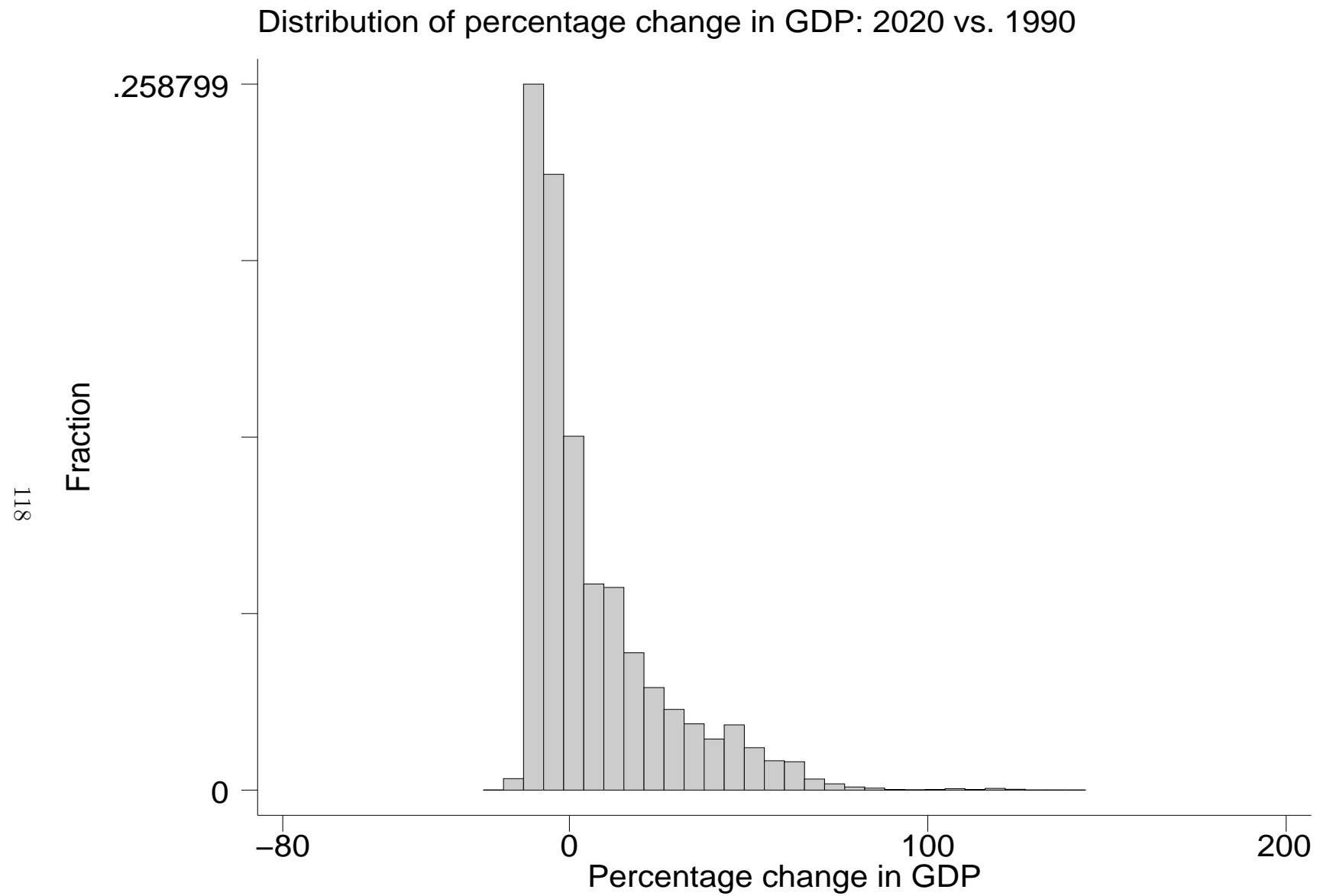


Figure 19c: Distribution of Percentage Changes in Regional GDP (relative to trend)
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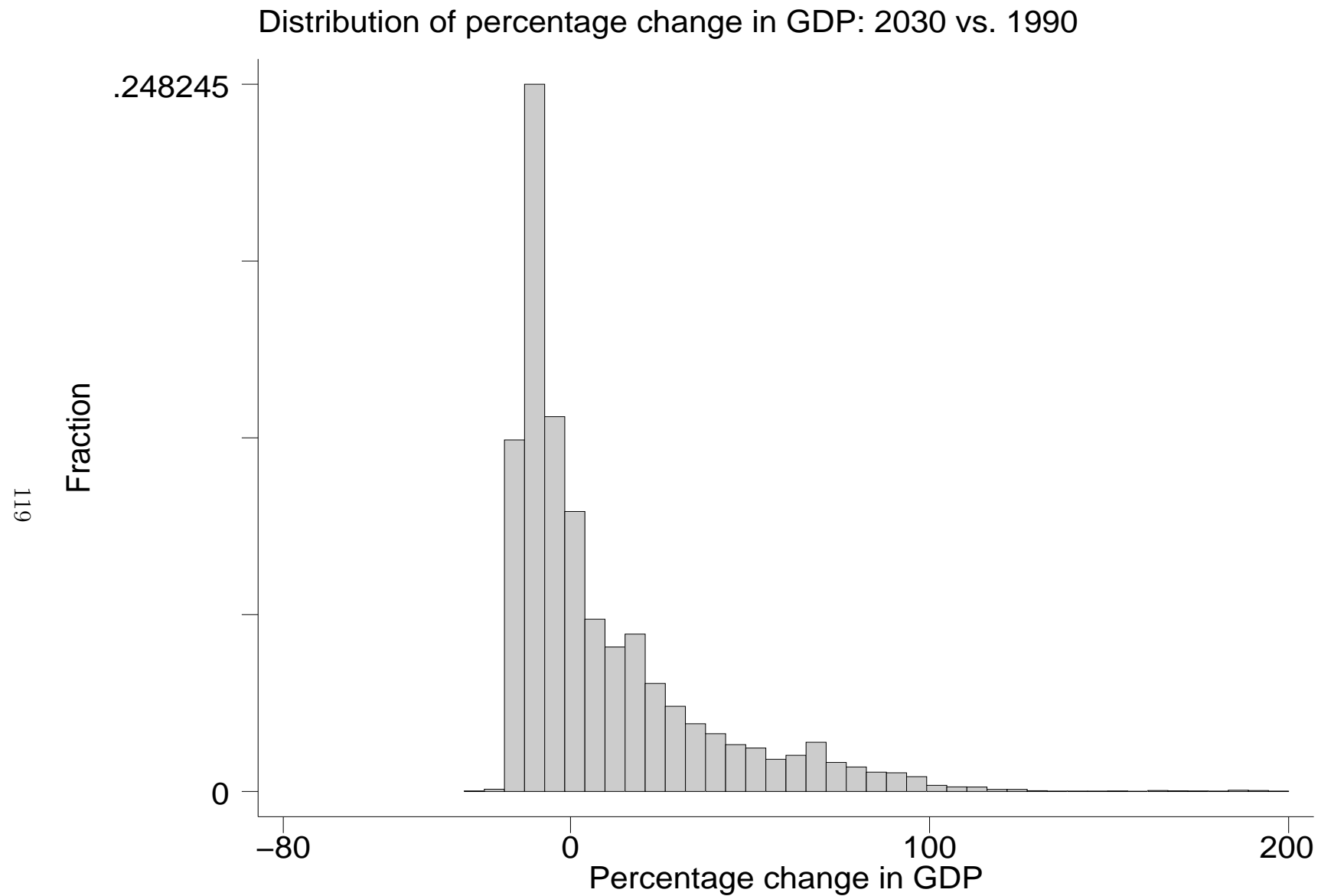


Figure 19d: Distribution of Percentage Changes in Regional GDP (relative to trend)
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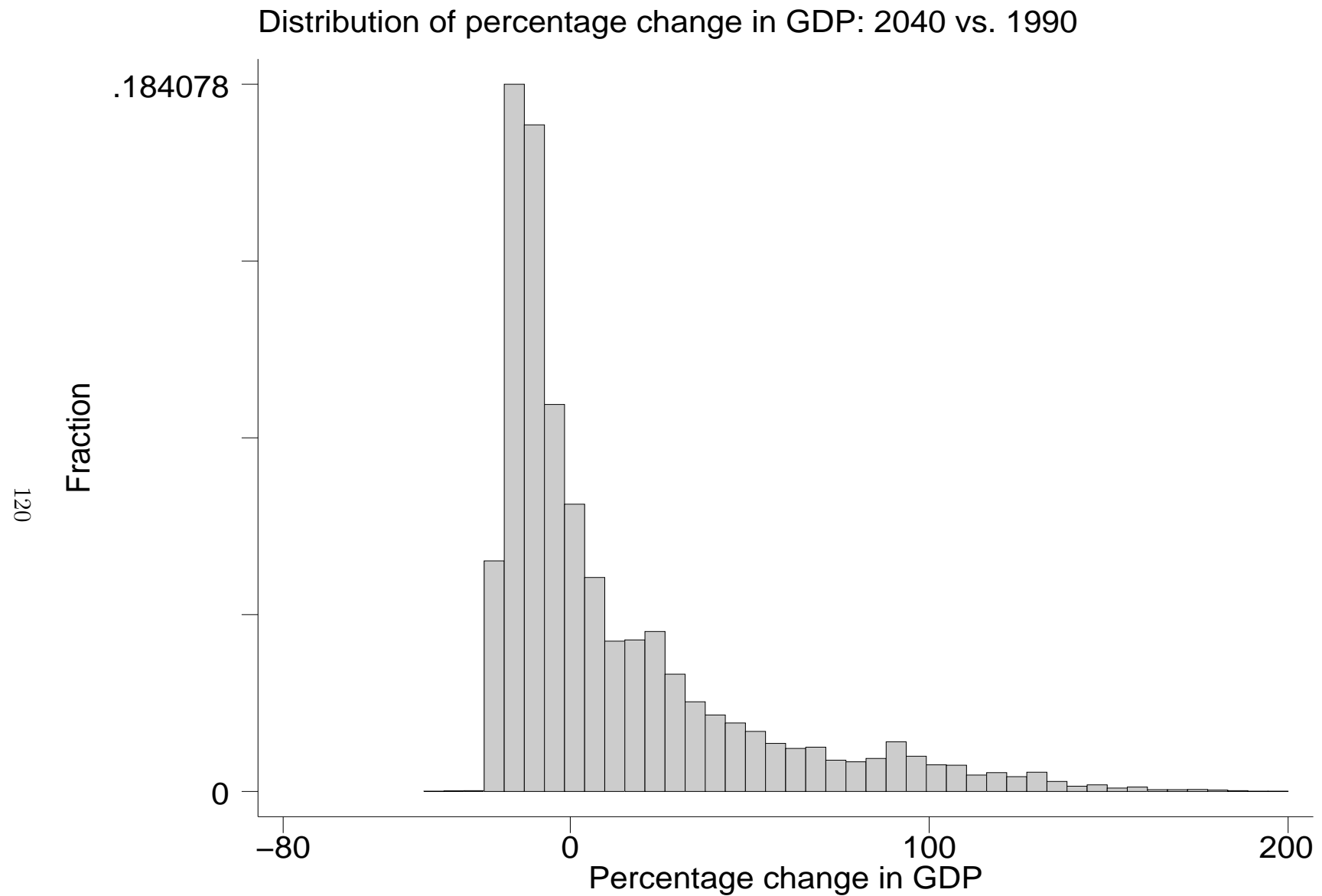


Figure 19e: Distribution of Percentage Changes in Regional GDP (relative to trend)
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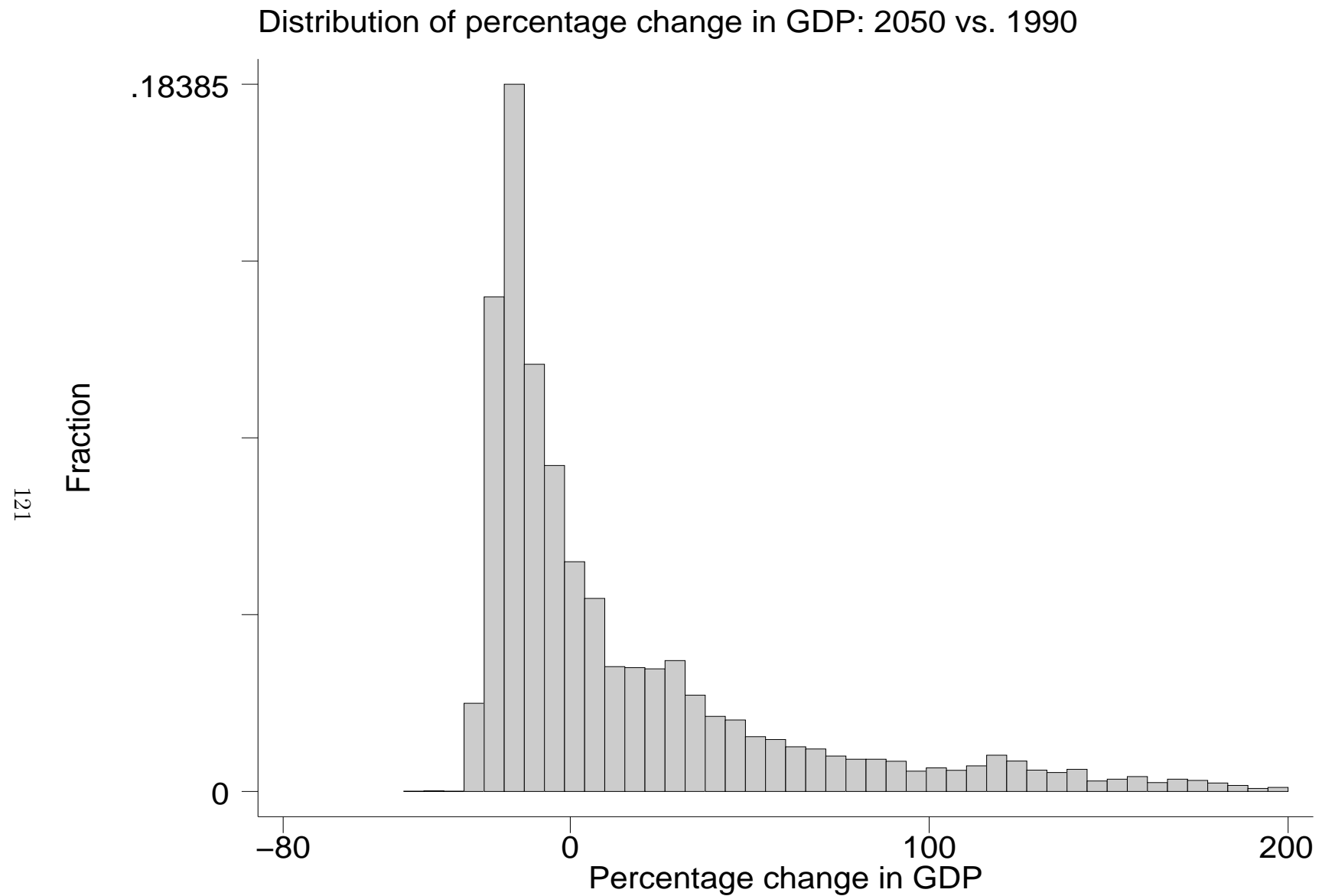


Figure 19f: Distribution of Percentage Changes in Regional GDP (relative to trend)
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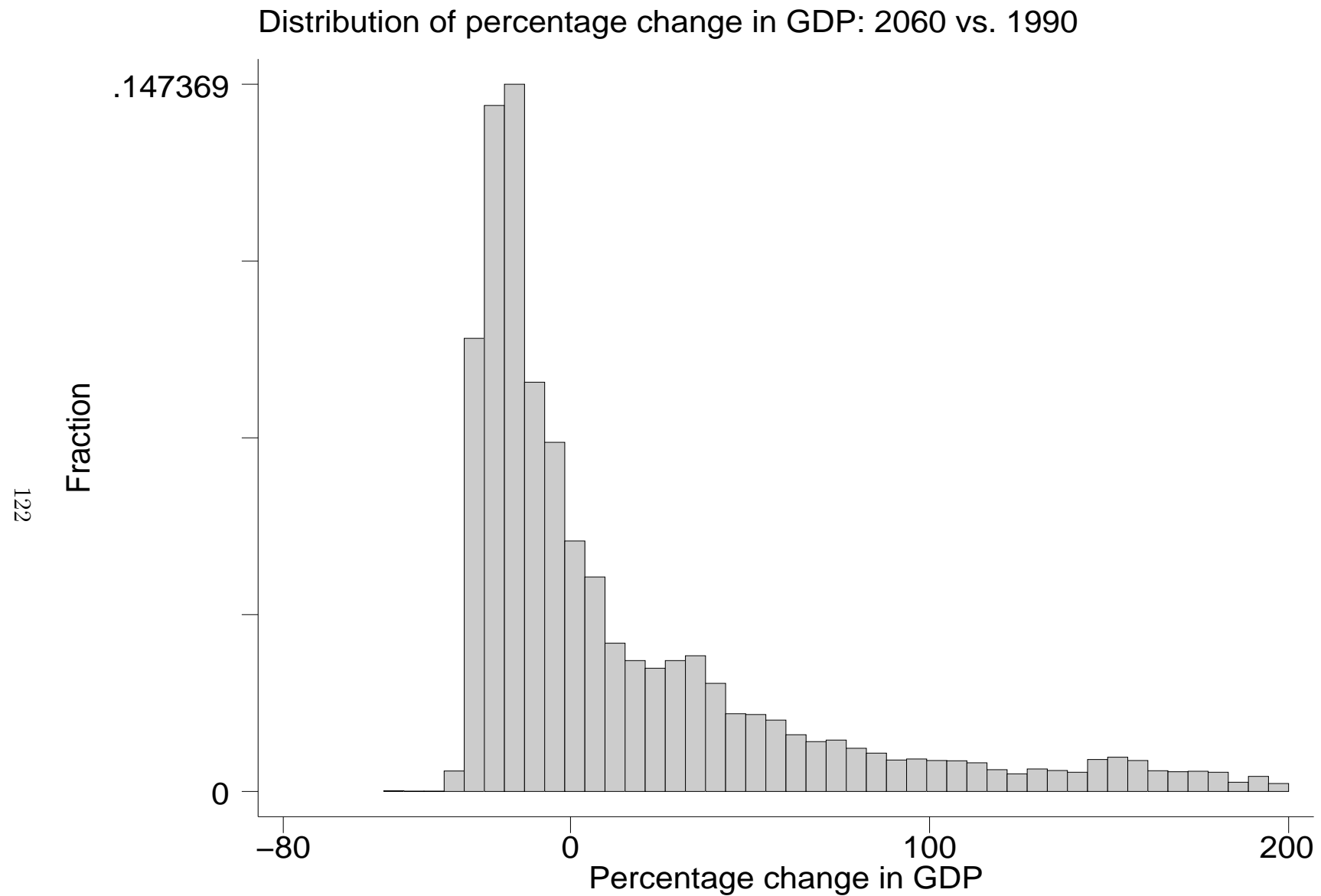


Figure 19g: Distribution of Percentage Changes in Regional GDP (relative to trend)
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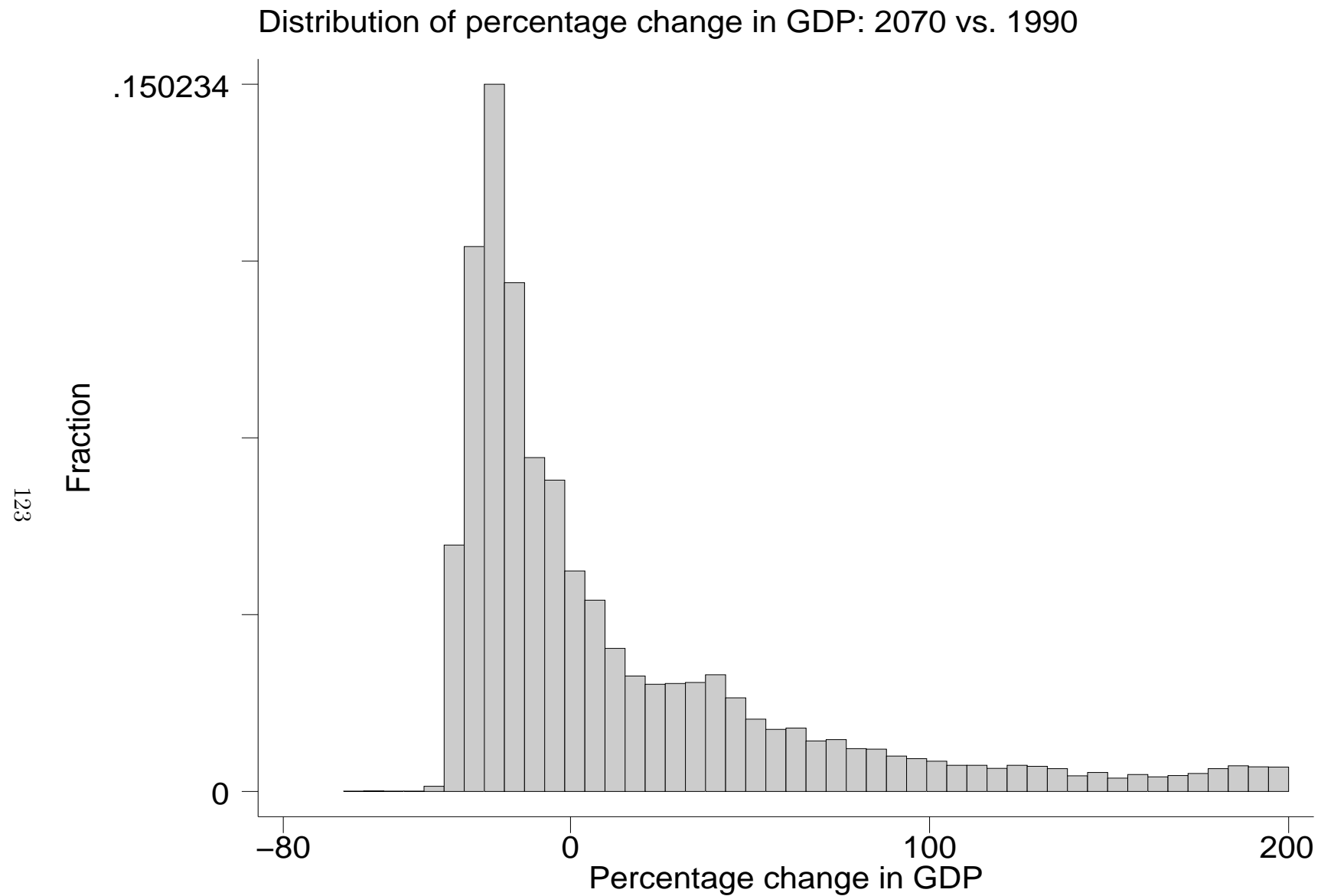


Figure 19h: Distribution of Percentage Changes in Regional GDP (relative to trend)
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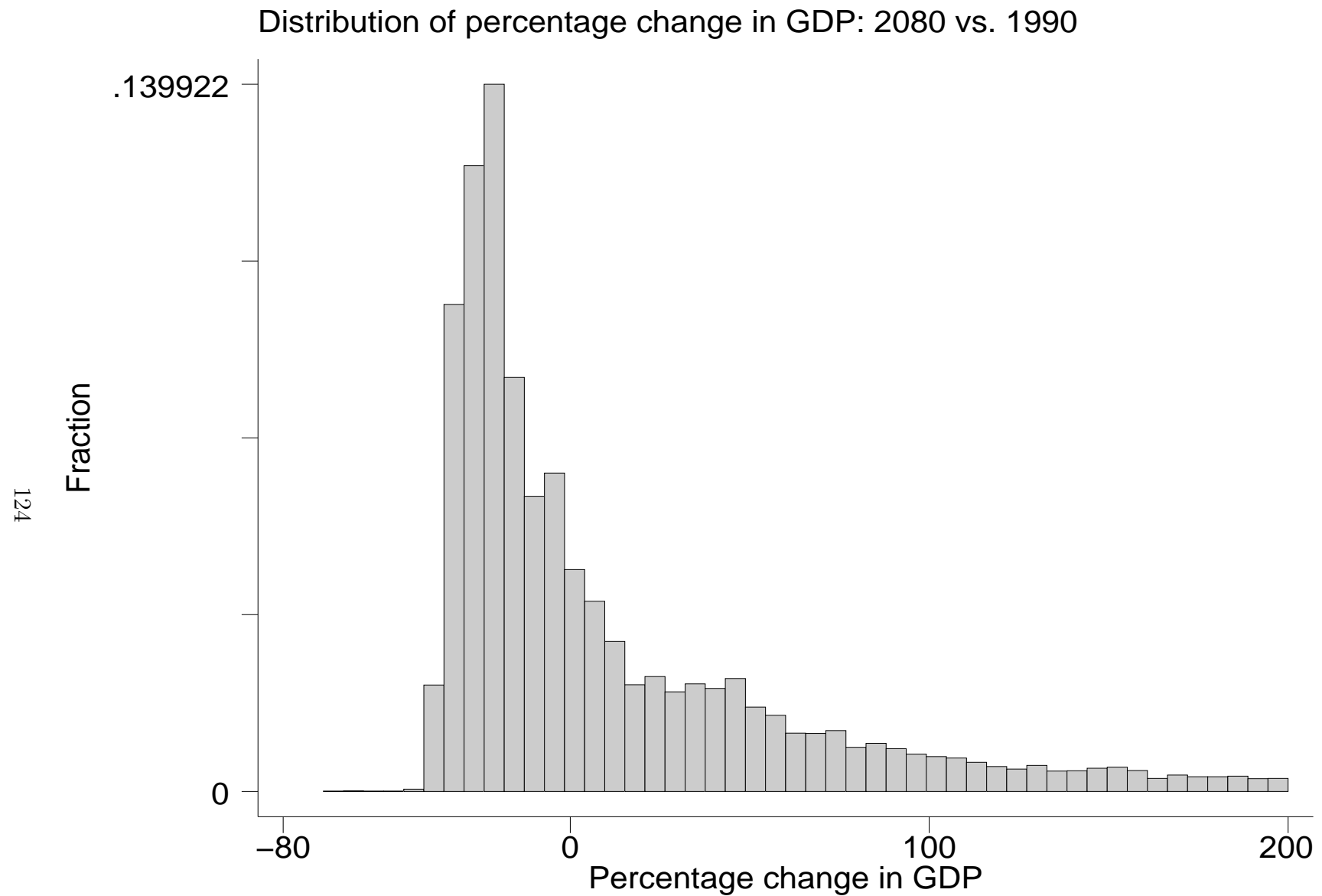


Figure 19i: Distribution of Percentage Changes in Regional GDP (relative to trend)
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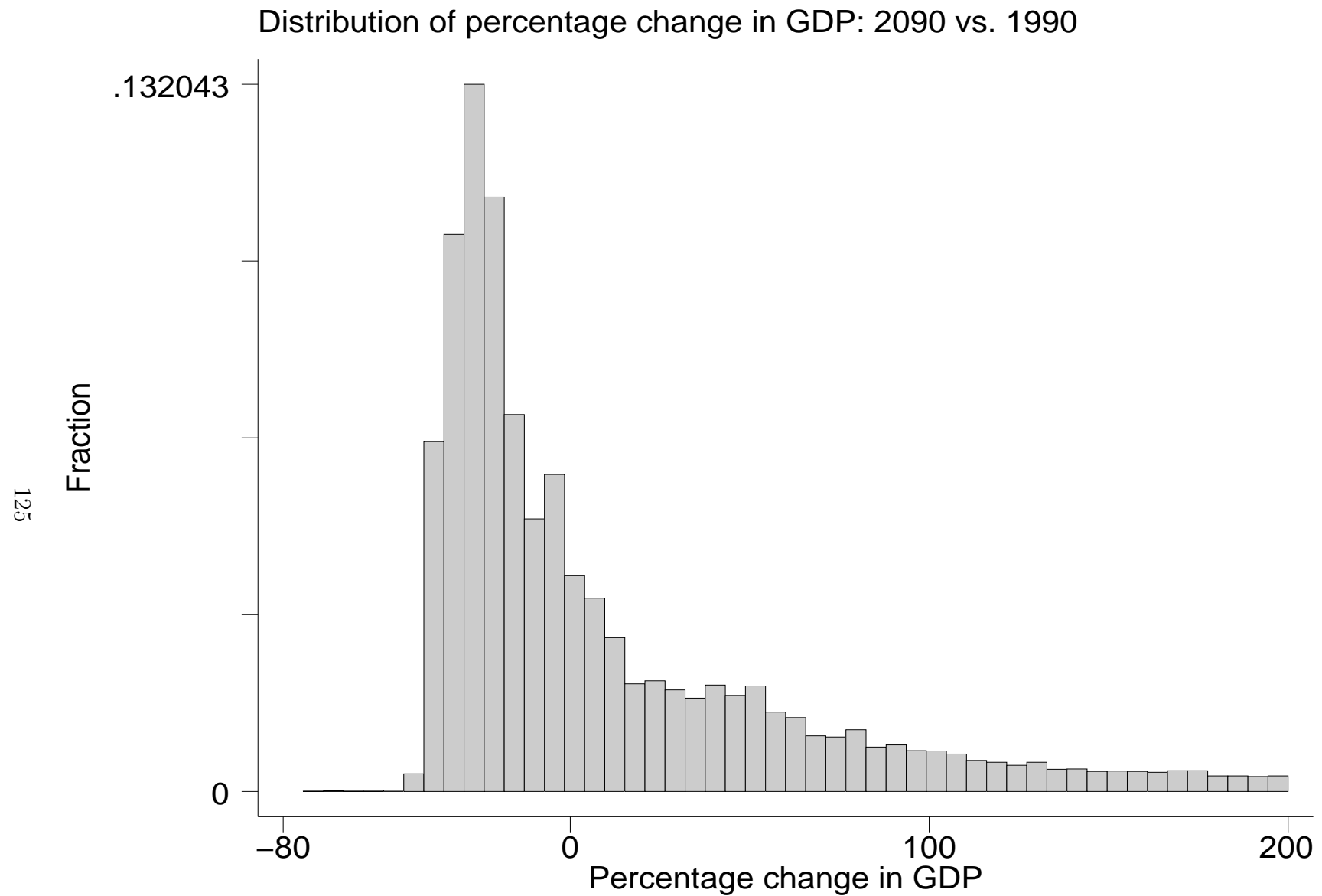


Figure 19j: Distribution of Percentage Changes in Regional GDP (relative to trend)
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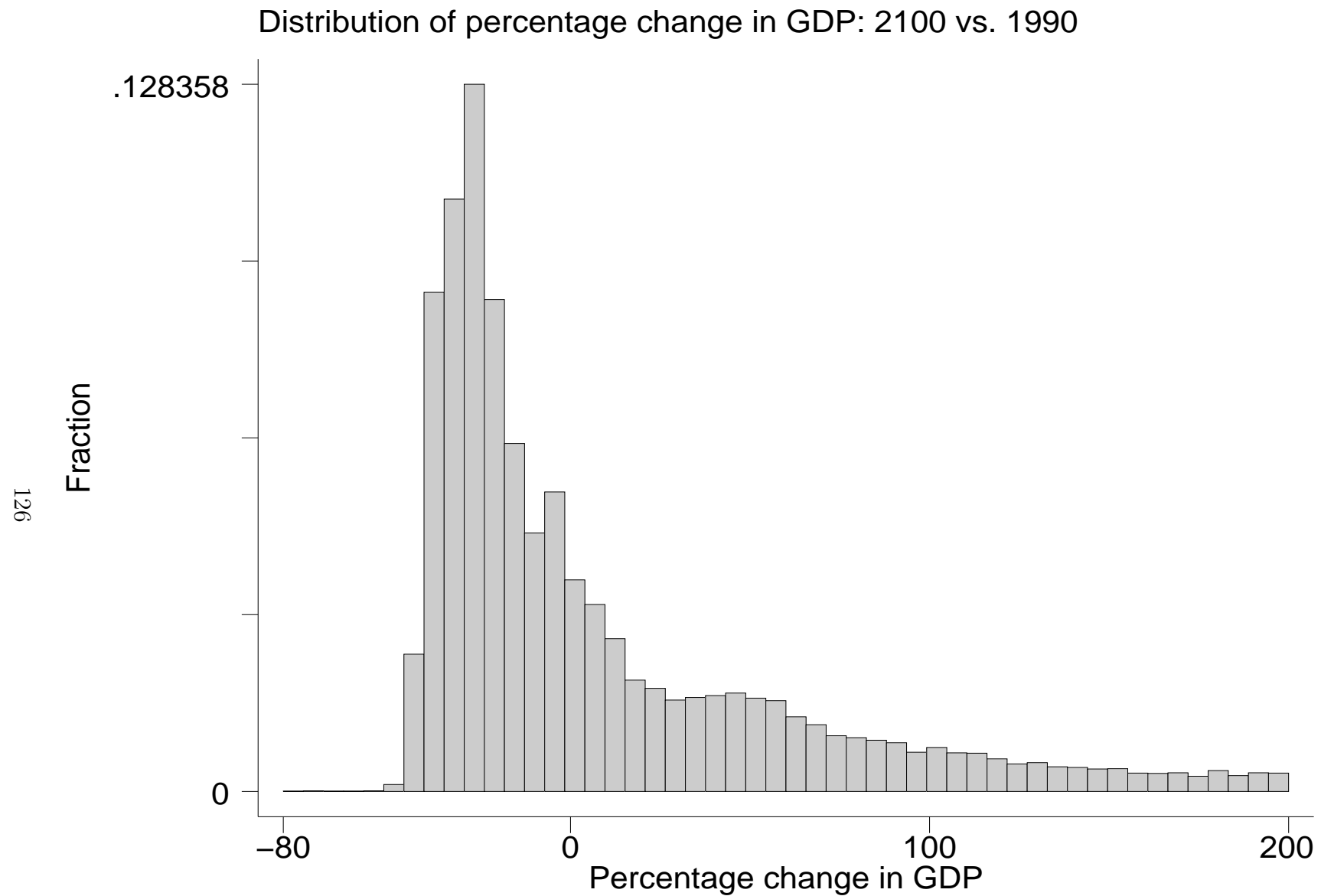


Figure 19k: Distribution of Percentage Changes in Regional GDP (relative to trend)
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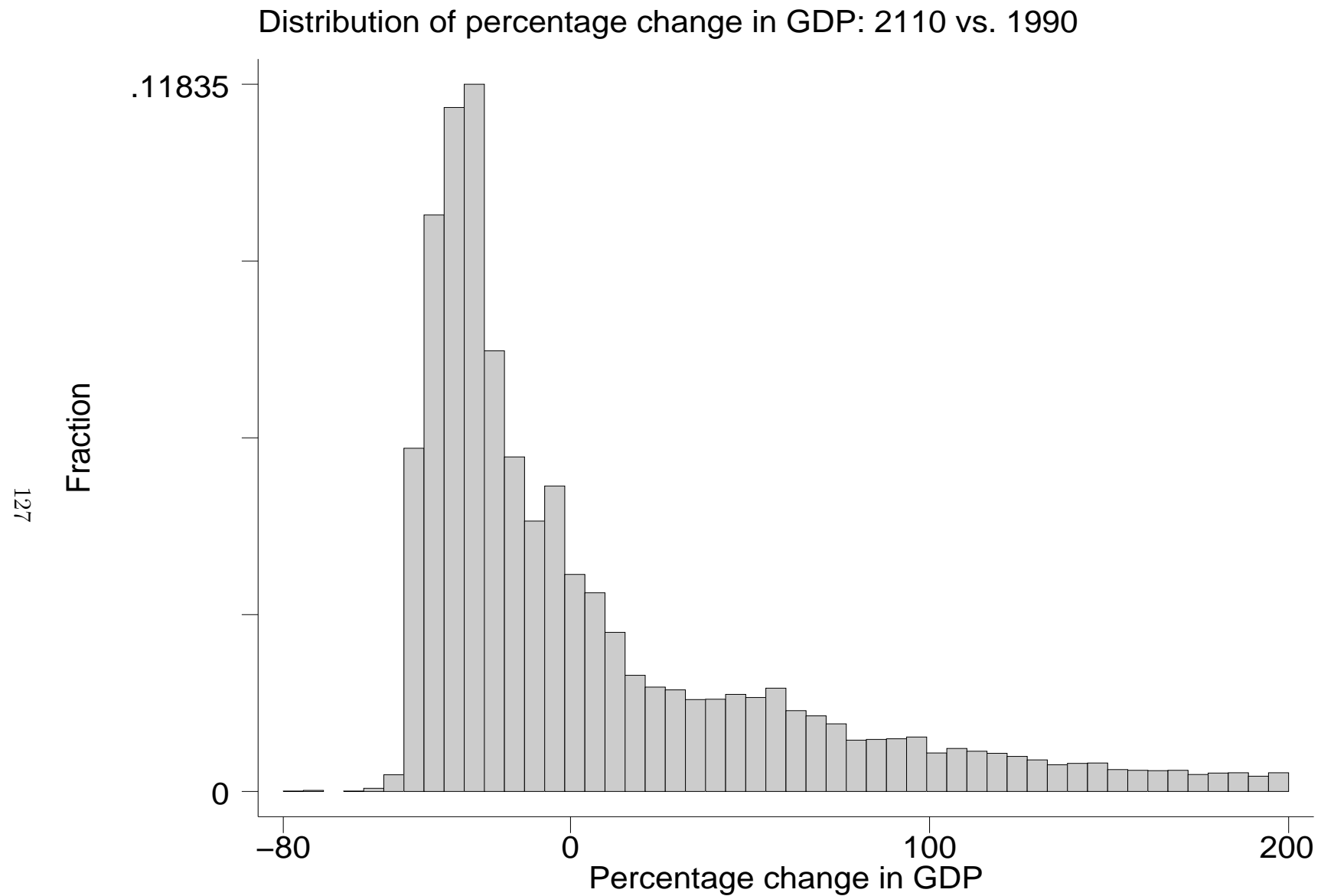


Figure 19l: Distribution of Percentage Changes in Regional GDP (relative to trend)
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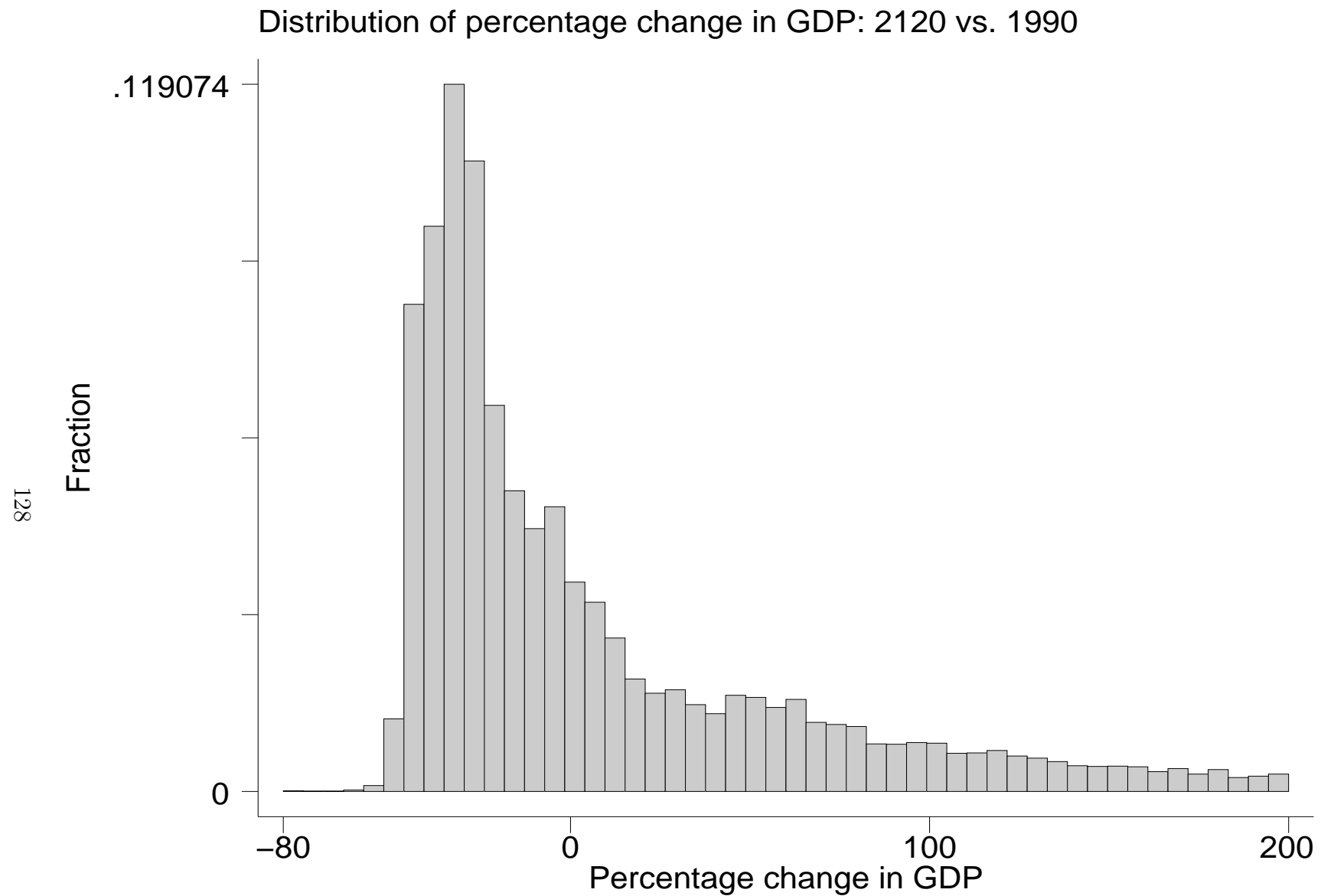


Figure 19m: Distribution of Percentage Changes in Regional GDP (relative to trend)
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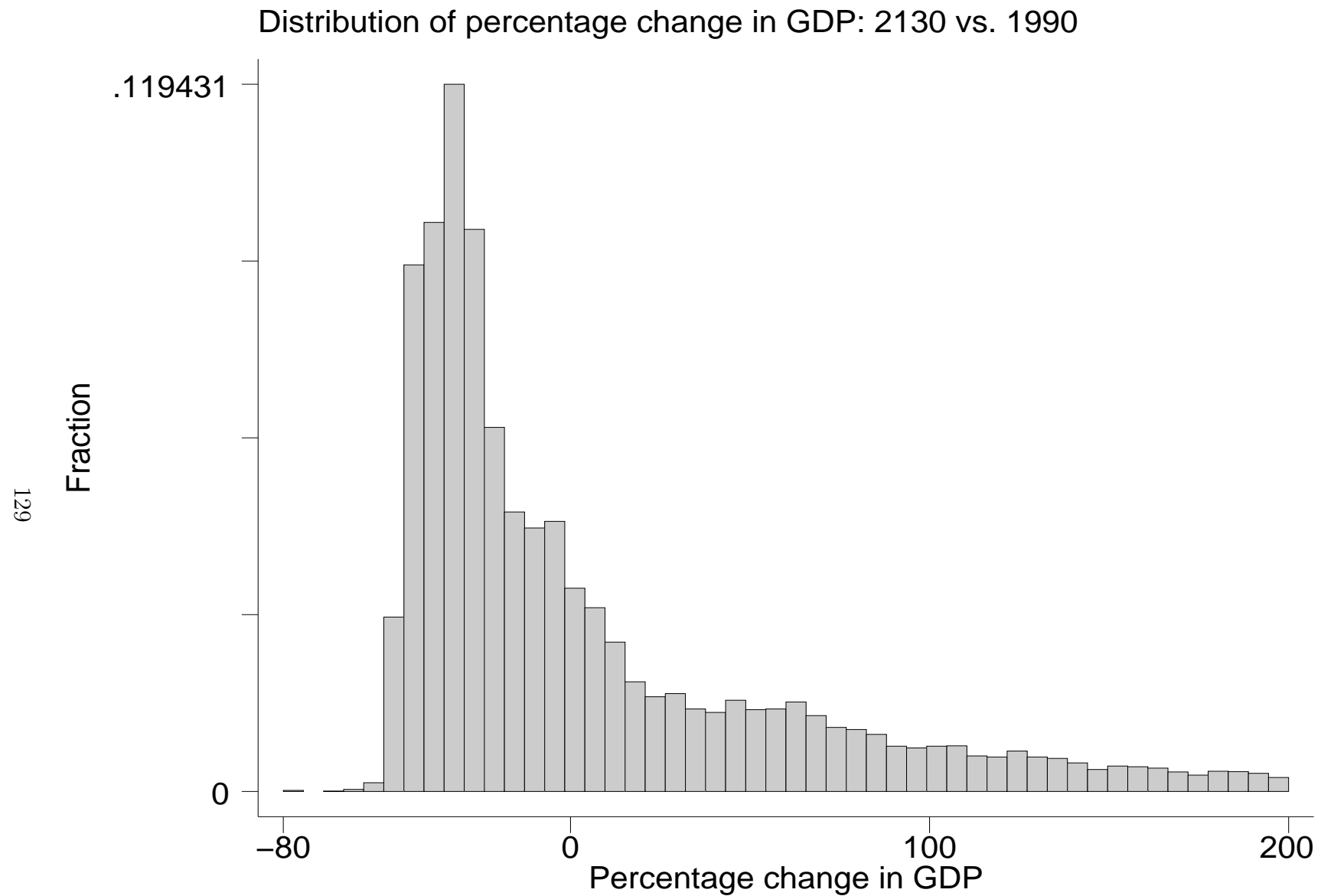


Figure 19n: Distribution of Percentage Changes in Regional GDP (relative to trend)
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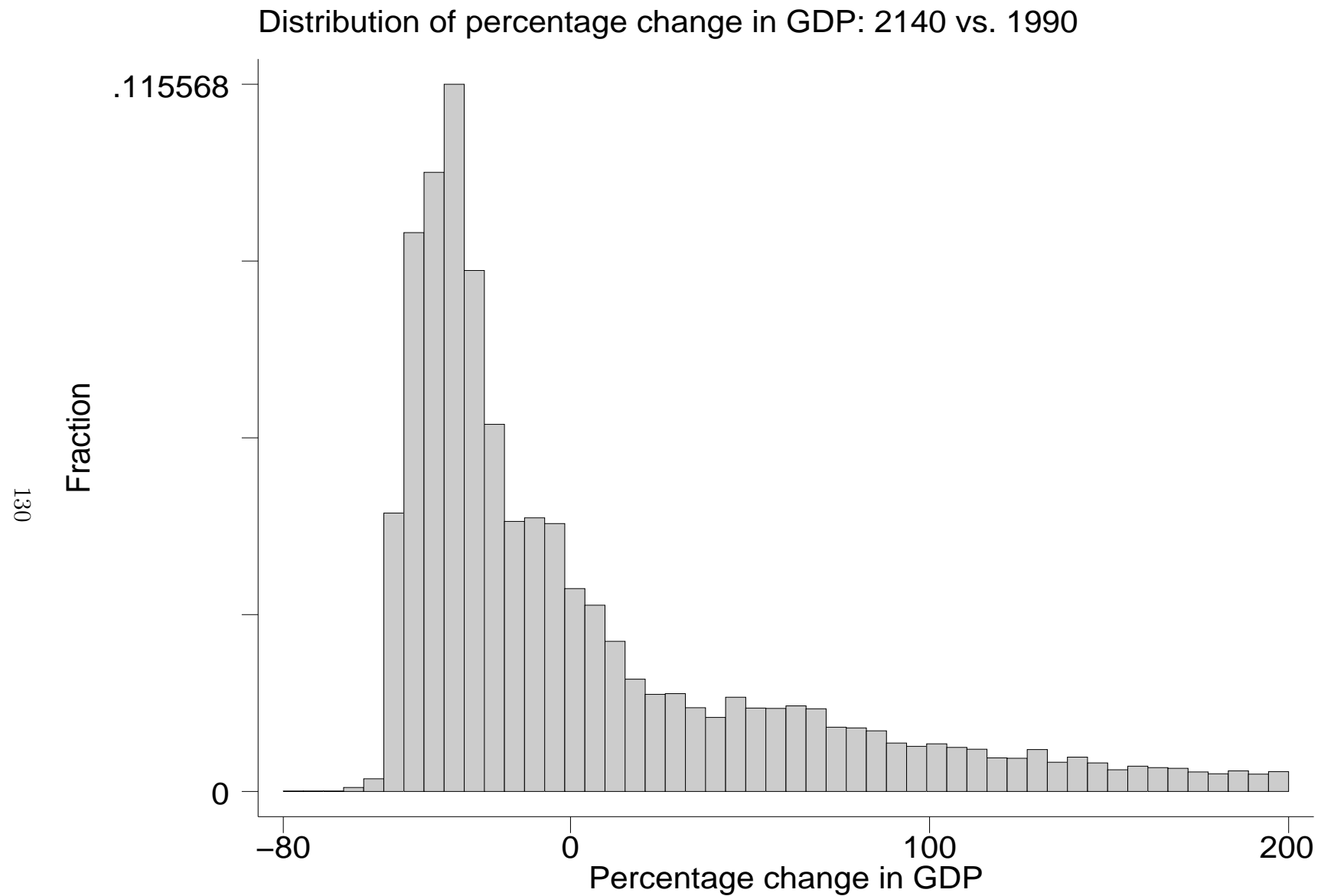


Figure 19o: Distribution of Percentage Changes in Regional GDP (relative to trend)
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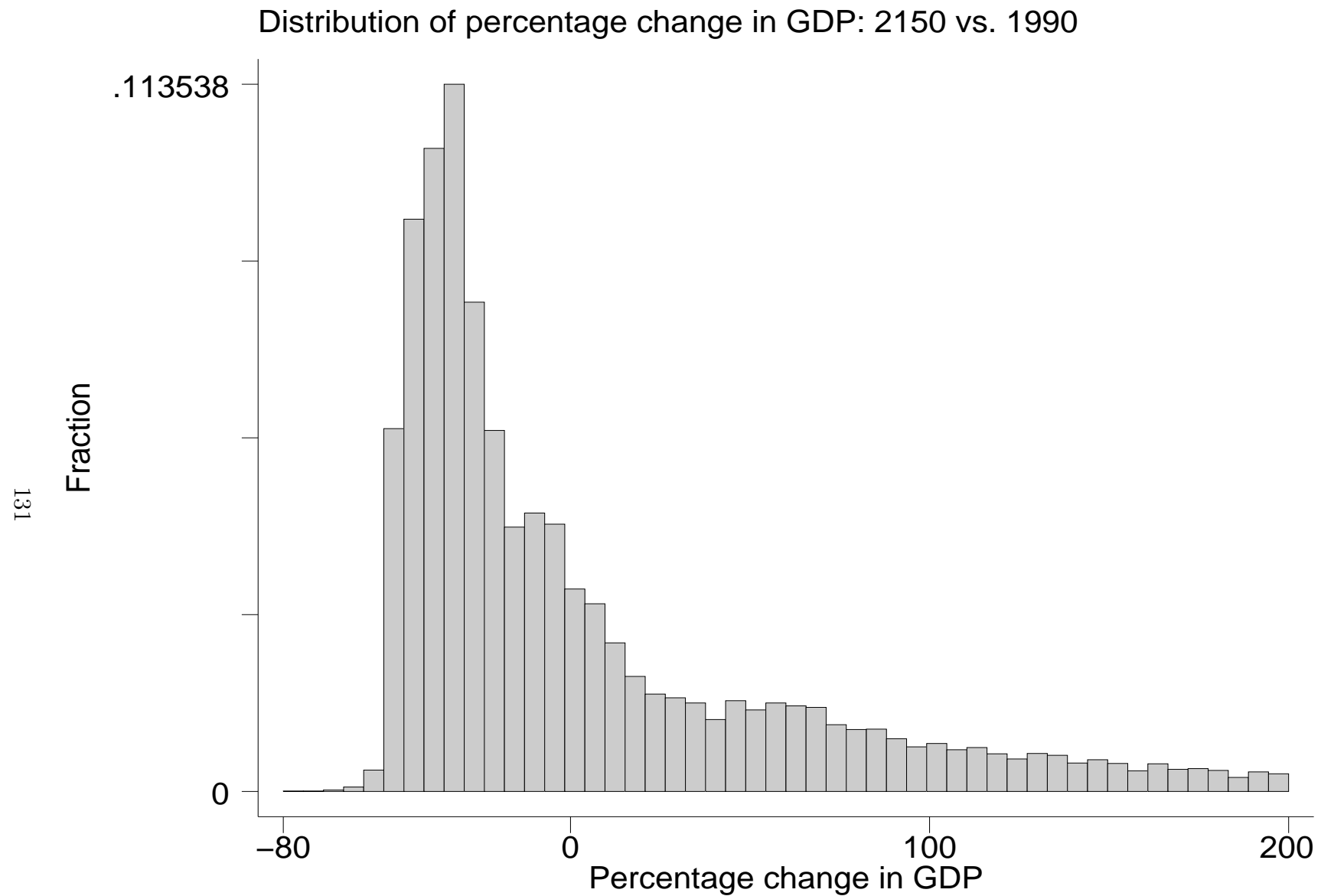


Figure 19p: Distribution of Percentage Changes in Regional GDP (relative to trend)
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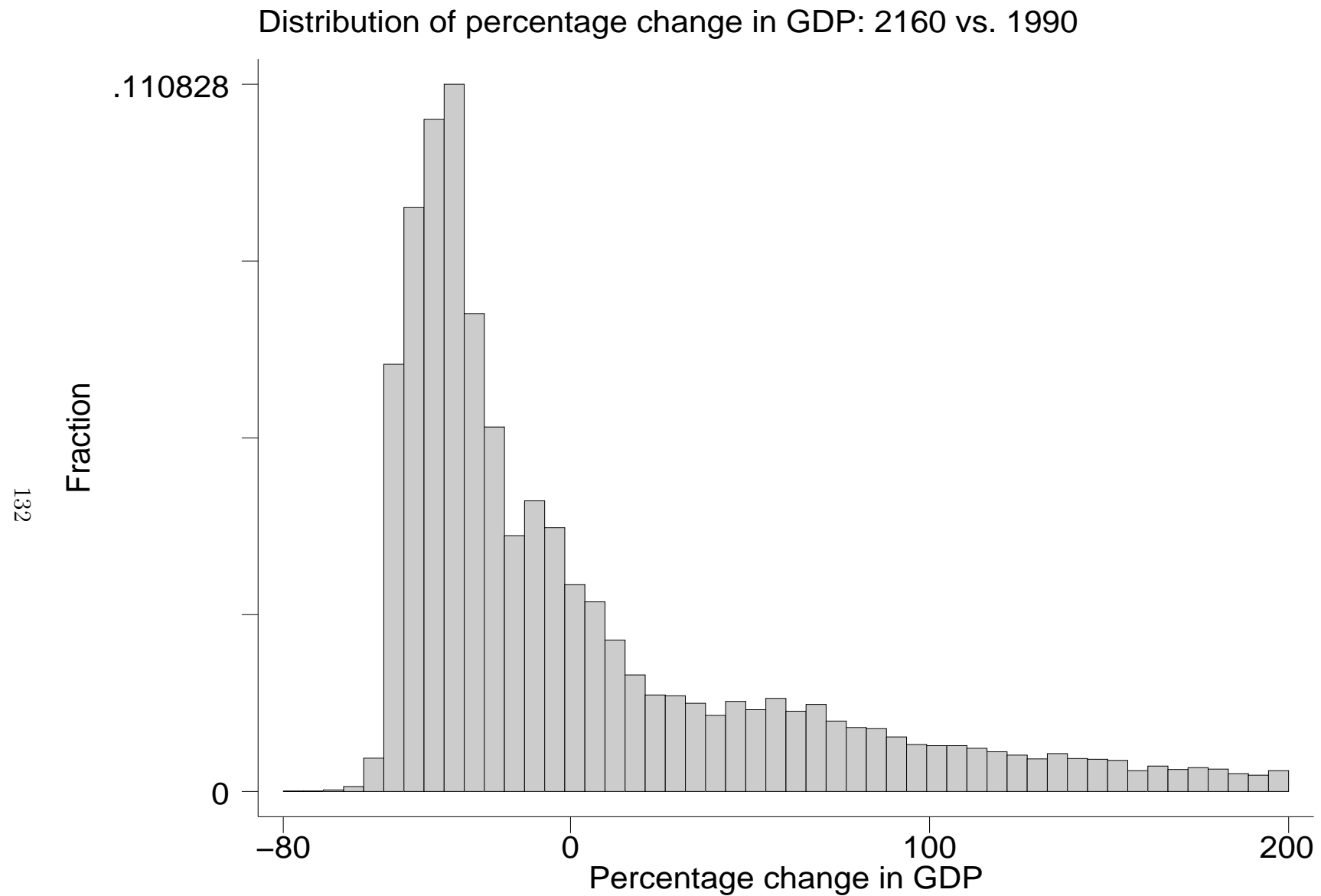


Figure 19q: Distribution of Percentage Changes in Regional GDP (relative to trend)
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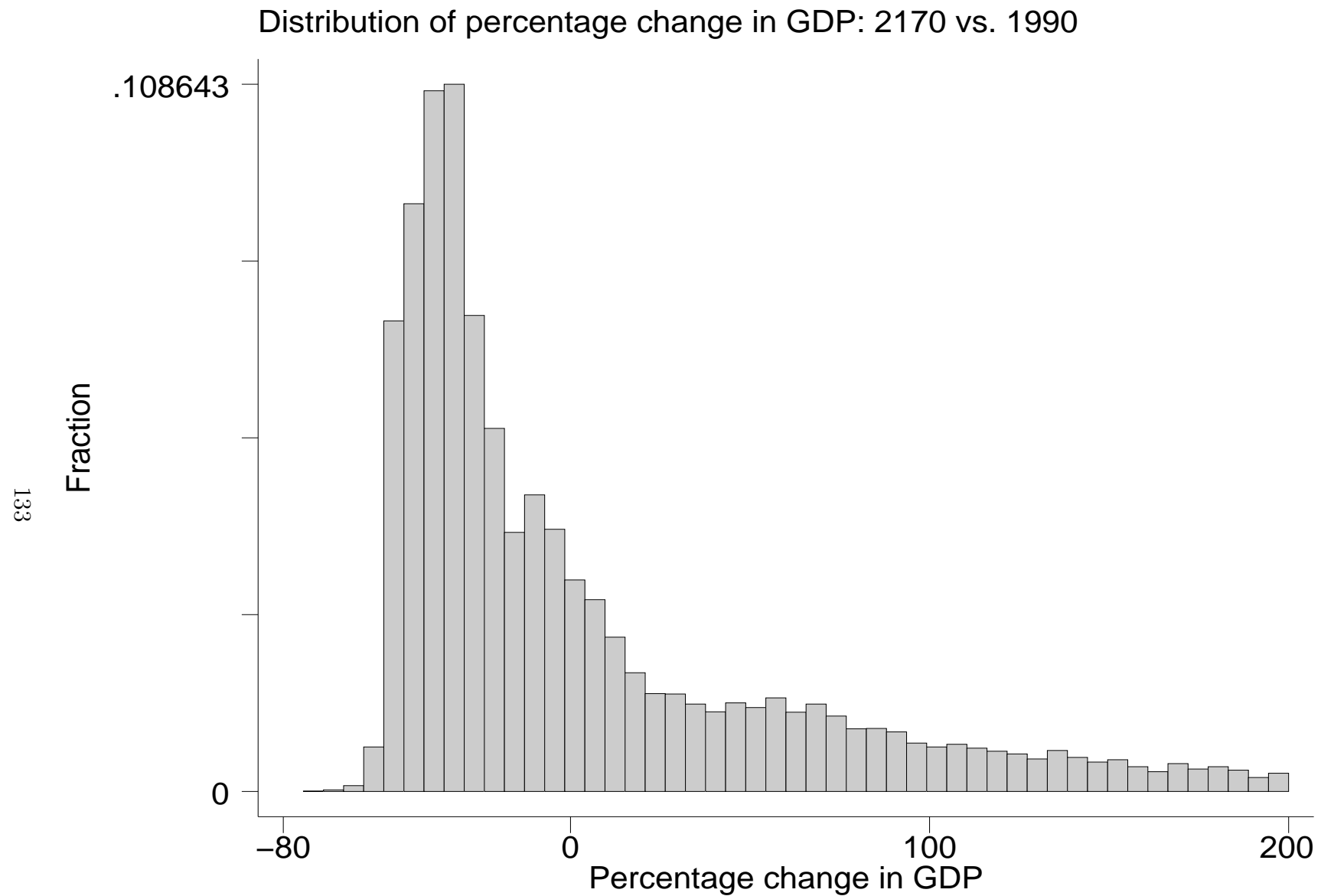


Figure 19r: Distribution of Percentage Changes in Regional GDP (relative to trend)
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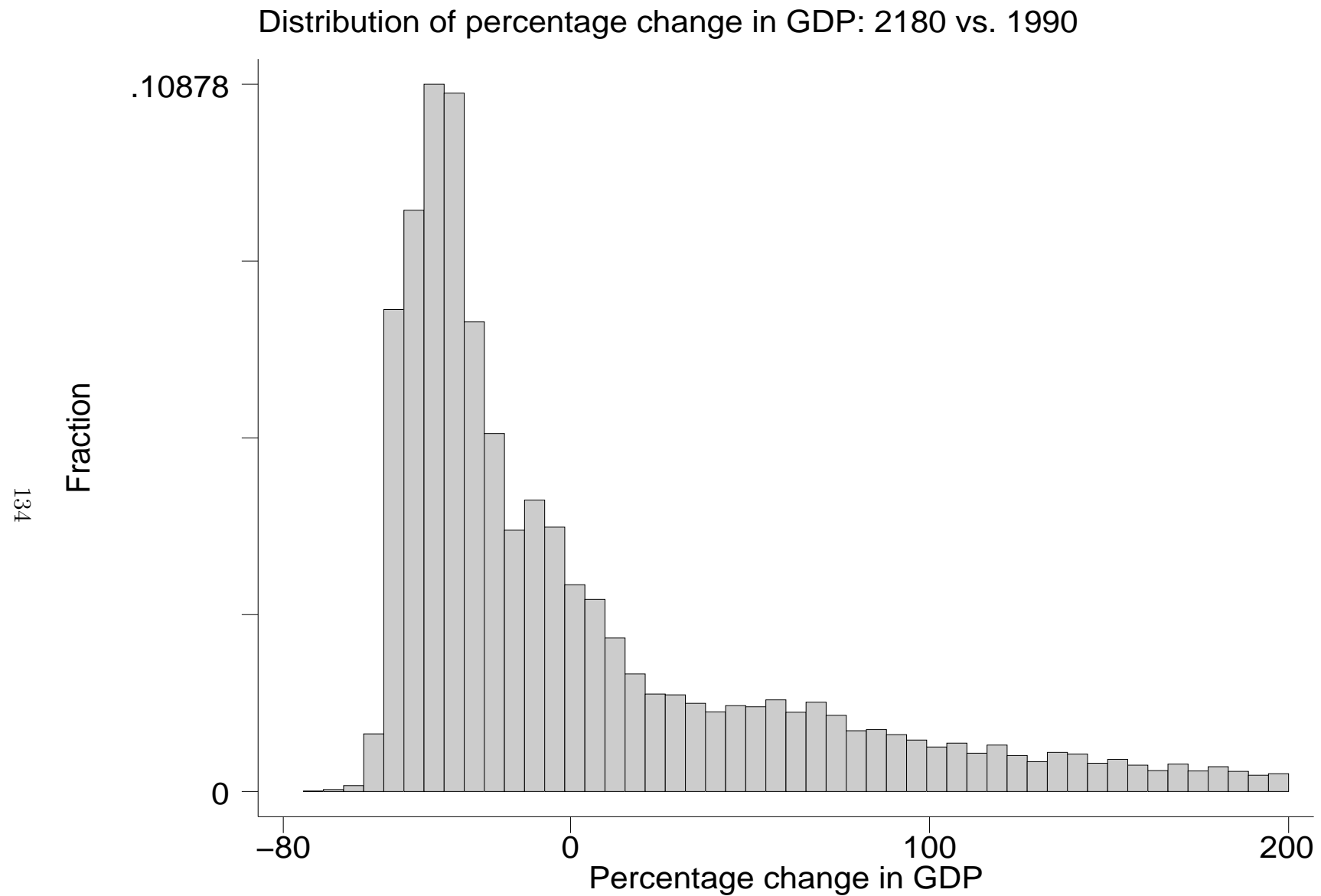


Figure 19s: Distribution of Percentage Changes in Regional GDP (relative to trend)
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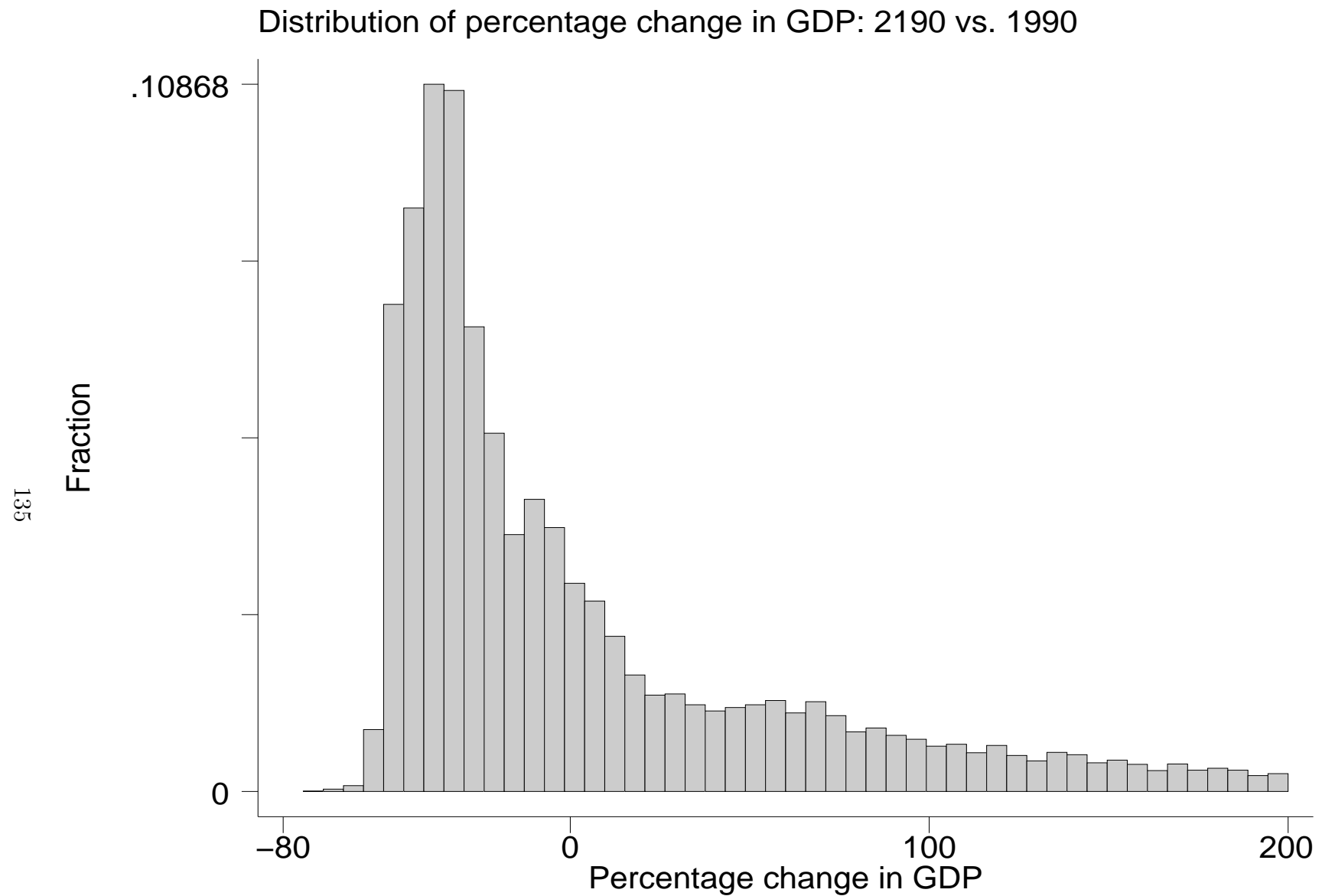


Figure 19t: Distribution of Percentage Changes in Regional GDP (relative to trend)
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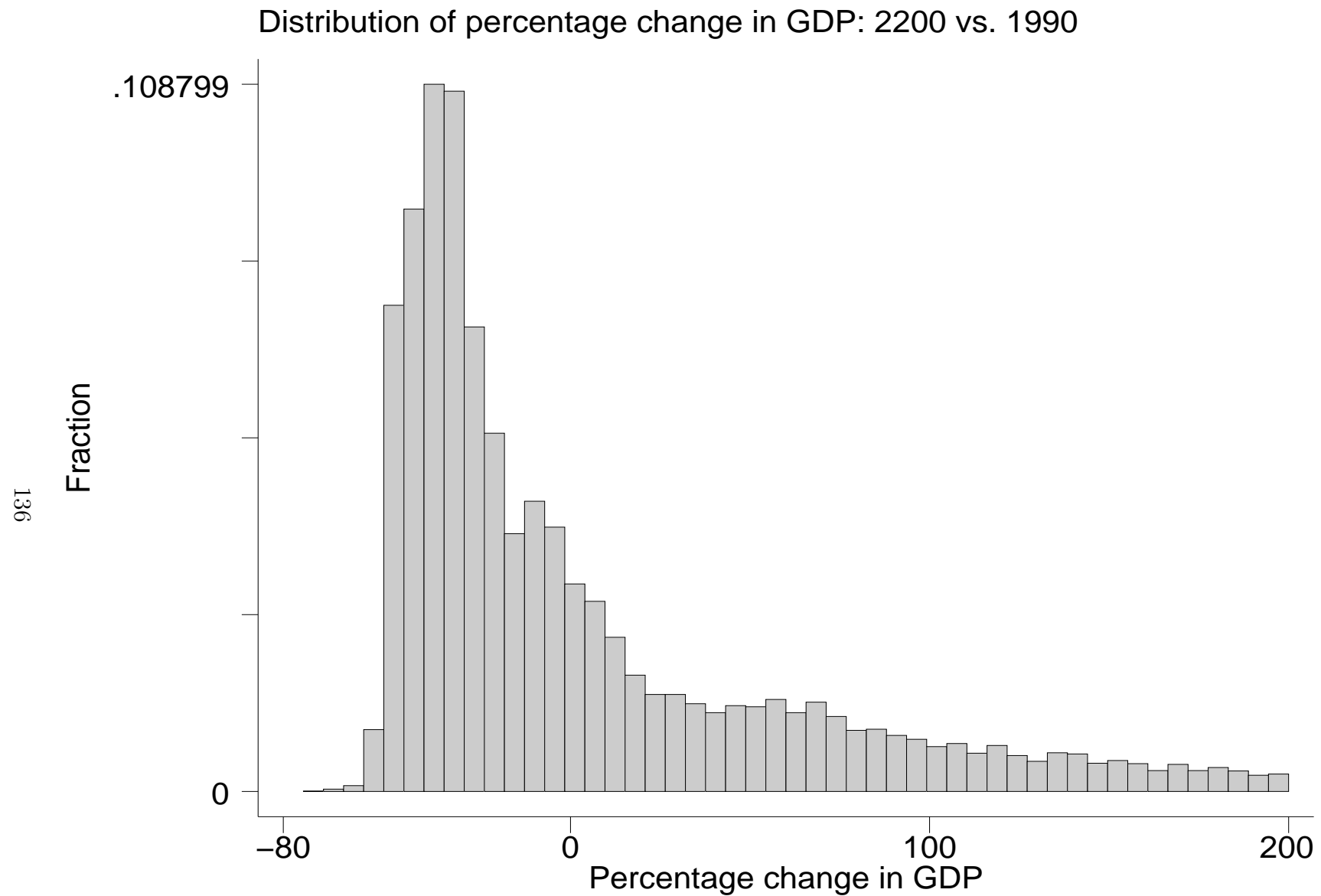


Figure 19u: Distribution of Percentage Changes in Regional GDP (relative to trend)
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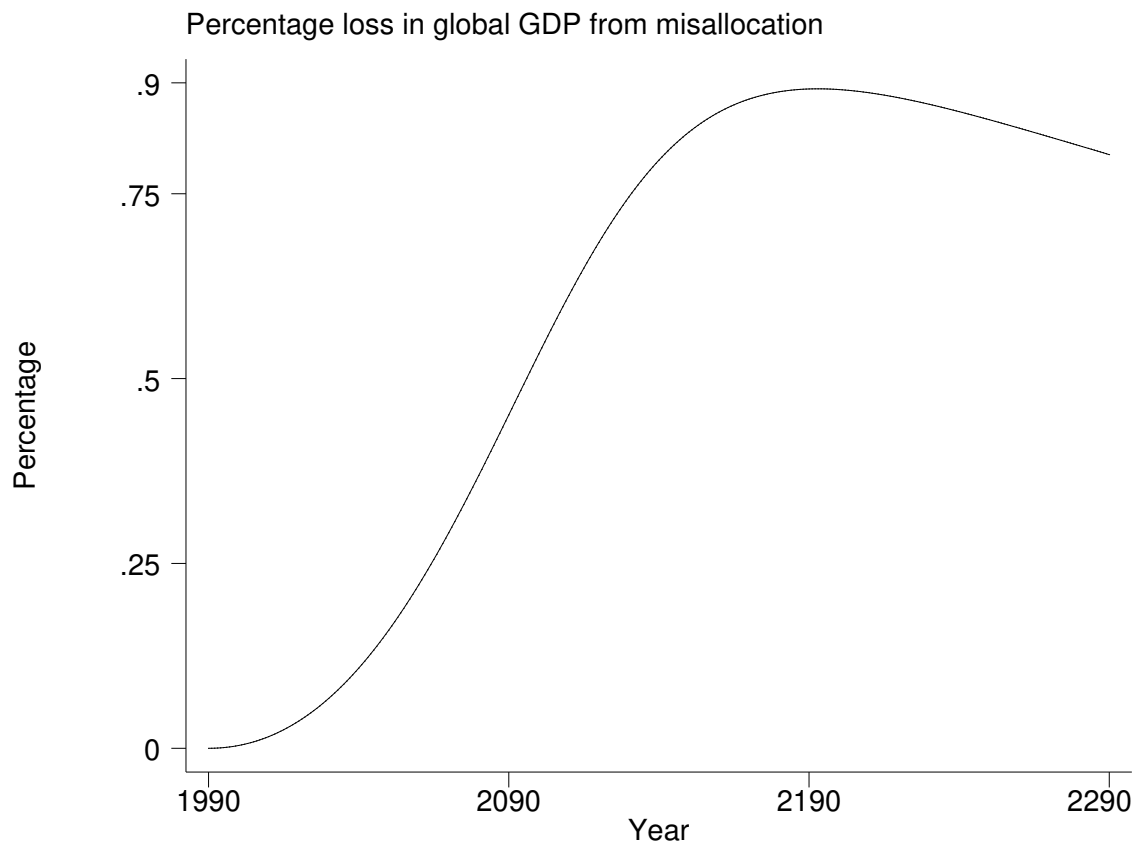


Figure 20: Percentage Loss in Global GDP from Misallocation
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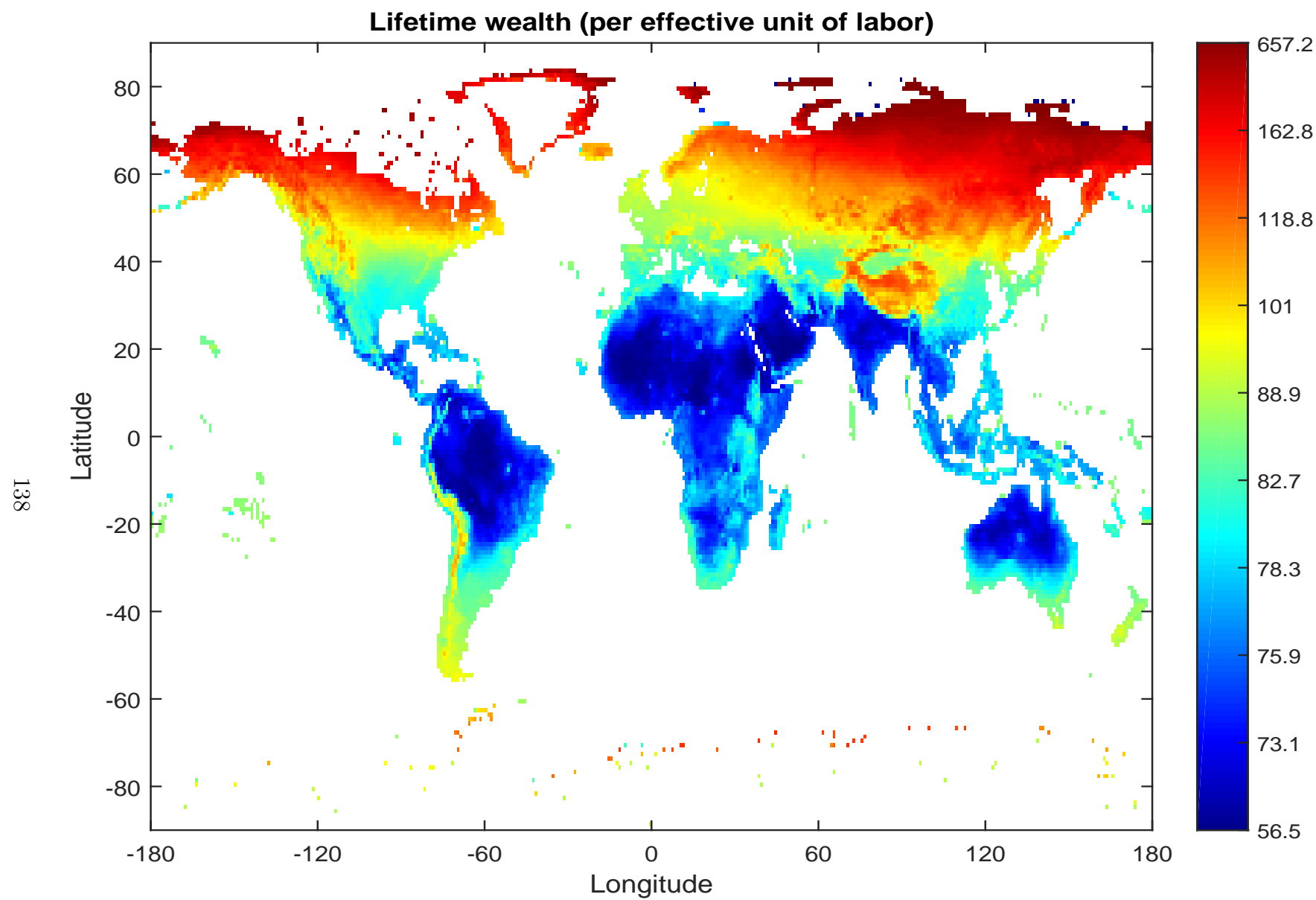


Figure 21: Log of Lifetime Wealth by Region (after removing permanent productivity)
([return to text](#))

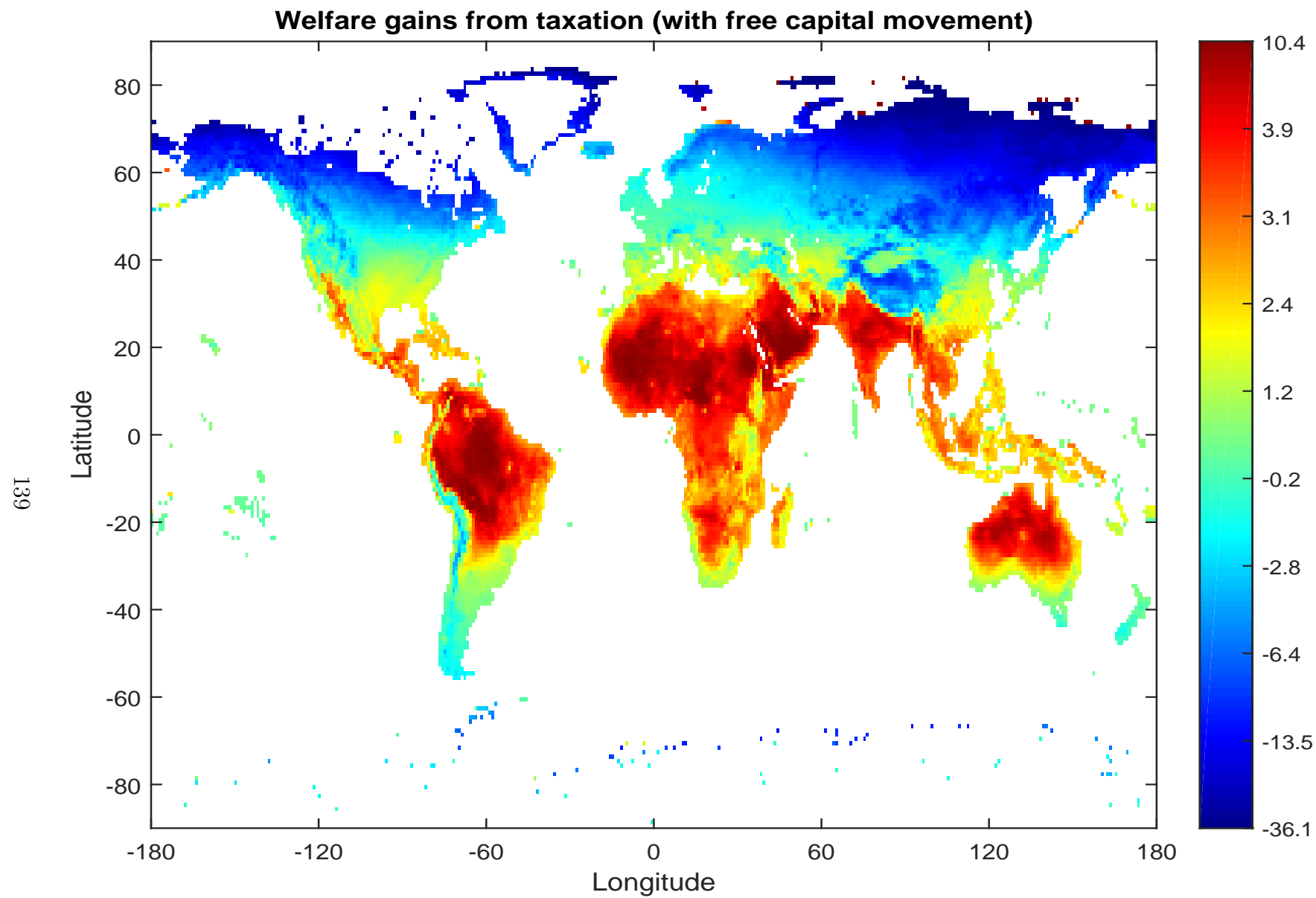


Figure 22: Consumption-Equivalent (%) Welfare Gains from Carbon Tax (with free capital mobility)
([return to text](#))

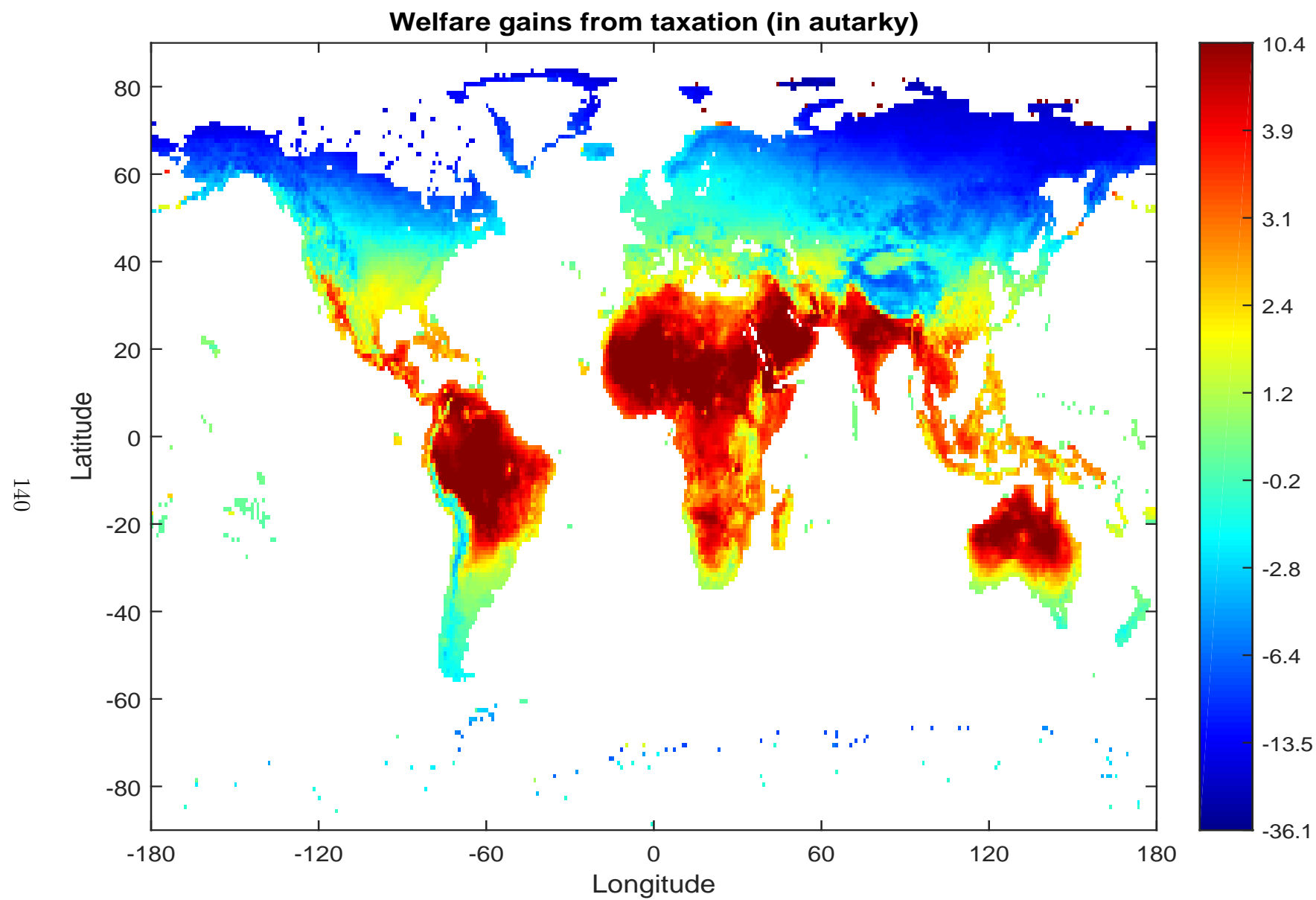


Figure 23: Consumption-Equivalent (%) Welfare Gains from Carbon Tax (in autarky)
([return to text](#))

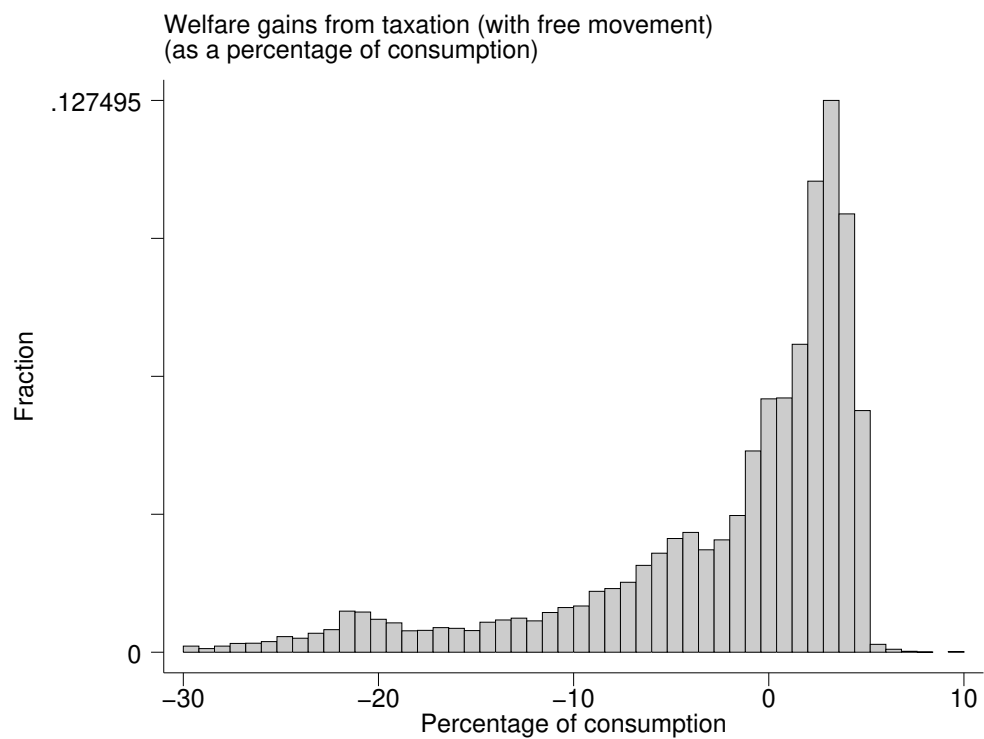


Figure 24: Distribution of Welfare Gains from Carbon Tax (with free capital mobility)
([return to text](#))

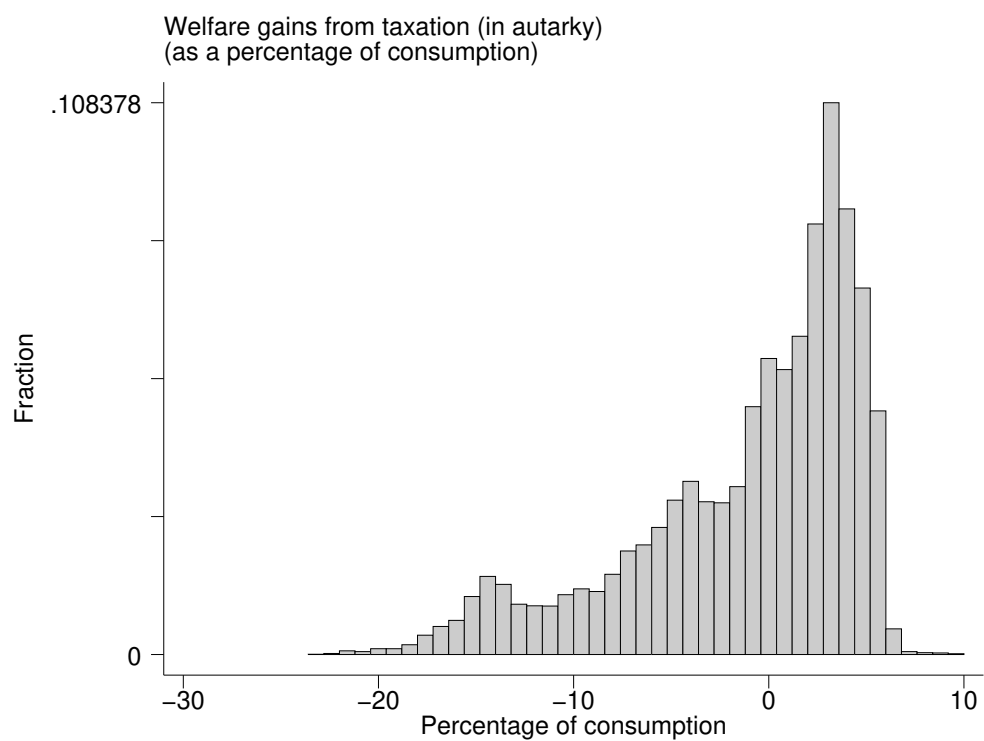


Figure 25: Distribution of Welfare Gains from Carbon Tax (in autarky)
([return to text](#))

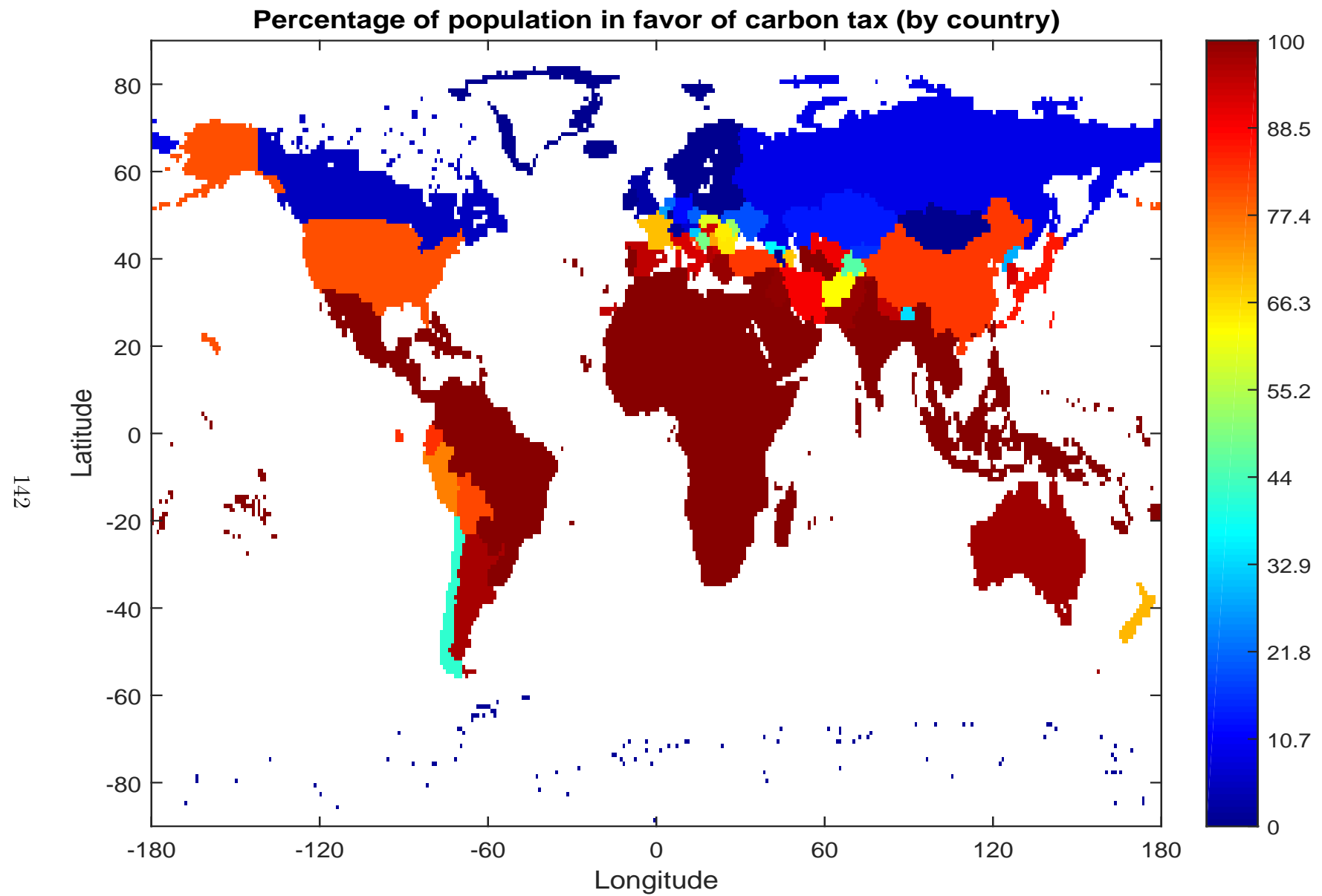


Figure 26: Percentage Voting in Favor of Carbon Tax by Country
([return to text](#))

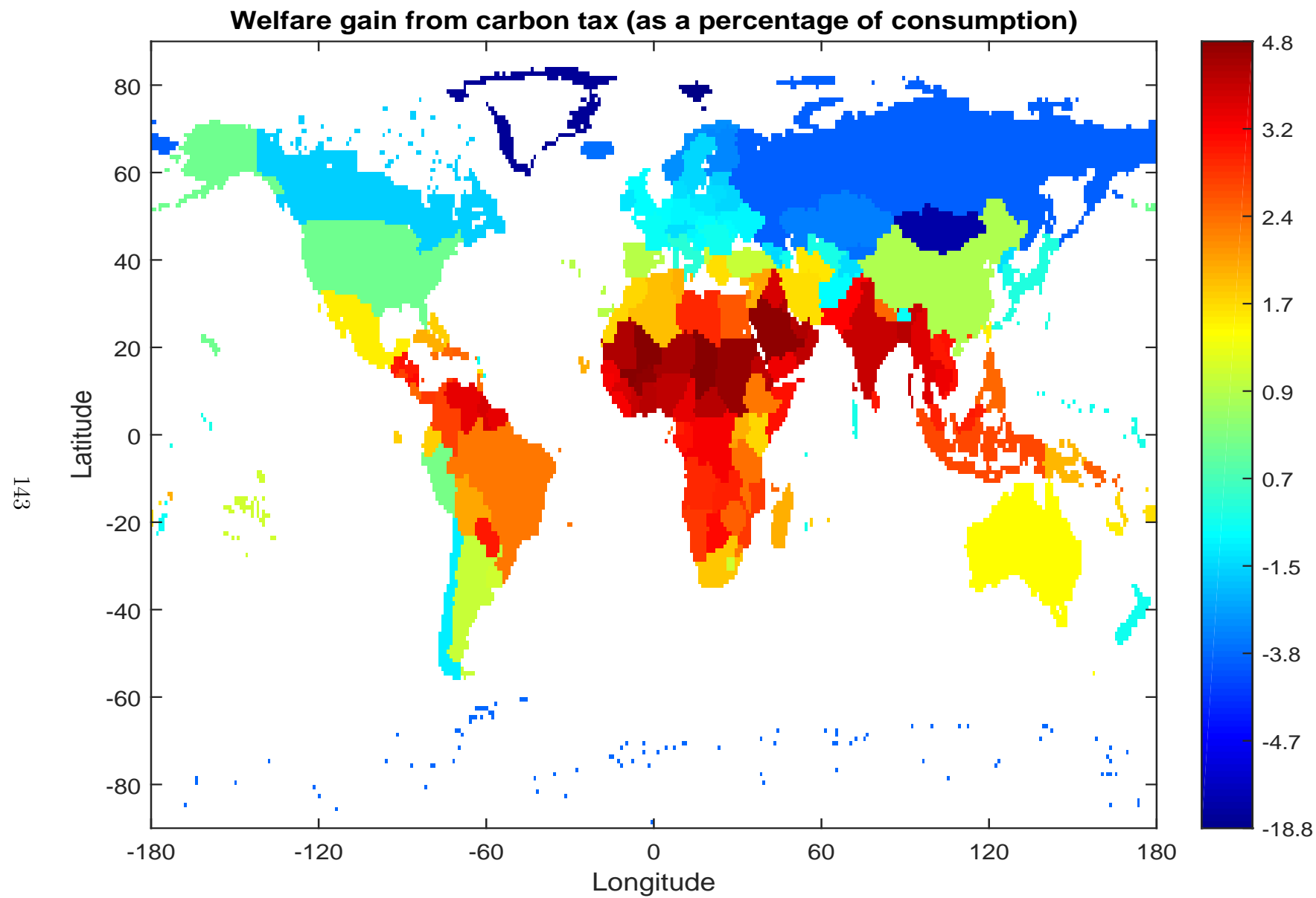


Figure 27: Consumption-Equivalent (%) Welfare Gain from Carbon Tax by Country
([return to text](#))

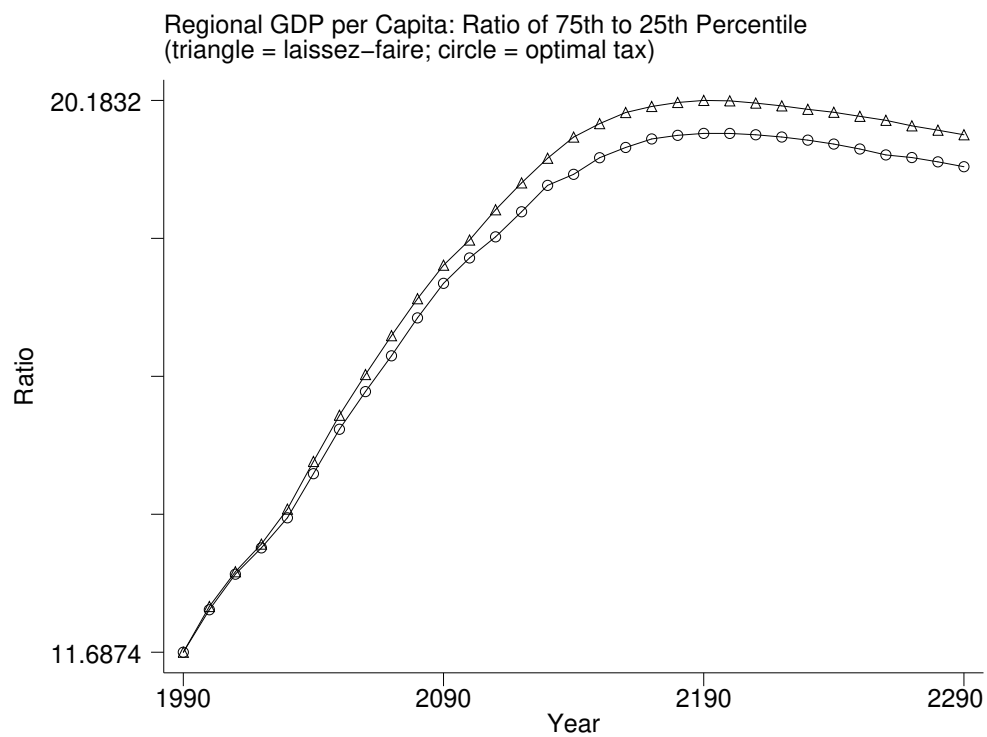


Figure 28: Inequality in GDP per Capita by Region: 75–25 ratio
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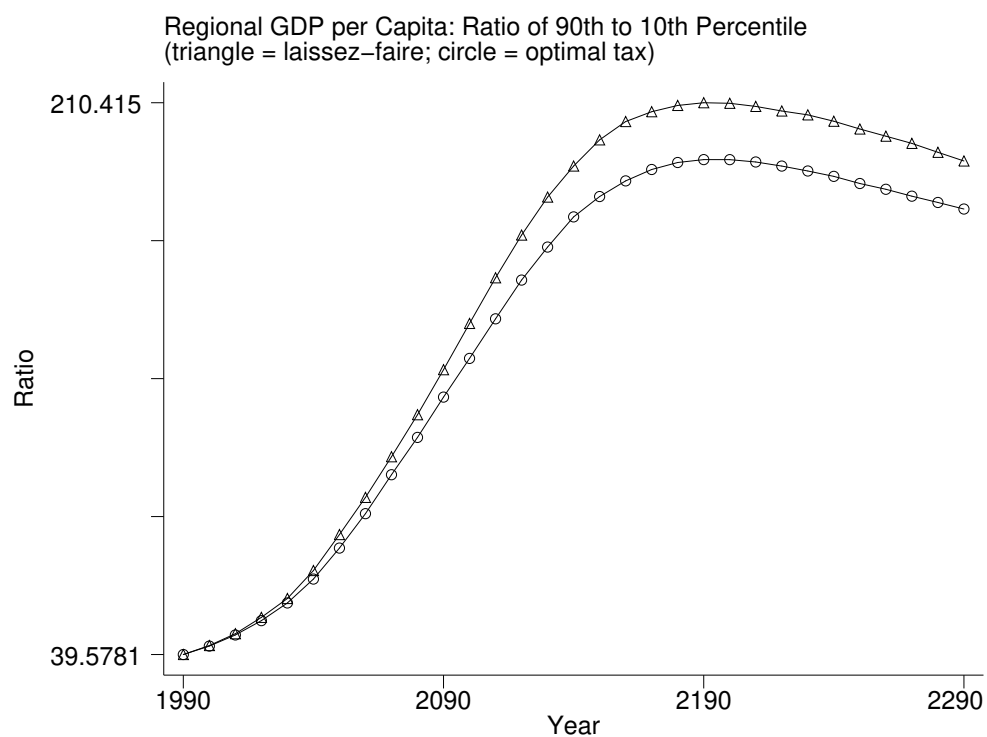


Figure 29: Inequality in GDP per Capita by Region: 90–10 ratio
([return to text](#))

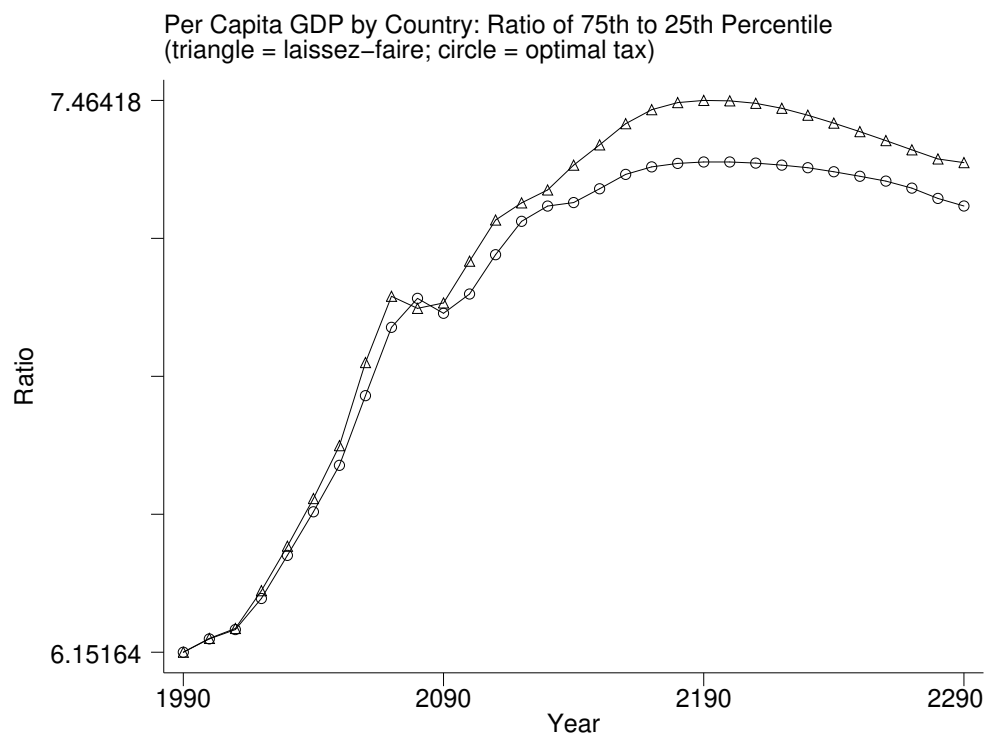


Figure 30: Inequality in GDP per Capita by Country: 75–25 Ratio
([return to text](#))

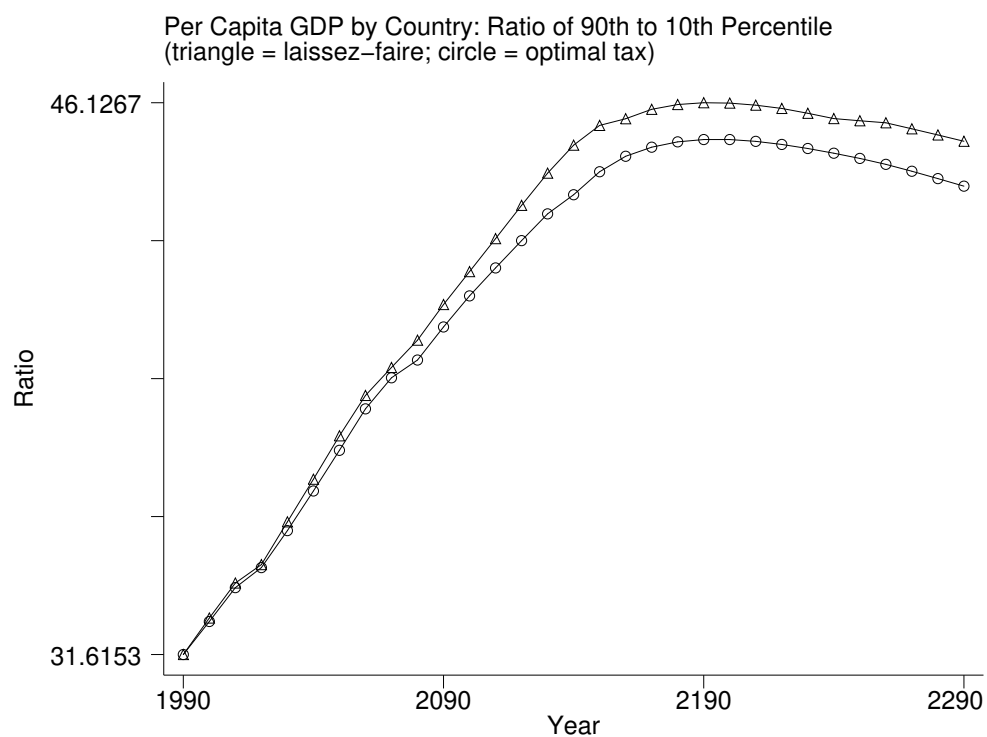


Figure 31: Inequality in GDP per Capita by Country: 90–10 Ratio
([return to text](#))

Technical Appendix A

This appendix demonstrates that the two definitions of equilibrium in Sections 2.8 and 2.9 are equivalent. The first step is to show that the consumer's budget constraint (1), together with the first-order conditions to the firms' problems described in Section 2.5, are equivalent to the budget constraint in the entrepreneur's dynamic programming problem (25), as summarized in the first, fifth, and sixth constraints to that problem. (The second, third, and fourth constraints describe the evolution of the entrepreneur's effective units of labor and are identical to those in the consumer's problem described in Section 2.3.) The second step is to show that the market for regional final goods clears in the recursive equilibrium in Section 2.9 when the regional government balances its budget.

Dropping subscripts, the first-order conditions for a typical firm in the final-goods sector are:

$$\begin{aligned} F_k(k^c, \ell^c, e^c) &= r \\ F_\ell(k^c, \ell^c, e^c) &= w \\ F_e(k^c, \ell^c, e^c) &= p + \tau\chi\psi \end{aligned}$$

Similarly, the first-order conditions for a typical firm in the energy sector are:

$$\begin{aligned} p\zeta^{-1}F_k(k^e, \ell^e, e^e) &= r \\ p\zeta^{-1}F_\ell(k^e, \ell^e, e^e) &= w \\ p\zeta^{-1}F_e(k^e, \ell^e, e^e) &= p + \tau\chi\psi \end{aligned}$$

Taking ratios of first-order conditions and using the fact that the partial derivatives of F are homogeneous of degree zero (because F is homogeneous of degree one) yields:

$$\begin{aligned} \frac{F_k(1, \ell^c/k^c, e^c/k^c)}{F_\ell(1, \ell^c/k^c, e^c/k^c)} &= \frac{F_k(1, \ell^e/k^e, e^e/k^e)}{F_\ell(1, \ell^e/k^e, e^e/k^e)} \\ \frac{F_k(1, \ell^c/k^c, e^c/k^c)}{F_e(1, \ell^c/k^c, e^c/k^c)} &= \frac{F_k(1, \ell^e/k^e, e^e/k^e)}{F_e(1, \ell^e/k^e, e^e/k^e)}, \end{aligned}$$

implying that $\ell^c/k^c = \ell^e/k^e$ and $e^c/k^c = e^e/k^e$. From these equalities it follows that $\ell/k = \ell^c/k^c = \ell^e/k^e$ and that $e/k = e^c/k^c = e^e/k^e$, where $k \equiv k^c + k^e$ is aggregate regional capital and $e \equiv e^c + e^e$ is aggregate regional energy use. These equalities also imply that $F_i(k^c, \ell^c, e^c) = F_i(k^e, \ell^e, e^e)$ for $i = k, \ell, e$; as a result, $p\zeta^{-1} = 1$, i.e., the price of one Gtoe (in units of the numéraire consumption good) is equal to the ratio of the productivity of the final-goods sector to that of the energy sector. Moreover, for $i = k, \ell, e$ and $j = k, c$, $F_i(k^j, \ell^j, e^j) = F_i(1, \ell^j/k^j, e^j/k^j) = F_i(1, \ell/k, e/k) = F_i(k, \ell, e)$. Aggregate regional energy use can then be pinned down in terms of

the regional aggregate factors of production by the condition:

$$F_e(k, \ell, e) = \zeta + \tau\chi\psi, \quad (13)$$

Using the firm's first-order conditions and first-degree of homogeneity of F ,

$$\begin{aligned} rk + w\ell &= F_k(k, \ell, e)k + F_\ell(k, \ell, e)\ell \\ &= F(k, \ell, e) - F_e(k, \ell, e)e \\ &= F(k, \ell, e) - \zeta e - \tau\chi\psi e \\ &= G(k, \ell, e) - \tau\chi\psi e, \end{aligned}$$

where $G(k, \ell, e) \equiv F(k, \ell, e) - \zeta e$. The consumer's budget constraint (1) can then be rewritten as:

$$c + k + qb' = G(k, \ell, e) - \tau\chi\psi e + (1 - \delta)k + b + R,$$

where e satisfies equation (13), which is precisely the first-order condition to the energy-choice problem (7) faced by the entrepreneur that defines the function g_e determining the entrepreneur's optimal choice of energy. The consumer's budget constraint is therefore equivalent to the first, fifth, and sixth constraints in the entrepreneur's dynamic program.

Finally, regional output of final goods, $F(k^c, \ell^c, e^c)$, is equal to $k_c F(1, \ell/k, e/k)$. Similarly, $\zeta e = \zeta F(k^e, \ell^e, e^c) = \zeta F(1, \ell/k, e/k)$. Summing,

$$F(k^c, \ell^c, e^c) + \zeta e = (k^c + k^e)F(1, \ell/k, e/k) = F(k, \ell, e),$$

so that $G(k, \ell, e)$ as defined above is equal to regional output of final goods. This fact, together with the entrepreneur's budget constraint and the condition that the regional government balance its budget in every period, ensures that the regional market for final goods clears, i.e., output of final goods equals regional consumption plus regional investment less the region's net saving in the global bond market.

Technical Appendix B

This appendix shows that the equilibrium defined in Section 2.9 aggregates under the restriction on the production function described in Section 2.11. In particular, it shows that the behavior of the aggregate variables in the equilibrium defined in Section 2.9 satisfies the equilibrium conditions described in Section 2.11.

The key part of the argument is to show that the solution to dynamic programming problem (11) of the stand-in global consumer generates time paths for global aggregates (given time paths for the global temperature and regional transfers) that correspond to the aggregation of the time paths for regional variables generated by the solution to the regional entrepreneurs' problem described in (25).

In sequential form, the first-order conditions (with respect to choices for savings, either in regional capital and or in the global bond market) to a typical entrepreneur's problem in region i can be written:

$$\begin{aligned} U'(c_{it}) &= \beta U'(c_{i,t+1}) (\alpha \Pi_{i,t+1} (k_{i,t+1}/\ell_{i,t+1})^{\alpha-1} + 1 - \delta) \\ q_t U'(c_{it}) &= \beta U'(c_{i,t+1}), \end{aligned}$$

where $\Pi_{i,t+1} \equiv \Phi(x_{i,t+1}^*) - \tau_{it} \chi_t \psi x_{i,t+1}^*$. As noted in Section 2.11, entrepreneurs in each region therefore set the marginal net return to investing in a unit of capital in period $t+1$ equal to the common net return to investing instead in the risk-free bond, i.e.,

$$\alpha \Pi_{i,t+1} (k_{i,t+1}/\ell_{i,t+1})^{\alpha-1} - \delta = q_t^{-1} - 1, \quad (14)$$

This equalization of returns across regions implies that $k_{i,t+1}$ can be expressed as a fraction, $\theta_{i,t+1}/\theta_{t+1}$, of the global capital stock, K_{t+1} , where $\theta_{i,t+1} \equiv \ell_{it} \Pi_{i,t+1}^{\frac{1}{1-\alpha}}$ and $\theta_{t+1} \equiv \sum_{i=1}^M N_i \theta_{i,t+1}$.

Because the felicity function has a constant elasticity of intertemporal substitution, ν^{-1} , the second of the two first-order conditions can be written:

$$c_{it} = (\beta^{-1} q_t)^{\frac{1}{\nu}} c_{i,t+1}.$$

Multiplying both sides by regional population, N_i , and then summing over all regions i yields:

$$C_t = (\beta^{-1} q_t)^{\frac{1}{\nu}} C_{t+1}, \quad (15)$$

where global consumption $C_t \equiv \sum_{i=1}^M N_i c_{it}$ for all t .

Multiplying both sides of equation (14) by N_i , summing over all regions, and then substituting $(\theta_{i,t+1}/\theta_{t+1}) K_{t+1}$ for $k_{i,t+1}$ implies that $\alpha \theta_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - \delta = q_t^{-1} - 1$. Substituting this expression for q_t into equation (15) and rearranging yields

$$C_t^{-\nu} = \beta C_{t+1}^{-\nu} (\alpha \theta_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} + 1 - \delta),$$

which is precisely the first-order condition to the dynamic program (11) of the stand-in global consumer.

It remains to show that the aggregate resource constraint faced by the stand-in global consumer coincides with the aggregation of the regional budget constraints faced by entrepreneurs. Multiplying both sides of the first constraint to the entrepreneur's problem (25) by N_i and summing over regions (and now using subscripts explicitly to denote regions and time) yields:

$$\sum_{i=1}^M N_i \omega_{it} = \sum_{i=1}^M N_i c_{it} + \sum_{i=1}^M N_i k_{i,t+1} + q_t \sum_{i=1}^M N_i b_{i,t+1}.$$

Imposing the equilibrium condition that the global bond market clear (at zero) in every period, and noting that, by definition, global wealth in period t , Ω_t , equals $\sum_{i=1}^M N_i \omega_{it}$, this equation is then identical to the first constraint in the stand-in global consumer's problem. Similarly, multiplying both sides of the last constraint in the entrepreneur's problem by N_i and summing across regions yields:

$$\begin{aligned} \sum_{i=1}^M N_i \omega_{i,t+1} &= \sum_{i=1}^M N_i \Phi(x_{i,t+1}^*) k_{i,t+1}^\alpha \ell_{i,t+1}^{1-\alpha} - \sum_{i=1}^M N_i \tau_{i,t+1} \chi_{t+1} \psi e_{i,t+1}^* + (1-\delta) \sum_{i=1}^M N_i k_{i,t+1} + \\ &\quad \sum_{i=1}^M N_i b_{i,t+1} + \sum_{i=1}^M N_i R_{i,t+1} \\ &= \sum_{i=1}^M N_i \left(\Phi(x_{i,t+1}^*) - \tau_{i,t+1} \chi_{t+1} \psi e_{i,t+1}^* \right) k_{i,t+1}^\alpha \ell_{i,t+1}^{1-\alpha} + (1-\delta) \sum_{i=1}^M N_i k_{i,t+1} + \\ &\quad \sum_{i=1}^M N_i b_{i,t+1} + \sum_{i=1}^M N_i R_{i,t+1}, \end{aligned}$$

using equation (9) to substitute for the optimal energy choice, $e_{i,t+1}^*$. Substituting $(\theta_{i,t+1}/\theta_{t+1})K_{t+1}$ for $k_{i,t+1}$ and imposing again the market-clearing condition in the global bond market, it is straightforward to show that this equation then reduces to the second constraint in the global stand-in consumer's problem.

The time-varying sequence of decision rules to the global stand-in consumer's problem therefore generates the same aggregate dynamics, given time paths for global temperature and the set of regional transfers, as does the aggregation of the time paths implied by the solution to the problem of the regional entrepreneurs. It follows that the equilibrium time path for global temperature, i.e., the one that is in turn generated by the global stand-in consumer's decisions, coincides with the one generated by the regional entrepreneurs' decisions, provided that transfers in each region are set equal to tax revenues in region. This latter condition is itself imposed in the definition of equilibrium with a stand-in consumer outlined in Section 2.11.

Technical Appendix C

This appendix demonstrates the equivalence between the socially optimal allocation described in Section 2.12 and the competitive-equilibrium (aggregate) allocation defined in Section 2.11 under optimal taxes on carbon emissions. It also (partially) characterizes the optimal path for emissions taxes.

Substitute equation (12), the resource constraint, into the planner's objective and let η_{it} , $i = 1, 2$, be the Lagrangian multipliers on the two laws of motion for the stocks of atmospheric carbon, i.e., equations (5) and (6). A typical first-order condition with respect to aggregate (or global) capital, K_t , is then:

$$-\beta^t U'(C_t) + \beta^{t+1} U'(C_{t+1}) (\Phi(x_{t+1}) \Lambda(S_{t+1}) \alpha K_{t+1}^{\alpha-1} + 1 - \delta) = (\eta_{1t} \phi_1 + \eta_{2t} (1 - \phi_1) \phi_2) \chi_t \psi \alpha x_t \Lambda(S_t) K_t^{\alpha-1}. \quad (16)$$

A typical first-order condition with respect to the permanent stock of atmospheric carbon, S_{1t} , is:

$$\beta^t U'(C_t) \Phi(x_t) \Lambda'(S_t) K_t^\alpha + \eta_{1t} x_t (1 - \phi_1 \chi_t \psi) - \eta_{2t} (1 - \phi_1) \phi_2 \chi_t \psi \Lambda'(S_t) K_t^\alpha = \eta_{1,t+1}.$$

Similarly, a typical first-order condition with respect to the slowly-decaying stock of atmospheric carbon, S_{2t} , is:

$$\beta^t U'(C_t) \Phi(x_t) \Lambda'(S_t) K_t^\alpha - \eta_{1t} x_t \phi_1 \chi_t \psi + \eta_{2t} (1 - (1 - \phi_1) \phi_2 \chi_t \psi) \Lambda'(S_t) K_t^\alpha = \phi_3 \eta_{2,t+1}.$$

Finally, a typical first-order condition with respect to energy intensity, x_t , is:

$$\beta^t U'(C_t) \Phi'(x_t) \Lambda(S_t) K_t^\alpha - (\eta_{1t} \phi_1 + \eta_{2t} (1 - \phi_1) \phi_2) \chi_t \psi \Lambda(S_t) K_t^\alpha = 0,$$

so that:

$$\beta^t U'(C_t) \Phi'(x_t) = (\eta_{1t} \phi_1 + \eta_{2t} (1 - \phi_1) \phi_2) \chi_t \psi.$$

Let $\tilde{\eta}_{it}$, $i = 1, 2$, be the period- t Lagrangian multipliers scaled by the marginal utility of consumption in period t :

$$\tilde{\eta}_{it} = \frac{\eta_{it}}{\beta^t U'(C_t)}. \quad (17)$$

The first-order condition for energy intensity then reads:

$$\Phi'(x_t^*) = (\tilde{\eta}_{1t} \phi_1 + \tilde{\eta}_{2t} (1 - \phi_1) \phi_2) \chi_t \psi, \quad (18)$$

where x_t^* is the planner's optimal choice for energy intensity. Suppose that the tax on emissions

in period t in each region is set equal to a common value, τ_t , defined as follows:

$$\tau_t \equiv \tilde{\eta}_{1t}\phi_1 + \tilde{\eta}_{2t}(1 - \phi_1)\phi_2. \quad (19)$$

Then the planner's first-order condition for energy intensity is identical to the one in equation (10): in other words, in the competitive equilibrium defined in Section 2.11 every region chooses the same energy intensity as the planner, i.e., $x_{it}^* = x_t^*$ for all i .

Given this (common) choice for the tax on emissions, the first-order condition for capital can be rewritten:

$$(\Phi(x_t) - \tau_t \chi_t \psi x_t) \alpha \Lambda(S_t) K_t^{\alpha-1} + 1 - \delta = q_t^{-1},$$

where q_t is the intertemporal marginal rate of substitution between consumption in period t and consumption in period $t + 1$, evaluated at the (socially) optimal allocation:

$$q_t \equiv \frac{\beta U'(C_{t+1})}{U'(C_t)}. \quad (20)$$

This first-order condition for capital coincides with the one in the stand-in consumer's problem (11) in the competitive equilibrium defined in Section 2.11 because, when $\tau_{it} = \tau_t$ for all i , the coefficient Θ_t in the resource constraint in the stand-in consumer's problem equals $(\Phi(x_t) - \tau_t \chi_t \psi x_t) \Lambda_t$.

Imposing the condition that each region balances its budget in every period, the aggregate resource constraint faced by the stand-in consumer coincides with the planner's. In addition, the planner and the stand-in consumer face the same physical constraints governing the carbon cycle and the global and regional temperatures. Since, as shown above, the first-order conditions governing the optimal choices for capital and energy use are also identical for the planner and the stand-in consumer, the planning allocation and the competitive-equilibrium allocation coincide when the tax on emissions is set according to equation (19).

It remains to characterize the behavior of the two (scaled) Lagrangian multipliers, $\tilde{\eta}_{it}$, $i = 1, 2$, that pin down the optimal tax on emissions in terms of the aggregate allocation. The first-order conditions for the two stocks of atmospheric carbon are a pair of simultaneous equations that can be used to express the period- t (scaled) multipliers in terms of the period- $(t+1)$ (scaled) multipliers. To conserve on notation, define $a_{1t} \equiv \phi_1 \chi_t \psi$, $a_{2t} \equiv (1 - \phi_1) \phi_2 \chi_t \psi$, $\Phi_t \equiv \Phi(x_t)$, and $\Lambda'_t \equiv \Lambda'(S_t)$. Then

$$\Delta_t \begin{bmatrix} \tilde{\eta}_{1t} \\ \tilde{\eta}_{2t} \end{bmatrix} = -\Phi_t \Lambda'_t K_t^\alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix} + q_t \begin{bmatrix} 1 & 0 \\ 0 & \phi_3 \end{bmatrix} \begin{bmatrix} \tilde{\eta}_{1,t+1} \\ \tilde{\eta}_{2,t+1} \end{bmatrix},$$

where

$$\Delta_t \equiv \begin{bmatrix} 1 - a_{1t} x_t \Lambda'_t K_t^\alpha & a_{2t} x_t \Lambda'_t K_t^\alpha \\ a_{1t} x_t \Lambda'_t K_t^\alpha & 1 - a_{2t} x_t \Lambda'_t K_t^\alpha \end{bmatrix}.$$

In the calibrated model, a marginal addition to the stock of atmospheric carbon reduces (aggregate) productivity, so $\Lambda'_t < 0$. The term $-\Phi_t \Lambda'_t K_t^\alpha$ is then the marginal loss of output in period t stemming from a marginal addition to the stock of atmospheric carbon in period t . Similarly, the term $x_t \Lambda'_t K_t^\alpha$ is the marginal reduction in energy use in period t stemming from a marginal addition to the stock of atmospheric carbon in period t .

Note that

$$\Delta_t^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = (1 - \chi_t \psi x_t \phi \Lambda'_t K_t^\alpha)^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

where $\phi \equiv 1 - (1 - \phi_1)(1 - \phi_2)$, i.e., ϕ is the fraction of carbon emissions in period t that enters the atmosphere. The term $\chi_t \psi x_t \phi \Lambda'_t K_t^\alpha$ is then the marginal reduction in atmospheric carbon emissions stemming from a marginal addition to the stock of atmospheric carbon in period t .

Rewrite the law of motion for the vector of (scaled) multipliers:

$$\begin{bmatrix} \tilde{\eta}_{1t} \\ \tilde{\eta}_{2t} \end{bmatrix} = \frac{-\Phi_t \Lambda'_t K_t^\alpha}{1 - \chi_t \psi x_t \phi \Lambda'_t K_t^\alpha} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + q_t \Delta_t^{-1} \begin{bmatrix} 1 & 0 \\ 0 & \phi_3 \end{bmatrix} \begin{bmatrix} \tilde{\eta}_{1,t+1} \\ \tilde{\eta}_{2,t+1} \end{bmatrix}. \quad (21)$$

By repeated substitution, one can iterate forward on equation (21) to obtain an expression for the current vector of (scaled) multipliers as the sum of current and discounted future losses of output stemming from a marginal addition to the stock of atmospheric carbon today:

$$\begin{bmatrix} \tilde{\eta}_{1t} \\ \tilde{\eta}_{2t} \end{bmatrix} = \frac{-\Phi_t \Lambda'_t K_t^\alpha}{1 - \chi_t \psi x_t \phi \Lambda'_t K_t^\alpha} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \sum_{i=1}^{\infty} \left[\prod_{s=t}^{t+i-1} \left(q_s \Delta_s^{-1} \begin{bmatrix} 1 \\ \phi_3 \end{bmatrix} \right) \right] \frac{-\Phi_{t+i} \Lambda'_{t+i} K_{t+i}^\alpha}{1 - \chi_{t+i} \psi x_{t+i} \phi \Lambda'_{t+i} K_{t+i}^\alpha}.$$

In this formula, output losses in any given period t , i.e., $-\Phi_t \Lambda'_t K_t^\alpha$, are scaled by the term $1 - \chi_t \psi x_t \phi \Lambda'_t K_t^\alpha$, which is larger than one when $\Lambda'_t < 0$ because an additional unit of atmospheric carbon, while generating a marginal loss in output in period t , at the same time reduces carbon emissions into the atmosphere in period t . In addition, a future loss in output in period $t + i$ is discounted by the vector

$$\prod_{s=t}^{t+i-1} \left(q_s \Delta_s^{-1} \begin{bmatrix} 1 \\ \phi_3 \end{bmatrix} \right).$$

It is the product of two terms. The first, $\prod_{s=t}^{t+i-1} q_s$, expresses consumption in period $t + i$ in units of the period- t consumption good. The second,

$$\prod_{s=t}^{t+i-1} \left(\Delta_s^{-1} \begin{bmatrix} 1 \\ \phi_3 \end{bmatrix} \right),$$

is a product of i terms, each of which is itself the product of two terms. The second of these terms, $[1 - \phi_3]^\top$, recognizes that in period $t+i$ there remains only a fraction, ϕ_3^{i-1} , of a marginal additional to the depreciating stock of atmospheric carbon, S_{2t} , in period t . The first of these terms, Δ_s^{-1} , recognizes that marginal additions to either stock of atmospheric carbon in period $t+i$ lead to a (marginal) reduction in emissions in that period.

Technical Appendix D

This appendix shows how to compute competitive equilibria under both autarky and free capital mobility, as defined in Sections 2.10 and 2.11, respectively. It also shows how to compute the solution to the social planning problem defined in Section 2.12. The technical description of the computations in this appendix complements the verbal description provided in Section 2.13.

Consider first the case of autarky; the computation of competitive equilibrium under free capital mobility proceeds similarly. The first step in solving for competitive equilibrium under autarky is to convert the dynamic programming problem (25) faced by a typical entrepreneur into its stationary counterpart (imposing in addition the restriction on the production function in Section 2.11). Define $v_t^i(\omega, A) \equiv v_t^i(\omega, A; \bar{T}_i, \gamma_i)$, so that the superscript i denotes a region's temperature at time 0, \bar{T}_i , and its sensitivity to changes in the global temperature, γ_i . For any variable z , define the scaled variable:

$$\hat{z} \equiv \frac{z}{AD(\bar{T}_i)}.$$

Guessing that $v_t^i(\omega, A) = (AD(\bar{T}_i))^{1-\nu} \hat{v}_t^i(\hat{\omega})$ for all t , it is straightforward to show that the sequence of functions $\{\hat{v}_t^i\}_{t=0}^\infty$ satisfies the following stationary version of the entrepreneur's problem (under the assumption of autarky in which case the global bond market is shut down);

$$\hat{v}_t^i(\hat{\omega}) = \max_{\hat{c}, \hat{k}'} [U(\hat{c}) + \hat{\beta} \hat{v}_{t+1}^i(\hat{\omega}')] \quad (22)$$

subject to:

$$\begin{aligned} \hat{\omega} &= \hat{c} + (1+g)\hat{k}' \\ T'_i &= \bar{T}_i + \gamma_i(T_{t+1} - T_0) \\ \hat{\ell}' &= D(T'_i)/D(\bar{T}_i) \\ \hat{\omega}' &= \Pi_{i,t+1}(\hat{k}')^\alpha (\hat{\ell}')^{1-\alpha} + (1-\delta)\hat{k}' + \hat{R}_{i,t+1}, \end{aligned}$$

where $\hat{\beta} \equiv (1+g)^{1-\nu}\beta$ and $\Pi_{it} \equiv \Phi(x_{it}^*) - \tau_{it}\chi_t\psi x_{it}^*$ as in Section 2.11. To generate the optimal sequence of decisions in the original (nonstationary) problem, first use the methods described

below to obtain, for each $t \geq 0$, the optimal decision rules $\hat{k}' = \hat{g}_{it}^k(\hat{\omega})$ in the stationary problem. Next, iterate using these decision rules to generate an optimal sequence, $\{\hat{k}_{it}\}_{t=1}^{\infty}$, of decisions in the stationary problem. Finally, undo the scaling to obtain the optimal sequence of decisions in the original problem:

$$\begin{aligned} k_{it} &= A_t D(\bar{T}_i) \hat{k}_{it} \\ &= (1+g)^t \bar{A}_i D(\bar{T}_i) \hat{k}_{it} \\ &= (1+g)^t \bar{a}_i \hat{k}_{it}, \end{aligned} \tag{23}$$

where $\bar{a}_i \equiv \bar{A}_i D(\bar{T}_i)$ as in Section 3.8.

To solve for the sequence of decision rules in the stationary problem for any given region i , work backwards from an eventual steady state in which the global temperature has converged to a constant T_{∞} (once the stock of atmospheric carbon has stabilized). The value function \hat{v}_{∞}^i in this steady state solves the problem:

$$\hat{v}_{\infty}^i(\hat{\omega}) = \max_{\hat{c}, \hat{k}'} [U(\hat{c}) + \beta \hat{v}_{\infty}^i(\hat{\omega}')] \tag{24}$$

subject to:

$$\begin{aligned} \hat{\omega} &= \hat{c} + (1+g)\hat{k}' \\ T_{i,\infty} &= \bar{T}_i + \gamma_i(T_{\infty} - T_0) \\ \hat{\ell}_{\infty} &= D(T_{i,\infty})/D(\bar{T}_i) \\ \hat{\omega}' &= \Phi(x^*)(\hat{k}')^{\alpha}(\hat{\ell}_{\infty})^{1-\alpha} + (1-\delta)\hat{k}', \end{aligned}$$

noting that because all energy is eventually green there are no carbon emissions and hence no taxes (or revenues from taxes) in the eventual steady state. This problem is solved numerically using standard methods by representing the value function by a cubic spline on a fixed grid of points for $\hat{\omega}$ and then iterating on the Bellman equation to update the guess for \hat{v}_{∞}^i , using Howard's policy improvement algorithm to accelerate convergence to the fixed point.

To obtain the time-varying decision rules for each region i along the transition path, iterate backwards to time 0 on equation (22), using the same numerical methods as for the steady-state problem (24) to compute decision rules in each time period. These iterations start from a time period S_{∞} by which the global temperature has (very nearly) converged to T_{∞} (for these computations, $S_{\infty} = 3000$ is sufficiently large). When solving for this sequence of decision rules, a typical entrepreneur in region i takes as given a sequence for the global temperature, $\{T_t\}_{t=0}^{S_{\infty}}$, and a sequence for the lump-sum transfer, $\{\hat{R}_{it}\}_{t=0}^{S_{\infty}}$.

To economize on the number of regions for which decision rules must be computed, first solve for decision rules for a discrete set of pairs (\bar{T}_i, γ_i) defined by a two-dimensional grid, with 40 points spanning the range of observed values for \bar{T}_i and 25 points spanning the range of observed values for γ_i . In practice, there are no regions in some parts of this grid, so the number of pairs for which we compute decision rules can be reduced $40 \times 25 = 1000$ to approximately 700. To compute saving decisions for regions not on the grid, interpolate by fitting cubic polynomials across decisions for nearby points in the grid.

The next step in the computation of competitive equilibrium under autarky is to use the computed decisions (in the stationary problem) to generate paths for the global temperature and regional tax revenues up to period S_∞ . The global temperature at any point in time depends on the sum of the two stocks of atmospheric carbon. The paths for these two stocks in turn depend on global energy use in the nonstationary economy, which can be obtained by calculating optimal energy use (per capita) in each region in the stationary economy using equation (9), rescaling energy use as in equation (23, and then summing across all regions, weighting by regional population.

Finally, update the assumed paths for temperature and regional transfers using the newly-computed ones (setting regional transfers equal to regional tax revenues) and continue iterating on these paths until convergence (typically only a small number of such iterations is required).

Computing the competitive equilibrium under full capital mobility follows the same steps, but in this case only one dynamic program needs to be solved, that of the stand-in consumer given in (11). Once the aggregate equilibrium is computed, regional outcomes can be backed out as explained in Section 2.11.

To solve the planning problem associated with the stand-in consumer's problem (11), work instead with the first-order conditions to that problem, after removing the trend growth rate of productivity in order to make them stationary as in problem (22), rescaling afterwards as needed. The algorithm proceeds in several steps; there is an outer loop to calculate paths for the two scaled Lagrangian multipliers, which determine the path of the carbon tax, and an inner loop to compute the competitive equilibrium given this path. The competitive-equilibrium paths can then be used to update the Lagrangian multipliers.

To begin, guess on paths for the two Lagrangian multipliers. Next, use the first-order conditions for savings and energy use, together with the laws of motion for the two stocks of atmospheric carbon, to compute the competitive equilibrium outcome given the path for the carbon tax. To do so, guess on a path for the intertemporal marginal rate of substitution in equation (20). Given this path (and the one for the carbon tax), use the first-order condition for savings and the laws of motion for the two stocks of atmospheric carbon to generate time paths for aggregate (global) capital and the global stock of atmospheric carbon. That is, given K_t , S_{1t} , and S_{2t} , use the first-order condition (16), together with the laws of motion (5) and (6), to solve (numerically) for K_{t+1}

and S_{t+1} , using equation (18) to determine the optimal choice for energy intensity (and in turn the optimal choice for energy use, rescaled appropriately to incorporate trend growth). These paths determine a path for aggregate consumption, which can in turn be used to generate a new path for the intertemporal marginal rate of substitution. Calculate a weighted average of this new path and the one taken as given and continue iterating until convergence, thereby completing the inner loop.

Finally, armed with these paths, iterate backwards on (the stationary counterpart) of equation (21) from the eventual steady state to compute a new sequence of Lagrangian multipliers. Return to the outer loop with these paths (rescaled to account for trend growth) and continue iterating until convergence.

Technical Appendix E

This appendix describes a method for computing competitive equilibria when there are idiosyncratic shocks to regional temperatures, i.e., when the temperature in any given region is not deterministic but instead follows a stochastic process calibrated to observed data on regional temperatures. In such a calibration, the shocks to regional temperature do not wash out in the aggregate, even though the number of regions is large, because these shocks are correlated across space as well as time. Consequently, there is also aggregate uncertainty because the global temperature varies stochastically around a trend path for the global climate. Earlier versions of this work used these methods successfully to compute equilibria in which regions can engage in risk-free (one-period-ahead) borrowing and lending in an international bond market subject to a restriction on borrowing. Each entrepreneur thus faces a nontrivial portfolio problem in deciding whether to save locally or abroad.

In this setting, the problem of a typical entrepreneur in region i (leaving out carbon taxes for simplicity) is:

$$v_t(\omega, z, A, \Gamma, S; \bar{T}_i, \gamma_i) = \max_{c, k', b'} U(c) + \beta \mathbb{E}_{Z'|Z} [v_{t+1}(\omega', z', A', \Gamma', S'; \bar{T}_i, \gamma_i)] \quad (25)$$

subject to:

$$\begin{aligned} \omega &= c + k' + q_t(\Gamma, S)b' \\ A' &= (1 + g)A \\ T'_i &= \bar{T}_i + \gamma_i(T(S') - T_0) \\ \ell' &= D(A', T'_i, z') \\ e' &= g_{t+1}^e(k', \ell') \\ \omega' &= G(k', \ell', e') + (1 - \delta)k' + b' \\ b' &\geq \underline{b}(k') \\ \Gamma' &= H_t(\Gamma, S) \\ S' &= \Xi_t(\Gamma', S) \end{aligned}$$

In this formulation, z is a shock to region i 's temperature and is assumed to follow a zero-mean $AR(1)$ process common across regions.⁴⁶ Γ is the joint distribution of regions across region-specific states (ω, z) . Productivity (specifically, the number of efficiency units of labor) in region i now depends both on the region's climate (or average temperature), T_i , and on the current shock, z , to its temperature (viewed as a zero-mean deviation from its climate). The full set of shocks across all regions is denoted by the vector Z ; the innovations in the $AR(1)$ processes for each region are assumed to be correlated across space. International borrowing cannot exceed an amount, \underline{b} , that depends on regional capital. The state vector for each region now includes the distribution Γ (including therefore the marginal distribution over Z) and the sum of the two stocks of atmospheric carbon, S . The function $T(S)$ determining the global climate (i.e., the average global temperature) is given by equation (3).

Finally, the bond price, $q_t(\Gamma, S)$, now varies over time as the aggregate state varies. The (stochastic) evolution of the bond price depends on the evolution of Γ and S , governed by the two functions $H_t(\Gamma, S)$ and $S' = \Xi_t(\Gamma', S)$. These functions are time-varying because the global climate varies over time.

The stock of atmospheric carbon, S , varies stochastically because global energy use also varies stochastically (recall that the regional shocks to temperature do not wash out in the aggregate in a reasonable calibration). But the stochastic variations in S are very small in percentage terms mainly because global emissions on average are a small fraction of the global stock of carbon. In the

⁴⁶This assumption can be relaxed in a more realistic calibration.

numerical solution of the model it is therefore reasonable to assume that S follows a deterministic (but still time-varying) path as in the model without shocks to temperature.

Nonetheless, agents must still forecast stochastic fluctuations in the equilibrium bond price. To make this problem manageable, we adapt methods from our earlier work in Krusell & Smith (1998) and Krusell & Smith (1997) to this setting in which, unlike in that work, there is a (stochastic) transition path to an eventual stochastic steady state without climate change (though that steady state continues to display variations in the global temperature around a stable climate). We assume, and then verify, that approximate aggregation holds in this environment as it did in our earlier work, so that Γ can be represented, for purposes of (highly accurate) forecasting, by its mean, i.e., \bar{k} , the global capital stock, and \bar{z} , the average shock to regional temperatures.⁴⁷

With these simplifications, the problem of a typical entrepreneur becomes:

$$v_t(\omega, z, A, \bar{k}, \bar{z}; \bar{T}_i, \gamma_i) = \max_{c, k', b'} U(c) + \beta \mathbb{E}_{\bar{z}', z' | \bar{z}, z} [v_{t+1}(\omega', z', A', \bar{k}', \bar{z}'; \bar{T}_i, \gamma_i)] \quad (26)$$

subject to:

$$\begin{aligned} \omega &= c + k' + q_t(\bar{k}, \bar{z})b' \\ A' &= (1 + g)A \\ T'_i &= \bar{T}_i + \gamma_i(T(S_{t+1}) - T_0) \\ \ell' &= D(A', T'_i, z') \\ e' &= g_{t+1}^e(k', \ell') \\ \omega' &= G(k', \ell', e') + (1 - \delta)k' + b' \\ b' &\geq \underline{b}(k') \\ \bar{k}' &= H_t(\bar{k}, \bar{z}), \end{aligned}$$

where \bar{z} follows an $AR(1)$ process because it is the average of the z s (each of which, again, follows an $AR(1)$ process, with correlated innovations across regions).

As in Krusell & Smith (1998) and Krusell & Smith (1997), we further assume that the two “aggregate” functions q_t and H_t are linear in \bar{k} and \bar{z} , with a set of time-varying coefficients, μ_t . Given these coefficients, one can iterate backwards on equation (26) from the eventual stochastic steady state to compute time-varying decision rules for physical capital and bondholdings.⁴⁸ When solving this set of problems, entrepreneurs take as given (deterministic) paths for the global stock of atmospheric carbon, S_t , and the coefficients, μ_t , in the aggregate functions.

⁴⁷ \bar{k} is simply total global capital, whereas \bar{z} is weighted by regional GDP.

⁴⁸The equilibrium in the stochastic steady state is computed following the methods in Krusell & Smith (1997) in which the coefficients in the two aggregate functions are constant.

To update both paths, we adapt the methods in Krusell & Smith (1997), using the computed decision rules to simulate the global economy forwards in time, varying the bond price in each period to clear the international bond market. Unlike in Krusell & Smith (1997), it is necessary to simulate the behavior of the global economy N times because the coefficients μ_t vary with time. Let \bar{k}_t^j , \bar{z}_t^j , and \hat{q}_t^j denote aggregate capital, the average shock to temperature, and the market-clearing bond price, respectively, in the j th simulation in period t . For each t , use the N simulated paths to regress capital in period $t + 1$ on the period- t state variables. Similarly, regress the market-clearing bond price in period t on the period- t state variables. These regressions yield a new set of (estimated) coefficients $\hat{\mu}_t$. Calculate a weighted average of these coefficients and the ones taken as given, update the path for the global carbon stock with the new simulated path, and then continue iterating until both the coefficients and the path for the carbon stock converge.⁴⁹ As in our earlier work, this method can be generalized to incorporate additional features of the distribution in making forecasts and to allow for nonlinear laws of motion.

To conserve on computation time, we implement two modifications to this approach which permit accurate solutions even when $N = 2$. First, in each time period of a simulation, compute tomorrow's capital and the market-clearing bond price twice, once (as before) for the current distribution over (ω, z) and then a second time after a small perturbation to this distribution. Next, use the two outcomes for tomorrow's capital and today's bond price to calculate numerical derivatives, using a standard first-difference approximation, of the aggregate functions with respect to their two arguments; because these functions are assumed to be linear, these numerical derivatives are (highly accurate) estimates of the slope coefficients if approximate aggregation holds (as it does to a high degree of accuracy).⁵⁰

Nonetheless, it remains important to take an average across simulations at each point in time because the coefficients in the aggregate functions vary with time. The second modification, then, is to use antithetic variates across the two (in this case) simulations: i.e., let the simulated innovations in the z s in the second simulation be equal in absolute value, but of opposite sign, to the innovations in the first simulation. It turns out the estimated slope coefficients (computed via perturbation) are very nearly linear in the underlying innovations, in which case the average across the two simulations is very nearly equal to the (true) mean.

⁴⁹As per usual, when generating simulated paths hold the underlying random draws from the pseudo random generator fixed across iterations to eliminate jitter along the way to convergence.

⁵⁰We also verify that linearity yields an excellent fit to the simulated data.