E - Lighting the Street

Problemsetters: Nicolas / Bernhard

Problem — 69/75 solves

There are n street lights $p_1, \ldots, p_n$ on $[0, L]$ and m numbers $l_1, \ldots, l_m$. You need to pick the cheapest l_i such that $[p_j - l_i, p_j + l_i]$ cover the whole $[0, L]$.

Solution — $\mathcal{O}(n \log n)$

Sort $p_1, \ldots, p_n$. The minimum possible l_i is

$$\max(p_1, L - p_n, \left\lceil \frac{p_i - p_{i-1}}{2} \right\rceil)$$

Find the smallest l_i that is at least as big.

C - Concert Lineup

Problemsetters: Theodor

Problem 59/75 solves

Given an array n, and a sequence of even integers k_i. For each k_i, delete first $k_i/2$ odd numbers and reverse first $k_i/2$ even numbers.

Solution $\mathcal{O}(n)$

Use a linked list or reverse the array and use a stack.
Total runtime is at most $2n$.

D - LSB

Problemsetters: Theodor / Paula

Problem 41/75 solves

Given n, write a program using only $<<$, $>>$, ^ and $|$ to isolate the least significant bit of any input bitset of length n.

Solution $\mathcal{O}(\log n)$

Let i be the LSB. Make B that has 1 for $j \geq i$ and 0 otherwise. This is achieved by repeatedly doing

- $B_i = B_{i-1} << 2^k$;
- $B_{i+1} = B_i | B_{i-1}$.

In the end, the answer is $B \text{ \^{} } (B << 1)$.

J - Gibberish

Problemsetters: Theodor / Paula

Problem $34/75$ solves

Find the secret permutation p by querying $p(w)$ for some string w.

Solution $\mathcal{O}(n \log n)$

We can solve the problem using $\lceil \log_{26} n \rceil$ queries by setting

$$w_{i,j} = i\text{-th digit of } j \text{ in base } 26$$

F - AutoBahn Optimization

Problemsetters: Constantin

Problem 16/75 solves

There are n cars splitting into two lanes. After the split, each cars rides with the minimum speed of itself and all cars ahead of it. Find the splitting that minimizes slowdown over all cars.

Solution $\mathcal{O}(ns)$

Speed on the slower lane is always the minimum of all car speeds.

$$dp[\text{current car}][\text{speed on faster lane}] = dp[i][j] = \text{min slowdown}$$

Let x be the smallest speed so far. We have two options:

1. $j \leq s_i$: Put the car in the faster lane for $s_i - j$ slowdown.
2. $j > s_i$: Put the car in the faster lane for no slowdown (but its speed reduces to s_i), or in the slower lane for $s_i - x$ slowdown.

B - Digital Products

Problemsetters: Dmytro Fedoriaka

Problem — 15/75 solves

Find number of unique $D(x)$ for $x \leq n$, where $D(x)$ is the product of all digits of x.

Solution — $\mathcal{O}(\log^4 n)$

Consider how many distinct $D(x) = 2^a \cdot 3^b \cdot 5^c \cdot 7^d$ there are.
We can bound all pwers by $18 \cdot 3 = 54$, So there are at most 54^4 such numbers. Actually way less, 36101 below 10^{18}.
For each possible product, find the smallest number having it, then binary search how many numbers less than n.

H - Zurich Trams

Problemsetters: Bernhard

Problem 13/75 solves

Given a tree and some tram lines, we want to find the shortest path from A to B.

Solution $\mathcal{O}(nm)$

For each edge figure out the first time each tram comes to it t_i, and the length of the tram cycle c_i by brute force in $O(nm)$ time. Next we walk the path from A to B keeping track of our current time t, at each step checking which tram can get us to the next vertex the fastest from time t onward, then update t.

L - Drawing Rectangles

Problemsetters: Theodor / Oleksandr

Problem 11/75 solves

You paint k independent axis-aligned lattice rectangles on an $n \times m$ canvas. Find the expected number of rectangles not covered by any other rectangle.

Solution $\mathcal{O}(n^2 m^2 \log k)$

For each $[l_x, r_x] \times [l_y, r_y]$, find the probability of it being covered:

$$p = a + b - ab,$$

then add $1 + p + \cdots + p^{k-1} = \frac{1-p^k}{1-p}$ to the answer, where

$$a = \frac{\binom{l_x+1}{2} + \binom{n-r_x+1}{2}}{\binom{n+1}{2}}, \quad b = \frac{\binom{l_y+1}{2} + \binom{m-r_y+1}{2}}{\binom{m+1}{2}}.$$

K - Cheater Detector

Problemsetters: Theodor

Problem 4/75 solves

Check if graph can be split into a clique and an independent set.

Solution $\mathcal{O}((n+m)\log m)$

Sort vertices by degrees. If the partition exists, it also exists in a way that the independent set is the prefix, and the clique is the suffix, and there are at most 2 candidate splits in this case.

G - Contrived Intelligence

Problemsetters: Archit Manas

Problem — 2/75 solves

There is an integer polynomial $P(x)$. You may ask for k, how many $P(1), \ldots, P(k)$ are divisible by k.
You need to find for each k, how many $P(1), \ldots, P(k)$ are co-prime with k for all $k \leq 10^5$ in at most 10^4 queries.

Solution — $\mathcal{O}(n \log n)$

let $A(k)$ be the input, $B(k)$ the output. Then,

$$B(k) = \sum_{d|k} \mu(d) A(d) \frac{m}{d}$$

And $A(pq) = A(p)A(q)$ when $\gcd(p, q) = 1$, so ask $A(p)$ for all primes and use it to combine the answer.

A - Matrix Minors

Problemsetters: Oleksandr Kulkov

Problem 0/75 solves

Find all minors of a $n \times n$ matrix for $n \leq 500$.

Solution $\mathcal{O}(n^3)$

Minors are closely related to $(\operatorname{adj} A)_{ij} = (-1)^{i+j} M_{ji}$:

$$A \cdot \operatorname{adj} A = \operatorname{adj} A \cdot A = I \det A$$

Find rank r of the matrix:

1. $r = n$: $\operatorname{adj} A = A^{-1} \det A$.
2. $r < n - 1$: $\operatorname{adj} A = 0$.
3. $r = n - 1$: $\operatorname{adj} A = \alpha x y^\top$, where $Ax = 0$ and $A^\top y = 0$.

I - Periodic Recurrence

Problemsetters: Dmytro Fedoriaka

Problem $0/75$ solves

Find the period of $F_n = aF_{n-1} + bF_{n-2}$ modulo M.

Solution $\mathcal{O}(\sqrt{M})$

By CRT, this is the LCM of periods modulo p^k.
For each p^k, the period always divides $p^{k-1}(p^2 - 1)$ or $p^k(p - 1)$.