*Handout 1:* **Symmetric Matrices:** rows and columns are interchangeable. $A = A^T$. **Diagonal Matrix:** all off-diagonal elements are zero `diag()` func in R. **0 and I Matrices:** $AI = IA = A$. **Equal Matrices:** same dimensions and corresponding elements are equal. **Matrix Multiplication:** $AB \neq BA$ in general; inner dimensions must match. **Distributive Property:** $A(B+C) = AB + AC$. **Transpose of Products:** $(AB)^T = B^T A^T$.

*Handout 2:* **Sum of elements:** $\sum_{i=1}^n a_i = j^T a$ where $j$ is a vector of 1s. **Sum of squares:** $\sum_{i=1}^n a_i^2 = a^T a$ also called the dot product of $a$ and $a^T$. **Length of 2-dim vector:** $\sqrt{a^T a}$. **Quadratic Form:** $y^T A y$ where $A$ is symmetric matrix and $y$ is a nx1 vector and this returns a scalar. **Linearly Dependent:** $c_1 v_1 + \ldots + c_n v_n = 0$ where $c_i$ are not all zero. **Rank:** number of linearly independent columns/rows; $\leq$ min(n,p); = min(n,p) if full rank; get to row-echelon form and count pivot rows for rank. **Non-singular: square matrix with full rank; less than full rank is singular; non-singularity = inverse. Inverse:** $A^{-1}A = AA^{-1} = I$; $(AB)^{-1} = B^{-1}A^{-1}$; $(A^T)^{-1} = (A^{-1})^T$. **Positive Definite Matrix:** $x^T A x > 0$, $A = T^T T$ then T is square root matrix of A; PDM is also full ranks and has an inverse; $[x1x2]A[x1x2]^T > 0$ where A is `c(2, 0, 2, 0), ncol=2, nrow=2`. **Determinant:** Scalar; `det()` or $|A|$; **computed for var/cov matrix** which is always square, symmetric, and PDM; diagonal elements $s_i^2$ and off-diagonal elements $s_{ij}$; $|A| = ad - bc$ for `c(a, b, c, d)`; for 3x3 matrix $|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$. **Props of Det:** A is diagonal then $|A| = \prod_{i=1}^n a_{ii}$ (product of diagonals); singular then det = 0; non-singular then det $\neq 0$ **has an inverse**; PDM then det $> 0$; **Trace:** sum of diagonal elements for square matrix. **Orthogonal Matrix:** dot product = 0; correlation coeff. r = 0 if Orthogonal and linearly independent; numerator of Pearson should be 0, so $x_c^T y_c^T = 0$; **Normalizing a vector:** $x_c = x/\sqrt{x^T x}$. **Orthogonal Matrix Properties:** matrix A is orthogonal matrix if it is a square matrix with orthonormal (unit length) rows and columns; $A^T A = AA^T = I$ which means $A^T = A^{-1}$; **Eigenvalues and Eigenvectors:** $Ax = \lambda x$, where A defines a transformation and x is not affected by it; A is nxn then n distinct eigenvalues; solve $|A - \lambda I| = 0$; Got eigenvalues, plug in and solve for x, to make it length 1 divide by $\sqrt{2}$; **Eigen Properties:** eigenvalues of a PDM are all positive; eigenvectors of a symmetric matrix are all orthogonal: $x_i^T x_j = 0$ for $i \neq j$; trace(A) = sum of eigenvalues; det(A) = product of eigenvalues; present eigenvalues **in descending order**.

*Handout 3:* **Sample Mean Vector:** `colMeans()`; estimate of the population mean vector. **Sample Variance:** $s^2 = \frac{1}{n-1}\sum_{i=1}^n (x_i - \bar{x})$ so the distance from average, on average; `cov()` for var/cov matrix. **Population Variance:** $\sigma^2 = E[(y-\mu)^2]$. **Covariance:** $s_jk = \frac{1}{n-1}\sum_{i=1}^n (y_j - \bar{y}_j)(y_k - \bar{y}_k)$; large & positive strong, linear positive association; lose to 0 is a weak linear assoc. **Correlation Coeff and Corr Matrix:** corr coeff is a scaled covariance so it is more interpretable; sample corr coeff: $r_{jk} = \frac{s_{jk}}{\sqrt{s_{jj}}\sqrt{s_{kk}}}$ and same for population; `cor()` for correlation matrix, has 1 on diagonal, $\rho_{jk} = \rho_{kj}$, **between -1 and 1 and no units**;

*Handout 4:* **Linear Combination:** $z = c_1 x_1 + c_2 x_2$; $z = c^T x$; **Sample Mean :** Vector of z: $\bar{z} = c^T \bar{x}$. **Interpretation:** sample overall grade $\bar{z}$ across the n students is the linear combination of the sample means of the various different components. **Population Mean:** $E(z) = c^T \mu$. **Variance:** $Var(z) = c_1^2 var(x_1) + c_2^2 var(x_2) + 2c_1 c_2 cov(x_1, x_2)$; $s_z^2$ for sample. **Interpretation:** variance of sum of 2 random vars must take into account cov between all pairs of random vars: var(x1 + x2) = formula above. **Sample Variance:** $s_z^2 = A^T S A$ where S is the var/cov matrix. **Population Variance:** $\sigma_z^2 = c^T \Sigma c$. **Several Linear Combinations:** $Z = CX$; $E(Z) = C\mu$; $Var(Z) = C\Sigma C^T$; $z = Ay$ and same equations as above. **Partitioning:** sample covariance matrix has transpose of each other within the partitioned matrix. **Number of Elements:** $\frac{n(n+1)}{2}$ unique elements in a cov or corr matrix of n variables. **Generalized Sample Variance:** $|S|$ is product of eigenvalues of S, $|S| = 0$ if any eigenvalue is 0 (multi co-linearity). **Mahalanobis distance:** multivariate version of a z-score; $\delta^2 = (y_i - \mu)^T \Sigma^{-1} (y_i - \mu)$ is in quadratic form and return s a scalar; **variables that have a high degree of variation will contribute less to the overall Mahalanobis distance**; how far a measurement is from the mean vector relative to what a typical deviation from the mean is. **Multivariate Normal Distribution:** $f(y) = \frac{1}{(2\pi)^{p/2}|\Sigma|^{1/2}} exp(-\frac{1}{2}(y-\mu)^T \Sigma^{-1}(y-\mu))$; $y \sim N_p(\mu, \Sigma)$; returned value should be a scalar. **Properties of Multivariate Normal:** y is $N_p(y_i, \Sigma)$; $z \sim N(a^T\mu, a^T \Sigma a)$ and individual y's must be normal too; rank(A) = q $\leq$ p then $z \sim N_q(A\mu, A^T \Sigma A)$; if $\mu = 0$ and $\Sigma = I$ then $Ay \sim N(0, I)$. **Independent Random Variables:** $y_j$ and $y_k$ are independent if and only if covariance $s_{jk} = 0$ or stated in terms of correlation of row jk; **Implication goes both ways since property of MV normal distribution.**

*Handout 5:* **Disprove p vars are jointly MV normally distributed is to show that at least 1 y is not uni-variate normal.**; using qq-plot or rigorous tests; null hypothesis of these tests is that variables do follow a MV normal distribution. **Univariate CLT:** Typically $\bar{y}$ is within $\frac{\sigma}{\sqrt{n}}$ of $\mu$ and follows a normal distribution. **Multivariate CLT:** $\bar{y} \sim N_p(\mu, \frac{\Sigma}{n})$ for a large enough sample size $n$. **Sample Variance and Cov Matrix Distribution:** univariate follows a chi-squared distribution; sample var/cov matrix follows a Wishart distribution: $(n-1)S \sim Wishart(n-1, \Sigma)$ if y follows a MV normal distribution. **Univariate 1-Sample T Test:** $t = \frac{\bar{y} - \mu_0}{s/\sqrt{n}}$ nominator is how far $\bar{y}$ is from the hypothesized pop mean if $H_0$ were true; denominator is relative to a typical deviation of $\bar{y}$ from $\mu$. **Hotelling's 1-Sample $T^2$ Test:** $T^2 = n(\bar{y} - \mu_0)^T S^{-1} (\bar{y} - \mu_0)$; measures how far observed $\bar{y}$ is from its expected value, if the null were true, while taking into account the variance/cov of the sample mean vector. **Hotelling's $T^2$ Density:** no upper bound; reject when $T^2$ is large. **P-value accuracy:** data values must have been sampled from MV normal dist, S must be non-singular, and n > p (observations > variables). **Steps to Take After Rejecting Null:** check multivariate normality of data, conduct univariate tests on each variable. **Benefits of MV Tests:** using p univariate tests inflates the type I error rate (rejecting null when it is true); p = 4 and $\alpha = 0.05$ then $1 - (1 - 0.05)^4 = 0.19$ probability and quickly increases with p; does not ignore correlation structure between p vars; more power (prob of rejecting null when null is true) and high power is good; small deviations may be statistically significant when combined. **Limitations of MV Tests:** interpretations difficult without univariate tests when statistically significant MV test; **non-directional** so null hypothesis for MV is 2-tailed.

*Handout 6:*

- **Univariate Two-Sample T Test:** null and alt hypoth; t stat and pooled variance; assuming equal variances; validity of p-val is sampled from normal dist with equal var but possibly diff means
- **Multivariate Two-Sample Hotelling's T Test:** null and alt; t stat (maha dist); pooled sample cov; validity pval is MV normal dist with equal cov matrices but diff means; yij is the jth obs in group i and vectors with p entries; individual t test to see where differences lie.
- **Paired Multivariate Data:** Difference = Treatment - Control; multivariate and paired (same subject); apply 1-sample Hotelling's T test; null is mean diff 0, alt is mean diff not 0; t stat is maha between d-bar and 0; validity is mv normal; follow up univariate analysis to see which dist from 0 is significant;

*Handout 7:*

- **Univariate 1-Way ANOVA:** yi. is sum across all measurements in sammple i, y-bari. is sample mean of all measurements in sample i, y-bar.. is (grand) sample mean of all measurements across all samples; statistical model for data, assume ...; null and alt; estimate of $\sigma^2$ using within variation MSE (within group); MSE thought of as pooled estimate of $\sigma^2$; between sample variation is MSH; F stat eq, explained var / unexplained var (within is noise and between is signal); Reject when F large; look at graphs, sd of within group (noise).
- **1-Way MANOVA:** vector yij has yij1...yijp; statistical model with yi being estimate of mu; null and alt hypoth; between var is H eq, within var is E eq; E + H is total sample covariance; dets of each give generalized vars; Wilk's $\Lambda$ stat which is unexplained or noise / total; reject when $\Lambda$ is small; get F by transforming and and when large reject null. manova() then summary with Wilk's test, summary$SS gives H and $Residuals gives E; Follow up with univariate anovas to see which var has diff means across groups; TukeyHSD() with Bonferroni correction finds which groups are different, p adj is bumped up little so hard to reject null, protect from making type I error, compare p-vals to alpha / num of vars to see which are significant, can compare sample means this way.

*Handout 8:*

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