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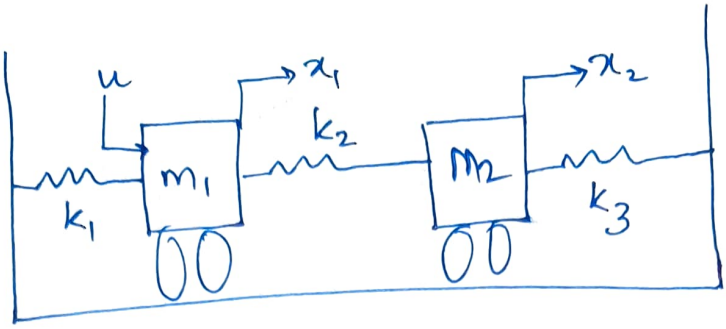
1.

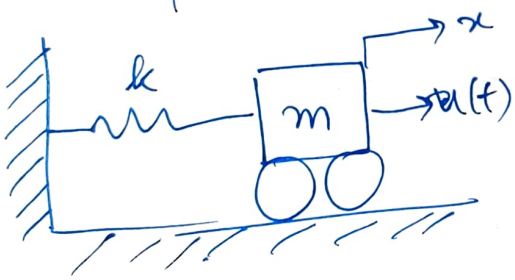


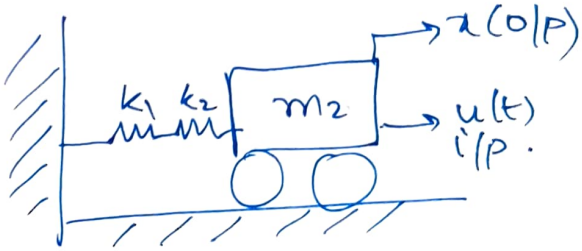
$P_1 = (x_1, 0, 0)^T$ $\qquad$ $V_1 = (\dot{x}_1, 0, 0)$

$P_2 = (x_2, 0, 0)^T$ $\qquad$ $V_2 = (\dot{x}_2, 0, 0)$

$$K.E = K(q, \dot{q}) = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

② while deriving for K.E we don't consider k.
∴ for both conditions:

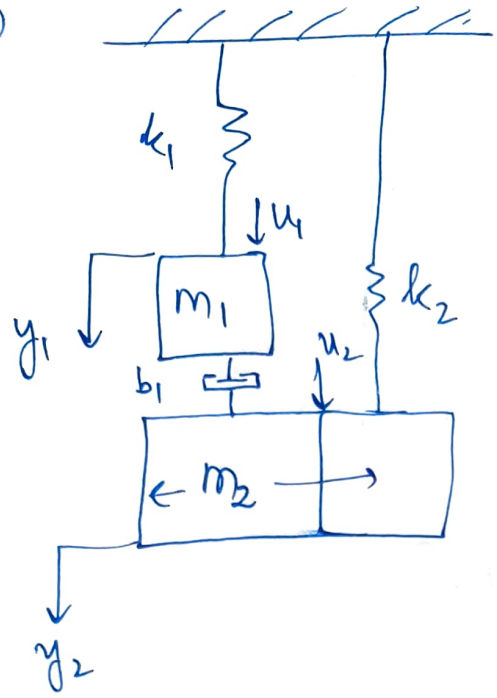




$P_m = (x, 0, 0)^T$ $\qquad$ $V_m = (\dot{x}, 0, 0)^T$

$$K.E = K(q, \dot{q}) = \frac{1}{2} m \dot{x}^2$$

③



$$P_1 = (0, y_1, 0)^T$$
$$P_2 = (0, y_2, 0)^T$$
$$V_1 = (0, \dot{y}_1, 0)^T$$
$$V_2 = (0, \dot{y}_2, 0)^T$$

$$K.E = K(q, \dot{q}) = \frac{1}{2} m_1 V_1^T V_1 + \frac{1}{2} m_2 V_2^T V_2$$
$$= \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2$$

④. $P_1 = (x_1, 0, 0)^T$ $\quad V_1 = (\dot{x}_1, 0, 0)^T$

$P_2 = (x_2, 0, 0)^T$ $\quad V_2 = (\dot{x}_2, 0, 0)^T.$

$$K.E = K(q, \dot{q}) = \frac{1}{2} m_1 V_1^T V_1 + \frac{1}{2} m_2 V_2^T V_2$$
$$= \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

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(5)

$P_1 = (x_1, 0, 0)^T$ $\quad$ $V_1 = (\dot{x}_1, 0, 0)^T$

$P_2 = (x_2, 0, 0)^T$ $\quad$ $V_2 = (\dot{x}_2, 0, 0)^T$

$P_3 = (x_3, 0, 0)^T$ $\quad$ $V_3 = (\dot{x}_3, 0, 0)^T.$

$$K.E = \frac{1}{2} \sum_{i=1}^{3} m_i V_i^T V_i$$

$m_1 = 4\,kg$ $\quad$ $m_2 = 5\,kg$ $\quad$ $m_3 = 5\,kg.$

$$KE = \frac{1}{2} m_1 V_1^T V + \frac{1}{2} m_2 V_2^T V + \frac{1}{2} m_3 V_3^T V_3$$

$$= \frac{1}{2} 4(\dot{x}_1^2) + \frac{1}{2}(5)(\dot{x}_2^2) + \frac{1}{2}(5)(\dot{x}_3^2).$$

(6)(i). $f(x) = -ax + b$ $\quad$ $a > 0,\ b > 0$

$f(x_1) = -ax_1 + b$

$f(x_2) = -ax_2 + b$

$f(x_1 + x_2) = -a(x_1 + x_2) + b = -ax_1 - ax_2 + b$

$f(x_1) + f(x_2) = -a(x_1 + x_2) + 2b$

$f(x_1) + f(x_2) \neq f(x_1 + x_2)$

$\therefore$ Non-linear.

6.(b) $f(x) = 3x$

$x = \{(1,1,a)^T : a \in \mathbb{R}\}$

$x = \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix}$. $\begin{pmatrix} 1 \\ 1 \\ a_1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ a_1+a_2 \end{pmatrix}$

$f[(1,1,a_1)^T] = 3[1,1,a_1]^T = 3\begin{bmatrix} 1 \\ 1 \\ a_1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3a_1 \end{bmatrix}$ — ①

$f[(1,1,a_2)^T] = 3[(1,1,a_2)]^T = 3\begin{bmatrix} 1 \\ 1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3a_2 \end{bmatrix}$ — ②

$f\left[\begin{pmatrix} 1 \\ 1 \\ a_1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ a_2 \end{pmatrix}\right] = 3\begin{bmatrix} 2 \\ 2 \\ a_1+a_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 3(a_1+a_2) \end{bmatrix}$

$\cancel{f}$ ① + ② $= \begin{bmatrix} 3 \\ 3 \\ 3a_1 \end{bmatrix} + \begin{pmatrix} 3 \\ 3 \\ 3a_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 3(a_1+a_2) \end{pmatrix}$

But we have $x_3 = x_1 + x_2$.

$x_i \in \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix}$

which is not the case:

$\boxed{\therefore \text{ Non linear}}$

Additivity : $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$

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(6.c) . $f(x) = cx$. $c > 0$ $x = \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$ $a \in \mathbb{R}, b \in \mathbb{R}$.

$x = \{(a,b,0)^T : a \in \mathbb{R}, b \in \mathbb{R}\} \cup \cancel{\{(a,0,b)^T\}}$

$\cup$

$\{(a,0,b)^T : a \in \mathbb{R}, b \in \mathbb{R}\}$

$\cup$

$\{(0,a,b)^T : a \in \mathbb{R}, b \in \mathbb{R}\}$

If $x = \{(a,b,0)^T, (a,0,b)^T, (0,a,b)^T\}$.

$x_1 = (a,b,0)^T$ $x_2 = (a,0,b)^T$

$x_1 + x_2 = (2a, b, b)^T$ $f(x_1 + x_2) = c(2a, b, b)^T$

$f(x_1) = c(a,b,0)^T$ $f(x_2) = c(a,0,b)^T$

$f(x_1) + f(x_2) = c(2a, b, b)$

$f(x_1) + f(x_2) \neq f(x_1 + x_2)$

Nonlinear.

Additivity : $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$

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(6.b). $f[(1,1,a_1)^T] = 3\begin{bmatrix}1\\1\\a_1\end{bmatrix}$

$f[(1,1,a_2)^T] = 3\begin{bmatrix}1\\1\\a_2\end{bmatrix}$.

$f(x_1) + f(x_2) = f(x_1+x_2)$.

6(c). $f(x) = cx$. $c>0$ . . . .

$f(x_1) = c\begin{bmatrix}a_1\\b_1\\0\end{bmatrix}$ $f(x_2) = c\begin{bmatrix}a_2\\b_2\\0\end{bmatrix}$

$f(x_1+x_2) = c\begin{bmatrix}a_1+a_2\\b_1+b_2\\0\end{bmatrix} = f(x_1)+f(x_2)$.

Same for all 3 cases $\begin{cases}(a,b,0)^T\\(0,a,b)^T\\(a,0,b)^T\end{cases}$

If we take from same category

$\therefore$ linear!

As $f(x_1) + f(x_2) = f(x_1+x_2)$

Homogenity $\alpha x(t) \rightarrow \alpha y(t)$

Additivity : $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$

1. Check if systems given by ODE is linear:

Linearity follows superposition Theory:

If $x_1(t) \rightarrow y_1(t) = \alpha_1 x_1(t)$

$x_2(t) \rightarrow y_2(t) = \alpha_2 x_2(t)$

$x_3(t) = x_1(t) + x_2(t) \rightarrow y_3(t) = \alpha(x_1(t) + x_2(t))$

$x_1(t) + x_2(t) = \alpha x_1(t) + \alpha x_2(t)$

$x_3(t) \rightarrow x_1(t) + x_2(t)$ $\quad y_3(t) \Rightarrow \alpha x_3(t)$

$= \alpha(x_1(t) + x_2(t))$

1. a). $3\frac{d^2y}{dt^2} + \frac{dy}{dt} + 5y = 9\frac{du}{dt} + u.$

Order = 2 $\quad$ Degree = 1

An nth order ODE is linear if it can be written as

$a_n(t)\frac{d^n x}{dt^n} + \dots + a_1(t)\frac{dx}{dt} + a_0(t)x = E(t) = b_0(t)u + \dots + b_n(t)\frac{d^n u}{dt^n}$

It is homogenous if $E(t) = 0$.

$\therefore$ The above equation can be written as:

$3\frac{d^2y}{dt^2} + \frac{dy}{dt} + 5y - 9\frac{du}{dt} - u = 0.$

Order = 2 $\quad$ Degree = 1

$\therefore$ The ODE is linear.

$y, u \rightarrow$ linear, co-efficients $\rightarrow$ linear

$\therefore$ linear.

(b) $-7\frac{d^3y}{dt^3} + 3\frac{dy}{dt} + 2y = 5\frac{du}{dt} + 7u.$

$-7\frac{d^3y}{dt^3} + 3\frac{dy}{dt} + 2y - 5\frac{du}{dt} - 7u = 0.$

Order = 3 Degree = 1

ODE is linear.

$u, y \rightarrow$ linear
coefficients $\rightarrow$ linear.

$\therefore$ ODE $\rightarrow$ linear.

Also,
for $f(u_1, y_1)$

$-7\frac{d^3y_1}{dt^3} + 3\frac{dy_1}{dt} + 2y_1 = 5\frac{du_1}{dt} + 7u_1$

for $f(u_2, y_2)$

$-7\frac{d^3y_2}{dt^3} + 3\frac{dy_2}{dt} + 2y_2 = 5\frac{du_1}{dt} + 7u_1.$

$f(u_1+u_2, y_1+y_2) \Rightarrow f(u_3, y_3).$

$-7\frac{d^3(y_1+y_2)}{dt^3} + 3\frac{d(y_1+y_2)}{dt} + 2(y_1+y_2) - 5\frac{d(u_1+u_2)}{dt} + 7(u_1+u_2)$

$= \cancel{f(x_1)} = \cancel{f(u_1)} + \cancel{f} = f(u_1, y_1) + f(u_2, y_2)$

$\therefore$ linear.

(c). $8\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 3\frac{du}{dt}$

(c). $8\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = 3\frac{du}{dt} + u + 3$

for $y_1(t)$ & $u_1(t)$

for $y_1(t) \rightarrow$ $8\frac{d^2y_1}{dt^2} + 2\frac{dy_1}{dt} - 3y_1 = 3\frac{du_1}{dt} + u_1 + 3$ — ①

for $y_2(t) \rightarrow$ $8\frac{d^2y_2}{dt^2} + 2\frac{dy_2}{dt} - 3y_2 = 3\frac{du_2}{dt} + u_2 + 3$ — ②

for ① & ②, adding them.

$$8\left(\frac{d^2y_1}{dt^2} + \frac{d^2y_2}{dt^2}\right) + 2\left(\frac{dy_1}{dt} + \frac{dy_2}{dt}\right) - 3(y_1+y_2) = 3\left(\frac{du_1}{dt} + \frac{du_2}{dt}\right) + (u_1+u_2) + (3+3).$$

$$\boxed{\begin{array}{l} f(u_1,y_1) + f(u_2,y_2) \neq f(u_1+u_2, y_1+y_2) \\ \therefore \text{ Not linear.} \end{array}}$$

$f(u_1+u_2, y_1+y_2)$

$$8\frac{d^2(y_1+y_2)}{dt^2} + 2\frac{d(y_1+y_2)}{dt} - 3(y_1+y_2) = 3\frac{d(u_1+u_2)}{dt} + (u_1+u_2) + 3$$

missing 3!

3. (d). $8\frac{d^2y}{dt^2} + 2t \cdot \frac{dy}{dt} = 3\frac{du}{dt} + 4u$

For $(y_1, u_1) \rightarrow 8\frac{d^2y_1}{dt^2} + 2t \cdot \frac{dy_1}{dt} = 3\frac{du_1}{dt} + 4u_1$

For $y(u_2, y_2) \rightarrow 8\frac{d^2y_2}{dt} + 2t \cdot \frac{dy_2}{dt} = 3\frac{du_2}{dt} + 4u_2$.

For ~~f(x, y~~ $f(u_1, y_1)$ & $f(u_2, y_2)$ we get

$f(u_1, y_1) + f(u_2, y_2)$

$$8\left(\frac{d^2y_1}{dt^2} + \frac{d^2y_2}{dt^2}\right) + 2t\left(\frac{dy_1}{dt} + \frac{dy_2}{dt}\right) = 3\left(\frac{du_1}{dt} + \frac{du_2}{dt}\right) + 4(u_1 + u_2).$$

$$f(u_1, y_1) + f(u_2, y_2) = f(u_1 + u_2, y_1 + y_2)$$

$\therefore$ linear

2.(e). $8y \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = \frac{3du}{dt} + u$

For $f(u_1, y_1)$

$$8y_1 \cdot \frac{d^2y_1}{dt^2} + 2\frac{dy_1}{dt} + y_1 = \frac{3du_1}{dt} + u_1 \quad \text{— (1)}$$

For $f(u_2, y_2)$

$$8y_2 \cdot \frac{d^2y_2}{dt^2} + 2\frac{dy_2}{dt} + y_2 = \frac{3du_2}{dt} + u_2 \quad \text{— (2)}$$

for $f(u_1, y_1) + f(u_2, y_2)$

we can't satisfy superposition.

If $f(u_1+u_2, y_1+y_2)$
we get.

$$8(y_1+y_2)\frac{d^2(y_1+y_2)}{dt} + 2\frac{d(y_1+y_2)}{dt} + (y_1+y_2) = \frac{3d(u_1+u_2)}{dt} + (u_1+u_2) \quad \text{— (3)}$$

$f(u_1, y_1) + f(u_2, y_2) \neq f(u_1+u_2, y_1+y_2)$.

$\therefore$ Non-linear.

Also, in $8y\left(\frac{d^2y}{dt^2}\right)$ y is a dependent variable & non-linear

$\therefore$ Non-linear

D.3 (a,b,c,d).

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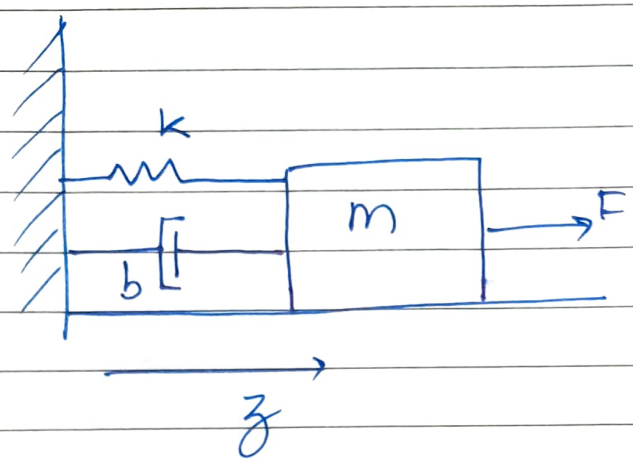

Data:

Spring Const = k

Damping const = b

Position of spring at rest = 0 (2)

Speed of mass = $\dot{z}$

External force = F.

a) To find P.E.

(b). Generalised coordinates => $q = z$

$\tau = F$

~~Friction~~ Damping $= -Bq = -Bz$

$$K.E = \frac{1}{2} m\dot{z}^2 = \frac{1}{2} m\dot{q}^2$$

(a) P.E (of Spring) $\Rightarrow P(q) = \frac{1}{2} kz^2 = \frac{1}{2} kq^2$

Lagrangian $\Rightarrow L(q,\dot{q}) = K - P = \frac{1}{2} m\dot{z}^2 - \frac{1}{2} kz^2$

$$\frac{\partial L}{\partial \dot{q}} = \frac{2}{2} m\dot{z} = mz \quad \Big| \quad \frac{\partial L}{\partial q} = \frac{1}{2} \times 2 \times kz = kz$$

Euler - Lagrange Eq:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = \tau - B\dot{q}$$

$$\boxed{m\ddot{z} - kz = F - b\dot{z}}$$

D(2) K.E of Spring Mass Damper $= \frac{1}{2} m\dot{z}^2$

$= \cancel{\frac{1}{2} mq}$

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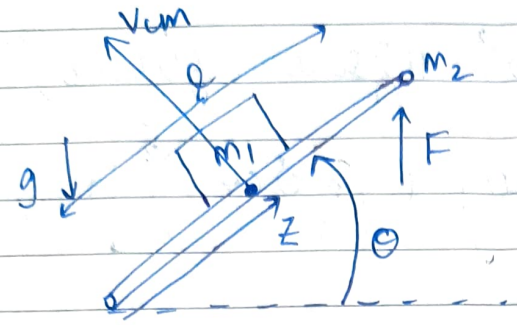
E.2 Block on Beam

(a).



Position of block from pivot $= z$

Speed of block $= \dot{z}$

Angle of beam from level $= \theta$

Angular velocity $= \dot{\theta}$

Mass of Rod $= m_2$

Mass of block $= m_1$

$$K = \sum_{i=1}^{N} \frac{1}{2} m_i V_{cm,i}^T V + \sum_{i=1}^{2} \frac{1}{2} \omega_i^T J_{cm,i} \omega \Big]$$

If block moves without rolling $\omega_{block} = 0$ $\boxed{\omega_{block} = 0}$ → Block.

$$K = \frac{1}{2} m_{block} V_{block}^T V_{block} + \frac{1}{2} \omega_{block}^T J_{block} \omega_{block} +$$

$$\frac{1}{2} m_{beam} V_{beam}^T V_{beam} + \frac{1}{2} \omega_{beam}^T J_{beam} \omega_{beam}$$

$$P_{block} = \begin{pmatrix} z(t)\cos\theta(t) \\ z(t)\sin\theta(t) \\ 0 \end{pmatrix} \qquad P_{beam} = \begin{pmatrix} \ell/2 \cos\theta(t) \\ \ell/2 \sin\theta(t) \\ 0 \end{pmatrix}$$

$$\cancel{V_{block} = \begin{pmatrix} \dot{z}\cos\theta \\ \dot{z}\sin\theta \\ 0 \end{pmatrix}} \qquad V_{beam} = \begin{pmatrix} -\ell/2\, \dot{\theta}\sin\theta \\ \ell/2 \cos\theta \\ 0 \end{pmatrix}$$

$$\cancel{V_{beam}} = V_{block} = \begin{pmatrix} \dot{z}\cos\theta - z\dot{\theta}\sin\theta \\ \dot{z}\sin\theta + z\dot{\theta}\cos\theta \\ 0 \end{pmatrix} \qquad \omega_{beam} = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}$$

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Substituting these in K.E equation

$$K = \frac{1}{2} m_1 \left[(\dot{z}\cos\theta - z\dot{\theta}\sin\theta)^2 + (\dot{z}\sin\theta + z\dot{\theta}\cos\theta)^2 \right]$$
$$+ \frac{1}{2} m_2 \left[\left(-\frac{l}{2}\dot{\theta}\sin\theta\right)^2 + \left(\frac{l}{2}\dot{\theta}\cos\theta\right)^2 \right] + \frac{1}{2} J_z \dot{\theta}^2$$

$$\therefore J_z = \frac{m_2 l^2}{12}$$

$$K = \frac{1}{2} m_1 \left[(\dot{z})^2\cos^2\theta + (z\dot{\theta})^2\sin^2\theta + \cancel{(-2)(\dot{z}\cos\theta)(z\dot{\theta}\sin\theta)} \right.$$
$$\left. + (\dot{z})^2(\sin^2\theta) + (z\dot{\theta})^2\cos^2\theta + \cancel{2(\dot{z}\sin\theta)(z\dot{\theta}\cos\theta)} \right]$$
$$+ \frac{1}{2} m_2 \left(\frac{l}{2}\dot{\theta}\right)^2 + \frac{1}{2} J_z \dot{\theta}^2$$

$$K.E = \frac{1}{2} m_1 \left[(\dot{z})^2 + (z\dot{\theta})^2 \right] + \frac{1}{2} m_2 \left(\frac{l}{2}\dot{\theta}\right)^2 + \frac{1}{2} J_z \dot{\theta}^2$$

$$= \cancel{\frac{1}{2} m_1 (\dot{z})^2} \; \frac{1}{2} m_1 \dot{z}^2 + \frac{1}{2} m_1 z \dot{\theta}^2 + \frac{1}{2} m_2 \frac{l^2}{4} \dot{\theta}^2 + \frac{1}{2} J_z \dot{\theta}^2$$

$$= \frac{1}{2} m_1 \dot{z}^2 + \frac{1}{2} m_1 z \dot{\theta}^2 + \frac{1}{2} m_2 \frac{l^2}{4} \dot{\theta}^2 + \frac{1}{2}\left(\frac{m_2 l^2}{12}\right)\dot{\theta}^2$$

$$= \cancel{\frac{1}{2} m_1 \dot{z}^2} \; = \frac{1}{2} m_1 \dot{z}^2 + \frac{1}{2}\left[\frac{m_2 l^2}{3} + m_1 z^2\right]\dot{\theta}^2$$

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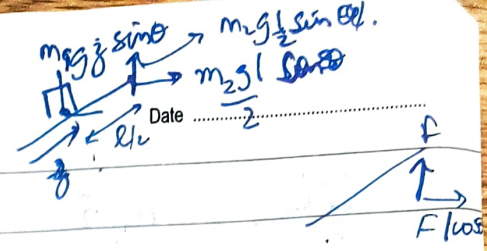

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E3) a) Potential Energy.

Generalised co-ordinates $\Rightarrow q = (z, \theta)^T$. [position]

P_0 = P.E @ $z = 0$, $\theta = 0$

Along Center of mass,

$$P = P_0 + m_1 g z \sin\theta + m_2 g \frac{l}{2} \sin\theta$$

External Forces along $z = \tau_1 = 0$
External " " $\theta = \tau_2 = Fl\cos\theta$

Generalised co-ordinates $\Rightarrow \tau = (0, Fl\cos\theta)^T$ [Forces]

No damping Forces.

$$K(q, \dot{q}) = \frac{1}{2} m_1 \dot{z}^2 + \frac{1}{2}\left[\frac{m_2 l^2}{3} + m_1 z^2\right]\dot{\theta}^2$$

$$= \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2}\left[\frac{m_2 l^2}{3} + m_1 q_1^2\right]\dot{q}_2^2$$

P.E $\Rightarrow$ $P(q) = P_0 = m_1 g q_1 \sin q_2 + m_2 g \frac{l}{2} \sin q_2$

$$L(q, \dot{q}) = \frac{1}{2} m_1 \dot{z}^2 + \frac{1}{2}\left[\frac{m_2 l^2}{3} + m_1 z^2\right]\dot{\theta}^2 - P_0 - m_1 g z \sin\theta$$

$$\cancel{-mg} - m_2 g \frac{l}{2} \sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = \tau_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \tau_2 = Fl\cos\theta.$$

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$$\frac{\partial L}{\partial \dot{z}} = m_1 \dot{z} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) = m_1 \ddot{z}$$

$$\frac{\partial L}{\partial z} = m_1 z \dot{\theta}^2 - m_1 g \sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = \left(\frac{m_2 l^2}{3} + m_1 z^2\right)\dot{\theta} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \left(\frac{m_2 l^2}{3} + m_1 z^2\right)\ddot{\theta} + 2 m_1 z \dot{z} \dot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -m_1 g z \cos\theta - m_2 g \frac{l}{2} \cos\theta$$

Equations of motion:

$$m_1 \ddot{z} - m_1 z \dot{\theta}^2 + m_1 g \sin\theta = 0.$$

$$\left(\frac{m_2 l^3}{3} + m_1 z^2\right)\ddot{\theta} + 2 m_1 z \dot{z} \dot{\theta} + m_1 g z \cos\theta + m_2 \frac{g l}{2} \cos\theta = F l \cos\theta.$$

$$\begin{pmatrix} m_1 & 0 \\ 0 & \frac{m_2 l^2}{3} + m_1 z^2 \end{pmatrix} \begin{pmatrix} \ddot{z} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} m_1 z \dot{\theta}^2 - m_1 g \sin\theta \\ F l \cos\theta - 2 m_1 z \dot{z} \dot{\theta} - m_1 g z \cos\theta - m_2 \frac{g l}{2} \cos\theta \end{pmatrix}$$

D.3 (a,b,c,d).

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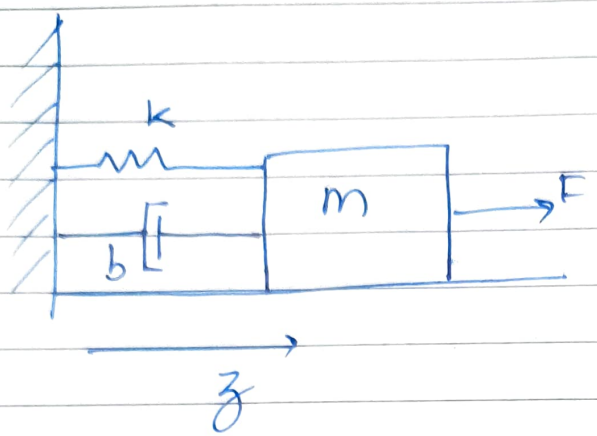


Data:

Spring Const = k

Damping Const = b

Position of spring at rest = 0 (2)

Speed of mass = $\dot{z}$

External force = F.

a) To find P.E.

(b). Generalised coordinates => $q = z$

$\tau = F$

~~Friction~~ Damping $= -Bq = -Bz$

$$K.E = \frac{1}{2} m\dot{z}^2 = \frac{1}{2} m\dot{q}^2$$

(a) P.E (of Spring) $\Rightarrow P(q) = \frac{1}{2} kz^2 = \frac{1}{2} kq^2$

Lagrangian $\Rightarrow L(q,\dot{q}) = K - P = \frac{1}{2} m\dot{z}^2 - \frac{1}{2} kz^2$

$$\frac{\partial L}{\partial \dot{q}} = \frac{2}{2} m\dot{z} = m\dot{z} \quad \Big| \quad \frac{\partial L}{\partial q} = \frac{1}{2} \times 2 \times kz = kz$$

Euler - Lagrange Eq:

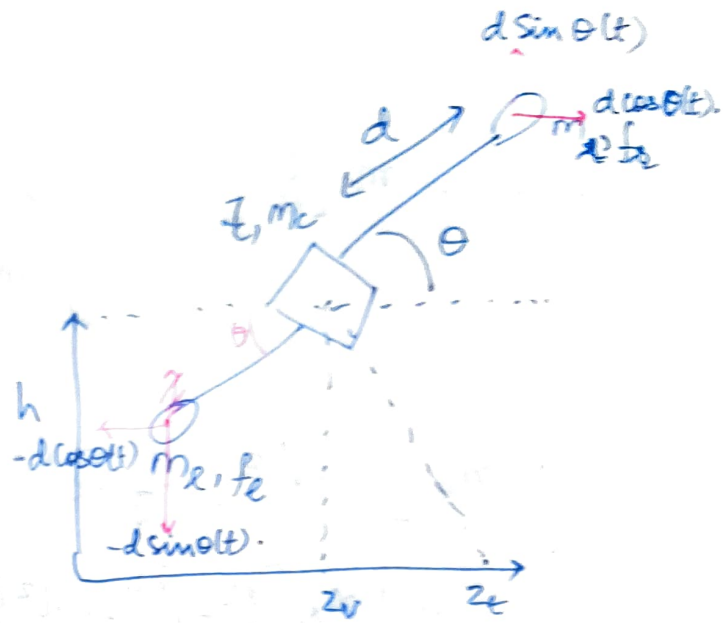
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = \tau - B\dot{q}$$

$$\boxed{m\ddot{z} - kz = F - b\dot{z}}$$

D(2) K.E of Spring Mass Damper $= \frac{1}{2} m\dot{z}^2$

$= \cancel{\frac{1}{2} mq}$

(T 2) a:



Data.

$m_c, J_c, m_r, f_r, m_l, f_l$

Horizontal velocity $= \dot{z}_v$

$F_{drag} = -\eta \dot{z}_v$

$m_r, m_l \rightarrow$ No rotation (fixed).

$m_c \rightarrow$ Can Rotate.

$$K.E = \frac{1}{2} m_c v_c^T v_c + \frac{1}{2} \omega_c^T J_c \omega_c + \frac{1}{2} v_r^T v_r m_r + \frac{1}{2} m_l v_l^T v_l$$

$$P_c = (z(t), h(t), 0)^T \qquad v_c = (\dot{z}, \dot{h}, 0)$$

$$P_r = \begin{pmatrix} z(t) + d\cos\theta(t) \\ z(t) + d\sin\theta(t) \\ 0 \end{pmatrix} \qquad P_r = \begin{pmatrix} \dot{z} + d\dot{\theta}(-\sin\theta) \\ \dot{z} + d\dot{\theta}(\cos\theta) \\ 0 \end{pmatrix}$$

$$P_l = \begin{pmatrix} z(t) + (-d\cos\theta(t)) \\ z(t) + (-d\sin\theta(t)) \\ 0 \end{pmatrix} \qquad P_l = \begin{pmatrix} \dot{z} + d\dot{\theta}(\sin\theta) \\ \dot{z} + d\dot{\theta}(-\cos\theta) \\ 0 \end{pmatrix}$$

$$\omega_c = (0, 0, \dot{\theta})^T$$

$$K.E = \frac{1}{2} m_c\left[\dot{z}^2 + \dot{h}^2\right] + \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2} m_r\left[(\dot{z} - d\dot{\theta}\sin\theta)^2 + (\dot{h} + d\dot{\theta}\cos\theta)^2\right]$$

$$+ \frac{1}{2} m_e\left[(\dot{z} + d\dot{\theta}\sin\theta)^2 + (\dot{h} - d\dot{\theta}\cos\theta)^2\right]$$

$$= \frac{1}{2} m_c\left[\dot{z}^2 + \dot{h}^2\right] + \frac{1}{2} J_c \dot{\theta}^2 + \cancel{\frac{1}{2} m_r\left[\dot{z}^2 + \dot{h}^2 - 2\dot{z}d\dot{\theta}\sin\theta + 2d\dot{\theta}\dot{h}\cos\theta\right]}$$

$$+ \frac{1}{2} m_r\left[\dot{z}^2 + \dot{h}^2 + d^2\dot{\theta}^2 - 2d\dot{z}\dot{\theta}\sin\theta + 2d\dot{h}\dot{\theta}\cos\theta\right]$$

$$+ \frac{1}{2} m_e\left[\dot{z}^2 + \dot{h}^2 + d^2\dot{\theta}^2 + 2d\dot{z}\dot{\theta}\sin\theta + (-2)d\dot{h}\dot{\theta}\cos\theta\right]$$

If $m_r = m_e$ (Same size).

$$K.E = \frac{1}{2}\dot{z}^2\left[m_c + 2m_r\right] + \frac{1}{2}\dot{h}^2\left[m_c + 2m_r\right] + \frac{1}{2}\dot{\theta}^2\left[J_c + 2m_r d^2\right]$$

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F3 (a, b, c, d).

Generalised co-ordinates position $\Rightarrow q = (z, h, \theta)^T$

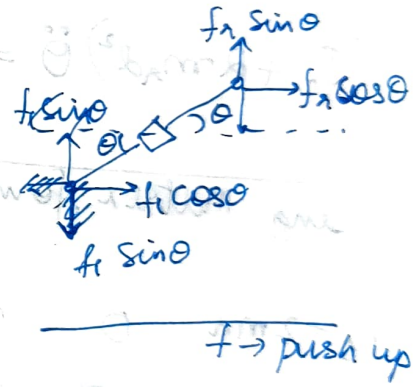
$P.E = \phi P_0$ @ $z=0, h=0, \theta=0$

$P.E = PE_c + PE_r + PE_l$ $(\because m_l = m_r)$

$P = P_0 + m_c g h + m_r g(h + d\sin\theta) + m_l g(h - d\sin\theta)$.

$= P_0 + m_c g h + m_r(2gh) = P_0 + gh(m_c + 2m_r)$.

External forces $\rightarrow f_l, f_r$



$\tau_1 = -f_r \sin\theta - f_l \sin\theta$

$\tau_2 = f_r \cos\theta + f_l \cos\theta$

$\tau_3 = d(f_r - f_l)$

$$-B\dot{q} = \begin{pmatrix} -\mu & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{z} \\ \dot{h} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} -\mu\dot{z} \\ 0 \\ 0 \end{pmatrix}$$

$$K.E = \frac{1}{2}\dot{z}^2[m_c + 2m_r] + \frac{1}{2}\dot{h}^2[m_c + 2m_r] + \frac{1}{2}\dot{\theta}^2[J_c + 2m_r d^2]$$

$$L = K - P = \frac{1}{2}\dot{z}^2[m_c + 2m_r] + \frac{1}{2}\dot{h}^2[m_c + 2m_r] + \frac{1}{2}\dot{\theta}^2[I + 2m_r d^2]$$

$$- P_0 - gh(m_c + 2m_r)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = \tau_1 - \mu \dot{z}$$

$$\frac{\partial}{dt}\left(\frac{dL}{\partial \dot{h}}\right) - \left(\frac{\partial L}{\partial h}\right) = \tau_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) = \tau_3$$

$$\frac{\partial L}{\partial \dot{z}} = (m_c + 2m_r)\dot{z}$$

$$\frac{d}{\partial t}\left(\frac{\partial L}{\partial \dot{z}}\right) = (m_c + 2m_r)\ddot{z}$$

$$\frac{\partial L}{\partial z} = 0$$

$$(m_c + 2m_r)\ddot{z} = -(f_r + f_l)\sin\theta - \mu\dot{z}$$

$$(m_c + 2m_r)\ddot{h} + (m_c + 2m_r)g = (f_r + f_l)\cos\theta$$

$$(J_c + 2m_r d^2)\ddot{\theta} = d(f_r - f_l)$$

$$\frac{\partial L}{\partial \dot{h}} = (m_c + 2m_r)\dot{h}$$

$$\frac{\partial}{dt}\left(\frac{\partial L}{\partial \dot{h}}\right) = (m_c + 2m_r)\ddot{h}$$

$$\cancel{\frac{\partial}{\partial t}}\ \frac{\partial L}{\partial h} = -(m_c + 2m_r)g$$

using matrix form:

$$\begin{pmatrix} m_c + 2m_r & 0 & 0 \\ 0 & m_c + 2m_r & 0 \\ 0 & 0 & J_c + 2m_r d^2 \end{pmatrix} \begin{pmatrix} \ddot{z} \\ \ddot{h} \\ \ddot{\theta} \end{pmatrix}$$

$$= \begin{pmatrix} -(f_r + f_l)\sin\theta - \mu\dot{z} \\ (f_r + f_l)\cos\theta - g(m_2 + 2m_r) \\ d(f_r + (-f_l) \end{pmatrix}$$

$$\frac{\partial L}{\partial \dot{\theta}} = J_c + 2m_r d^2$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = (J_c + 2m_r d^2)\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = 0$$

D-3 (a, b, c, d).

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(6.b). $f[(1,1,a_1)^T] = 3\begin{bmatrix}1\\1\\a_1\end{bmatrix}$

$f[(1,1,a_2)^T] = 3\begin{bmatrix}1\\1\\a_2\end{bmatrix}$.

$f(x_1) + f(x_2) = f(x_1+x_2)$.

6(c). $f(x) = cx$. $c>0$ . . . .

$f(x_1) = c\begin{bmatrix}a_1\\b_1\\0\end{bmatrix}$ $f(x_2) = c\begin{bmatrix}a_2\\b_2\\0\end{bmatrix}$

$f(x_1+x_2) = c\begin{bmatrix}a_1+a_2\\b_1+b_2\\0\end{bmatrix} = f(x_1)+f(x_2)$.

Same for all 3 cases $\begin{cases}(a,b,0)^T\\(0,a,b)^T\\(a,0,b)^T\end{cases}$

If we take from same category

$\therefore$ linear!

As $f(x_1) + f(x_2) = f(x_1+x_2)$

2.(e). $8y \cdot \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = \frac{3du}{dt} + u$

For $f(u_1, y_1)$

$$8y_1 \cdot \frac{d^2y_1}{dt^2} + 2\frac{dy_1}{dt} + y_1 = \frac{3du_1}{dt} + u_1 \quad \text{— (1)}$$

For $f(u_2, y_2)$

$$8y_2 \cdot \frac{d^2y_2}{dt^2} + 2\frac{dy_2}{dt} + y_2 = \frac{3du_2}{dt} + u_2 \quad \text{— (2)}$$

for $f(u_1, y_1) + f(u_2, y_2)$

we can't satisfy superposition.

If $f(u_1+u_2, y_1+y_2)$
we get.

$$8(y_1+y_2)\frac{d^2(y_1+y_2)}{dt} + 2\frac{d(y_1+y_2)}{dt} + (y_1+y_2) = 3\frac{d(u_1+u_2)}{dt} + (u_1+u_2) \quad \text{— (3)}$$

$$f(u_1, y_1) + f(u_2, y_2) \neq f(u_1+u_2, y_1+y_2).$$

$\therefore$ Non-linear.

Also, in $8y\left(\frac{d^2y}{dt^2}\right)$ y is a dependent variable & non-linear

$\therefore$ Non-linear

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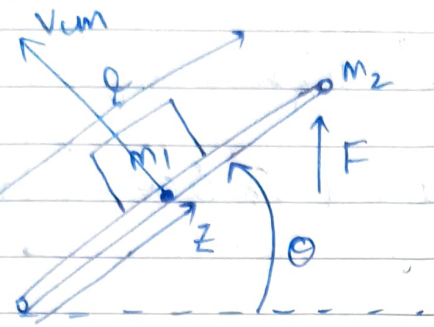
$\theta(t)$

E.2 Block on Beam

(a).



Position of block from pivot $= z$

Speed of block $= \dot{z}$

Angle of beam from level $= \theta$

Angular velocity $= \dot{\theta}$

Mass of Rod $= m_2$

Mass of block $= m_1$

$d\cos\theta(t)$.

f_2

$$K = \sum_{i=1}^{N} \frac{1}{2} m_i V_{cm,i}^T V + \sum_{i=1}^{n} \frac{1}{2} \omega_i^T J_{cm,i} \omega \Big]$$

If block moves without rolling $\omega_{block} = 0$ $\boxed{\omega_{block} = 0}$

$\rightarrow$ Block.

$$K = \frac{1}{2} m_{block} V_{block}^T V_{block} + \frac{1}{2} \omega_{block}^T J_{block} \omega_{block} +$$

$$\frac{1}{2} m_{beam} V_{beam}^T V_{beam} + \frac{1}{2} \omega_{beam}^T J_{beam} \omega_{beam}$$

$$P_{block} = \begin{pmatrix} z(t)\cos\theta(t) \\ z(t)\sin\theta(t) \\ 0 \end{pmatrix} \qquad P_{beam} = \begin{pmatrix} \ell/2 \cos\theta(t) \\ \ell/2 \sin\theta(t) \\ 0 \end{pmatrix}$$

$$\cancel{V_{block} = \begin{pmatrix} \dot{z}\cos\theta \\ \dot{z}\sin\theta \\ 0 \end{pmatrix}} \qquad V_{beam} = \begin{pmatrix} -\ell/2\, \dot{\theta}\sin\theta \\ \ell/2 \cos\theta \\ 0 \end{pmatrix}$$

$$\cancel{V_{beam}} = V_{block} = \begin{pmatrix} \dot{z}\cos\theta - z\dot{\theta}\sin\theta \\ \dot{z}\sin\theta + z\dot{\theta}\cos\theta \\ 0 \end{pmatrix} \qquad \omega_{beam} = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}$$

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Substituting these in K.E equation

$$K = \frac{1}{2} m_1 \left[(\dot{z}\cos\theta - z\dot{\theta}\sin\theta)^2 + (\dot{z}\sin\theta + z\dot{\theta}\cos\theta)^2 \right]$$
$$+ \frac{1}{2} m_2 \left[\left(-\frac{l}{2}\dot{\theta}\sin\theta\right)^2 + \left(\frac{l}{2}\dot{\theta}\cos\theta\right)^2 \right] + \frac{1}{2} J_z \dot{\theta}^2$$

$$\therefore J_z = \frac{m_2 l^2}{12}$$

$$K = \frac{1}{2} m_1 \left[(\dot{z})^2\cos^2\theta + (z\dot{\theta})^2\sin^2\theta + \cancel{(-2)(\dot{z}\cos\theta)(z\dot{\theta}\sin\theta)} \right.$$
$$\left. + (\dot{z})^2(\sin^2\theta) + (z\dot{\theta})^2\cos^2\theta + \cancel{2(\dot{z}\sin\theta)(z\dot{\theta}\cos\theta)} \right]$$
$$+ \frac{1}{2} m_2 \left(\frac{l}{2}\dot{\theta}\right)^2 + \frac{1}{2} J_z \dot{\theta}^2$$

$$\underline{\underline{K.E}} = \frac{1}{2} m_1 \left[(\dot{z})^2 + (z\dot{\theta})^2 \right] + \frac{1}{2} m_2 \left(\frac{l\dot{\theta}}{2}\right)^2 + \frac{1}{2} J_z \dot{\theta}^2$$

$$= \cancel{\frac{1}{2} m_1 (\dot{z})^2} \; \frac{1}{2} m_1 \dot{z}^2 + \frac{1}{2} m_1 z^2 \dot{\theta}^2 + \frac{1}{2} m_2 \frac{l^2}{4} \dot{\theta}^2 + \frac{1}{2} J_z \dot{\theta}^2$$

$$= \frac{1}{2} m_1 \dot{z}^2 + \frac{1}{2} m_1 z^2 \dot{\theta}^2 + \frac{1}{2} m_2 \frac{l^2}{4} \dot{\theta}^2 + \frac{1}{2}\left(\frac{m_2 l^2}{12}\right)\dot{\theta}^2$$

$$= \cancel{\frac{1}{2} m_1 \dot{z}^2} = \frac{1}{2} m_1 \dot{z}^2 + \frac{1}{2}\left[\frac{m_2 l^2}{3} + m_1 z^2\right]\dot{\theta}^2$$