Lab 1: Introduction to Matlab

Objectives

The purpose of this laboratory is to overview the basic Matlab functions in linear systems and control. At the end of this lab, you will be able to plot the step response and the frequency response of a rational transfer function.

1 Modeling linear systems in Matlab

The first step in the design of a control system is to identify a mathematical model of the system. Such a mathematical model could be derived either from physical laws or from experimental data. For LTI systems, we only need to identify the impulse response or it's frequency-domain representation—the transfer function. In this lab, we introduce how to model transfer functions in Matlab.

1.1 Defining a transfer function

There are various methods to define a transfer function in Matlab. As an example, suppose we have a system described by the following constant coefficient linear differential equation:

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_1 \frac{du(t)}{dt} + b_0 u(t)$$

say for $a_2 = 1$, $a_1 = 3$, $a_0 = 2$, $b_1 = 2$, $b_0 = 6$.

We know that the transfer function of this system is

$$H(s) = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

1. The first method of specifying the function is to literally type out this expression. We first define the symbol s using the `tf` function and then define the transfer function H in terms of s.

```
a2 = 1;
a1 = 3;
a0 = 2;
b1 = 2;
b0 = 6;

% Define a system with transfer function s.
s = tf('s');
H1 = (b1*s + b0) / (a2*s^2 + a1*s + a0)
```

2. The second method is to directly specify the polynomials of the numerator and the denominators.

```
num = [b1 b0];
den = [a2 a1 a0];

H2 = tf(num, den)
```

There are various other forms of the `tf` function. Type

```
doc tf
```

to view the complete documentation of the `tf` function.

3. Another way to define the transfer function is using the poles and zeros. Let's reconsider the above transfer function in pole-zero form:

$$H(s) = \frac{2s+6}{s^2+3s+2} = 2\frac{s+3}{(s+1)(s+2)}$$

We can specify the transfer function in the pole zero form using the `zpk` function.

```
k = 2;          % The gain
z = [ -3 ]      % The set of zeros
p = [ -1 -2]    % The set of poles

H3 = zpk(z,p,k);
```

There are other forms of using the `zpk` function. Type

```
doc zpk
```

to view the complete documentation of the `zpk` function.

It is possible to identify the poles and zeros of a rational transfer function using `tf2zp`:

```
[z,p,k] = tf2zp(num,den);
```

1.2 Plotting time response of an LTI system

1. To plot the impulse response of a transfer function, use

```
impulse(H);
```

As an example, plot the impulse response of the system defined in the previous section.

2. To plot the step response of a transfer function, use

```
step(H);
```

As an example, plot the impulse response of the system defined in the previous section.

There are other forms of both the `impulse` and `step` functions. For example,

```
impulse(H,Tfinal)
```

plots the impulse response from $t = 0$ to $t = \text{Tfinal}$ and

```
t = 2:0.01:4;
impulse(H,t)
```

plots the impulse response for the specified t values.

When invoked with output arguments:

```
[y, t] = impulse(H);
[y, t] = impulse{H, Tfinal);
y = impulse(H, t)
```

the function returns the output response y and no plot is drawn. Type

```
doc impulse
```

to view the complete documentation of the `impulse` function.

3. To simulate the time response of the system to an arbitrary input u, use

```
lsim(H, u, t)
```

Type

```
doc lsim
```

to view the complete documentation of the `lsim` function.

1.3 Plotting frequency response of an LTI system

1. To plot the Bode plot of the system, use

```
bode(H)
```

When invoked with output arguments:

```
[mag, phase, wout] = bode(sys)
```

the function returns the magnitude and phase of the response at each frequency in the vector `wout`. The function automatically determines `wout` based on the system dynamics. No plot is drawn.

2. To plot the pole-zero map of the system, use

```
pzmap(H)
```

1.4 Interconnecting systems

1. To get the transfer function for two systems connected in series, use

   ```
   H = series(H1,H2);
   ```

2. To get the transfer function for two systems connected in parallel, use

   ```
   H = parallel(H1,H2);
   ```

3. To get the transfer function for two systems connected in feedback, use

   ```
   H = feedback(H1,H2);
   ```

1.5 Drawing 2D plot

Given two vectors `X` and `Y` of equal length, one can plot the line plot of `X` verus `Y` using

```
plot(X, Y)
```

Consider the following example:

```
t = 0:0.01:4;
u = sin(2*pi*t/4);
plot(t, u);
grid on;
```

The last line adds a grid to the plot. For plotting data with logarithmic scales for the x-axis, use

```
lsim(H,u,t)
```

To generate logarithmically spaced vector with n points between decades 10^a and 10^b, use

```
pzmap(H)
```

2 Lab experiment

Consider a system described by the following differential equation

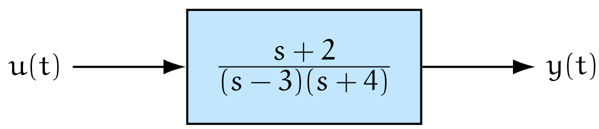
$$\frac{d^2y(t)}{dt^2} + 10.1\frac{dy(t)}{dt} + 101y(t) = 100u(t)$$

1. Write the transfer function of the above system.
2. Using MATLAB find the zeros, poles, and the gain of this system.
3. Find the impulse response and step response of the system.

4. Using MATLAB, identify phase and magnitude of the transfer function using `bode` function. Store the phase and magnitude as vectors. Using MATLAB, plot $20\log(\text{magnitude})$ vs frequency on a semilog plot and plot phase vs frequency on a semilog plot.
5. Using MATLAB, plot the output from $t = 0$ to $t = 5$ when the input is $u(t) = \sin(\omega t)$, where $\omega = \pi$. Note that the output is of the form $A\sin(\omega t + \varphi)$. A is called the gain of the system and φ is called the phase shift. Identify the gain and the phase shift from the MATLAB output.
6. Change the values of the poles, zeros, and the gain of the system and use MATLAB to see what happens to the step response and the bode plot.

3 Lab assignment

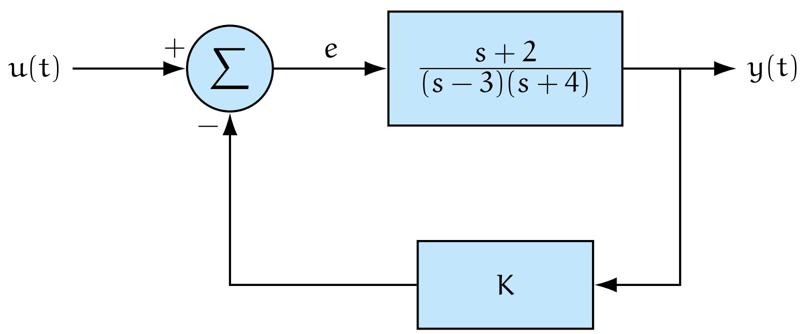
Consider the LTI system shown below:



Open loop system

1. Using MATLAB, draw the step response, impulse response, and the Bode plot of the system. Using these plots, identify whether the system is BIBO stable or not. Explain your answer.

   Now, suppose we connect this system in feedback with a constant gain feedback, as shown below:



Closed loop system

2. Using MATLAB, draw the zero pole map of the closed loop system for values $K \in \{1, 2, 3, \dots, 10\}$.
3. In part 2, there is a critical interval of K of the form $[K_0, K_0 + 1]$ such that for $K = K_0 + 1$ all the poles are in the left hand plane (and therefore the system is stable), but for $K = K_0$ that is not the case. Identify (for accuracy up to the first decimal place), the smallest value of $K \in [K_0, K_0 + 1]$ for which all the poles are in the left hand plane.
4. For the critical value of K identified in part 3, use MATLAB to draw the step response, impulse response, and Bode plot of the closed loop system.