Cournot market competition simulations

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April 23, 2021

https://github.com/ispanos/CournotGame

This file also serves as documentation for the python script contained in the above link and is published under the MIT licence. The format of the script's output is subject to change.

1 Defining our market

We assume the linear Inverse Demand Curve,

$$P = A - B * Q$$

where P is the Price and Q is the total demand.

We also assume that the first derivatives of the Cost Curves of our companies are:

$$MC_i = K_i + M_i q_i$$

where **i** is the number of the company. In our case, in the beginning there are 3 companies.

2 Calculating production levels

2.1 Best Responses

Since the companies are in a **Cournot market game**, each one of them is going to maximize its profits, by adjusting its production, according to the demand and its competitor's production.

$$max\Pi_i(q_i) => MR_i = MC_i \ (1)$$

...

$$for\ i = 1,\ (1)\ =>\ (M_1 + 2 * B) * q_1 + B * q_2 + B * q_3 = A - K_1$$
$$for\ i = 2,\ (1)\ =>\ B * q_1 + (M_2 + 2 * B) * q_2 + B * q_3 = A - K_2$$
$$for\ i = 3,\ (1)\ =>\ B * q_1 + B * q_2 + (M_3 + 2 * B) * q_3 = A - K_3$$

$$Or$$

$$\begin{bmatrix} (M_1 + 2 * B) & B & B \\ B & (M_2 + 2 * B) & B \\ B & B & (M_3 + 2 * B) \end{bmatrix} * \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} A - K_1 \\ A - K_2 \\ A - K_3 \end{bmatrix}$$

3 Merger

3.1 Marginal Cost

After companies i and j merge, we add the two marginal cost curves horizontally:

$$q_m = q_{i(MC_i)} + q_{j(MC_j)},$$

where $q_{m(MC_m)}$ is the inverse marginal cost curve of the new company, named m.

The new company, has now the following marginal cost:

$$MC_m = \frac{M_j * K_i + M_i * K_j}{M_i + M_j} + \frac{M_i * M_j}{M_i + M_j} * q_m$$

$$Or$$

$$MC_m = K_m + M_m * q_m$$

3.2 Best Responses

The new company, and the one that wasn't included in the merger, compete again on the amount of output they will produce.

$$max\Pi_i(q_i) => MR_i = MC_i \ (1)$$

...

$$for\ i = 1,\ (1)\ =>\ (M_m + 2 * B) * q_m + B * q_2 = A - K_m$$
$$for\ i = 2,\ (1)\ =>\ B * q_1 + (M_2 + 2 * B) * q_2 = A - K_2$$

$$Or$$

$$\begin{bmatrix} (M_m + 2 * B) & B \\ B & (M_2 + 2 * B) \end{bmatrix} * \begin{bmatrix} q_m \\ q_2 \end{bmatrix} = \begin{bmatrix} A - K_m \\ A - K_2 \end{bmatrix}$$

3.3 Edge Cases

To reduce the complexity of the python function, that calculates the marginal cost curve of the new company, I'm working under the assumption that the demand is relatively high. The script does not work in cases where the demand is so low that the company is better off using only one of the two facilities at its disposal.

Furthermore, if one of the companies has a constant marginal cost, and the other one has a linear marginal cost, the script terminates. In cases like that, its likely that the new company manufactures only in the facilities with the constant marginal cost. Still, a very high constant marginal cost could be suboptimal, compared to a low, yet variable marginal cost.

These checks can't be done in the `merge_companies()` function without increasing the complexity of the `Company` object. Then, for every 2 companies that have merged, with that specific combination of marginal costs, our calculations would have to increase exponentially.

To manually find if the company is going to produce in both facilities or not, calculate q_m, solve for MC_m and run the function `set_cournot_production(demand, companies)` for all 3 possible MC_m curves. Out of the three possible outcomes, select the marginal cost curve that maximizes the profits of company m.

4 Cournot market game for N companies

We observe that the matrix that solves for the production units of each company, follows a clear pattern.

On the left side, the diagonal, is

$$(MC_i + 2 * B),\ where\ i = 1, 2, 3...N,\ for\ N\ companies$$

On the right side,

$$(A - K_i),\ where\ i = 1, 2, 3...N,\ for\ N\ companies$$

In order to create a function *(in Python)* to calculate the units of production for any number of companies we:

1. Create an N x N matrix, X, where every element is B, the slope of the inverse demand curve
2. Two N x 1 arrays that are composed of the elements mentioned above,

$$X: \begin{bmatrix} B\ B\ B\ ...B \\ B\ B\ B\ ...B \\ ...\ ...\ ... \\ B\ B\ B\ ...B \end{bmatrix},\ D: \begin{bmatrix} MC_1 + 2 * B \\ MC_2 + 2 * B \\ MC_3 + 2 * B \\ ... \\ MC_N + 2 * B \end{bmatrix}\ and\ U: \begin{bmatrix} (A - K_1) \\ (A - K_2) \\ (A - K_3) \\ ... \\ (A - K_N) \end{bmatrix}$$

3. Replace the diagonal of matrix X, with matrix D to create matrix H.
4. Finally, if we solve $H * q = U$ for q, we get the production units for every company competing in the market.

5 Simulations

5.1 Three-way game with one merger

Let's start with 3 companies with the following marginal cost curves:

Company 1: $MC_1 = 2.71 + 5.34 * q_1$,

Company 2: $MC_2 = 6.13 + 1.11 * q_2$,

Company 3: $MC_3 = 4.75 + 1.53 * q_3$

With an inverse demand curve : $P = 2221.08 - 15.81 * Q$

```
Company 1 with Mc = 2.71 + 5.34 * q
        Produces 29.25 units  with €13529.37 profit.

Company 2 with Mc = 6.13 + 1.11 * q
        Produces 36.36 units  with €20906.58 profit.

Company 3 with Mc = 4.75 + 1.53 * q
        Produces 35.56 units  with €19995.47 profit.

Total production is 101.18 units @ €621.41.
HHI:3363
```

* * *

Now lets simulate the mergers of the above companies, one by one.

```
The sum of the profits of companies 1 and 2
         before the merger were: €34435.96

Company 1&2 with Mc = 5.54 + 0.92 * q
        Produces 46.34 units  with €33954.67 profit.

Company 3 with Mc = 4.75 + 1.53 * q
        Produces 44.76 units  with €31668.42 profit.

Total production is 101.18 units @ €780.81.
HHI:5002
The new price is 26.0% higher.
```

* * *

```
The sum of the profits of companies 1 and 3
         before the merger were: €38063.87
```

```
Company 1&3 with Mc = 4.3 + 1.19 * q
        Produces 45.56 units  with €32817.94 profit.

Company 2 with Mc = 6.13 + 1.11 * q
        Produces 45.67 units  with €32969.39 profit.

Total production is 101.18 units @ €778.79.
HHI:5000
The new price is 25.0% higher.
```

* * *

```
The sum of the profits of companies 2 and 3
         before the merger were: €50317.11

Company 2&3 with Mc = 5.55 + 0.64 * q
        Produces 49.67 units  with €39004.7 profit.

Company 1 with Mc = 2.71 + 5.34 * q
        Produces 38.77 units  with €23769.22 profit.

Total production is 101.18 units @ €822.78.
HHI:5076
The new price is 32.0% higher.
```

5.2 Merger Paradox

As we simulate the mergers of two companies, by adding their $q_{(mc)}$ horizontally, we can observe that the resulting companies produce fewer units. The competing companies are now fewer, thus the HHI index increases after the merger. The new equilibrium is closer to the equilibrium of a monopoly. However, the profits of the newly created company are less than the sum of the profits of the companies that merged. None of the above mergers are profitable, and the companies would rather compete than merge.

The company that benefits from the merger is the one that did not take part in it. This happens because both its market share, and the market price, increase.

The conclusion is that neither the consumers, nor the companies that took part in the merger, benefit from the merger. The only beneficiary is the company that did not take part in the merger.

5.3 Consecutive mergers with 7 companies

Now, lets simulate a market comprized of more comprised, given that we are able to add as many companies as we want in the simulation. The demand in the market is the same as before.

```
Company 1 with Mc = 2.71 + 5.34 * q
        Produces 14.85 units  with €3484.77 profit.

Company 2 with Mc = 6.13 + 1.11 * q
        Produces 18.36 units  with €5326.99 profit.
```

```
Company 3 with Mc = 4.75 + 1.53 * q
        Produces 17.99 units  with €5117.23 profit.

Company 4 with Mc = 1.0 + 3.4 * q
        Produces 16.43 units  with €4270.29 profit.

Company 5 with Mc = 4.0 + 2.0 * q
        Produces 17.56 units  with €4874.07 profit.

Company 6 with Mc = 5.0 + 1.6 * q
        Produces 17.9 units  with €5068.04 profit.

Company 7 with Mc = 4.0 + 2.2 * q
        Produces 17.36 units  with €4766.41 profit.

Total production is 120.45 units @ €316.71.
HHI:1435
```

* * *

Company 1 and Company 2 merge:

```
The sum of the profits of companies 1 and 2
        before the merger, were: €8811.76

Company 1&2 with Mc = 5.54 + 0.92 * q
        Produces 20.77 units  with €6822.04 profit.

Company 3 with Mc = 4.75 + 1.53 * q
        Produces 20.09 units  with €6378.68 profit.

Company 4 with Mc = 1.0 + 3.4 * q
        Produces 18.33 units  with €5309.77 profit.

Company 5 with Mc = 4.0 + 2.0 * q
        Produces 19.6 units  with €6072.53 profit.

Company 6 with Mc = 5.0 + 1.6 * q
        Produces 19.99 units  with €6318.41 profit.

Company 7 with Mc = 4.0 + 2.2 * q
        Produces 19.38 units  with €5938.4 profit.

Total production is 118.16 units @ €353.05.
HHI:1669
The new price is 11.0% higher.
```

* * *

Company 1 and Company 2 and 3 merge:

```
The sum of the profits of companies 1&2 and 3
        before the merger, were: €13200.72

Company 1&2&3 with Mc = 5.24 + 0.57 * q
        Produces 24.7 units  with €9642.67 profit.

Company 4 with Mc = 1.0 + 3.4 * q
        Produces 21.28 units  with €7162.31 profit.

Company 5 with Mc = 4.0 + 2.0 * q
        Produces 22.79 units  with €8210.76 profit.

Company 6 with Mc = 5.0 + 1.6 * q
        Produces 23.26 units  with €8550.1 profit.

Company 7 with Mc = 4.0 + 2.2 * q
        Produces 22.54 units  with €8029.42 profit.

Total production is 114.56 units @ €409.87.
HHI:2005
The new price is 16.0% higher.
```

* * *

Company 1 and Company 2, 3 and 4 merge:

```
The sum of the profits of companies 1&2&3 and 4
        before the merger, were: €16804.98

Company 1&2&3&4 with Mc = 4.63 + 0.49 * q
        Produces 29.27 units  with €13545.94 profit.

Company 5 with Mc = 4.0 + 2.0 * q
        Produces 26.83 units  with €11378.06 profit.

Company 6 with Mc = 5.0 + 1.6 * q
        Produces 27.39 units  with €11857.11 profit.

Company 7 with Mc = 4.0 + 2.2 * q
        Produces 26.53 units  with €11126.76 profit.

Total production is 110.01 units @ €481.78.
HHI:2504
The new price is 18.0% higher.
```

* * *

Company 1 and Company 2, 3, 4 and 5 merge:

```
The sum of the profits of companies 1&2&3&4 and 5
        before the merger, were: €24924.01

Company 1&2&3&4&5 with Mc = 4.51 + 0.39 * q
        Produces 36.36 units  with €20906.78 profit.

Company 6 with Mc = 5.0 + 1.6 * q
        Produces 33.82 units  with €18081.11 profit.

Company 7 with Mc = 4.0 + 2.2 * q
        Produces 32.75 units  with €16953.88 profit.

Total production is 102.93 units @ €593.77.
HHI:3340
The new price is 23.0% higher.
```

* * *

Company 1 and Company 2, 3, 4, 5 and 6 merge:

```
The sum of the profits of companies 1&2&3&4&5 and 6
        before the merger, were: €38987.89

Company 1&2&3&4&5&6 with Mc = 4.6 + 0.32 * q
        Produces 48.08 units  with €36541.23 profit.

Company 7 with Mc = 4.0 + 2.2 * q
        Produces 43.08 units  with €29343.08 profit.

Total production is 91.16 units @ €779.89.
HHI:5015
The new price is 31.0% higher.
```

5.3.1 Table of total profits

Companies	Not merged	Merged
1, 2	€8811.76	€6822.04
1, 2, 3	€13928.99	€9642.67
1, 2, 3, 4	€18199.28	€13545.94
1, 2, . . . , 5	€23073.35	€20906.78
1, 2, . . . , 6	€28141.39	€36541.23

Only after 5 consecutive mergers did we see an increase in the profitability of the new company, compared to the pre-merge conditions. The new company, named "1&2&3&4&5&6" in the above code-block, has a €36541.23 profit. The above mentioned event is mainly attributable to the significant decline of competition. The HHI index is now three times higher than before. The price is more than two times higher, and the production is 25% lower.

Furthermore, the mergers that include less than 6 companiesare not profitable, hence the companies would rather compete than merge.

In a real market however, a merger like that would create a huge dead weight loss and such a price increase, that no committee would ever allow such a merger to take place.

5.4 Non symmetrical costs

A final simulation we can study would be one with companies that have asymmetric costs.

```
Company 1 with Mc = 26.71 + 8.34 * q
        Produces 26.08 units  with €10755.49 profit.

Company 2 with Mc = 4.0 + 2.0 * q
        Produces 36.64 units  with €21227.61 profit.

Company 3 with Mc = 4.1 + 2.2 * q
        Produces 36.23 units  with €20752.41 profit.

Total production is 98.95 units @ €656.6.
HHI:3406
```

* * *

When Company 1 and Company 2 merge:

```
The sum of the profits of companies 1 and 2
         before the merger were: €31983.1

Company 1&2 with Mc = 8.39 + 1.61 * q
        Produces 45.52 units  with €32757.21 profit.

Company 3 with Mc = 4.1 + 2.2 * q
        Produces 44.27 units  with €30990.0 profit.

Total production is 98.95 units @ €801.47.
HHI:5001
The new price is 22.0% higher.
```

* * *

When Company 1 and Company 3 merge:

```
The sum of the profits of companies 1 and 3
         before the merger were: €35331.78
```

```
Company 1&3 with Mc = 8.82 + 1.74 * q
        Produces 45.12 units  with €32179.76 profit.

Company 2 with Mc = 4.0 + 2.0 * q
        Produces 44.73 units  with €31631.46 profit.

Total production is 98.95 units @ €800.63.
HHI:5000
The new price is 22.0% higher.
```

* * *

When Company 2 and Company 3 merge:

```
The sum of the profits of companies 2 and 3
         before the merger were: €49765.39

Company 2&3 with Mc = 4.05 + 1.05 * q
        Produces 51.07 units  with €41232.02 profit.

Company 1 with Mc = 26.71 + 8.34 * q
        Produces 34.71 units  with €19046.77 profit.

Total production is 98.95 units @ €864.94.
HHI:5182
The new price is 32.0% higher.
```

* * *

If companies 1 and 2 merge, the merger is profitable. However, the constant part of $MC_{1\&2}$ is almost double than that of MC_2, so we have to make sure that the merged company is actually producing only in the facilities of company 2. So we use $MC_2 = MC_{1\&2}$

```
Company 2 with Mc = 4.0 + 2.0 * q
        Produces 45.01 units  with €32036.01 profit.

Company 3 with Mc = 4.1 + 2.2 * q
        Produces 44.51 units  with €31320.63 profit.

Total production is 89.52 units @ €805.71.
HHI:5000
```

Company "1&2" produces in both facilities, because the profits are now lower than before.

The reason that Company 1 and Company 2's merger is profitable, is due to the asymmetry of the marginal costs. The price increased by 22%, and the third company is almost 50% more profitable. The total production is exactly the same. All in all, the merger is probably going to be prevented by the competition committee.