B V Khadiravana

Theory Assignment - 01

Game - Theory

Q1) Given Pay-off Matrix:-

	B1	B2
A1	-1, 1	0, 0
A2	3, -3	2, -2
A3	4, -4	-1, 1

Pay off Matrix.

Observations : zero- sum game.

→ For utilities given in the above pay-off matrix, ~~neither~~ their is $\boxed{NO}$ Dominant strategy equilibrium since, there ~~dos~~ is no ~~Dominant~~ ~~stro~~ Strongly or even weakly or very weakly dominating strategy for either row player or column player.

→ Pure Mini-Max strategies.:-

(*) Saddle point.

	B1	B2	min of row i
A1	-1	0	-1
A2	3	2	2
A3	4	-1	-1
max of col j	4	2	

$\boxed{(A_2, B_2)}$ is a pure - mini-max strategy since A_2 is the saddle point for the matrix that represents ~~as~~ row players utility alone.

The row player is trying to maximize his gain while the column player is trying to minimize his loss.

Q5) MSNE for the following game :-

		q B1	$1-q$ B2
p	A1	5, 3	3, 4
$1-p$	A2	4, 5	4, 3

MSNE :-

COND 1) :- $3p + 5(1-p) = 4p + 3(1-p)$

$\Rightarrow -2p + 5 = p + 3$

$\boxed{p = \frac{2}{3}}$

COND 2 :-

$5q + 3(1-q) = 4q + 4(1-q)$

$2q + 3 = 4$

$\boxed{q = 1/2}$

∴ strategy for ~~Row player~~

MSNE = $((2/3, 1/3), (1/2, 1/2))$

Q2) Prove that every dominant strategy equilibrium of a two-player zero-sum game is a saddle point of the matrix.

Sol^n:-

let (S_i^*, S_j^*) be a dominant strategy equilibria.

$$\Rightarrow \; u_1(S_i^*, S_j) > u_1(S_i, S_{-i}) \quad \forall i \in N \setminus i^*$$

$$\Rightarrow \; u_1(S_i^*, S_j) = \max_i (S_i, S_{-i}) \quad \forall i \in N$$

$$\Rightarrow \; u_1(S_i^*, S_j^*) = \max_i (S_i, S_j^*) \rightarrow (1)$$

where (i) represents number of strategies for the row player

Similarly,

$$u_2(S_j^*, S_j) > u_2(S_j, S_j) \; ; \; \forall j \in N \setminus j^*$$

But we know,

$$u_1(S) + u_2(S) = 0.$$

$$\Rightarrow \; -u_2(S_j^*, S_{-j}) < -u_2(S_j, S_{-j}) \quad \forall j \in N / j^*$$

$$\Rightarrow \; u_1(S_i, S_j^*) < u_1(S_i, S_j)$$

$$\Rightarrow \; u_1(S_i^*, S_j^*) = \min_j (u_1(S_i, S_j)) \rightarrow (2).$$

Since there exists a dominant strategy equilibria

(1), (2) $\Rightarrow$ that (S_i^*, S_j^*) are a saddle point.

Q3) T.P.

$$u_i(\sigma_i, \sigma_{-i}) = \sum_{s_i \in S_i} \sigma_i(s_i) u_i(s_i, \sigma_{-i})$$

Sol^n

We know that,

$$u_i(s_i, \sigma_{-i}) = \sum_{s_{-i} \in S_{-i}} \left(\prod_{j \neq i} \sigma_j(s_j) \right) u_i(s_i, s_{-i}) \rightarrow (1)$$

$$\Rightarrow u_i(\sigma_i, \sigma_{-i}) = \sum_{(s_1, s_2, \ldots s_N) \in S} \left(\prod_{j \in N} \sigma_j(s_j) \right) u_i(s_i, s_{-i})$$

$$= \sum_{s_1 \in S_1} \sum_{s_2 \in S_2} \cdots \sum_{s_n \in S_n} \left(\prod_{j \in N} \sigma_j(s_j) \right) u_i(s_i, s_{-i})$$

$$= \sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \left(\prod_{j \neq i} \sigma_j(s_j) \right) \sigma_i(s_i) u_i(s_i, s_{-i})$$

$$= \sum_{s_i \in S_i} \sigma_i(s_i) \left\{ \sum_{s_{-i} \in S_{-i}} \left(\prod_{j \neq i} \sigma_j(s_j) \right) u_i(s_i, s_{-i}) \right\}$$

$\therefore$ From (1) $\Rightarrow$

$$u_i(\sigma_i, \sigma_{-i}) = \sum_{s_i \in S_i} \sigma_i(s_i) u_i(s_i, \sigma_{-i}).$$

Q4) Robot Game :-

$N = \cancel{Q}\{1, 2\}$ (2 players) (Robots)

Actions :- {GO, Divert}

Pay off - Matrix :-

	GO	DIVERT
GO	-COC, -COC	0, -COD
DIVERT	-COD, 0	-COD, -COD

Cost of Action 'GO' = 0.

Cost of Action 'DIVERT' = COD

Cost of ~~Action~~ Collision = COC.

Also,

$\boxed{COC > COD}$

Check if (GO, GO) is PSNE :

$U_1(GO, GO) = -COC$

$U_1(DIVERT, GO) = -COD.$

$U_1(GO, GO) < U_1(Divert, GO) \Rightarrow$ NOT PSNE.

But (GO, DIVERT) :-

$U_1(GO, DIVERT) > U_1(Divert, Divert)$ &

$U_2(GO, Divert) > U_2(GO, GO)$

∴ (GO, Divert) is a PSNE.

~~For Divis~~ Similarly (Divert, GO) is a PSNE (since utilities are symmetric)

For (Divert, Divert) :-

$U_2(Divert, Divert) < U_2(Divert, GO)$

∴ NOT PSNE.

Therefore PSNEs for the Robots are: (GO, Divert), (Divert, GO)