Radial Gap Conversion

Given a radial gap G left over from Gap Simplification, and the two terminal points are known with polar coordiante ζ_l, ζ_r in egoframe. Convert polar coordiante to coordiante p_l, p_r. Compose $g_l, g_r \in SE(2)$ with identity rotation component, and translation component equals respective coordinate points.

Given $\eta = \arctan(\epsilon^2, \epsilon^1) \Rightarrow \mathbb{J} = R(\eta) \in \mathfrak{se}(2)$. Note that for anchored centre of rotation on the left, η need to be taken its negative value. Make T be the $SE(3)$ element with $\mathbb{J}$ as its rotation component and zero translation. For the case where rotation is centered around the right gap point:

$$g = g_r \circ T \circ g_r^{-1} \circ g_l \tag{1}$$

The translation component of g denotes the location of rotated point, convert to new polar coordinate with r_g, θ_g. Find corresponding scan index

$$i_g = n \frac{\theta_g}{2\pi} + n_{\text{offset}}$$

where n is the number of scan points in $\mathcal{L}$.

Similarly, retrieve the index (i_r) for anchor of rotation, then do linear search for the following

$$j = \arg\min_{k \in [i_g, i_r]} d(\zeta_k, \zeta_r)$$

Then for the found ζ_j, find $r_{\text{arc}} = d(\zeta_j, \zeta_r)$. Normalize the translation component of $g_r^{-1} \circ g_l$ to this length, and recompute equation 1 to get the true rotated gap point taken the possible additional scan inbetween.

Radial Extension Given a radial gap G left over from Gap Simplification, and the two terminal points are known with polar coordiante ζ_l, ζ_r in egoframe. Convert polar coordiante to coordiante p_l, p_r. The minimum safe distance be s. Define `pol2car()` and `car2pol()` converts (x, y) and (r, θ) back and forth.

$$\begin{aligned}
\mathbf{e}_l &= \mathbf{p}_l/||\mathbf{p}_L||, \mathbf{e}_r/||\mathbf{p}_r|| \\
\mathbf{e}_b &= (\mathbf{e}_l + \mathbf{e}_r), \mathbf{e}_b/ = ||\mathbf{e}_b|| \\
\mathbf{q}_b &= -s\mathbf{e}_b
\end{aligned}$$

We can now shift back the frame using $\mathbf{q}_b$ and conduct operations.

$$\begin{aligned}
qLp &= \mathbf{p}_l - \mathbf{q}_b, qRp = \mathbf{p}_r - \mathbf{q}_b \\
pLp &= \texttt{car2pol}(qLp), pRp = \texttt{car2pol}(qRp) \\
\phi_b &= (pRp - pLp)_\theta \\
\mathbf{p}_b &= \texttt{car2pol}(-\mathbf{q}_b) \\
\theta_l &= \mathbf{p}_{b,\theta} - \arccos(\mathbf{e}_l \cdot \mathbf{e}_r)/4, \theta_r = \mathbf{p}_{b,\theta} + \arccos(\mathbf{e}_l \cdot \mathbf{e}_r)/4,
\end{aligned}$$

Now define function

$$\texttt{pTheta}(\theta, \phi, pRp, pLp) = pLp(\theta - pRp_\theta)/\phi + pRp(pLp_\theta - \theta)/\phi$$

Apply function `pTheta` over $\theta_{l/r}$ and ϕ_b, pLp, pRp to get polar coordiantes pLn, pRn in shifted frame.

$$p_{L/R} = \texttt{car2pol}(\texttt{pol2car}(p\{L/R\}n) + \mathbf{q}_b)$$

$p_{L/R}$ are the radial extended gap points in polar coordinates in the original robot frame.

Algorithm 1: Gap Simplification

Input: $\mathcal{G}, c_a, c_d$
Result: Set of Reduced Gaps
$\mathcal{G}' \leftarrow \varnothing$;
$s \leftarrow$ True;
for $G \in \mathcal{G}$ **do**
  if s ***and*** g *is Swept* ***and*** g *is Left* **then**
    $s =$ False;
    $\mathcal{G}' \leftarrow \mathcal{G}' + G$;
  else
    if s ***or*** g *is Left* **then**
      $\mathcal{G}' \leftarrow \mathcal{G}' + G$;
    else
      if g *is Swept* **then**
        if $\mathcal{G}'_{end}$ *is Left Radial* ***and*** $|G.l_l - \mathcal{G}'_{end}.l_r| < c_d$ **then**
          Merge G with $\mathcal{G}'_{\text{end}}$;
        else
          $\mathcal{G}' \leftarrow \mathcal{G}' + G$;
        end
      else
        $i \leftarrow -1$;
        for $j \leftarrow Reversed(range(\mathcal{G}'.size))$ **do**
          $G' \leftarrow \mathcal{G}'[j]$;
          $\Delta\theta = |G'.\theta_l - \mathcal{G}'_{\text{end}}.\theta_r|$, $\Delta r = |G'.l_l - \mathcal{G}'_{\text{end}}.l_r|$;
          if $\Delta\theta < c_a$ ***and*** $\Delta r < c_d$ **then** $i \leftarrow j$;
        end
        if $i \neq -1$ **then**
          Erase $\mathcal{G}'$ between i and end;
          Merge G with $\mathcal{G}'_{\text{end}}$;
        else
          $\mathcal{G}' \leftarrow \mathcal{G}' + G$;
        end
      end
    end
  end
end
return $\mathcal{G}'$