Intuition behind the scaling of durations

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Let $\{X(t)\}_{t\geq 0}$ be a continuous H-sssi process. Define the crossing duration as

$$w_n(t) = \inf\bigl\{s \geq t : |X(s) - X(t)| \geq 2^n \delta\bigr\} - t\,,$$

for some $\delta > 0$ and $t \geq 0$. By stationarity of increments and self-similarity of $\{X(t)\}$ for any $n \geq 0$:

$$X(t+w_n) - X(t) \underset{\text{si}}{\overset{\mathscr{D}}{\sim}} X(w_n) \underset{\text{ss}}{\overset{\mathscr{D}}{\sim}} w_n^H X(1)\,.$$

(do ss ans si generalise to stopping times?).

Since $\{X(t)\}$ is continuous

$$X(t+w_n) - X(t) = \delta 2^n \text{ a.s}\,,$$

whence almost surely

$$\frac{1}{\delta 2^n}\bigl(X(t+w_n) - X(t)\bigr) = \frac{1}{\delta}\bigl(X(t+w_0) - X(t)\bigr)\,.$$

Thus we get

$$\frac{1}{\delta}2^{-n} w_n^H X(1) \overset{\mathscr{D}}{\sim} \frac{1}{\delta} w_0^H X(1)\,,$$

which implies that (not sure here as well...)

$$2^{-n} w_n^H \overset{\mathscr{D}}{\sim} w_0^H$$

and

$$w_0 \overset{\mathscr{D}}{\sim} 4^{-\frac{n}{2H}} w_n$$