Chapter 1

Symmetry and reversibility in hydrodynamic dispersion

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30. October

This report describes the master project for Ivar Svalheim Haugerud, supervised by professor Eirik Grude Flekkøy at PoreLab, university of Oslo. The work described will be performed in the period from January 2020 to May 2021. The thesis will be presented for the degree of master of science.

The work will look into the reversibility of diffusion in fluid flow in complex geometries. Low Reynolds numbers allows for reversible velocity fields which combined with diffusion makes it possible to introduce a notion of reversibility for the movement of a passive tracer in the fluid flow. This reversibility is called the reciprocity relation, and will be studied in this project. The conditions, limitations and applications of the reciprocity relation will be investigated analytically and numerically. Analytically the convection–diffusion equation will be studied, a continuation of results derived by E. G. Flekkøy [1]. This will be tested numerically for different situations with the Lattice-Boltzmann algorithm. The numerical and analytical results can be compared.

Description

Motivation

Hydrodynamic dispersion is the irreversible spreading of a passive tracer by the combined effect of diffusion and fluid flow. In the case of dispersion on a reversible velocity field, the concentration measured at one point in the flow in response to a concentration pulse at another point will be the same if the two points are interchanged while simultaneously reversing the direction of the velocity field [1]. This is called the reciprocity relation and is true regardless of the magnitude of molecular diffusion.The reciprocity relation technique may have interesting medical, biological and industrial applications. The reciprocity relation also allows for prediction and placement of a desired amount of a medical preparation within some tissue of the body which is otherwise inaccessible. For example within the lymph system or any closed cavity where a slow elastic expansion and contraction would drive a reversible flow. As the withdrawn profile from an injection of a tracer in a system can give information on it's interior structure the reciprocity relation can also be utilized as an echo experiment.

Methods

The work performed will be both theoretical and numerical. The analytical investigation will be done on the convection-diffusion equation, from which the reciprocity relation is derived [1]. The derivation done by E.G. Flekkøy [1] will be investigated further.
The Lattice Boltzmann method solves the discretized Boltzmann equation by evolving the one-particle distribution functions. This allows for the evolution of the convection-diffusion equation to be investigated numerically. Using the Lattice Boltzmann method the reciprocity relation can be studied for different geometries, diffusivities and velocity fields.

Points to investigate

The limitations of the reciprocity relation on reynolds number can be studied both numerically and analytically. Numerically one can simulate the concentration profile for a variety of velocity fields. How the concentration peak, width and error depends on the reynolds numbers and geometries can be investigated using the Lattice Boltzmann algorithm. This can possibly be studied analytically as well, by working with Navier Stokes for non-zero reynolds numbers, resulting in a non-reversible flow. Comparisons between the analytical and numerical result can be drawn and analyzed.

A question to investigate can be if the reciprocity relation holds true for different diffusivities when swapping the sign of the velocity field. If not can it be compensated by altering the magnitude of the velocity field as well? Maybe changing the advective and diffusive transport rates, while keeping their ratio the same?

Different methods for injection can be tested to find which maximize the concentration and minimize the spread at the receptor point.

The derivation by E.G. Flekkøy can be studied further looking for generalizations. The result can be studied for different diffusivities, reynolds number and geometries. How the reciprocity relation deviates for non-reversible flows for different reynolds number can be examined. A microscopic description using statistical mechanics can also be explored.

The reciprocity relation allows for the variance in position of a concentration in the fluid flow to decrease. The entropy process during this process can be investigated.

Work plan

Fall 2019

- FYS4420 - Experimental techniques in condensed matter physics
  FYS4430 - Condensed matter physics II
  FYS4465 - Dynamics of complex media
- Writing project description and literature study

Spring 2020

- FYS4130 – Statistical Mechanics
  FYS4160 – The General Theory of Relativity
- Literature study
- Implementing the Lattice Boltzmann algorithm, and reproducing the reciprocity relation

Fall 2020

- FYS4110 – Modern quantum mechanics
- Analytical investigation into the dependency on reynolds number

Spring 2021

- Convection
- Writing

Supervision suggestions

Signatures

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