Symmetry and reversibility in hydrodynamic dispersion

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This report describes the master project for Ivar Svalheim Haugerud, supervised by professor Eirik Grude Flekkøy at PoreLab, university of Oslo. The work described will be performed in the period from January 2020 to May 2021 and will be presented for the degree of master of science.

The work will look into the reversibility of diffusion in fluid flow in complex geometries. Low Reynolds numbers allows for reversible velocity fields which combined with diffusion makes it possible to introduce a notion of reversibility for the movement of a passive tracer in the fluid flow. This reversibility is called the reciprocity relation, and will be studied in this project. The conditions, limitations and applications of the reciprocity relation will be investigated analytically and numerically. Analytically the convection–diffusion equation will be studied, a continuation of the results derived by E. G. Flekkøy [1]. This will be combined with a numerical investigation with the Lattice-Boltzmann algorithm. The numerical and analytical results will be compared.

Description

Motivation

Hydrodynamic dispersion is the irreversible spreading of a passive tracer by the combined effect of diffusion and fluid flow. In the case of dispersion on a reversible velocity field, the concentration measured at one point in the flow in response to a concentration pulse at another point will be the same if the two points are interchanged while simultaneously reversing the direction of the velocity field [1]. This is called the reciprocity relation and is true regardless of the magnitude of molecular diffusion.
The reciprocity relation technique may have interesting medical, biological and industrial applications, as it allows for prediction and placement of a tracer inside an otherwise inaccessible region. In medicine this can be the placement of a medical preparation within some tissue of the body. For example within the lymph system or any closed cavity where a slow elastic expansion and contraction would drive a reversible flow.
As the withdrawn profile from an injection of a tracer in a system can give information on it's interior structure the reciprocity relation can also be utilized as an echo experiment due to it's time symmetry.
The spreading of concentrations in a fluid is a fundamental and frequently occurring phenomena. A better understanding of the symmetry and time reversibility in such a process has an intrinsic value.

Methods

The work performed will be both theoretical and numerical. The analytical investigation will be done on the convection-diffusion equation, from which the reciprocity relation is derived. The derivation done by E.G. Flekkøy [1] will be investigated further.
The Lattice Boltzmann method solves the discretized Boltzmann equation by evolving the one-particle distribution functions. This allows for the evolution of the convection-diffusion equation to be investigated numerically. Using the Lattice Boltzmann method the reciprocity relation can be studied for different geometries, diffusivities and velocity fields.

Points to investigate

The limitations of the reciprocity relation on reynolds number can be studied both numerically and analytically. How the concentration peak, width and error depends on the reynolds numbers and geometry can be investigated using the Lattice Boltzmann algorithm. This can be studied analytically as well by working with Navier Stokes for non-zero reynolds numbers, resulting in a non-reversible flow. How the reciprocity relation deviates for non-reversible flows for different reynolds number can be examined. The reciprocity relations relationship to Onsager symmetries can be investigated. A microscopic description using statistical mechanics can also be explored. The derivation by E.G. Flekkøy [1] can be studied further looking for generalizations. Comparisons between the analytical and numerical result can be drawn and analyzed.

A question to investigate can be if the reciprocity relation holds true for different diffusivities when swapping the sign of the velocity field. If not it would be interesting to see if it can be compensated, by for example altering the magnitude of the velocity field. Perhaps changing the advective and diffusive transport rates, while keeping their ratio the same will produce the same concentration profile?

The reciprocity relation has only been studied numerically in two dimensions. If it translates to three dimensions can be studied using the lattice Boltzmann method. Which effects the increase in the number of dimensions have on the concentration profile can be investigated.

The reciprocity relation allows for the variance in position of a concentration in the fluid flow to decrease. The role of entropy during this process can be studied. Different methods for injection can be tested to find which maximize the concentration and minimize the spread at the receptor point.

Work plan

Fall 2019

- FYS4420 - Experimental techniques in condensed matter physics
  FYS4430 - Condensed matter physics II
  FYS4465 - Dynamics of complex media

- Writing project description and literature study

Spring 2020

- FYS4130 – Statistical Mechanics
  FYS4160 – The General Theory of Relativity
  10 credits in master thesis

- Literature study
- Implementing the Lattice Boltzmann algorithm, and reproducing previous results
- Begin testing the reciprocity relation for different velocity fields

Fall 2020

- FYS4110 – Modern quantum mechanics
  20 credits in master thesis
- Analytical investigation into the dependency of the reciprocity relation on reynolds number
- Simulate the reciprocity relation for different diffusivities, geometries and reynolds numbers
- Study the role of entropy
- Begin writing thesis

Spring 2021

- 30 credits in master thesis
- Compare analytical and numerical results
- Conclude
- Finish writing thesis

Signatures

Flekkøy will be the main supervisor, with Måløy and Linga as co-supervisors.

Ivar Svalheim Haugerud: __________

Eirik Grude Flekkøy: __________

Knut Jørgen Måløy: __________

Gaute Linga: __________

Sources

1. Symmetry and reversibility in mixing fluids - E.G.Flekkøy, Physics of Fluids 9, 3595 (1997); https://doi.org/10.1063/1.869497