The reciprocity relation: A study of symmetry and reversibility in hydrodynamic dispersion

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This report describes the master project for Ivar Svalheim Haugerud, supervised by professor Eirik Grude Flekkøy and veileder2 at PoreLab, university of Oslo. The work described will be performed in the period from January 2020 to May 2021. The thesis will be presented for the degree of master of science.

The work will look into the reversibility of diffusion in miscible fluid flow in complex geometries. The study is both analytical and numerical. Analytically the Boltzmann equation will be used to ... The Boltzmann equation will be studied numerically with the Lattice Boltzmann algorithm.

Description

Motivation

Hydrodynamic dispersion is the irreversible spreading of a passive tracer by the combined effect of diffusion and fluid flow. In the case of dispersion on a reversible velocity field, the concentration measured at one point in the flow in response to a concentration pulse at another point will be the same if the two points are interchanged while simultaneously reversing the direction of the velocity field [1]. This is called the reciprocity relation and is true regardless of the magnitude of molecular diffusion. An analytic derivation of this relation was done by E.G. Flekkøy [1], and has been measured experimentally [2,3,4]. As the withdrawn profile from an injection of a tracer in a system can give information on it's interior structure the reciprocity relation can be used as an echo experiment, which has been studied experimentally [2]. This technique may have interesting medical, biological and industrial applications. The reciprocity relation also allows for prediction and placement of a desired amount of a medical preparation within some tissue of the body which is otherwise inaccessible. For example within the lymph system or any closed cavity where a slow elastic expansion and contraction would drive a reversible flow.

Methods

The work performed will be both theoretical and numerical. The analytical investigation will be done with the Boltzmann equation, which describes the statistical behaviour of a thermodynamic system outside of equilibrium. This is done by working with the time evolution of the system in phase space.
A discredited version of the Boltzmann equation will be studied numerically using the Lattice Boltzmann algorithm. The numerical study will be a proof of concept, and gives us the possibility to study the reciprocity relation for various geometries and flow fields.

Points to investigate

What happens to the entropy? How is it effected by Raynolds number? How is it effected by the The Péclet number? Can it be achieved in three dimensions? Can this be applied to biological systems? Does the reciprocity relation hold for pore lengths of different sizes? Does the reciprocity relation hold for different injection and receptor points? Is it independent of distance and time of diffusion? If not, in what way? Can it be used to study trapping and release processes in recirculation zones?

Work plan

Fall 2019

- FYS4420 - Experimental techniques in condensed matter physics
  FYS4430 - Condensed matter physics II
  FYS4465 - Dynamics of complex media

- Writing project description and literature study

Spring 2020

- FYS4130 – Statistical Mechanics
  MENA5010 – Nanophysics
- Implementing the Lattice Boltzmann algorithm
- Literature study

Fall 2020

- FYS4110 – Modern quantum mechanics
- Diffusion
- Advection

Spring 2021

- Convection
- Writing

Supervision suggestions

Signatures

Ivar Svalheim Haugerud: ______________________

Eirik Grude Flekkøy: ______________________

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