CSC 5350 Assignment 3

Due date: 8 December 2008

1. Consider the following game that is played by two players with a red box and a green box. First, player 1 puts a ball into either the red box (action R) or the green box (action G). Player 2 is not allowed to see when player 1 puts the ball into the box, so player 2 does not know which box contains the ball and which is empty. Then, player 1 tells player 2 whether the ball is in the red box (action r) or the green box (action g). However, player 1 can choose to tell the truth or tell a lie, and then player 2 can choose to believe (action y) or not to believe (action n).

   The rule is as follows.

   - Case 1: player 1 puts the ball into the red box. In this case, player 1's utility is **ten (10)** and player 2's is **zero (0)** if player 1 successfully cheats player 2, or player 1's utility is **minus ten (–10)** and player 2's is **one (1)** if player 1 does not succeed in cheating player 2.
   - Case 2: player 1 puts the ball into the green box. In this case, player 1's utility is **five (5)** and player 2's is **zero (0)** if player 1 successfully cheats player 2, or player 1's utility is **minus five (–5)** and player 2's is **one (1)** if player 1 does not succeed in cheating player 2.
   - In both cases, if player 1 decides not to cheat player 2, but to tell the truth to player 2, then both players' utilities are **three (3)** if player 2 believes the truth, but both players' utilities are **minus one (–1)** if player 2 does not believe the truth.

   (a) This game can be modelled as an extensive game with incomplete information $G = \langle N, H, P, f_c, (\mathcal{I}_i), (\succsim_i) \rangle$.

      i. **(2 marks)** Write down N in the game G.

      ii. **(2 marks)** Write down P in the game G.

      iii. **(4 marks)** Write down $\mathcal{I}_1$ in the game G.

      iv. **(4 marks)** Write down $\mathcal{I}_2$ in the game G.

   (b) **(4 marks)** Is this game a game with perfect recall? Justify your answer.

   (c) **(4 marks)** How many pure strategies does player 1 have? What are they?

   (d) **(4 marks)** How many pure strategies does player 2 have? What are they?

(e) If player 2 thinks that it is more likely that player 1 puts the ball into the red box, discuss how player 2 should respond.

i. **(4 marks)** Discuss under what condition player 2 should choose to believe.

ii. **(4 marks)** Discuss under what condition player 2 should choose not to believe.

(f) **(4 marks)** If player 1 thinks that player 2 will use the behavioural strategy β_2, which is to simply purely randomly choose to believe or not, what should be player 1's best behavioural strategy β_1 in response to β_2?

(g) **(4 marks)** Describe a consistent assessment that contains the strategy profile (β_1, β_2) in question (f).

(h) **(4 marks)** Is the assessment in question (g) a sequential equilibrium? Justify your answer.

(i) **(4 marks)** Suppose you, as an observer, know that player 1 puts the ball into the red box and tells a lie. Assume that the probability that player 2 believes is 0.8, then what is the outcome of the game?

2. Player 1 and player 2 play the following game.

- First, player 1 decides whether to continue ('*C*') or to stop ('*S*'). If player 1 decides to stop, both players get the same score of 0. Otherwise (that is, player 1 decides to continue), the game goes on as follows.
- Player 1 writes an integer on a piece of paper. He can choose to write either '1', '2' or '3' on the piece of paper. After writing the number, he folds the piece of paper, so that player 2 does not know what he writes.
- Then player 2 also writes a number on another piece of paper. He can choose to write either '1', '2' or '3' on the piece of paper.
- After player 2 writes a number, both players compare the numbers they write, and the score is determined by the following rules:
  - If both players write the same number, then both get the same score of 0.
  - **<u>Otherwise</u>**, if a player writes '1' and the other writes '3', then the former player gets a score of 1 and the latter player a score of 0.

- **Otherwise**, if a player writes m and the other writes n, where $m > n$, then the former player gets a score of 1, and the latter player a score of 0.

- Every player prefers high scores to low scores.

(a) **(8 marks)** Formulate this game as an extensive game with imperfect information. You may present your solution using a game tree.

(b) **(8 marks)** Is this game a game with perfect recall? Justify your answer.

(c) **(8 marks)** Suppose player 2 plays the following behavioural strategy if he is invited to play: the probability of writing '1' is ½; the probability of writing '2' is ⅛; the probability of writing '3' is ⅜. What is player 1's best behavioural strategy in response to player 2's behavioural strategy?

(d) **(8 marks)** Let β denote the behavioural strategy profile described in (c). Find a belief system μ such that (β, μ) is a consistent assessment.

(e) **(8 marks)** Is the assessment (β, μ) in (d) a sequential equilibrium? Justify your answer.

3. Consider a group of seven (7) players, each having one card. Players 1 and 2 each has a red card, and the other players each has a green card. There is a rule saying that players can form groups, such that a group can receive \$1 if the group members have one red card *and* one green card, and, in general, \$*n* if they have *n* red cards *and* *n* green cards.

(a) This scenario can be modelled as a conational game with transferable payoff $\langle N, v \rangle$.

   i. **(1 mark)** Write down N in the game.

   ii. **(1 mark)** Write down v in the game.

(b) **(4 marks)** Consider the following set of imputations

$$Y_1 = \{(x, x, y, y, y, y, y) : 2x + 5y = 2, x \geq 0, y \geq 0\}$$

That is, the utilities of players 1 and 2 are *x*, and the utilities of other players are *y*. Discuss whether or not Y_1 is a stable set. Prove your answer.

(c) **(2 marks)** Give a stable set different from Y_1 (if Y_1 is a stable set). Prove your answer.

(d) **(2 marks)** What is the 'standard of behaviour' in your answer to question (c)?

(e) **(2 marks)** Identify one imputation that is in the core of the game.

— End of Assignment —