CSC 5350 Assignment 1

XIAO Zigang
zgxiao@cse.cuhk.edu.hk
Department of Computer Science and Engineering
The Chinese University of Hong Kong

October 2, 2008

1 (Exercise 24.1)

a. Let u_1 be player $1'$s payoff function in G, and v_1 be her payoff function in G'. According to the question statement, for every action profile (x, y) in G and G', $v_1(x, y) \geq u_1(x, y)$. Suppose there are n action available to player 1. Then for each action x_i:

$$\min_y v_1(x_i, y) \geq \min_y u_1(x_i, y), i = 1, 2, \dots, n \tag{1}$$

Moreover:

$$\max_x \min_y v_1(x, y) \geq \max_x \min_y u_1(x, y) \tag{2}$$

Since $\max_x \min_y v_1(x, y)$ is a maxminimizer for both players in G', which means $v_1(x^*, y^*) = \max_x \min_y v_1(x, y)$, i.e. (x^*, y^*) is the Nash Equilibrium of G'. Similarly we got (s^*, t^*), the Nash Equilibrium of G.Then we got $v_1(x^*, y^*) \geq u_1(s^*, t^*)$, which means the Nash Equilibrium in G' is no worse than that in G.

b. Let $u_1(x, y)$ be the first player's payoff function. Let G' be the game where one of player $1'$s action is deleted, i.e. $(A_1)' \subseteq (A_1)$. The payoff function is the same. Let X, X' the set produced by $\min_y u_1(x, y)$ in G and G'. Then we got $X' \subseteq X$,hence $\max(X') \leq \max(X)$. That is,

$$\max_{x \in (A_1)'} \min_y u_1(x, y) \leq \max_{x \in (A_1)} \min_y u_1(x, y) \tag{3}$$

(They are the Nash Equilibrium of G and G'.)

c. For a, let G and G' be

	y_1	y_2
x_1	(1,3)	(2,2)
x_2	(2,2)	**(4,4)**

$\Rightarrow$

	y_1	y_2
x_1	**(3,3)**	(5,2)
x_2	(2,2)	(4,4)

The Nash Equilibrium is (x_2, y_2). If we change (x_1, y_1) to $(3, 3)$, (x_1, y_2) to $(5, 2)$ then the Nash Equilibrium is (x_1, y_1) in G', where player 1 has a worse payoff 3 than the payoff 4 in (x_2, y_2) of G.

For b, use the second table as G. If we prohibit player 1 from using x_1, then $(x_2, y_2) = (4, 4)$ is the Nash Equilibrium such that player 1 will have a higher payoff than the Nash Equilibrium in G.

2 (Exercise 35.1)

Assume that there exists a mixed strategy Nash Equilibrium $\alpha^* = (\alpha_i^*)$. For every player i, his action profile α_1^* is in the form $(1(p_1^1), 2(p_2^1) \ldots K(p_K^1))$.

1. If other player's actions are fixed, player i can always calculate and choose the action x^* which is closest to 2/3 of

   average as some other people may be, and this is his payoff, which is greater than 0,or he gets payoff value 0.

2. Let m^* be the largest number that has a probability greater than 0 for player i in α_i^*. Since for every action in the support of his mixed strategy Nash Equilibrium , mixed strategy yields the same payoff for him. Also the payoff should be positive because of (1). When he chooses m^*, it means that he is using m^* to achieve the "closest to 2/3 of average" goal. Also there are some other players who also choose m^*. Then these people as well as i get the split prize. Others get 0.

3. If player i chooses m^*, he would rather choose $m^* - 1$ in order to get a better payoff because at that time he becomes the only one who is closest to 2/3 average, and thus getting better payoff. However, if he chooses $m^* - 1$, others will also adjust their strategy to get better payoff, which results an iterate decreasing of m^*, and finally $m^* = 1$ for all the players. Since we assume that m^* is the largest number that has a positive probability, the probability of m^* becomes 1 and we got the unique mixed strategy Nash Equilibrium in which every player's strategy is pure.

3 (Exercise 94.2)

Prove it as two parts.

1. Without loss of generality, let $(a_1', a_2') \sim_i (a_1', a_2'')$ for $i = 1, 2$. Let u_i be their payoff function, then we got $u_1(a_1', a_2') = u_1(a_1', a_2'')$ and $u_2(a_1', a_2') = u_2(a_1', a_2'')$. Then we can construct a extensive game with perfect information Γ as in Figure **??**.

2. Suppose G is the strategic form of an extensive game Γ with perfect information. There are only two strategy for each player in the strategic form, and each strategy contains only one action, which means that both player have only one chance to move on a non-terminal history. Without loss of generality, let player 1 move first. From the empty sequence, he could choose to play a_1' or a_1''. Because player 2 only have one chance to move on a non-terminal history, say, a_1'', then player $1's$ action a_1' leads to a terminal history, so the strategy profile between both player yields the same outcome. Hence we got $(a_1', a_2') \sim_i (a_1', a_2'')$ for $i = 1, 2$ in this case.

	a_2'	a_2''
a_1'	(1,2)	(1,2)
a_1''	(3,4)	(5,6)

$\Rightarrow$

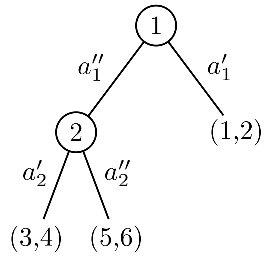

Figure 1: extensive game with perfect information construct from G

By combining 1 and 2, we prove the if and only if condition.

— End of Assignment —