

# 1. Levels and goals

## III. From Navigation to Analysis

---

Giacomo Bergami

Newcastle University

# Objectives

- Characterizing non-stochastic games.
- Characterizing agent planning via completion of intermediate goals.
- Exploiting rule-based planning techniques for checking whether a game is solvable or not.
- Exploiting theorem-proving based techniques for:
  - ① debugging unsolvable games, and
  - ② identifying different strategies for winning the game
- Characterizing game balance as an interplay of solvability and difficulty.
- Ranking strategies by difficulty.

# Procedural Generation: a pseudocode

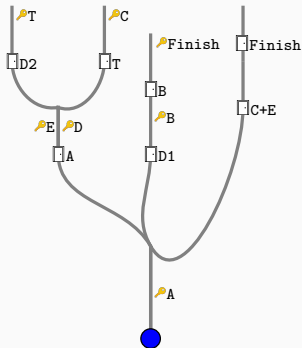
Procedural Generation for locks-and-keys game often start with generating a map. The generation of such a map resembles the generation of a *planar* graph.

`ProceduralDungeon::generate_data(double)` shows how to randomly generate a level.

```
1: function GENERATEROOMSANDCORRIDORS(#room, prob)
2:   for i  $\leftarrow$  1 to #room do                                     ▷ Generating #room rooms.
3:     do r  $\leftarrow$  RANDOMDOOR() while INTERSECTS(r, rooms)
4:     rooms.add(r)
5:   for i  $\leftarrow$  1 to #room do
6:     for j  $\leftarrow$  1 to i - 1 do
7:       if random.prob() < prob then continue                       ▷ Randomly discarding an edge.
8:       if not INTERSECTS((i  $\leftrightarrow$  j), corridors) then corridors.add(i  $\leftrightarrow$  j)  ▷ Making the level a planar graph
```

After this, we might follow any preferred way to generate locks and keys. But how to guarantee the game's solvability?

# Solvability: Locks and Keys (1/2)



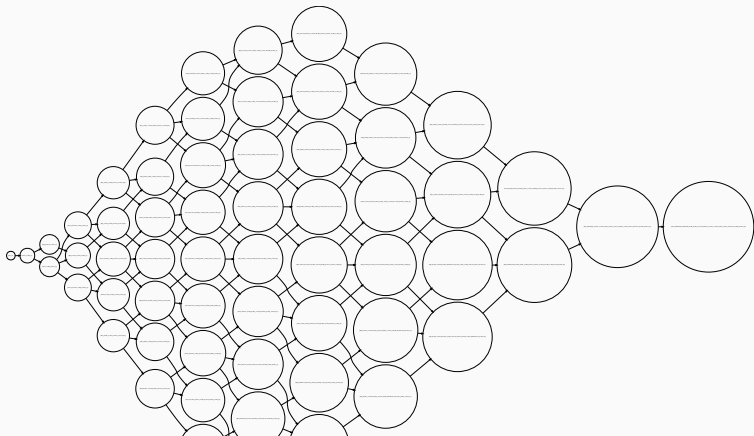
We can encode our level using predicate logic through Horn clauses:

- 1 Set all the resources accessible from the start as the initial state ( $\{Key_A\}$ ).
- 2 Set the final door as the goal ( $Door_{Finish}$ ).
- 3 If I have access to the key  $Key_B$  and to all of the doors leading to  $Door_B$ , then I can open the door: e.g.,  $Key_B \wedge Door_D \Rightarrow Door_B$
- 4 If I have access to a door, then I have access to all the resources between it and the next obstacles: e.g.,  $Door_A \Rightarrow Key_E$ ,  $Door_A \Rightarrow Key_D$

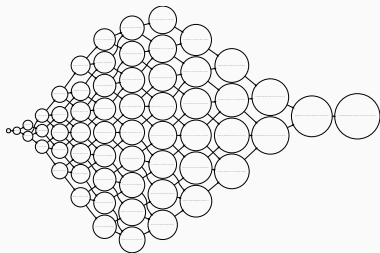
Solvability is generally independent from the player's skills and probabilistic chances. Let's see how we can exploit the navigation rules to reach *Finish*.

## State Eplosion with State Machines

Given the first configuration for the Locks and Key generation, if we decide to represent each node as a possible state configuration reached by applying a set of rules, we obtain the following (GenerateUnweightedPossibleStates<T>::generateUnweightedPossibleStates):



# State Explosion with State Machines



- While traditional State Machine techniques require to generate all the possible states, logical rules might generate one single graph per strategy.
- Determining the costs over this graph is more challenging, as many other different combinations must be considered.
- In the next example, we will show how exploiting this representation for even a simplistic (but realistic) scenario will lead to an explosion to the number of the possible states.

## Solution #1: “Forward” Rule-Based Planning for Solvability

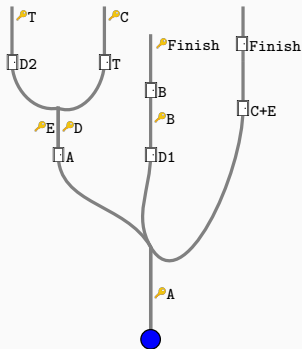
ClosedWorldInference<T>::grounding:

- Each node of the “graph” represents the set of collected resources and solved tasks at a given time in the game. We apply one single strategy, and therefore we might produce one single path!
- The previous approach proves whether the game is solvable by identifying a possible solution.
- We do not generate multiple possible strategies, rather than *we apply systematically each rule* until we can meet our goal.
- This same strategy could be adopted to enact rational reasoning to a NPC agent.
- The game cost is not necessarily the one with optimal cost (**Exercise:** *greedy heuristic over costs* for short-sighted strategies)!

## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$



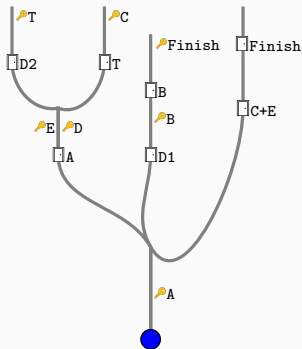
Resources: Key<sub>A</sub>



# Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

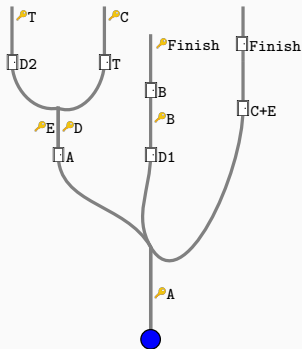


Resources:  $\text{Key}_A$ ,  $\text{Door}_A$

## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E$ ,  $\text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}$ ,  $\text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

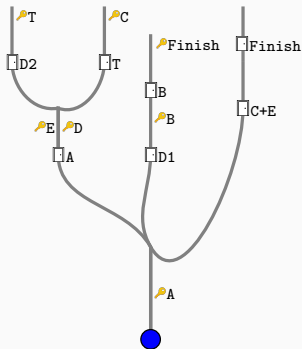


Resources:  $\text{Key}_A$ ,  $\text{Door}_A$ ,  $\text{Key}_D$

## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E$ ,  $\text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}$ ,  $\text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

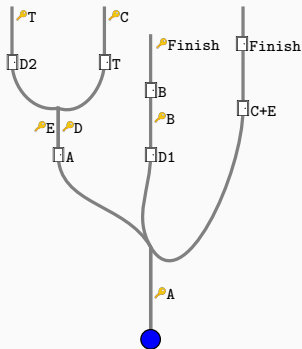


Resources:  $\text{Key}_A$ ,  $\text{Door}_A$ ,  $\text{Key}_D$ ,  $\text{Key}_E$

## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

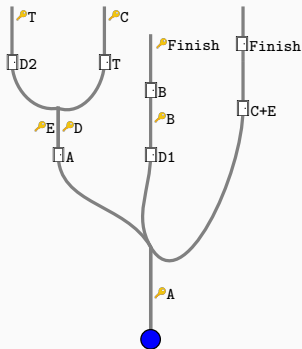


Resources:  $\text{Key}_A, \text{Door}_A, \text{Key}_D, \text{Key}_E, \text{Door}_{D1}$

## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

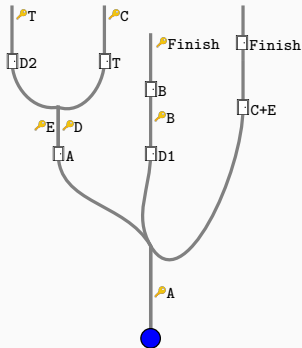


Resources:  $\text{Key}_A, \text{Door}_A, \text{Key}_D, \text{Key}_E, \text{Door}_{D1}, \text{Door}_{D2}$

## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

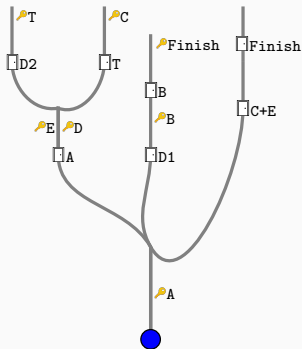


Resources:  $\text{Key}_A$ ,  $\text{Door}_A$ ,  $\text{Key}_D$ ,  $\text{Key}_E$ ,  $\text{Door}_{D1}$ ,  $\text{Door}_{D2}$ ,  $\text{Key}_T$

## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

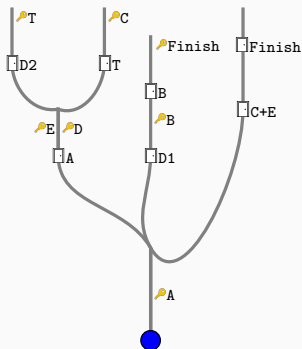


Resources:  $\text{Key}_A$ ,  $\text{Door}_A$ ,  $\text{Key}_D$ ,  $\text{Key}_E$ ,  $\text{Door}_{D1}$ ,  $\text{Door}_{D2}$ ,  $\text{Key}_T$ ,  $\text{Key}_B$

## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$



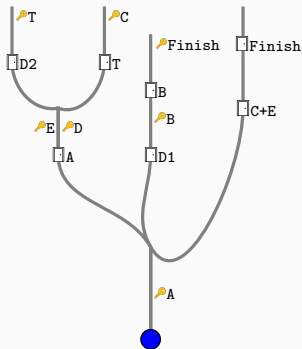
Resources:  $\text{Key}_A, \text{Door}_A, \text{Key}_D, \text{Key}_E, \text{Door}_{D1}, \text{Door}_{D2}, \text{Key}_T, \text{Key}_B, \text{Door}_T$



## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

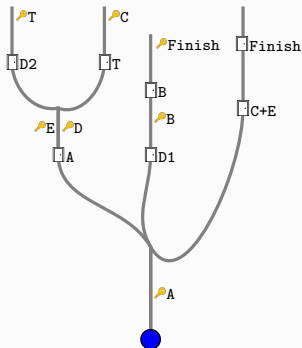


Resources:  $\text{Key}_A, \text{Door}_A, \text{Key}_D, \text{Key}_E, \text{Door}_{D1}, \text{Door}_{D2}, \text{Key}_T, \text{Key}_B, \text{Door}_T, \text{Door}_B$

## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

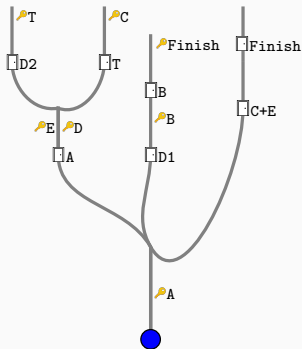


Resources:  $\text{Key}_A, \text{Door}_A, \text{Key}_D, \text{Key}_E, \text{Door}_{D1}, \text{Door}_{D2}, \text{Key}_T, \text{Key}_B, \text{Door}_T, \text{Door}_B, \text{Key}_C$

## Solvability: Locks and Keys (2/2)

Rules (*Horn Clauses*):

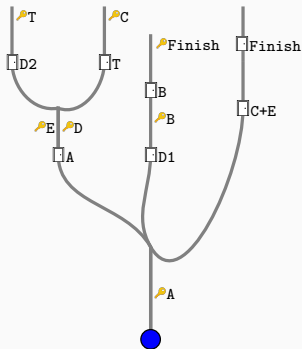
- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$



Resources:  $\text{Key}_A, \text{Door}_A, \text{Key}_D, \text{Key}_E, \text{Door}_{D1}, \text{Door}_{D2}, \text{Key}_T, \text{Key}_B,$   
 $\text{Door}_T, \text{Door}_B, \text{Key}_C, \text{Door}_{C+E}$

## Solvability: Locks and Keys (2/2)

### Rules (*Horn Clauses*):



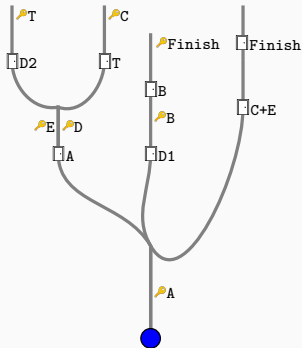
- 1  $\text{Key}_A \Rightarrow \text{Door}_A$
- 2  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- 3  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- 4  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- 5  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- 6  $\text{Door}_T \Rightarrow \text{Key}_C$
- 7  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- 8  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- 9  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- 10  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- 11  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

Resources: Key<sub>A</sub>, Door<sub>A</sub>, Key<sub>D</sub>, Key<sub>E</sub>, Door<sub>D1</sub>, Door<sub>D2</sub>, Key<sub>T</sub>, Key<sub>B</sub>, Door<sub>T</sub>, Door<sub>B</sub>, Key<sub>C</sub>, Door<sub>C+E</sub>, Key<sub>Finish</sub>

## Solvability: Locks and Keys (2/2)

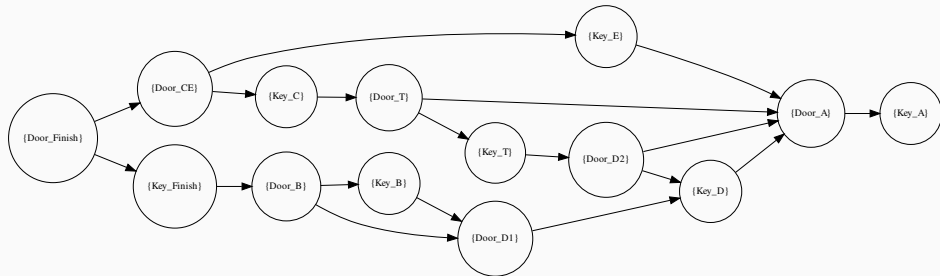
Rules (*Horn Clauses*):

- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

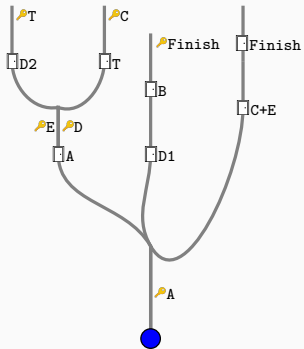


Resources:  $\text{Key}_A, \text{Door}_A, \text{Key}_D, \text{Key}_E, \text{Door}_{D1}, \text{Door}_{D2}, \text{Key}_T, \text{Key}_B,$   
 $\text{Door}_T, \text{Door}_B, \text{Key}_C, \text{Door}_{C+E}, \text{Key}_{\text{Finish}}, \text{Door}_{\text{Finish}}$

## Solution #2: “Backward” Rule-Based Planning: Stalemates and Difficulty



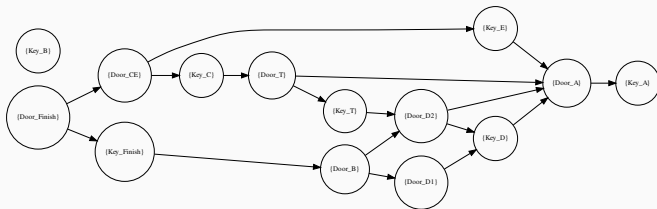
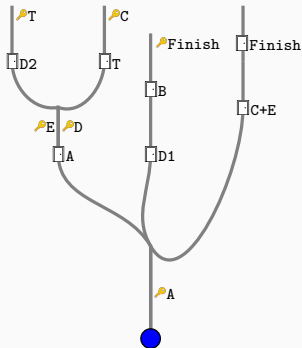
- **Each state does not represent** a possible gameplay state as in GOAP, rather than a **resource/subgoal to be obtained/solved**.
- Nodes reachable in one step determine tasks/resources that *should be done/obtained simultaneously*.
- One single graph is generated *for each possible solution* independently from the order of the actions; each solution is different only if different strategies are used to reach a same (possibly intermediate) goal.
- If not all the *Init* preconditions or resources/goals are used to solve the game, *some initial resources might be discarded*, as irrelevant for solving the final riddle with the given strategy.



Is this game level solvable? Why?

# Stalemates: Missing Resources

The previous algorithm will generate a graph with one isolated node (the source of the error).



Some debugging errors are returned:

Error: Cannot apply rule for:  $\text{Key}_B$

Error: Cannot generate precondition  $\text{Key}_B$  for  
 $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$



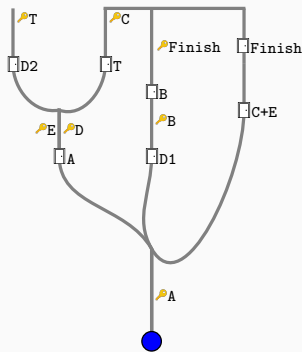
## Solution #2: “Backward” Rule-Based Planning: Solvability

GenerateBacktrackStates<T>::generateGraphs:

- Is it possible to detect the reasons that bring to a stalemate?
  - ① We need to navigate backwards from the goal,
  - ② try to apply all the rules backwards (from the result towards the prerequisites),
  - ③ and detect at which step we halt the navigation towards the initial configuration.
- Each node of the graph provides only one resource and task at a time. The path represent dependencies among the tasks, that might be visited to enact a strategy!
- By doing so, please observe that we might generate multiple graphs, where each graph represents a possible strategy.

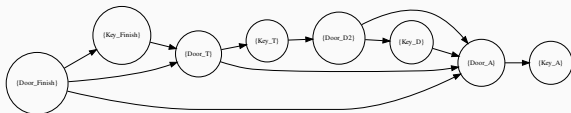
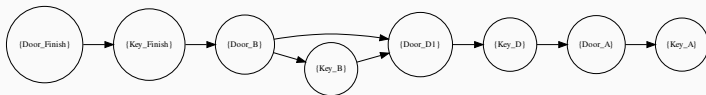
## Multiple Solutions: Example (1/2)

Let us further change the game level so to allow multiple possible ways to solve the game:  
what is the difference?



- ①  $\text{Key}_A \Rightarrow \text{Door}_A$
- ②  $\text{Door}_A \Rightarrow \text{Key}_E, \text{Door}_A \Rightarrow \text{Key}_D$
- ③  $\text{Door}_A \wedge \text{Key}_D \Rightarrow \text{Door}_{D2}, \text{Key}_D \Rightarrow \text{Door}_{D1}$
- ④  $\text{Door}_A \wedge \text{Key}_T \Rightarrow \text{Door}_T$
- ⑤  $\text{Door}_{D2} \Rightarrow \text{Key}_T$
- ⑥  $\text{Door}_T \Rightarrow \text{Key}_C, \text{Door}_T \Rightarrow \text{Key}_{\text{Finish}}$
- ⑦  $\text{Door}_{D1} \Rightarrow \text{Key}_B$
- ⑧  $\text{Door}_{D1} \wedge \text{Key}_B \Rightarrow \text{Door}_B$
- ⑨  $\text{Door}_B \Rightarrow \text{Key}_{\text{Finish}}, \text{Door}_B \Rightarrow \text{Key}_C$
- ⑩  $\text{Key}_C \wedge \text{Key}_E \Rightarrow \text{Door}_{C+E}$
- ⑪  $\text{Door}_{C+E} \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}},$   
 $\text{Door}_A \wedge \text{Door}_T \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}},$   
 $\text{Door}_{D1} \wedge \text{Door}_B \wedge \text{Key}_{\text{Finish}} \Rightarrow \text{Door}_{\text{Finish}}$

## Multiple Solutions: Example (2/2)



- 1 The algorithm might generate nodes that are irrelevant, as they are not directly reachable from the goal (those could be pruned):
- 2 as a result, the algorithm might generate duplicate solutions, but those are easily detectable.
- 3 Given that these are “dependency graphs”, we might exploit a topological sort for getting the best order to perform the actions to reach the goal.

# Checking whether a current Dungeon Level is solvable

For each possible path between start and final room (`ProceduralDungeon::isFeasable()`):

- Collect all of the keys before the first door appearing in the path as an “initial state”.
- Track all the doors met while scanning the path in a list  $\ell$ :
- When a new door is met  $d_i$  associated to a key  $k_i$ , create a new rule  $k_i \wedge \bigwedge_{d_j \in \ell} d_j \Rightarrow d_i$ , as well as the requirement that, for opening the door, you need its key:  $k_i \Rightarrow d_i$ .
- For each key  $\kappa$  met between the last door  $\delta \in \ell$  and the current  $d_i$ , generate a new rule  $\delta \Rightarrow \kappa$

For each backward graph having no computation errors:

- Sort the vertices in inverse topological order
- Return the associated strategy by collecting the node labels.

The game is unsolvable if there are no graphs or if the graphs have errors or empty strategies.

Game balance is the study of the mathematical properties of the game so to assess its *fairness* and *engagingness*. Some of the key points for Game balance are:

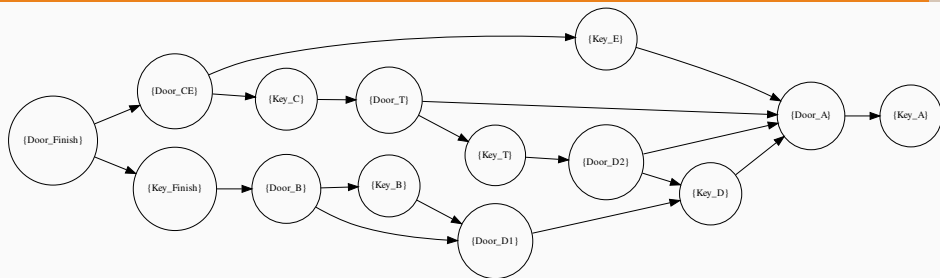
## ① Solvability

- By analyzing the game mechanics, we need to assess whether we might reach the end of the game or not (e.g., winning).
- No stalemates should arise, in which nobody can win or lose.
- We also want to avoid *dominant strategies* that always lead to winning the game.

## ② Difficulty

- Difficulty should be proportional to the player's skills
- Difficulty should increase with the progress with the game

# Difficulty in Non-Stochastic Games (1/2)



- We might also generate graphs with costs/probabilities associated to both nodes and edges:  
`GenerateWeightedPossibleStates<T>`.
- We use all the graphs with no errors for handling difficulty within a game level. Suppose that each rule is associated to a cost coming from:
  - 1 the  $[min, max]$  cost of reaching a door or a resource from the previous position.
  - 2 eventually, the cost  $s$  for collecting the resource or opening the door.
  - 3 Some resources might be available only from a given XP level.
- This cost, namely  $c_e$ , is then associated to each edge  $e$ .

## Difficulty in Non-Stochastic Games (2/2)

If the algorithm returns one single graph, the user has only one choice (up to performing actions in a different order), and therefore we can only assess its hardness.

The maximum  $c_e$  of an edge  $e$  gives the maximum cost  $M$  that is expected to be solved (e.g.) by the maximum level of expertise  $X$ :

- Given the current user level of expertise  $\chi \leq X$ ,
- Remove all the unavailable resources for level  $\chi$  ( $\Rightarrow$  rule update)
- Remove edges having minimum cost  $\min + s$  greater than  $M/\chi$
- Remove the isolated nodes

If there are no paths connecting a *Init* resource to a *Goal* resource/goal, the resulting game might be hard (if not impossible) to solve for an user of XP  $\chi$ , and such scenario might be discarded.

The suitable solutions for  $\chi$  are the remaining graphs.

## Solvability:

- by representing a game with a set of rules, we might detect if the game is solvable and,
- if not, we might get where the bug is while generating the game.

## Difficulty:

- In non-stochastic games, we might exploit solvability techniques to generate and rank possible alternatives to solve the game.