1. Levels and goals

II. From Probability to Navigation

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Pacing & Statistic Analysis

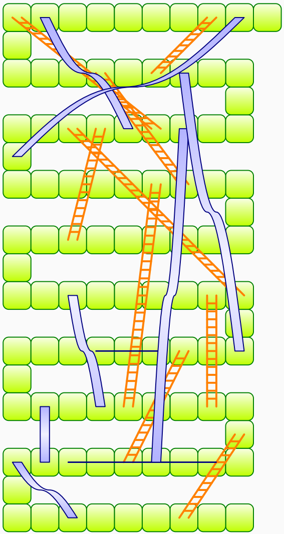
Pacing is a fundamental analysis in Game Balancing:

- The aim of the game is to put obstacles and power-ups with trade-offs.
- Obstacles should never be insurmountable or unfair.

Statistical Analysis is often used to analyse games for balancing:

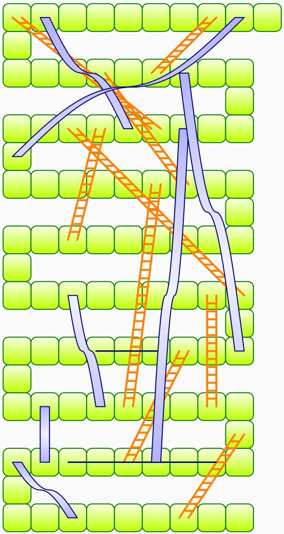
- We can identify faults (e.g., unbalanced areas) and make corrections
- The usual approach is to collect *logs* from real plays, and then infer statistics via *post-mortem analysis*.
- Still, is it possible to analyse the game play before sending the product to testers and/or sending it to production?

Snakes and Ladders (1/3)



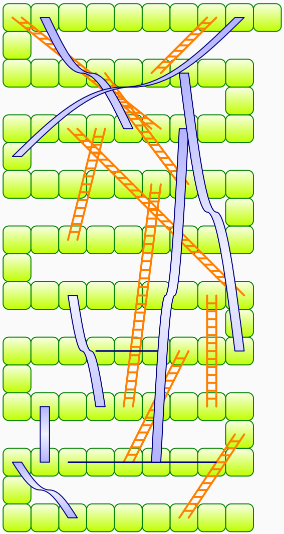
- The game contains 100 cells:
- The player starts the game from the first cell.
- **Actions**:
  1. At each turn, the player rolls a die:
  2. if they step into the lower end of a ladder, the player progresses towards its upper end;
  3. if they step into the upper end of a snake, the player backtracks towards its lower end;
  4. otherwise, the player stays in the reached state.
- The game ends when the last cell is reached.

Snakes and Ladders (2/3)



- The game is *memoryless*: at a given point in the game, the player's progression from the current square is independent of how they arrived at that square.
- Each edge weight is now independent from each player's skills.

Snakes and Ladders (3/3)



Without Snakes and Ladders:

- Each non-rigged die generates numbers in $I = \{ 1, 2, 3, 4, 5, 6 \}$ with probability $\mathbb{P}(I = i) = 1/6$ for each $i \in I$.
- Given a initial cell i, I can only move from i to one of the following states: $\{ i + 1, \ldots, i + 6 \} \setminus \{ n \in \mathbb{N} \mid n > 100 \}$.

With Snakes and Ladders:

- arriving to a starting point of a snake/ladder from cell i has 0 probability,
- while the probability of reaching the end of the snake/ladder from cell i is increased by $1/6$.

Balancing Process

While assessing the game balance, we should proceed as follows:

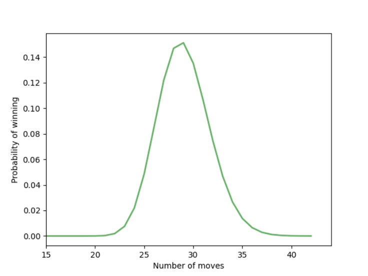
1. First, we should create a balanced game:
   - *E.g., we should generate a board with neither snakes nor ladders.*
2. Next, we might consider the average running time (*average hitting time*) and the probability of winning the game in a given number of steps.
   - *Collect such values as reasonable outcomes for the game.*
3. Introduce minimal changes, so to always determine how single changes affect the overall system.
4. Compare the values obtained with the previous configuration, which can be thought as reasonable if returns similar or better values than the previous configuration.
5. If the game configuration is reasonable, go back to the 3rd item and continue to refine the game.

The effect of power-ups and penalties

This simple game allows us to determine how power-ups and penalties might affect the duration and the probability of reaching the end of the game in a given amount of steps:

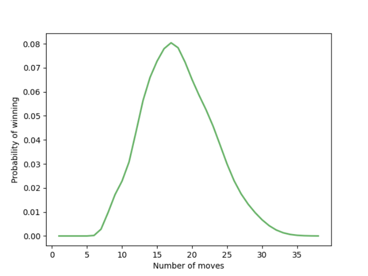
- By only adding ladders, the time for the game-play is considerably reduced:
  1. We generate fewer and shorter paths, but there might be multiple possible way to get paths of the same length:
  2. The probability of reaching the final state in a given amount of steps is reduced.
- By only adding snakes, the time of the game-play is considerably increased:
  1. Even if it is extremely improbable, players might get stuck in loops, thus increasing the length of the possible paths;
  2. Therefore, the probability of reaching the end of the game in a given amount of steps if considerably decreased.
- Intuitively, adding both snakes and ladder will provide a trade-off situation between possible length of game-play and probability of reaching the end of the game.

Snakes and Ladders: Probability of winning in n turns (1/2)



Without Snakes and Ladders.

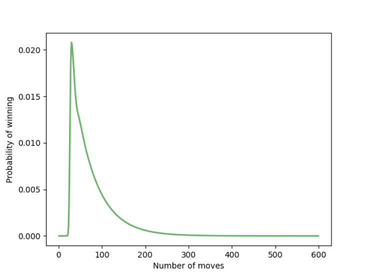
- We need at least 17 turns to win the game.
- It is more likely to win the game with probability 0.15 after 29 turns.



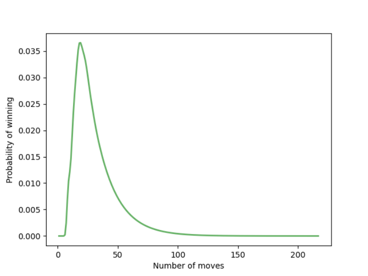
With Ladders.

- The game fastens up, we now need at least 6 turns to win the game.

Snakes and Ladders: Probability of winning in n turns (2/2)



With Snakes.



With Snakes and Ladders.

- The game slows down, as we might get stuck in a loop.
- Furthermore, the overall chances of winning decreases.
- Adding back the ladders, the chances of winning increase as well as the length of loops decrease.

Average Hitting Time

Given that our problem has only one initial state, cell$=1$, we are interested in x_1.

The following result confirm the intuitive results in length of game-play from performing *random walks* lover the stochastic process:

- Without Snakes and Ladders: ~ 29 turns.
- With only Ladders: ~ 18 turns.
- With only Snakes: ~ 46 turns.
- With both Snakes and Ladders: ~ 25 turns.

Conclusions

This section introduced the key concepts to understand the relevance of determining the *average hitting times* and the *probability of terminating a game in a given amount of steps*.

- We defer the theoretical analysis of such games to the *Players and adversaries* set of lectures: **Modelling Winning Probabilities in Stochastic Games**.
- In that occasion, we are also going to determine how changes in probability affect the overall game duration.