# 1. Levels and goals

## III. From Navigation to Analysis

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# Objectives

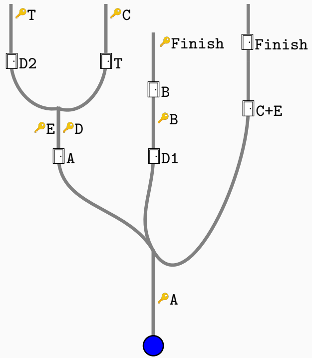
- Characterizing game balance as an interplay of solvability and difficulty.
- Characterizing agent planning via completion of intermediate goals.
- Exploiting rule-based planning techniques for checking whether a game is solvable or not.
- Exploiting theorem-proving based techniques for:
    1. debugging unsolvable games, and
    2. identifying different strategies for winning the game
- Ranking strategies by difficulty.
- Positive and Negative feedback in games as a premise to Reinforcement Learning.

# Game balance

Game balance is the study of the mathematical properties of the game so to assess its *fairness* and *engagingness*. Some of the key points for Game balance are:

1. Solvability
   - By analyzing the game mechanics, we need to assess whether we might reach the end of the game or not (e.g., winning).
   - No stalemates should arise, in which nobody can win or lose.
   - We also want to avoid *dominant strategies* that always lead to winning the game.
2. Difficulty
   - Difficulty should be proportional to the player's skills
   - Difficulty should increase with the progress with the game
3. Positive and Negative Feedback
   - They are essential game mechanics for giving the players' suggestions on which the best strategy to win the game might be.

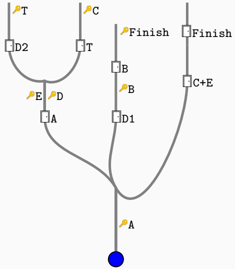
# Solvability: Locks and Keys (1/3)




As a use case for solvability, let us take a game randomly generating a game level for locks and keys. We have a “declarative” generator, which keeps track of:

- all the collectible keys/clues,
- the requirements for solving parts of the game (e.g., opening a door), and
- the resources required to open the doors (e.g., possessing the right key).

# Solvability: Locks and Keys (2/3)

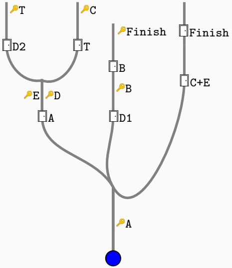



Alternatively, we can easily generate such rules given from the generated result:

1. Set all the resources accessible from the start as the initial state ($\{\mathrm{Key}_{\mathtt{A}}\}$).
2. Set the final door as the goal ($\mathrm{Door}_{\mathtt{Finish}}$).
3. If I have access to the key $\mathrm{Key}_{\mathtt{B}}$ and to all of the doors leading to $\mathrm{Door}_{\mathrm{B}}$, then i can open the door: e.g., $\mathrm{Key}_{\mathtt{B}} \wedge \mathrm{Door}_{\mathrm{D}} \Rightarrow \mathrm{Door}_{\mathrm{B}}$
4. If I have access to a door, then I have access to all the resources between it and the next obstacles: e.g., $\mathrm{Door}_{\mathtt{A}} \Rightarrow \mathrm{Key}_{\mathtt{E}}$, $\mathrm{Door}_{\mathtt{A}} \Rightarrow \mathrm{Key}_{\mathtt{D}}$

Solvability is generally independent from the player's skills and probabilistic chances. Let's see how we can exploit the navigation rules to reach `Finish`.

# Solvability: Locks and Keys (3/3)




Resources: $\mathrm{Key}_{\mathtt{A}}$

Rules (*Horn Clauses*):

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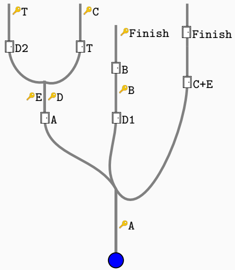


Resources: $\mathrm{Key}_{\mathtt{A}}$, $\mathrm{Door}_{\mathtt{A}}$

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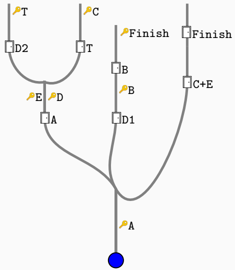


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Resources: $\mathrm{Key}_{\mathtt{A}}$, $\mathrm{Door}_{\mathtt{A}}$, $\mathrm{Key}_{\mathtt{D}}$

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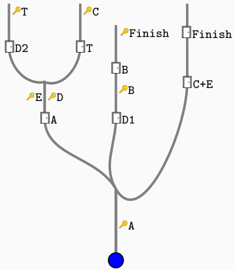


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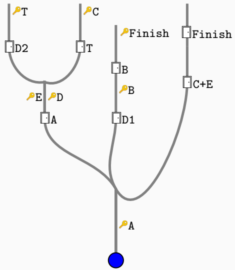


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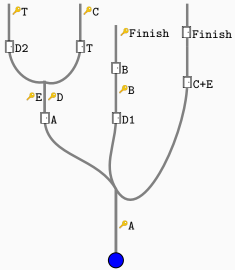


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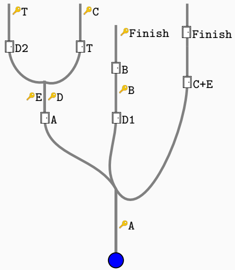


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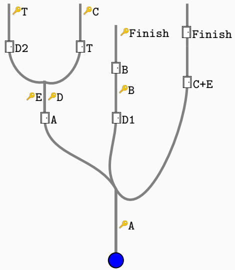


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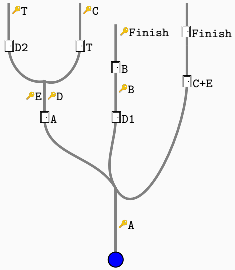


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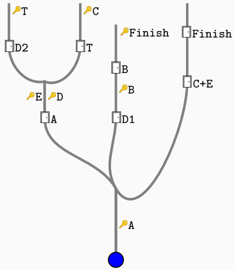


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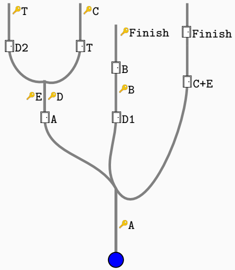


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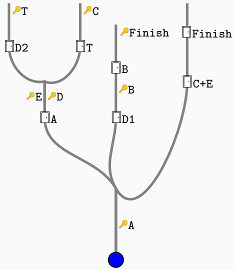


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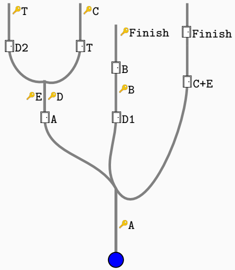


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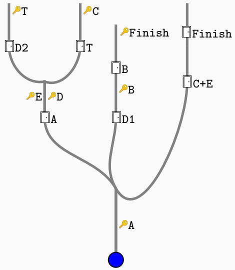
4. $\mathrm{Door}_{\mathtt{A}} \wedge \mathrm{Key}_{\mathtt{T}} \Rightarrow \mathrm{Door}_{\mathtt{T}}$
5. $\mathrm{Door}_{\mathtt{D2}} \Rightarrow \mathrm{Key}_{\mathtt{T}}$
6. $\mathrm{Door}_{\mathtt{T}} \Rightarrow \mathrm{Key}_{\mathtt{C}}$
7. $\mathrm{Door}_{\mathtt{D1}} \Rightarrow \mathrm{Key}_{\mathtt{B}}$
8. $\mathrm{Door}_{\mathtt{D1}} \wedge \mathrm{Key}_{\mathtt{B}} \Rightarrow \mathrm{Door}_{\mathtt{B}}$
9. $\mathrm{Door}_{\mathtt{B}} \Rightarrow \mathrm{Key}_{\mathtt{Finish}}$
10. $\mathrm{Key}_{\mathtt{C}} \wedge \mathrm{Key}_{\mathtt{E}} \Rightarrow \mathrm{Door}_{\mathtt{C+E}}$
11. $\mathrm{Door}_{\mathtt{C+E}} \wedge \mathrm{Key}_{\mathtt{Finish}} \Rightarrow \mathrm{Door}_{\mathtt{Finish}}$

Resources: $\mathrm{Key}_{\mathtt{A}}$, $\mathrm{Door}_{\mathtt{A}}$, $\mathrm{Key}_{\mathtt{D}}$, $\mathrm{Key}_{\mathtt{E}}$, $\mathrm{Door}_{\mathtt{D1}}$, $\mathrm{Door}_{\mathtt{D2}}$, $\mathrm{Key}_{\mathtt{T}}$, $\mathrm{Key}_{\mathtt{B}}$, $\mathrm{Door}_{\mathtt{T}}$, $\mathrm{Door}_{\mathtt{B}}$, $\mathrm{Key}_{\mathtt{C}}$, $\mathrm{Door}_{\mathtt{C+E}}$, $\mathrm{Key}_{\mathtt{Finish}}$

# Solvability: Locks and Keys (3/3)




Rules (*Horn Clauses*):

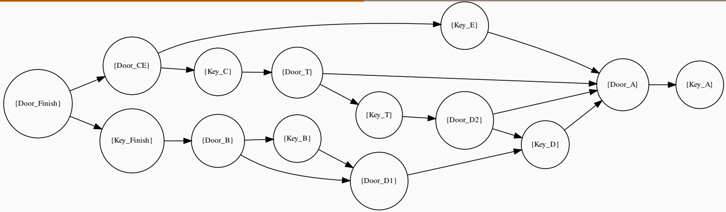
1. $\text{Key}_{\mathtt{A}} \Rightarrow \text{Door}_{\mathtt{A}}$
2. $\text{Door}_{\mathtt{A}} \Rightarrow \text{Key}_{\mathtt{E}}$, $\text{Door}_{\mathtt{A}} \Rightarrow \text{Key}_{\mathtt{D}}$
3. $\text{Door}_{\mathtt{A}} \wedge \text{Key}_{\mathtt{D}} \Rightarrow \text{Door}_{\mathtt{D2}}$, $\text{Key}_{\mathtt{D}} \Rightarrow \text{Door}_{\mathtt{D1}}$
4. $\text{Door}_{\mathtt{A}} \wedge \text{Key}_{\mathtt{T}} \Rightarrow \text{Door}_{\mathtt{T}}$
5. $\text{Door}_{\mathtt{D2}} \Rightarrow \text{Key}_{\mathtt{T}}$
6. $\text{Door}_{\mathtt{T}} \Rightarrow \text{Key}_{\mathtt{C}}$
7. $\text{Door}_{\mathtt{D1}} \Rightarrow \text{Key}_{\mathtt{B}}$
8. $\text{Door}_{\mathtt{D1}} \wedge \text{Key}_{\mathtt{B}} \Rightarrow \text{Door}_{\mathtt{B}}$
9. $\text{Door}_{\mathtt{B}} \Rightarrow \text{Key}_{\mathtt{Finish}}$
10. $\text{Key}_{\mathtt{C}} \wedge \text{Key}_{\mathtt{E}} \Rightarrow \text{Door}_{\mathtt{C+E}}$
11. $\text{Door}_{\mathtt{C+E}} \wedge \text{Key}_{\mathtt{Finish}} \Rightarrow \text{Door}_{\mathtt{Finish}}$

Resources: $\text{Key}_{\mathtt{A}}$, $\text{Door}_{\mathtt{A}}$, $\text{Key}_{\mathtt{D}}$, $\text{Key}_{\mathtt{E}}$, $\text{Door}_{\mathtt{D1}}$, $\text{Door}_{\mathtt{D2}}$, $\text{Key}_{\mathtt{T}}$, $\text{Key}_{\mathtt{B}}$, $\text{Door}_{\mathtt{T}}$, $\text{Door}_{\mathtt{B}}$, $\text{Key}_{\mathtt{C}}$, $\text{Door}_{\mathtt{C+E}}$, $\text{Key}_{\mathtt{Finish}}$, $\text{Door}_{\mathtt{Finish}}$

## “Forward” Rule-Based Planning for Solvability

- The previous approach proves whether the game is solvable by identifying a possible solution.
- The game cost is not necessarily the one with optimal cost (**Exercise**: *greedy heuristic over costs* for short-sighted strategies).
- Is it possible to detect the reasons that bring to a stalemate?
  1. We need to navigate backwards from the goal,
  2. try to apply all the rules backwards (from the result towards the prerequisites),
  3. and detect at which step we halt the navigation towards the initial configuration.
- By doing so, please observe that we might generate multiple graphs.

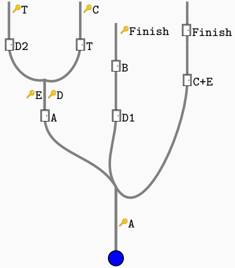
# Stalemates and Difficulty in Non-Stochastic Games




In the resulting graphs:

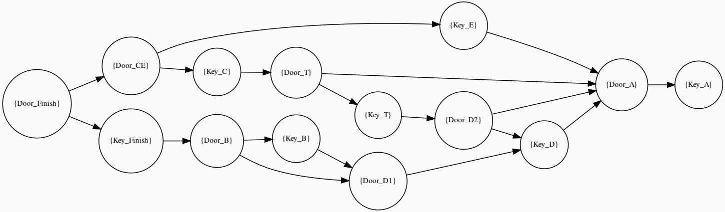
- **Each state does** not **represent** a possible gameplay state as in GOAP, rather than **a resource/subgoal to be obtained/solved**.
- Nodes reachable in one step determine tasks/resources that *should be done/obtained simultaneously*.
- One single graph is generated *for each possible solution* independently from the order of the actions; each solution is different only if different strategies are used to reach a same (possibly intermediate) goal.
- If not all the $Init$ preconditions or resources/goals are used to solve the game, *some initial resources might be discarded*, as irrelevant for solving the final riddle with the given strategy.

# Stalemates: Missing Resources




Is this game level solvable? Why?

# Difficulty in Non-Stochastic Games (1/2)




- We use all the graphs with no errors for handling difficulty within a game level. Suppose that each rule is associated to a cost, e.g., a combination of:
  1. the [*min*,*max*] cost of reaching a door or a resource from the previous position.
  2. eventually, the cost $s$ for collecting the resource or opening the door.
  3. Some resources might be available only from a given XP level.
- This cost, namely $c_e$, is then associated to each edge $e$.

# Difficulty in Non-Stochastic Games (2/2)

If the algorithm returns one single graph, the user has only one choice (up to performing actions in a different order), and therefore we can only assess its hardness.

The maximum $c_e$ of an edge $e$ gives the maximum cost $M$ that is expected to be solved (e.g.) by the maximum level of expertise $X$:

- Given the current user level of expertise $\chi \leq X$,
- Remove all the unavailable resources for level $\chi$ ($\Rightarrow$ rule update)
- Remove edges having minimum cost $min + s$ greater than ${}^{M}/_{\chi}$
- Remove the isolated nodes

If there are no paths connecting a $Init$ resource to a $Goal$ resource/goal, the resulting game might be hard (if not impossible) to solve for an user of XP $\chi$, and such scenario might be discarded.

The suitable solutions for $\chi$ are the remaining graphs.

# Power and Costs

**Power**:

- is everything providing an advantage
  - $\Rightarrow$ it gives positive feedback to the player.
- Success leads to more power within a positive loop, and increase the chances of winning.

**Costs**:

- are everything causing a disadvantage
  - $\Rightarrow$ it gives negative feedback to the player.
- For winning the game, I might pay some initial costs for getting, at the end, greater rewards (e.g., *quests*)
  - $\Rightarrow$ the agent should be able to plan strategies!
- *Shadow costs*: a game does not even provide feedback on the disadvantages, that will resolve into a greater chance for loosing.

# Positive Feedback Loops

- Please observe that the notion of feedback is tightly related with the one of game progression.
- Do you remember how we used to model game progression in the previous slides?

# Positive Feedback Loops

- Please observe that the notion of feedback is tightly related with the one of game progression.
- Do you remember how we used to model game progression in the previous slides? Through a transition model!

# Positive Feedback Loops

- Please observe that the notion of feedback is tightly related with the one of game progression.
- Do you remember how we used to model game progression in the previous slides?
- Collecting positive feedback and making no use of it will not help the user advancing the game:
  1. E.g., in a strategic game, capturing an enemy territory leads on the long run to greater resources.
  2. If the collected resources are not exploited for advancing the civilization or in conquering new territories, the user will never reach the goal test condition.
  3. Therefore, the player should enact a strategy for performing sensible actions in different game configuration/states.

# Conclusions (1/2)

Solvability:

- by representing a game with a set of rules, we might detect if the game is solvable and,
- if not, we might get where the bug is while generating the game.

Difficulty:

- In non-stochastic games, we might exploit solvability techniques to generate and rank possible alternatives to solve the game.

# Conclusions (2/2)

This section introduced the key concepts to understand Reinforcement Learning.

- We defer the in-depth discussion on the interplay between actions, probabilities, and rewards in the *Players and adversaries* set of lectures: **Correlating Skills with Strategies**.
- In that occasion, we are going to provide additional details on how probabilities, feedback, and skills might provide different strategies for solving one single game.
- We will focus on **PvE** games.