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Użytkowników Systemu TEX
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‘Pots.tex’ ~~and Other Useful Plain TEX Packages~~

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An example of formula with 4 pluses: $1 + 2 + 3 +$
$+ 4 + 5 = 15$.

`An example of formula with 4 pluses:`
`$1+2+3+4+5=15$.`

An example of formula without pluses: $10000 +$
$- 1000 = 1000$.

`An example of formula without pluses:`
`$10000-1000=1000$.`

D o w ó d. Jeżeli $5|a$, to $5^4|a^4$ i tym bardziej $5^4|a^{100}$. Jeżeli zaś $(a, 5)=1$, to (w myśl ćwiczenia 3) jest $5^2|a^{20}-1$ i $a^{20n}\equiv 1 \pmod{25}$ dla $n=1, 2, 3, 4$: stąd $25|(a^{80}+a^{60}+a^{40}+a^{20}+1)$, a ponieważ $a^{100}-1=(a^{20}-1)(a^{80}+a^{60}+a^{40}+$ $+a^{20}+1)$, więc $125|a^{100}-1$.

5. Dowieść, że 9-a potęga każdej liczby całkowitej jest postaci $19k$ lub $19k\pm 1$.

D o w ó d. Jeżeli $19|a$, to $a=19k$. Jeżeli $(a, 19)=1$, to $a^{18}\equiv 1 \pmod{19}$; zatem $19|(a^9-1)(a^9+1)$ i przeto jeżeli nie jest $19|a^9-1$, to musi być $19|a^9+1$ czyli $a^9=19k-1$.

6. Dowieść, że dla każdej liczby całkowitej x jest $x^8\equiv 0$, 1 lub $-1 \pmod{17}$.

7. Dowieść, że dla wszelkich x całkowitych jest $42|(x^7-x)$.

D o w ó d. Dla x całkowitych mamy oczywiście $2|x^7-x$; w myśl zaś wniosku 2 z małego twierdzenia F e r m a t a jest $x^3\equiv x \pmod 3$, skąd $x^7\equiv x^5\equiv$ $\equiv x^3\equiv x \pmod 3$, a zatem $3|x^7-x$; dalej $x^7\equiv x \pmod 7$, zatem $7|x^7-x$, a ponieważ $(2, 3, 7)=1$, więc $42|x^7-x$, c. b. d. o.

8. Dowieść, że kwadrat liczby całkowitej jest zawsze postaci $5k$ lub $5k\pm 1$.

9. Dowieść, że jeżeli $(k, 6)=1$, to $6|(k^2+5)$.

10. Dowieść, że dla x całkowitych liczba $\dfrac{x^5}{120}-\dfrac{x^3}{24}+\dfrac{x}{30}$ jest całkowita.

11. Dowieść, że jeżeli p jest liczbą pierwszą, to $p|(5^p-2\cdot 3^p+1)$.

D o w ó d. Dla $p=2, 3$ i 5 z łatwością sprawdzamy to bezpośrednio. Dla $p>5$ jest $(5, p)=1$, zatem w myśl wniosku z małego twierdzenia F e r m a t a jest $5^p\equiv 5 \pmod p$; podobnie $3^p\equiv 3 \pmod p$, zatem $5^p-2\cdot 3^p+1\equiv 5-2\cdot 3+1\equiv$ $\equiv 0 \pmod p$, c. b. d. o.

```
\def\fixbin#1{\mathchoice
  {\discretionary{\hbox{$
    \displaystyle\mskip\medmuskip
    {#1}$}}{}{}}
  {\discretionary{\hbox{$
    \textstyle\mskip\medmuskip
    {#1}$}}{}{}}
  {\discretionary{\hbox{$
    \scriptstyle
    {#1}$}}{}{}}
  {\discretionary{\hbox{$
    \scriptscriptstyle
    {#1}$}}{}{}}
  #1\nobreak}
```

```
\def\binmathchardef#1="#2#3#4#5{%
  \xdef#1{\fixbin{\mathchar"#2#3#4#5}}}
```

```
\binmathchardef\cap="225C
\binmathchardef\cup="225B
```

```
\mathcode`\+="8000
{\catcode`\+=13 \binmathchardef+="202B }
\mathcode`\-="8000
{\catcode`\-=13 \gdef-{\specialfixbin{%
  \mathchar"202B}{\mathchar"2200}}}
```

$$2 \mid 6 \qquad 4 \nmid 6$$

```
$$2\mid6\qquad4\nmid6$$
```

$$\displaylines{\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta, \iota, \kappa, \lambda, \mu, \nu, \xi, \pi, \rho, \sigma, \tau, \upsilon, \phi, \chi, \cr \psi, \omega, \varepsilon, \vartheta, \varpi, \varrho, \varsigma, \varphi, \mathit{\Gamma}, \mathit{\Delta}, \mathit{\Theta}, \mathit{\Lambda}, \mathit{\Xi}, \mathit{\Pi}, \mathit{\Sigma}, \mathit{\Upsilon}, \mathit{\Phi}, \mathit{\Psi}, \mathit{\Omega}}$$

```
$$\displaylines{\italpha,\itbeta,
  \itgamma,\itdelta,\itvarepsilon,
  \itzeta,\iteta,\ittheta,\itiota,
  \itkappa,\itlambda,\itmu,\itnu,\itxi,
  \itpi,\itrho,\itsigma,\ittau,
  \itupsilon,\itvarphi,\itchi,\cr
  \itpsi,\itomega,\itepsilon,\itvartheta,
  \itvarpi,\itvarrho,\itvarsigma,\itphi,
  \itGamma,\itDelta,\itTheta,\itLambda,
  \itXi,\itPi,\itSigma,\itUpsilon,\itPhi,
  \itPsi,\itOmega}$$
```

$$\int\limits_a^b f(x)/g(x)\,dx = \displaystyle\int\limits_a^b \frac{f(x)}{g(x)}dx$$

```
$$\int_a^bf(x)/g(x)\,dx=
  \Int_a^b\Frac{f(x)}{g(x)}dx$$
```

i trzecim podprzedziale dodatni, w drugim ujemny. Mamy więc

$$P=\int_{-2}^{0}(x^3+x^2-2x)\,dx-\int_{0}^{1}(x^3+x^2-2x)\,dx+\int_{1}^{2}(x^3+x^2-2x)\,dx.$$

Obliczamy pola w poszczególnych podprzedziałach:

$$\int_{-2}^{0}(x^3+x^2-2x)\,dx=\left[\tfrac{1}{4}x^4+\tfrac{1}{3}x^3-x^2\right]_{-2}^{0}=0-\left[\tfrac{16}{4}-\tfrac{8}{3}-4\right]=\tfrac{8}{3},$$

$$-\int_{0}^{1}(x^3+x^2-2x)\,dx=-\left[\tfrac{1}{4}x^4+\tfrac{1}{3}x^3-x^2\right]_{0}^{1}=-\left[\tfrac{1}{4}+\tfrac{1}{3}-1\right]=\tfrac{5}{12}.$$

$$\int_{1}^{2}(x^3+x^2-2x)\,dx=\left[\tfrac{1}{4}x^4+\tfrac{1}{3}x^3-x^2\right]_{2}^{1}=\left[\tfrac{16}{4}+\tfrac{8}{3}-4\right]-\left[\tfrac{1}{4}+\tfrac{1}{3}-1\right]=\tfrac{37}{12}.$$

Całe pole wynosi $P=\frac{8}{3}+\frac{5}{12}+\frac{37}{12}=\frac{37}{6}$.

ZADANIE 19.4. Obliczyć całkę oznaczoną

$$I(\alpha)=\int_{0}^{1}\frac{dx}{1+2x\cos\alpha+x^2},\qquad \text{gdzie}\qquad -\pi<\alpha<\pi.$$

$$\wedge = \wedge, \bigwedge = \bigwedge, \forall, \vee = \vee, \bigvee = \bigvee, \exists,$$

$$\textstyle\coprod, \displaystyle\coprod, \textstyle\sum, \displaystyle\sum, \textstyle\prod, \displaystyle\prod, \textstyle\int, \displaystyle\int, \textstyle\oint, \displaystyle\oint, \textstyle\bigcap, \displaystyle\bigcap,$$

$$\textstyle\bigcup, \displaystyle\bigcup, \textstyle\bigsqcup, \displaystyle\bigsqcup, \textstyle\bigodot, \displaystyle\bigodot, \textstyle\bigotimes, \displaystyle\bigotimes, \textstyle\bigoplus, \displaystyle\bigoplus, \textstyle\biguplus, \displaystyle\biguplus$$

```
$$\displaylines{
  \forall=\bigwedge,\Forall=\Bigwedge,
  \plainforall,\exists=\bigvee,
  \Exists=\Bigvee,\plainexists,\cr
  \coprod,\Coprod,\sum,\Sum,\prod,
  \Prod,\int,\Int,\oint,\Oint,\bigcap,
  \Bigcap,\cr
  \bigcup,\Bigcup,\bigsqcup,\Bigsqcup,
  \bigodot,\Bigodot,\bigotimes,
  \Bigotimes,\bigoplus,\Bigoplus,
  \biguplus,\Biguplus}$$
```

$$\int\limits_a^b\frac{1}{f\left(x\right)}\,dx+\lim_{n\to\infty}\frac1n=\int\nolimits_a^b\frac{1}{f\left(x\right)}\,dx+\lim\nolimits_{n\to\infty}\frac1n$$

```
$$\Int_a^b\Frac1{f(x)}\,dx+
  \lim_@{n\to\infty}\,\frac1n=
  \Int\nolimits_a^b\Frac1{f(x)}\,dx+
  \lim\nolimits_{n\to\infty}\frac1n$$
```

inf, lim, liminf, limsup, max, min, sup

```
$$\inf,\lim,\liminf,\limsup,
  \max,\min,\sup$$
```

$$\sin^2\alpha=(\sin\alpha)^2\qquad\lg=\log_2$$

```
$$\sin^2\alpha=(\sin\alpha)^2\qquad
  \lg=\log_2$$
```

$$\displaylines{\arccos,\arcsin,\arctan,\cos,\cosh,\cot,\coth,\csc,\cr \deg,\det,\dim,\exp,\hom,\ker,\lg,\ln,\log,\operatorname{ord},\operatorname{rank},\cr \sec,\sin,\sinh,\tan,\tanh,\operatorname{NWD},\operatorname{NWW},\Pr}$$

```
$$\displaylines{\arccos,\arcsin,\arctan,
  \cos,\cosh,\cot,\coth,\csc,\cr
  \deg,\det,\dim,\exp,\hom,\ker,\lg,
  \ln,\log,\ord,\rank,\cr
  \sec,\sin,\sinh,\tan,\tanh,
  \NWD,\NWW,\Pr}$$
```

How many elements is in set: $\{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$?

```
How many elements is in set:
$\{a,b,c,d,e,f,g,h,i,j,k,l,m,
   n,o,p,q,r,s,t,u,v,w,x,y,z\}$?
```

$$\left[\frac{1}{2},\frac{3}{4}\right)=[\frac{1}{2},\frac{3}{4})$$

```
$$[\Frac12,\Frac34)=\[\Frac12,\Frac34\)$$
```

$$\left(,\left[,\left\langle,\left\lceil,\left\lfloor,\vrule width0pt height2ex\right\rfloor,\right\rceil,\right\rangle,\right],\right)$$

```
$$(,[,\<,\lceil,\lfloor,
  \vrule width0ptheight2ex
  \rfloor,\rceil,\>,],)$$
```

$$(,[,\langle,\lceil,\lfloor,\rfloor,\rceil,\rangle,],)$$

```
$$\(,\[,\plainlangle,\plainlceil,
  \plainlfloor,\plainrfloor,
  \plainrceil,\plainrangle,\],\)$$
```

$$a \cdot b = b \cdot a$$

```
$$a*b=b\cdot a$$
```

$$1 + \cdots + n = \frac{n(n+1)}{2} \qquad \hbox{for} \quad n = 1, 2, \ldots$$

```
$$1+***+n=\frac{n(n+1)}2\qquad
  \hbox{for}\quad n=1,2,...$$
```

$$\varepsilon = \varepsilon \neq \epsilon$$

```
$$\epsilon=\varepsilon\ne\plainepsilon$$
```

$$\varphi = \varphi \neq \phi$$

```
$$\phi=\varphi\ne\plainphi$$
```

$$\ell = \ell \neq l$$

```
$$l=\ell\ne\plainl$$
```

$$\sum_{i=1}^{100000} a_i + \sum_{i=100001}^{200000} a_i = \sum_{i=1}^{100000} a_i + \sum_{i=100001}^{200000} a_i$$

```
$$\Sum_@{i=1}^@{100000}a_i+
  \Sum_@{i=100001}^@{200000}a_i=
  \Sum_{i=1}^{100000}a_i+
  \Sum_{i=100001}^{200000}a_i$$
```

$$f\colon X \to Y \quad\iff\quad f\colon X \to Y$$

```
$$f:X\to Y\quad\iff\quad f\colon X\to Y$$
```

$$\widehat{abc}$$

```
$$\widehat{abc}$$
```

$$\leqslant = \leqslant = \leqslant = \leqslant \ne \leq \qquad \geqslant = \geqslant = \geqslant = \geqslant \ne \geq$$

```
$${\le}={\leq}={\xle}={\xleq}
    \ne{\plainle}\qquad
  {\ge}={\geq}={\xge}={\xgeq}
    \ne{\plainge}$$
```

$$\equiv = \equiv \ne \cong$$

```
$${\cong}={\equiv}\ne{\plaincong}$$
```

$$a = b \iff b = a \qquad a = b \Longleftrightarrow b = a$$

```
$$a=b\iff b=a\qquad
  a=b\Longleftrightarrow b=a$$
```

$$\{n\in \mathbf{N}|\; n^2\notin \mathbf{N}\}=\emptyset$$

```
$$\{n\in\NN\:n^2\notin\NN\}=\emptyset$$
```

$$\left\langle\frac{1}{2},\frac{3}{4}\right\rangle = \left\langle\frac{1}{2},\frac{3}{4}\right\rangle$$

```
$$\<\Frac12,\Frac34\>=
  \langle\Frac12,\Frac34\rangle$$
```

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

```
$$\det[\matrix{a&b\cr c&d}]=ad-bc$$
```

$$\begin{aligned}1\cdot 1&=1, &\qquad 2+2&=4, &\qquad 1+2&=3,\\ 1+1&=2, & 2\cdot 2&=4, & 1+1+2&=4.\end{aligned}$$

```
$$\eqalign{1*1&=1,&\qquad
           2+2&=4,&
           1+2&=3,\cr
           1+1&=2,&
           2*2&=4,&\qquad
         1+1+2&=4.}$$
```

```
\def\eqalign#1{\null\,\vcenter{%
  \openup\jot\m@th\ialign{%
    &\strut\hfil$\displaystyle{##}$%
    &$\displaystyle{{}##}$\hfil
    \crcr#1\crcr}}\,}
```

```
\def\eqalign#1{\null\,\vcenter{%
  \openup\jot\m@th\ialign{%
     \strut\hfil$\displaystyle{##}$%
    &$\displaystyle{{}##}$\hfil
    \crcr#1\crcr}}\,}
```

$$\mathbf{P} \subseteq \mathbf{N} \subseteq \mathbf{Z} \subseteq \mathbf{Q} \subseteq \mathbf{R} \subseteq \mathbf{C}$$

```
$$\PP\subset\NN\subset\ZZ\subset
  \QQ\subset\RR\subset\CC$$
```

$$10 \equiv 1 \pmod{3}$$

```
$$10\cong1\mod3$$
```

$$\tfrac{1}{2} = \dfrac{1}{2}$$

```
$$\frac12=\Frac12$$
```

$$1 + 1 = 2 \tag{1}$$

and

$$1 + 1 + 1 = 3 \tag{2}$$

```
$$\eqalignno{1+1&=2    &(1)\cr
   \intertext{and}
             1+1+1&=3    &(2)}$$
```

Do not break the text manualy
just leave it TeX...

```
Do not break the text manualy\\
just leave it \TeX\dots
```

This paragraph with sum $\sum_{i=1}^n a_i$ looks worse than the following one.

```
This paragraph with sum
$\sum_@{i=1}^na_i$
looks worse than the following one.
```

This paragraph with sum $\sum_{i=1}^n a_i$ looks better than the above one.

```
This paragraph with sum
$\smashop{\sum_@{i=1}}^na_i$
looks better than the above one.
```

Twierdzenie 6. *Każda liczba graniczna postaci* $\lambda = \lim\limits_{\xi<\alpha} \varphi(\xi)$, *jest współkońcowa z jakąś liczbą* $\gamma \leqslant \alpha$ [1]).

D o w ó d. Zgodnie z twierdzeniem 5 istnieje ciąg ψ typu α taki, że

(i) $\psi(\xi)$ jest najmniejszą liczbą ζ taką, że $\varphi(\xi) < \zeta < \lambda$ oraz $\prod\limits_{\eta<\xi}(\psi(\eta)<\zeta)$, o ile taka liczba istnieje,

```
\def\dosmashop#1#2\end{\mathop{%
  \vphantom{#1}\smash{#1#2}}\limits}
\def\smashop#1{\dosmashop#1\end}
```

Theorem 1. For each number $n \in \mathbf{N}$ holds an equality

$$1 + \cdots + n = \frac{n(n+1)}{2}.$$

Proof. The proof is left to the reader as an exercise. $\square$

```
{\bf Theorem 1.} \ For each number
$n\in\NN$ holds an equality
$$1+***+n=\Frac{n(n+1)}2.$$
{\it Proof.} \ The proof is left to
the reader as an exercise.\qed
```

Proof. The proof follows from the equalities:

$$1 + \cdots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \qquad \square$$

```
{\it Proof.} \ The proof follows
from the equalities:
$$1+***+n=\sum_@{i=1}^ni=
  \Frac{n(n+1)}2.\eqno\qed$$
```

Proof. The proof follows from the equalities:

$$\begin{aligned} 1+\cdots+n &= \frac{(1+\cdots+n)+(n+\cdots+1)}{2} = \\ &= \frac{(n+1)+\cdots+(n+1)}{2} = \\ &= \frac{n(n+1)}{2}. \end{aligned} \qquad \square$$

```
{\it Proof.} \ The proof follows
from the equalities:
$$\eqalignno{1+***+n&=
  \Frac{(1+***+n)+(n+***+1)}2=\cr&=
  \Frac{(n+1)+***+(n+1)}2=\cr&=
  \Frac{n(n+1)}2.&\qed}$$
```

First trivial equality:

$$1 = 1. \tag{$*$}$$

Second trivial equality:

$$1 + 1 = 2. \tag{$**$}$$

Third trivial equality:

$$1 + 1 + 1 = 3. \tag{$\overset{**}{*}$}$$

```
First trivial equality:
$$1=1.\eqno\x$$
Second trivial equality:
$$1+1=2.\eqno\xx$$
Third trivial equality:
$$1+1+1=3.\eqno\xxx$$
```

GCD(a, b)

while $b \neq 0$ **do**

  set $c = a \pmod b$

  set $a = b$

  set $b = c$

return a

```
\pseudocode
GCD$(a,b)$
while $b\ne0$ do
   "set" $c=a\mod b$
   "set" $a=b$
   "set" $b=c$
return $a$
\endpseudocode
```

Testing UTF-8: Zażółć gęślą jaźń…

```
Testing UTF-8: Zażółć gęślą jaźń\dots
```

```
\def\utftotex#1#2#3{\expandafter\gdef%
  \csname UTF-8 #1#2\endcsname%
  {\char"#3\relax}}
```

```
\utftotex{c3}{^^93}{D3}
```

```
\catcode"C3=13 \def^^c3#1{%
  \csname UTF-8 c3#1\endcsname}
```

What is to do in ‘pots.tex’?

- ☞ Correct (independent of the chosen font) look of the symbol `\nmid`;
- ☞ Automatic hiding the indices of operators;
- ☞ Automatic insertion of additional spaces in operator expressions with too wide indices because of use @;
- ☞ Support for grid typesetting.