Reinforcement Learning

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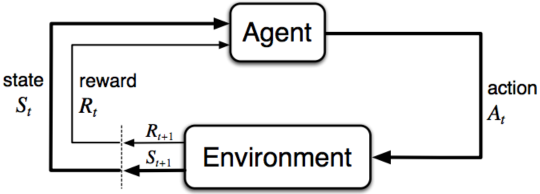
Overview

1. Reinforcement Learning
2. Markov Decision Processes

Reinforcement Learning

- **Framing of the problem of learning from interaction to achieve a goal.**
- **Agent**: learner and decision maker
- **Environment**: what the learner interacts with (everything outside the agent)
- Agent selects actions and the environment responds to those actions and presents new situations

Reinforcement Learning



- At each time step t, the agent receives the environment **state** $S_t \in \mathcal{S}$, and the agent then selects an **action** $A_t \in \mathcal{A}(S_t)$
  - $\mathcal{S}$ is the set of possible states (whatever information is available to the agent).
  - $\mathcal{A}(S_t)$ is set of actions available in state S_t
- One time step later, the agent receives a **reward**, $R_{t+1} \in \mathcal{R} \subset \mathbb{R}$, and ends up in a new state S_{t+1}

Reinforcement Learning

- At each time step, the agent implements a mapping π_t from states to probabilities of selecting each possible action, where π_t is called a **policy**
  - $\pi_t(a|s) =$ probability that $A_t = a$ if $S_t = s$

Reinforcement Learning Objective

The agent's goal is to maximize the total amount of reward it receives over the long run by changing its policy as a result of its experience

- Let the sequence of rewards after time step t is $R_{t+1}, R_{t+2}, R_{t+3}, \ldots$, then we want to maximize the return G_t
- The agent chooses A_t to maximize the discounted return:

$$G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

  where γ is the discount rate and $0 \geq \gamma \leq 1$
- The closer γ is to 1, the more the agent accounts for future rewards

Markov Property

- Probability of transitioning to new state s:

$$p(s', r|s, a) = Pr\{S_{t+1} = s', R_{t+1} = r|S_0, A_0, R_1, ..., S_{t-1}, A_{t-1}, R_t, S_t, A_t\}$$

- Probability with Markov assumption

$$p(s', r|s, a) = Pr\{S_{t+1} = s', R_{t+1} = r|S_t = s, A_t = a\}$$

- Markov assumption allows us to predict the next state and expected rewards from knowledge of only the current state
  - Assume that the current state tells us everything we need to know for future (e.g. current state of checker board)

Markov Decision Processes

- A R.I. task that satisfies the Markov property is called a Markov Decision Process (MDP)
- Given any state s and action a, the probability of each possible pair of next state and reward s', r, is denoted

$$p(s', r|s, a) = Pr\{S_{t+1} = s', R_{t+1} = r|S_t = s, A_t = a\}$$

From this probability representation, we can compute anything else we might need to know about the environment

Markov Decision Processes

- Expected rewards for state–action pairs:

$$r(s,a) = \mathbb{E}[R_{t+1} | S_t = s, A_t = a] = \sum_{r \in \mathcal{R}} r \sum_{s' \in \mathcal{S}} p(s', r | s, a)$$

- State-transition probabilities:

$$p(s' | s, a) = Pr\{S_{t+1} = s' | S_t = s, A_t = a\} = \sum_{r \in \mathcal{R}} p(s', r | s, a)$$

- Expected rewards for state–action–next-state triples:

$$r(s,a,s') = \mathbb{E}[R_{t+1} | S_t = s, A_t = a, S_{t+1} = s'] = \frac{\sum_{r \in \mathcal{R}} r * p(s', r | s, a)}{p(s' | s, a)}$$

References

Richard S. Sutton and Andrew G. Barto. Reinforcement Learning: An Introduction. MIT Press 2015.