Electron Diffraction

P. Cardegna

Revised L. Barton March 11, 2008

Introduction

This experiment observes the wave-like properties of electrons. The wavelength of the electrons is measured, and is compared to the wavelength predicted by the DeBroglie relation.

X-ray Diffraction

The wavelength of an electromagnetic wave can determined by analyzing the diffraction pattern resulting from the wave passing through a grating of suitable spacing (spacing comparable to the wavelength of the wave). For x-rays, this spacing is of the order of the interatomic spacing.

Consider the two-dimensional model of a crystalline solid. Assume a square crystal structure in which the lattice spacing; that is, the spacing between adjacent atoms in the solid, is a. The model is shown in Figure 1.

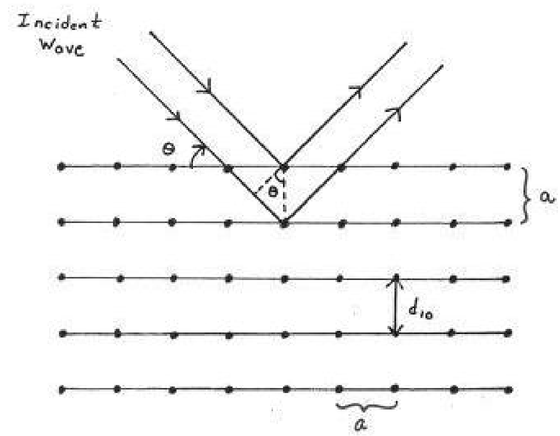

Figure 1: Geometry for Bragg's Law assuming a square lattice. The black dots represent the equilibrium positions of the atoms.

When an incident beam strikes the surface at an angle θ with respect to the surface, constructive interference will occur if the additional path length traveled by the wave reflecting from the adjacent surface is an integer multiple of the wavelength; specifically

$$2d_{hk}\sin\theta = m\lambda \qquad m = 1, 2, 3\ldots \qquad (1)$$

where d_{hk} is the spacing between adjacent planes and θ is the angle between the incident beam and the reflecting plane. The angle between the incident and the reflected beam is therefore 2θ. Equation 1 is referred to as Bragg's law. For the planes shown in Figure 1, the spacing d_{10} is simply a.

However, there are many other possible planes from which the diffraction pattern can be constructed. For the square lattice, another possible set of planes is shown in Figure 2. For these planes, the spacing d_{11} is $a/\sqrt{2}$. The subscripts on the d_{hk} refer to the standard crystallographic identification of the planes used.

If a beam consisting of a broad range of wavelengths is incident on a single crystal, the interference pattern consists of a set of diffraction points. The points correspond to different wavelengths constructively interfering after reflecting from different planes.

If a monochromatic beam is incident on a

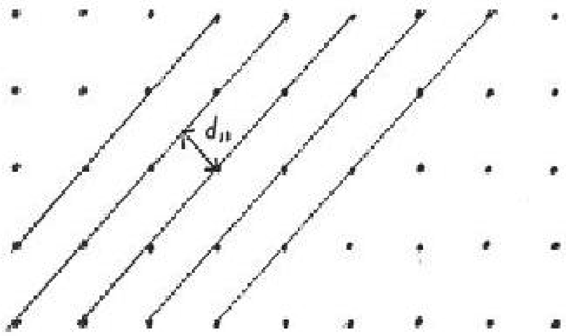

Figure 2: A second possible set of planes for Bragg reflection from a square lattice.

polycrystalline solid, the resulting pattern consists of concentric diffraction rings. Each ring corresponds to diffraction from a set of planes of different spacing. The diameter of one ring D is related to the diffraction angle θ and distance L between the plane of the target and the viewing screen; specifically,

$$\tan 2\theta = \frac{D/2}{L} \tag{2}$$

The geometry is shown in Figure 3.

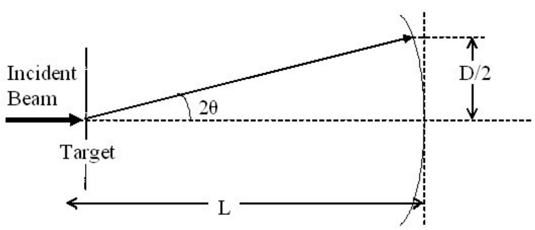

Figure 3: Relation between diffraction angle θ and ring diameter D.

Electron Diffraction

By the early 1920's, it was clearly established that electromagnetic radiation possessed both wave- and particle-like characteristics. In 1924, Louis deBroglie argued that symmetry encouraged, and the laws of physics did not forbid, that particles also possess both wave- and particle-like characteristics. The fundamental relation relating the magnitude of the particle's momentum p to its deBroglie wavelength λ is

$$\lambda = \frac{h}{p} \tag{3}$$

where h is Planck's constant. Davisson and Germer, scattering electrons from a nickel target, quantitatively verified this expression in 1925.

Apparatus: The TelAtomic 2555 Electron Diffraction Tube

The electron diffraction tube consists of an electron source, a grid to accelerate the electrons to a known energy, and a thin polycrystalline graphite target contained inside an evacuated glass tube. The source of the electrons is an indirectly heated cathode. An anode voltage V_a accelerates the electrons toward the target. The electrons diffract through the graphite target and impinge on a luminescent surface that has been deposited on the inside surface of the glass. A schematic of the hexagonal graphite lattice and the two possible diffraction planes is shown in Figure 4.

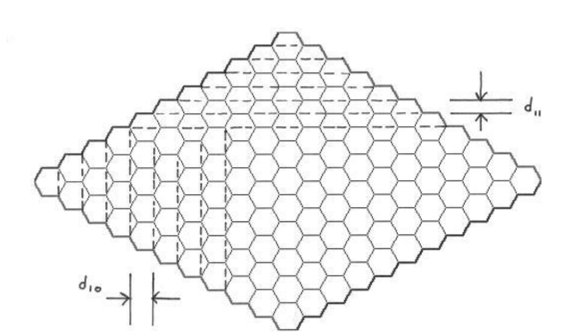

Figure 4: Two-dimensional hexagonal lattice with the two possible diffraction planes indicated.

Procedure

Wait a few minutes after turning on the heater supply to allow the cathode to reach thermal equilibrium. Set the anode voltage to about 2.5 kV and observe the two diffraction rings. They are "fuzzy" bright rings. Measure the diameter for each of the two diffraction rings using vernier calipers. The "fuzzy" rings are difficult to measure accurately. Take at few measurements of the diameter of the smaller ring at different positions around the ring, using the innermost edge of the bright smaller ring, and record those values. Repeat using the middle of the bright band. Do the same measurements (innermost edge and middle) for the larger ring. Repeat all these measurements for several different accelerating anode voltages between 2.0 and 5.0 kV. Thus at each anode voltage you should have readings for the diameter at the innermost edge of the smaller ring, the middle of the smaller ring, the innermost edge of the larger ring, and the middle of the larger ring. Tabulate your raw data results.

Analysis

The measured diameter of the diffraction rings D must be corrected for the curvature and thickness of the glass. According to the manufacturer, the radius of curvature r of the inside surface of the glass is 6.6 cm, the thickness t of the glass is 1.5 mm, and the distance L between the target and the outside surface of the glass envelope is (0.130 + 0.002) m. See Figure 5.

The known interplanar spacings for hexagonal graphite are $d_{11} = 0.123$ nm and $d_{10} = 0.213$ nm.

For each anode voltage, use Equation 3 to calculate the deBroglie wavelength. Calculate the uncertainty in the wavelength. For each anode voltage, use Equation 2 and the corrected diameter to calculate the diffraction angle θ and its uncertainty. Notice that the angle θ is small, so that $sin\theta = tan\theta \cong \theta$. Then, use Equation 1, the calculated value of θ, and the known interplanar spacings for hexagonal graphite, to calculate the Bragg wavelength (and its uncertainty). Note that all your observations are of first order Bragg diffraction, that is, $m = 1$ in Equation 1. The two rings are visible due to the two different lattice spacings d_{hk}. You must decide which ring corresponds to which d_{hk}; that is, decide whether a ring with smaller diameter (angle θ) has the smaller, or the larger, lattice spacing d_{hk}.

Compare the values of the wavelengths determined from the deBroglie relationship and those determined from the Bragg relationship. Do this for your measurements using the innermost edges of the rings as well as those from the middle of the bright band. Which of these Bragg wavelengths agrees better with the deBroglie wavelengths, the ones determined from the diameter measured at the innermost edge of each diffraction ring or the ones determined from the diameter of the middle of each of the fuzzy rings? Would you advise the next student to measure the innermost edge of the ring or the middle?

One useful way to determine how well the Bragg wavelengths agree with the deBroglie wavelengths is to compute the ratio of the two, for each Bragg wavelength. Based on the average and the standard deviation of these values, how well do your Bragg and deBroglie wavelengths agree? (3%? 10%? 30%?)

References

Bragg's Law and Diffraction: How waves reveal the atomic structure of crystals Applet created by Konstantin Lukin. Website, 2008

`http://www.eserc.stonybrook.edu/ProjectJava/Bragg/`

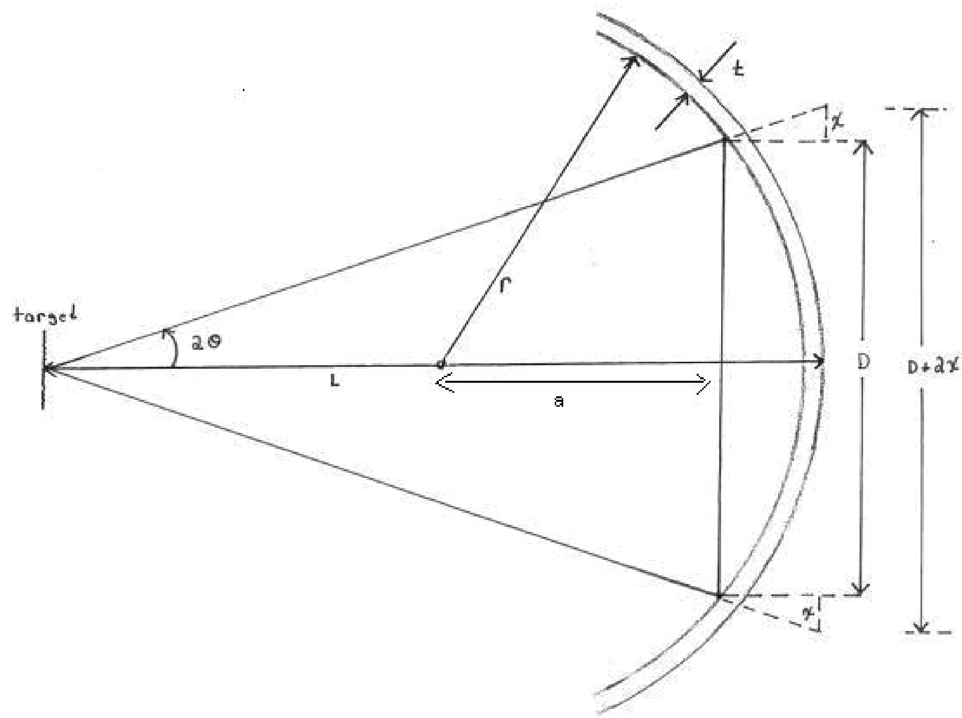

Figure 5: Geometry for diameter correction. If D is the measured diameter, then the corrected diameter is $(D + 2x)$