Electrons in a Magnetic Field

Introduction:

When an electron is accelerated from rest through a potential difference of magnitude V, it gains kinetic energy given by

$$\frac{1}{2}mv^2 = eV \qquad \text{(Equation 1)}$$

The Lorentz force on the electron moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$ is given by

$$\vec{F} = -e(\vec{v} \times \vec{B}) \qquad \text{(Equation 2)}$$

When the velocity is perpendicular to the field, the electron moves in a circular path with the centripetal force provided by the Lorentz force:

$$\frac{mv^2}{R} = evB \qquad \text{(Equation 3)}$$

Combining Equations 3 and 1, and rearranging, yields the radius of the path

$$R = \frac{\sqrt{2\mathrm{V}}}{B} \frac{1}{\sqrt{e/m}} \qquad \text{(Equation 4)}$$

A measurement of the radius of the electron's path at various values of the accelerating voltage V and the applied magnetic field B gives a direct measurement of the charge to mass ratio of the electron (e/m).

Apparatus:

The Leybold-Heraeus apparatus consists of a gas focused cathode ray tube in the form of a spherical evacuated glass bulb. Within the bulb, a filament is heated to generate electrons, which are subjected to a variable accelerating potential. The bulb contains a low pressure of hydrogen atoms which glow a faint blue when excited by the electron beam; this makes the path of the electrons visible.

The vacuum tube is mounted within a pair of coils that provide the applied magnetic field. This coil configuration is a common one when a uniform adjustable magnetic field is needed: it is called a Helmholz coil pair. Helmholz coils consist of two identical circular coils, driven with the same current, placed a distance apart that is equal to their radius.

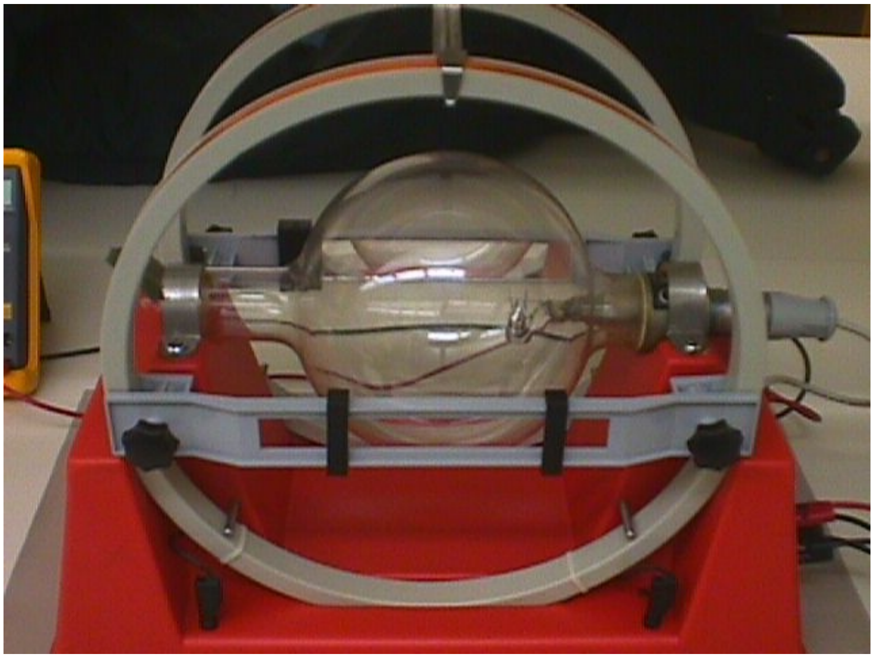

Figure 1: The Leybold-Heraeus e/m apparatus, showing the spherical evacuated tube and the Helmholz coil pair used to supply the magnetic field. The two horizontal (grey) assemblies mounted on the Helmholz coils are used to measure the radius of the path.

Any university physics text will show the field of a loop of wire, where the loop is in the x-y plane, and the field is measured along the symmetry axis out of that plane, the z axis. For a single loop of radius R_o, driven by current I, the magnitude of the field is given by

$$B(z) = \frac{\mu_o I R_0^2}{2(R_{o^2} + z^2)^{3/2}} \quad . \qquad \text{(Equation 5)}$$

For a loop with multiple turns, this result is multiplied by N, the number of turns in the coil. For the Helmholz configuration, two such loops are placed a distance R_o apart, so that the center point of the assembly is at a distance $z = R_o/2$, and the fields generated by the two coils add. It is straightforward to show that the field of this configuration is given by

$$B_{helmholz} = \frac{\mu_o N I}{(5/4)^{3/2} R_o} \quad . \qquad \text{(Equation 6)}$$

The wiring for this apparatus is simple. Operators of this equipment should familiarize themselves with these connections; where they go and why. You will have to wire it later, for now just look it over.

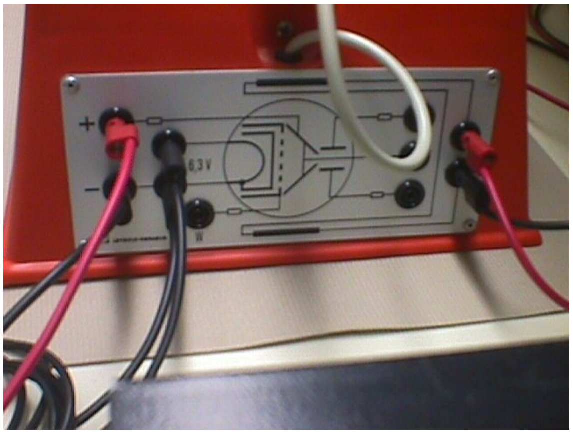

Figure 2: Wiring for the e/m tube. The left-most jacks provide the high voltage used to accelerate the electrons. The second pair of jacks, labeled 6.3V, provide power to the filament; which causes thermionic emission of electrons. To the right, the current to the Helmholz coil magnet is supplied.

An ordinary bench power supply powers the Helmholz coil magnet. It should be operated in Constant Current mode, and the current delivered monitored with an inline ammeter. Typically currents ranging from zero to about 3 amps are used.

A regulated high voltage dc power supply unit is used to provide the accelerating voltage for the electron beam. This unit also provides the 6.3Vac for the filament.

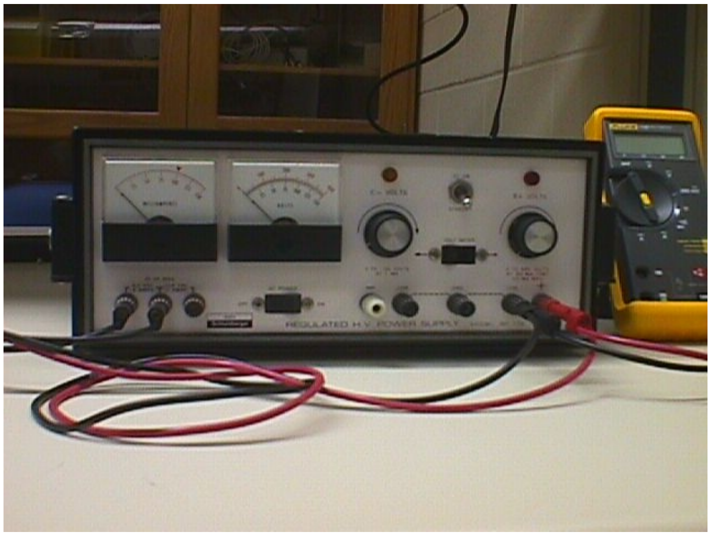

Figure 3: The power supply for e/m apparatus. The filament voltage is supplied by the left taps, the voltage labeled "B" is the high voltage output for the accelerating potential, on the right, controlled by the rightmost knob. Note that the unit has two switches, an AC Power rocker switch to turn it on, and then a toggle switch (upper right) to turn the high voltage stage from standby to on.

In order to determine the diameter of the circular path of the electrons in the electron beam tube, a measuring device consisting of two parts is used. Behind the tube, a rail with a mirror is mounted. In front of the tube, another rail is mounted with two movable "riders" that can be positioned side-to-side. These are grey, in the photos. When a circular beam path is observed one must visually align three things: the beam path, its reflection in the back mirror, and the "rider" in front. This is done by moving the rider, for both the left and the right side of the circular path. Then the distance between the riders can be measured with an ordinary ruler, yielding the diameter of the electron path. See Figure 4.

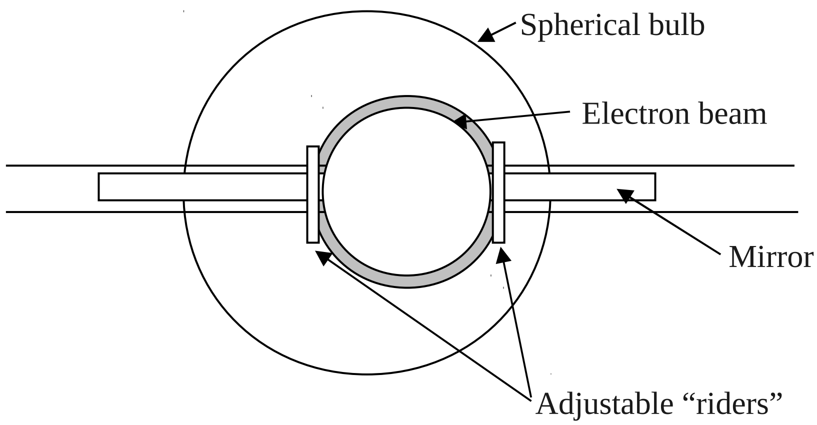

Figure 4: The riders used to measure the diameter of the electron beam circular path. The beam is aligned with its reflection in the mirror and with the rider, which minimizes parallax in the measurement.

Field Calibration:

Before beginning the actual experiment, you need to calibrate the field as a function of current and position for the Helmholtz coil.

A second apparatus with the e/m tube removed is provided. Use the FW Bell gaussmeter to calibrate the magnetic field strength. Turn it on, and place the probe in the zero field chamber, and zero the meter. Attach the bench power supply with ammeter to the apparatus with the tube missing. Mount the gaussmeter probe in the center of the coil and measure field strength as a function of current up to 3.0 Amps. Plot this and fit a straight line.

Set a current of 2.0 Amps, center the gaussmeter probe fore-and-aft, and measure the field strength as a function of position side to side. Plot this.

Helmholtz coils are known to provide spectacularly uniform B along their axis. Verify this by centering the probe side to side, and measuring field strength as a function of fore-and-aft (*z*-direction) position. Plot this. [Note: You can also prove this uniformity theoretically; both dB/dz and d^2B/dz^2 vanish at the center for the Helmholtz configuration of coil spacing equal to coil radius (derivatives of Equation 5). It's a very messy derivative you don't need to do.]

Analysis of Field Calibration Data:

Plot B at the center versus I. Is your data linear, as predicted by Equation 6? The equipment manufacturer gives nominal values of

N= 130 turns
R_o = 0.15 m

Are your results consistent with this? Compare the slope of your B(I) data with that predicted by Equation 6 and the nominal values for R_o and N.

Plot B vs position for both the fore and aft and side to side positions. For the size electron beam paths you observe, how uniform is B? (by what percentage does it vary, over this size?) Does the tremendous field uniformity along the z-direction (fore and aft) of the Helmholtz geometry matter for this experiment?

Procedure:

Wire the real equipment, and put aside the apparatus with the tube removed.

Turn the equipment on with the magnet current set to zero and the accelerating voltage also set to zero. Allow the filament to warm up for a few minutes.

Slowly ramp up the accelerating voltage until you see a straight blue line emanating from the filament; this should occur at around 100 V. You will need to work in a dark room in order to see anything. When you have an electron beam visible, turn up the magnet current until the beam goes in a circular path that fits inside the spherical bulb without hitting the bulb walls. Get a feel for how changing the accelerating voltage or the magnet current affects the diameter of the circle. Qualitatively, what does changing the accelerating potential or the magnet current do?

At a fixed voltage, vary the magnet current and measure diameter of the path at each magnet current. Repeat for several voltages. Accelerating voltages typically range from about 75 V to 350 V, while magnet currents vary from about ½ amp to 3 amps. At each accelerating voltage try to span the entire range of beam paths from as-big-as-fits to quite small.

If you have time, investigate what other magnetic fields (not perpendicular to the initial velocity of the electrons) do to the path, qualitatively.

Analysis:

Plot all of your data for diameter as a function of the variable $V^{1/2}/I$. From the slope, determine the ratio (e/m), with uncertainty. First, use the slope and your calibration constant that relates B and I. Then do it again, using the slope and the nominal constant (from the N and R given). Which is better?

Does your data pass through the origin? Should it? What would an intercept mean (a diameter of a circular orbit, at zero accelerating voltage)? Should you be fitting a line, or a simple proportionality with the fit “forced” through the origin?

Compare the e/m you determined with the one computed from tablulated known values of the fundamental constants e and m. Do they agree within uncertainty? Compute a percent difference between your measurement and the known.