TU WIEN Informatics

Adaptor Signature Based Atomic Swaps Between Bitcoin and a Mimblewimble Based Cryptocurrency

MASTER'S THESIS

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by

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Erklärung zur Verfassung der Arbeit

Jakob Abfalter, BSc

Hiermit erkläre ich, dass ich diese Arbeit selbständig verfasst habe, dass ich die verwendeten Quellen und Hilfsmittel vollständig angegeben habe und dass ich die Stellen der Arbeit – einschließlich Tabellen, Karten und Abbildungen –, die anderen Werken oder dem Internet im Wortlaut oder dem Sinn nach entnommen sind, auf jeden Fall unter Angabe der Quelle als Entlehnung kenntlich gemacht habe.

Wien, 6. April 2020

Jakob Abfalter

Acknowledgements

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Abstract

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Contents

CHAPTER 1

Introduction

Pedro: We need to discuss a structure for the introduction. Proposal:
- Introduce why coin exchanges are interesting
- Explain why atomic swaps protocols (e.g., one could use a trusted server for this and problem solved, right?)
- Why coin exchanges between Bitcoin and Mimblewimble?
- Why what you are proposing in this thesis is challenging?
- What are the main contributions of these thesis?
- What do you think is an interesting future research direction?

Mimblewimble The Mimblewimble protocol was introduced in 2016 by an anonymous entity named Jedusor, Tom Elvis [Jed16]. The author's name, as well as the protocols name, are references to the Harry Potter franchise. [1] In Harry Potter, Mimblewimble is a tongue-typing curse which reflects the goal of the protocol's design, which is improving the user's privacy. Later, Andrew Poelstra took up the ideas from the original writing and published his understanding of the protocol in his paper [Poe16]. The protocol gained increasing interest in the community and was implemented in the Grin [2] and Beam [3] Cryptocurrencies, which both launched in early 2019. In the same year, two papers were published, which successfully defined and proved security properties for Mimblewimble [FOS19, BCL+19].

Pedro: I would not add a line break at the end of each paragraph. The template should do that

Pedro: If you are going to compare to Bitcoin, you need to introduce Bitcoin before

[1] https://harrypotter.fandom.com/wiki/Tongue-Tying_Curse
[2] https://grin.mw/
[3] https://beam.mw/

Compared to Bitcoin, there are some differences in Mimblewimble:

- Use of Pedersen commitments instead of plaintext transaction values

  Pedro: The reader does not know what Pedersen commitments are at this point. Perhaps say transaction values are hidden from a blockchain observer while this is not the case in Bitcoin

- No addresses. Coin ownership is given by the knowledge of the opening of the coins Pedersen commitment.

  Pedro: This is also unclear. Could one see the commitment as the "address" in Mimblewimble? Perhaps you want to say that there is no scripting language supported?

- Spend outputs are purged from the ledger such that only unspent transaction outputs remain.

- No scripting features.

Pedro: Use "we" for contributions that you do in the thesis and "they" for parts that are borrowed from other works

Pedro: An intuition of these two terms is required here

Pedro: another sentence that shows that you need to explain before how Bitcoin works (the basics)

By utilizing Pedersen commitments in the transactions, we hide the amounts transferred in a transaction, improving the systems user privacy, but also requiring additional range proofs, attesting to the fact that actual amounts transferred are in between a valid range. Not having any addresses enables transaction merging and transaction cut through, which we will explain in section 3.3.3. However, this comes with the consequence that building transactions require active interaction between the sender and receiver, which is different than in constructions more similar to Bitcoin, where a sender can transfer funds to any address without requiring active participation by the receiver. Through transaction merging and cut-through and some further protocol features, which we will see later in this section, we gain the third mentioned property of being able to delete transaction outputs from the Blockchain, which have already been spent before. This ongoing purging in the Blockchain makes it particularly space-efficient as the space required by the ledger only grows in the number of UTXOs, in contrast to Bitcoin, in which space requirement increases with the number of overall mined transactions. Saving space is especially relevant for Cryptocurrencies employing confidential transactions because the size of the range proofs attached to outputs can be significant.

Pedro: What comes next is hard to read. It requires better organization: Advantages of Mimblewimble are: (i) .., (ii)...; Disadvantages are: (i)..., (ii),...).

Another advantage of this property is that new nodes joining the system do not have to verify the whole history of the Blockchain to validate the current state, making it much easier to join the network. Another limitation of Mimblewimble- based Cryptocurrencies is that at least the current construction does not allow scripts, such as they are available in Bitcoin or similar systems. Transaction validity is given solely by a single valid signature

plus the balancedness of inputs and outputs. This shortcoming makes it challenging to realize concepts such as multi signatures or conditional transactions which are required for Atomic Swap protocols. However, as we will see in 3.4 there are ways we can still construct the necessary transactions by merely relying on cryptographic primitives [FOS19].

CHAPTER 2

Motivation & Objectives

TODO

CHAPTER 3

Preliminaries

Pedro: Although not strictly required, IMO it is nice to have some text here introducing what the reader should expect in the rest of the section. For instance:
In this section, we first introduce the notation and definitions used hereby in this thesis. Then, we ..... Finally, we introduce.....

3.1 General Notation and Definitions

Notation We first define the general notation used in the following chapters to formalize procedures and protocols. Let $\mathbb{G}$ denote a cyclic group of prime order p and $\mathbb{Z}_p$ the ring of integers modulo p with identity element 1_p. $\mathbb{Z}_p^*$ is $\mathbb{Z}_p \setminus \{0\}$. g, h are adjacent generators in $\mathbb{G}$, where adjacent means the discrete logarithm of h in regards to g is not known. Exponentiation stands for repeated application of the group operation. We define the group operation between two curve points as $g^a \cdot g^{g^b} \stackrel{?}{=} g^{a\ +\ b}$.

Pedro: I think macro \ was broken here. I have updated to use \ instead. Please check that this is what you expected

Definition 3.1 (Hard Relation[AEE+20])**.** Given a language $L_R := \{A \mid \exists a \text{ s.t. } (A, a) \in R\}$ then the relation R is considered hard if the following three properties hold:

1. $\mathsf{genRel}((1^n))$ is a PPT sampling algorithm which outputs a statement/witness of the form $(A, a) \in R$.
2. Relation R is poly-time decidable.
3. For all PPT adversaries $\mathcal{A}$ the probability of finding a given A is negligible.

Pedro: We normally do not use the tilde to add spaces in math mode

Pedro: I would include these two relations below as your own definitions because I imagine that you would like to refer to them afterwards in the thesis

In this thesis we find two types of hard relations:

1. The output of a secure hash function (as defined in 3.3) and it's input $(I, \mathsf{H}(I))$.
2. The discrete logarithm x of g^x in the group $\mathbb{G}$.

Pedro: Link to paper/book where you got this definition from is missing

Pedro: What is valid here? You have not defined it before

Definition 3.2 (Signature Scheme)**.** A valid Signature Scheme must provide three procedures:

Pedro: I would write this sentence as: A signature scheme Φ is a tuple of algorithms $(\mathsf{keyGen},\ \mathsf{sign},\ \mathsf{verf})$ defined as follows:

$$\Phi = (\mathsf{keyGen},\ \mathsf{sign},\ \mathsf{verf})$$

Pedro: write the API of the algorithms in bullet points

$(sk, pk) \leftarrow \mathsf{keyGen}(1^n)$ takes as input a security parameter 1^n and outputs a keypair (sk, pk), consisting of a secret key sk and a public key pk, whereas the secret key has to be kept private and the public key is shared with other parties. sk can be used together with a message m to call the $\mathsf{sign}(sk,\ m)$ procedure to create a signature σ over the message m. Parties knowing pk can then test the validity of the signature by calling $\mathsf{verf}(pk,\ \sigma,\ m)$ with the same message m. The procedure will only output 1 if the message was indeed signed with the correct secret key sk of pk and therefore proves the possesion of sk by the signer . A valid signature scheme have to fulfill two security properties

Pedro: "proving that the sender had the sk" is a property that no all signature schemes may have

Pedro: Choose one unforgeability form, the one that you require later in the thesis

- Correctness: For all messages m and valid keypairs (sk, pk) the following must hold $\mathsf{verf}(pk,\ \mathsf{sign}(sk,\ m),\ m) \stackrel{?}{=} 1$
- Unforgeability: Note that there are different levels of Unforgability: [GMR88]
  - Universal Forgery: The ability to forge signatures for any message.
  - Selective Forgery: The ability to fogre signatures for messages of the adversary's choice.
  - Existential Forgery: The ability to forge a valid signature / message pair not previously known to the adversary.

Pedro: Except with negligible probability

Pedro: nice that you have defined the three properties. I would keep the one that you need later

minor
we normally
ital H

Definition 3.3 (Cryptographic Hash Function)**.** A cryptographic hash function H is defined as $\mathsf{H}(I) \rightarrow \{0,1\}^n$ for some fixed number n and some input I. A secure hashing function has to fullfill the following security properties: [AKDB11]

- Collision-Resistence (CR): Collision-Resistance means that it is computationally infeasible to find two inputs I_1 and I_2 such that $\mathsf{H}(I_1) := \mathsf{H}(I_2)$ with $I_1 \neq I_2$.
- Pre-image Resistence (Pre): In a hash function H that fulfills Pre-image Resistance it is infeasible to recover the original input I from its hash output $\mathsf{H}(I)$. If this security property is achieved, the hash function is said to be non-invertible.
- 2nd Pre-image Resistence (Sec): This property is similar to Collision-Resistance and is sometimes referred to as *Weak Collision-Resistance*. Given such a hash function H and an input I, it should be infeasible to find a different input I^* such that $I \neq I^*$ and $\mathsf{H}(I) \stackrel{?}{=} \mathsf{H}(I^*)$.

Pedro: I think here the Open operation of the commitment is missing, which you need later for the binding property

Definition 3.4 (Commitment Scheme [BBB+18])**.** A cryptographic Commitment Scheme *COM* is defined by a pair of functions $(\mathsf{keyGen}(1^n), \mathsf{commit}(I,k))$. keyGen is the setup procedure, it takes as input a security parameter 1^n and outputs public parameters PP. Depending on PP we define a input space $\mathbb{I}_{PP}$, a randomness space $\mathbb{K}_{PP}$ and a commitment space $\mathbb{C}_{PP}$.

The function commit takes an arbitrary input $I \in \mathbb{I}_{PP}$, and a random value $k \in \mathbb{K}_{PP}$ and generates an output $C \in \mathbb{C}_{PP}$.

Secure commitments must fullfill the *Binding* and *Hiding* security properties:

- *Binding:* If a Commitment Scheme is binding it must hold that for all *PPT* adversaries $\mathcal{A}$ given a valid input $I \in \mathbb{I}_{PP}$ and randomness $k \in \mathbb{K}_{PP}$ the probabilty of finding a $I^* \neq I$ and a k^* with $\mathsf{commit}(I,k) \stackrel{?}{=} \mathsf{commit}(I^*,k^*)$ is negligible.
- *Hiding:* For a *PPT* adversary $\mathcal{A}$, commitment inputs $I \in \mathbb{I}_{PP}$, $k \in \mathbb{K}_{PP}$ and a commitment output $C := \mathsf{commit}(I,k,)$ the probabilty of the adversary choosing the correct input $\{I, I^*\}$ must not be higher then $\frac{1}{2} + \mathsf{negl}(P)$.

Pedro: this definition seems wrong to me? Where does I^* come from?

Pedro: Add reference from where you took this definition. You may want to add that it is an "Additive" Homomorphic Commitment

Definition 3.5 (Homomorphic Commitment)**.** If a Commitment Scheme as defined in 3.4 is homomorphic then the following must hold

$$\mathsf{commit}(I_1,k_1) \cdot \mathsf{commit}(I_2,k_2) \stackrel{?}{=} \mathsf{commit}(I_1 + I_2, k_1 + k_2)$$

First, a Pedersen Commitment is an instance of Commitment Scheme as in Def 3.4; Second it has the homomorphic property as in 3.5. Clarify that, for instance, by explaining exactly how the algorithms are implemented, as you did below.

Definition 3.6 (Pedersen Commitment)**.** A Pedersen Commitment is an instantiation of a Homomoprhic Commitment Scheme as defined in 3.5:

$$\mathbb{C}_{PP} := \mathbb{G}$$

of order p, $\mathbb{I}_{PP}$, $\mathbb{K}_{PP} := \mathbb{Z}_p$. the procedures $(\mathsf{keyGen}, \mathsf{commit})$ are then instantiated as:

$$\mathsf{keyGen}(1^n) := g, h \leftarrow\!\!\$\, \mathbb{G}$$

$$\mathsf{commit}(I, k) := g^k h^I$$

3.2 Bitcoin

3.2.1 Bitcoin Transaction Protocol

3.2.2 Bitcoin Scaling and Layer Two Solutions

3.3 Privacy-enhancing Cryptocurrencies

3.3.1 Zero Knowledge Proofs

3.3.2 Range Proofs

3.3.3 Mimblewimble

In this section we will outline the fundamental properties of the protocols employed in Mimblewimble which are relevant for the thesis and particularily the construction of the Atomic Swap protocol defined in 5.

Transaction Structure

Pedro: I think that throughout this section, you have nice explanations of the different parts of the transaction. It would be also possible to add definitions for the different things that you use

- For two adjacent elliptic curve generators g and h a coin in Mimblewimble is a tuple of the form $(\mathcal{C}, \pi)$, where $\mathcal{C} := g^v \cdot h^k$ a Pedersen Commitment [Ped91] to the value v with blinding factor k. π is a range proof attesting to the fact that v is in a valid range in zero-knowledge.

  Pedro: not sure whether the point below is required

Pedro: you might want to specify what range is used here. Also I rewrote some part, so please check.

- As already pointed out, there are now addresses in Mimblewimble. Ownership of a coin is equivalent to the knowledge of its opening, so the blinding factor takes the role of the secret key.
- A transaction consists of $\mathcal{C}_{inp} := (\mathcal{C}_1, \ldots, \mathcal{C}_n)$ input coins and $\mathcal{C}_{out} := (\mathcal{C}'_1, \ldots, \mathcal{C}'_n)$ output coins.

A transaction is considered valid iff $\sum v'_i - \sum v_i \stackrel{?}{=} 0$ so the sum of all input values has to be 0. (Not taking transaction fees into account)

Pedro: doesn't need to check the range proofs as well?

From that we can derive the following equation:

$$\sum \mathcal{C}_{out} - \sum \mathcal{C}_{inp} := \sum (h^{v'_i} \cdot g^{k'_i}) - \sum (h^{v_i} \cdot g^{k_i})$$

So if we assume that a transaction is valid then we are left with the following so called excess value:

$$\mathcal{E} := g^{(\sum k'_i - \sum k_i)}$$

Knowledge of the opening of all coins and the validity of the transaction implies knowledge of $\mathcal{E}$. Directly revealing the opening to $\mathcal{E}$ would leak too much information, an adversary knowing the openings for input coins and all but one output coin, could easily calculate the unknown opening given $\mathcal{E}$. Therefore knowledge of $\mathcal{E}$ instead is proven by providing a valid signature for $\mathcal{E}$ as public key. Coinbase transactions (transactions creating new money as part of a miners reward) additionally include the newly minted money as supply s in the excess equation:

Pedro: You mean knowledge of the exponent of $\mathcal{E}$?

$$\mathcal{E} := g^{(\sum k'_i - \sum k_i)} - h^s$$

Finally a Mimblewimble transaction is of form:

$$tx := (s, \mathcal{C}_{inp}, \mathcal{C}_{out}, K) \text{ with } K := (\{\pi\}, \{\mathcal{E}\}, \{\sigma\})$$

where s is the transaction supply amount, $\mathcal{C}_{inp}$ is the list of input coins, $\mathcal{C}_{out}$ is the list of output coins and K is the transaction Kernel. The Kernel consists of $\{\pi\}$ which is a list of all output coin range proofs, $\{\mathcal{E}\}$ a list of excess values and finally $\{\sigma\}$ a list of signatures [FOS19].

Pedro: the $\mathcal{E}$ is a single value? or a set?

Transaction Merging

An essential property of the Mimblewimble protocol is that two transactions can easily be merged into one, which is essentially a non-interactive version of the CoinJoin protocol on Bitcoin [Max13] Assume we have the following two transactions:

$$tx_0 := (s_0, \mathcal{C}^0_{inp}, \mathcal{C}^0_{out}, (\{\pi_0\}, \{\mathcal{E}_0\}, \{\sigma_0\}))$$

$$tx_1 := (s_1,\ \mathcal{C}^1_{inp},\ \mathcal{C}^1_{out},\ (\{\pi_1\},\ \{\mathcal{E}_1\},\ \{\sigma_1\}))$$

Then we can build a single merged transaction:

$$tx_m := (s_0 + s_1,\ \mathcal{C}^0_{inp} \,||\, \mathcal{C}^1_{inp},\ \mathcal{C}^0_{out} \,||\, \mathcal{C}^1_{out},\ (\{\pi_0\} \,||\, \{\pi_1\}),\ \{\mathcal{E}_0\} \,||\, \{\mathcal{E}_1\},\ \{\sigma_0\} \,||\, \{\sigma_1\})$$

We can easily deduce that if tx_0 and tx_1 are valid, it follows that tx_m also has to be valid: If tx_0 and tx_1 are valid that means $\mathcal{C}^0_{inp} - \mathcal{C}^0_{out} - h^{s_0} := \mathcal{E}_0$, $\{\pi_0\}$ contains valid range proofs for the outputs $\mathcal{C}^0_{out}$ and $\{\sigma_0\}$ contains a valid signature to $\mathcal{E}_0 - h^{s_0}$ as public key, the same must hold for tx_1.

By the rules of arithmetic it then must also hold that

$$\mathcal{C}^0_{inp} \,||\, \mathcal{C}^1_{inp} - \mathcal{C}^0_{out} \,||\, \mathcal{C}^1_{out} - h^{s_0 + s_1} := \mathcal{E}_0 \cdot \mathcal{E}_1,\ \{\pi_0\} \,||\, \{\pi_1\}$$

must contain valid range proofs for the output coins and $\{\sigma_0\} \,||\, \{\sigma_1\}$ must contain valid signatures to the respective Excess points, which makes tx_m a valid transaction.

Subset Problem

Pedro: I think the content below is not fully clear yet. If needed for the rest, we need to clarify (e.g., add an example?)

A subtle problem arises with the way transactions are merged in Mimblewimble. From the shown construction, it is possible to reconstruct the original separate transactions from the merged one, which can be a privacy issue. Given a set of inputs, outputs, and kernels, a subset of these will recombine to reconstruct one of the valid transaction which were aggregated since Kernel Excess values are not combined. (which would invalidate the signatures and therefore break the security of the system) This problem has been mitigated in Cryptocurrencies implementing the protocol by including an additional variable in the Kernel, called offset value. The offset is randomly chosen and needs to be added back to the Excess values to verify the sum of the commitments to zero.

$$\sum \mathcal{C}_{out} - \sum \mathcal{C}_{inp} - h^s := \mathcal{E}^o$$

Every time two transactions are merged, the offset values are combined into a single value. If offsets are picked truly randomly, and the possible range of values is broad enough, the probability of recovering the uncombined offsets from a merged one becomes negligible, making it infeasable to recover original transactions from a merged one [Poe16].

Cut Through

Pedro: This requires further explanation and maybe an example?

From the way transactions are merged together, we can now learn how to purge spent outputs securely. Let's assume $\mathcal{C}_i$ appears as an output in tx_0 and as an input in tx_1, which are being merged. Remembering the equation for transaction balancedness, $\mathcal{C}_{inp} - \mathcal{C}_{out} := \mathcal{E}$ if $\mathcal{C}_i$ appears both in the inputs and outputs, and we erase it on both sides, the equation will still hold. Therefore every time a transaction spends an output, it can be virtually forgotten to improve transaction unlinkability as well as yielding saving space.

The Ledger

Pedro: Do we need this subsection?

The ledger of the Mimblewimble protocol itself is a transaction of the already discussed form. Initially, the ledger starts empty, and transactions are added and aggregated recursively.

- Only transactions in which input coins are contained in the output coins of the ledger will be valid.
- The supply of the ledger is the sum of the supplies of all transactions added so far. Therefore we can easily read the total circulating supply from the ledger state.
- Due to cut through, the input coin list of the ledger is always empty, and the output list is the set of UTXOs.

Transaction Building

As already pointed out, building transactions in Mimblewimble is an interactive process between the sender and receiver of funds. Jedusor, Tom Elvis originally envisioned the following two-step process to build a transaction: [Jed16]

Assume Alice wants to transfer coins of value p to Bob.

1. Alice first selects input coins $\mathcal{C}_{inp}$ of total value $v \geq p$ that she controls. She then creates change coin outputs $\mathcal{C}^A_{out}$ (could be multiple) of total value $v - p$ and then sends $\mathcal{C}_{inp}$, $\mathcal{C}^A_{out}$, a valid range proofs for $\mathcal{C}^A_{out}$, plus the opening $(-p,\ x)$ of $\sum \mathcal{C}^A_{out} - \sum \mathcal{C}_{inp}$ to Bob.

   Pedro: we need a way to express this clearer

2. Bob creates himself additional output coins $\mathcal{C}^B_{out}$ plus range proofs of total value p with keys (x^*_i) and computes a signature σ with the combined secret key $x + \sum x^*_i$ and finalizes the transaction as

   $$tx := (0,\ \mathcal{C}_{inp},\ \mathcal{C}^A_{out} \,||\, \mathcal{C}^B_{out},\ (\pi,\ \mathcal{E} := \sum \mathcal{C}^A_{out} \cdot \sum \mathcal{C}^B_{out} - \sum \mathcal{C}_{inp},\ \sigma))$$

   and publishes it to the network.

Figure 3.1 depicts the original transaction flow.
This protocol however turned out to be vulnerable. The receiver can spend the change coins $\mathcal{C}^A_{out}$ by reverting the transaction. Doing this would give the sender his coins back, however as the sender might not have the keys for his spent outputs anymore, the coins could then be lost.

Pedro: For security, privacy or both?

In detail this reverting transaction would look like:

$$tx_{rv} := (0,\ \mathcal{C}^A_{out} \,||\, \mathcal{C}^B_{out},\ \mathcal{C}_{inp},\ (\pi_{rv},\ \mathcal{E}_{rv},\ \sigma_{rv}))$$

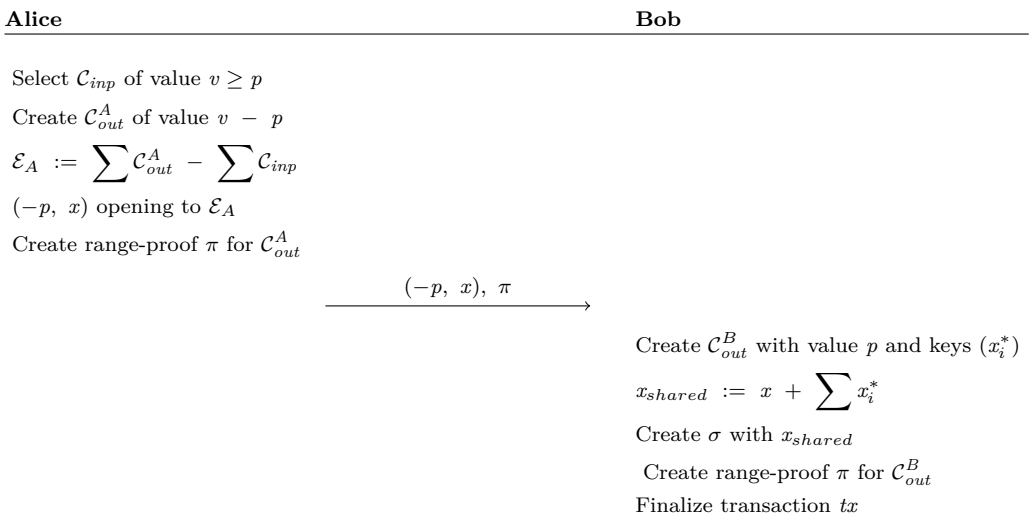

Figure 3.1: Original transaction building process

Pedro: Really nice that you created this protocol :)

Again remembering the construction of the Excess value of this construction would look like this:

$$\mathcal{E}_{rv} := \sum \mathcal{C}^A_{out} \,||\, \mathcal{C}^B_{out} - \mathcal{C}_{inp}$$

The key x originally sent by Alice to Bob is a valid opening to $\sum \mathcal{C}_{inp} - \sum \mathcal{C}^A_{out}$. With the inverse of this key x_{inv} we get the opening to $\sum \mathcal{C}^A_{out} - \mathcal{C}_{inp}$. Now all Bob has to do is add his keys $\sum x^*_i$ to get:

$$x_{rv} := -x + \sum x^*_i$$

Pedro: Why range proof is not correct here in the first place?

which is the opening to $\mathcal{E}_{rv}$. Furthermore obtaining a valid range proofs is trivial, as it once was a valid output the ledger will cointain a valid proof for this coin already.

This means Bob spends the newly created outputs and sends them back to the original input coins, chosen by Alice. It might at first seem unclear why Bob would do that. An example situation could be if Alice pays Bob for some good which Bob is selling. Alice decides to pay in advance, but then Bob discovers that he is already out of stock of the good that Alice ordered. To return the funds to Alice, he reverses the transaction instead of participating in another interactive process to build a new transaction with new outputs. If Alice already deleted the keys to her initial coins, the funds are now lost. The problem was solved in the Grin Cryptocurrency by making the signing process itself a two-party process which will be explained in more detail in chapter 4.

Fuchsbauer et al. [FOS19] proposed the following alternative way to build transactions which would not be vulnerable to this problem.

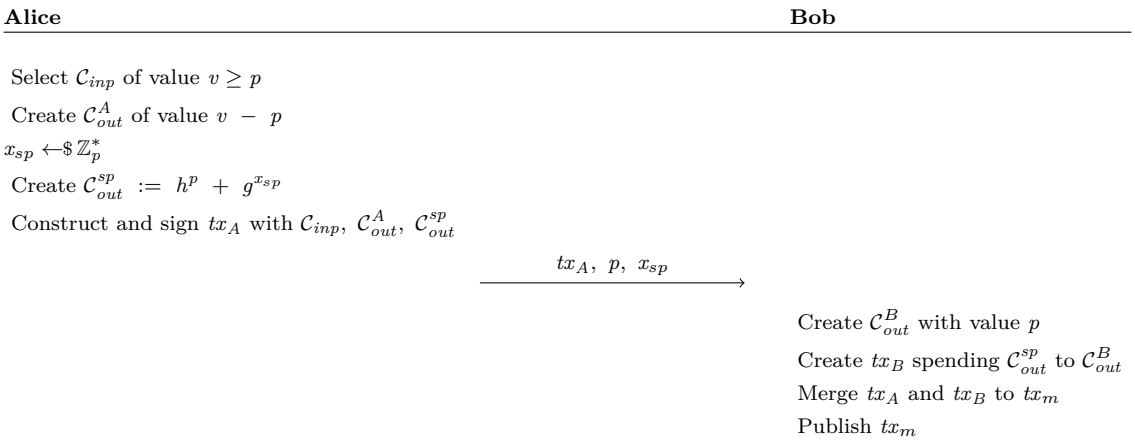

Figure 3.2: Salvaged tranction protocol by Fuchsbauer et al. [FOS19]

Pedro: put labels inside captions

1. Alice constructs a full-fledged transaction tx_A spending her input coins $\mathcal{C}_{inp}$ and creates her change coins $\mathcal{C}^A_{out}$, plus a special output coin $\mathcal{C}^{sp}_{out} := h^p \cdot g^{x_{sp}}$, where p is the desired value which should be transferred to Bob and x_{sp} is a randomly choosen key. She proceeds by sending tx_A as well as $(p,\ x_{sp})$ and the necessary range proofs to Bob.

2. Bob now creates a second transaction tx_B spending the special coin $\mathcal{C}^{sp}_{out}$ to create an output only he controls $\mathcal{C}^B_{out}$ and merges tx_A with tx_B into tx_m. He then broadcasts tx_m to the network. Note that when the two transactions are merged the intermediate special coin $\mathcal{C}^{sp}_{out}$ will be both in the coin output and input list of the transaction and therfore will be discarded.

The only drawback of this approach is that we have two transaction kernels instead of just one because of the merging step, making the transaction slightly bigger. A figure showing the protocol flow is depicted in Figure 3.2.

3.4 Scriptless Scripts

3.5 Adaptor Signatures

3.5.1 Schnorr Signature Construction

3.5.2 ECDSA Signature Construction

CHAPTER 4

Two Party Fixed Witness Adaptor Signatures

In this chapter, we will define a variant of the adaptor signature scheme as explained in section 3.5. The main difference in the protocol outlined in this thesis is that one of the two parties does know the fixed secret witness before the start of the protocol. The aim of the protocol will then be that the other person is able to extract the witness from the final signature. This feature can then be leveraged to build an Atomic Swap protocol as we will show in 5.

First we will define the general two-party signature creation protocol as it is currently implemented in Mimblewimble-based Cryptocurrencies. We reduce the generated signatures to the general case [Sch89] and prove its correctness. From this two-party protocol, we then derive the adapted variant, which allows hiding a fixed witness value in the signature, which can be revealed only by the other party after attaining the final signature.

We start by defining our extended signature scheme in section 4.1, proceed by providing a schnorr-based instantiation of the protocol in section 4.2 and finally prove its security in section 4.2.1.

4.1 Definitions

A two-party signature scheme is an extension of a signature scheme as defined in definition 3.2, which allows us to distribute signature generation for a composite public key shared between two parties Alice and Bob. Alice and Bob want to collaborate to generate a signature valid under the composite public key $pk := pk_A \cdot pk_B$ without having to reveal their secret keys to each other.

Definition 4.1 (Two Party Signature Scheme)**.** A *two party signature scheme* Φ_{MP} extends a signature scheme Φ with a tuple of protocols and algorithms $(\mathsf{dKeyGen}, \mathsf{signPrt}, \mathsf{vrfPt}, \mathsf{finSig})$ defined as follows:

- $((sk_A, pk_A, k_A, \Lambda), (sk_B, pk_B, k_B, \Lambda)) \leftarrow \mathsf{dKeyGen}\langle 1^n, 1^n\rangle$: The distributed key generation protocol takes as input the security parameter from both Alice and Bob and returns the tuple $(sk_A, pk_A, k_A, \Lambda)$ to Alice (similar to Bob) where (sk_A, pk_A) is a pair of private and corresponding public keys, k_A a secret nonce and Λ is the signature context containing parameters shared between Alice and Bob. We introduce Λ for the participants to share as well as update parameters with each other during the protocol execution.
- $(\tilde{\sigma_A}) \leftarrow \mathsf{signPrt}(m, sk_A, k_A, \Lambda)$: The partial signing algorithm is a DPT function that takes as input the message m and the share of the secret key sk_A and nonce k_A (similiar for Bob) as well as the shared signature context Λ. The procedure outputs $(\tilde{\sigma_A})$, that is, a share of the signature to a participant.
- $\{1, 0\} \leftarrow \mathsf{vrfPt}(\tilde{\sigma_A}, m, pk_A)$: The share verification algorithm is a DPT function that takes as input a signature share $\tilde{\sigma_A}$, a message m, and the other participants public key pk_A (similiar pk_B for Bobs partial signature). The algorithm returns 1 if the verification was successfull or 0 otherwise.
- $\sigma_{fin} \leftarrow \mathsf{finSig}(\tilde{\sigma_A}, \tilde{\sigma_B})$: The finalize signature algorithm is a DPT function that takes as input two shares of the signatures and combines them into a final signature valid ander the shared public key pk.

We require the two party signature scheme to be correct as well as unforgeable against chosen message attacks (EUF-CMA). EUF-CMA for a two-party signature scheme was defined for instance in [**?**]. For the correctness of the distributed key-generation protcol $\mathsf{dKeyGen}$, special care needs to be taken to gurantee a uniformly random distribution of generated keys as pointed out by Lindell and Yehuda in [**?**].

Definition 4.2 (Two Party Fixed Witness Adaptor Schnorr Signature Scheme)**.** From the definition 4.1, we now derive an adapted signature scheme Φ_{Apt}, which allows one of the participants to hide the discrete logarithm x of a statement $X := g^x$ chosen at the beginning of the protocol. Again we extend our previously defined signature scheme with new functions:

$$\Phi_{Apt} := (\Phi_{MP} \,||\, \mathsf{adaptSig} \,||\, \mathsf{verifyAptSig} \,||\, \mathsf{extWit})$$

- $\hat{\sigma_B} \leftarrow \mathsf{adaptSig}(\tilde{\sigma_B}, x)$: The adapt signature algorithm is a DPT function that takes as input a partial signature $\tilde{\sigma}$ and a secret witness value x. The procedure will output a adapted partial signature $\hat{\sigma}$ which can be verified to contain x using the $\mathsf{verifyAptSig}$ function, without having to reveal x.

- $\{1,0\} \leftarrow \mathsf{verifyAptSig}(\hat{\sigma_A},\ m,\ pk_A,\ X)$: The verification algorithm is a DPT function that takes as input an adapted partial signature $\hat{\sigma}$, the other participants public keys and a statement X. The function will verify the partial signature's validity as well that it contains the secret witness x.
- $x \leftarrow \mathsf{extWit}(\sigma_{fin},\ \tilde{\sigma_A},\ \hat{\sigma_B})$: The witness extraction algorithm is a DPT function that lets Alice extract the secret witness x from the final composite signature. Note that to extract the witness x the partial signatures shared between the participants beforehand and the statement X needs to be provided as inputs. This means that for executing this function one needs to first learn the partial signatures exchanged between the parties.

Note that our definition of the adaptor signature scheme is different then the definition seen in 3.5. This has the reason that we require our scheme to be able to hide a secret chosen before the signing protocol has been started. One of the participants will be able to hide this secret during the distributed signing protocol which the other party can extract after completion of the protocol. This feature is a requirement for our signature scheme such that we can build the atomic swap protocol which will be layed out in 5.3.

Definition 4.3 (Secure Adaptor Signature Scheme)**.** As defined by Aumayr et al. in [AEE+20], a secure adaptor signature scheme needs two security properties to be fulfilled:

1. $\mathsf{aEUF-CMA}$
2. Witness Extractability

We proceed by redefining these properties as well as adapted correctness for our adapted two-party fixed witness signature scheme defined in definition 4.2:

Similar to how it is defined in [AEE+20] additionally to Correctness as defined in 3.2 we require our signature scheme to satisfy Adaptor Signature Correctness . This property is given when every adapted partial signature generated by adaptSig can be completed into a final signature for all pairs $(x,\ X) \in R$, from which it will be possible to extract the witness computing extWit with the required parameters.

Definition 4.4 (Adaptor Signature Correctness)**.** More formally Adaptor Signature Correctness is given if for every security parameter $n \in \mathbb{N}$, message $m \in \{0,1\}^*$,
keypairs $((sk_A, pk_A, k_A, \Lambda), (sk_B, pk_B, k_B, \Lambda)) \leftarrow \mathsf{dKeyGen}\langle 1^n,\ 1^n\rangle$ with their composite public key $\Lambda.pk$ and every statement/witness pair $(X,\ x)\mathsf{genRel}(1^n)$ in a relation R it must hold that:

$$\Pr\left[\begin{array}{c} \mathsf{verf}(m,\ \sigma_{fin},\ \Lambda.pk) \ =\ 1 \\ \wedge \\ \mathsf{verifyAptSig}(\hat{\sigma_B},\ m,\ ,\ pk_B)X \ =\ 1 \\ \wedge \\ (X,\ x^*) \in R \end{array} \middle| \begin{array}{l} \tilde{\sigma_A} \leftarrow \mathsf{signPrt}(m,\ sk_A,\ k_A,\ \Lambda) \\ \tilde{\sigma_B} \leftarrow \mathsf{signPrt}(m,\ sk_B,\ k_B,\ \Lambda) \\ \hat{\sigma_B} \leftarrow \mathsf{adaptSig}(\tilde{\sigma_B},\ x) \\ \sigma_{fin} \leftarrow \mathsf{finSig}(\tilde{\sigma_A},\ \tilde{\sigma_B}) \\ x^* \leftarrow \mathsf{extWit}(\sigma_{fin},\ \tilde{\sigma_A},\ \hat{\sigma_B}) \end{array}\right] = 1.$$

Additionally to the regular definition of *existential unforgeability under chosen message attacks* as defined for example in [**?**] or [Vau06] we require that it is hard to produce a forged partial signature $\tilde{\sigma}$ if the adversary $\mathcal{A}$ gets to know a valid adapted signature $\hat{\sigma}$ w.r.t. some message m and a statement X.

Definition 4.5 ($\mathsf{aEUF-CMA}$)**.** For the definition of $\mathsf{aEUF-CMA}$ -security we define the experiment $\mathsf{forgeAptSig}_{\mathcal{A}}$ for a PPT adversary $\mathcal{A}$ with a keypair $(sk_A,\ pk_A)$, meaning the attacker plays the role of Alice in the protocol as follows:

$\mathsf{forgeAptSig}_{\mathcal{A}}(n)$

$\mathbb{S} := \emptyset$
$((sk_A, pk_A, k_A, \Lambda), (sk_B, pk_B, k_B, \Lambda)) \leftarrow \mathsf{dKeyGen}\langle 1^n,\ 1^n\rangle$
$m \leftarrow \mathcal{A}^{\mathcal{O}_{ps}(\cdot,\cdot,\cdot,\cdot)}(pk_B)$
$(x,\ X) \leftarrow \mathsf{genRel}(1^n)$
$\tilde{\sigma_A} \leftarrow \mathsf{signPrt}(m,\ sk_A,\ k_A,\ \Lambda)$
$\tilde{\sigma_B} \leftarrow \mathsf{signPrt}(m,\ sk_B,\ k_B,\ \Lambda)$
$\hat{\sigma_B} \leftarrow \mathsf{adaptSig}(\tilde{\sigma_B},\ x)$
$\tilde{\sigma_A}^* \leftarrow \mathcal{A}^{\mathcal{O}_{ps}(\cdot,\cdot,\cdot,\cdot)}(\tilde{\sigma_A}, \hat{\sigma_B})$
$\sigma_{fin} \leftarrow \mathsf{finSig}(\tilde{\sigma_A}^*,\ \tilde{\sigma_B})$
return $((m) \notin \mathbb{S} \ \wedge\ \tilde{\sigma_A}^* \neq \tilde{\sigma_A} \ \wedge\ \mathsf{verf}(m,\ \sigma_{fin},\ \Lambda.pk))$

$\mathcal{O}_{ps}(m, pk_A, pk_B, \Lambda)$

$(x,\ X) \leftarrow \mathsf{genRel}(1^n)$
$\tilde{\sigma_A} \leftarrow \mathsf{signPrt}(m,\ sk_A,\ k_A,\ \Lambda)$
$\tilde{\sigma_B} \leftarrow \mathsf{signPrt}(m,\ sk_B,\ k_B,\ \Lambda)$
$\hat{\sigma_B} \leftarrow \mathsf{adaptSig}(\tilde{\sigma},\ x)$
$\sigma_{fin} \leftarrow \mathsf{finSig}(\tilde{\sigma_A},\ \hat{\sigma_B})$
$\mathbb{S} := \mathbb{S} \cup \{m\}$
return $(\sigma_{fin},\ X)$

The adapted signature scheme Φ_{Apt} is called $\mathsf{aEUF-CMA}$ -secure if

$$\Pr[\mathsf{forgeAptSig}_{\mathcal{A}}(n) = 1] \leq \mathsf{negl}(n)$$

Definition 4.6 (Witness Extractability)**.** Informally the Witness Extractability property holds for an adapted signature scheme Φ_{Apt} computed for the statement X when we

can always extract the witness $(x,\ X)$ from the final signature σ_{fin}, given the partial signatures of the participants. To formalize this statement we describe an experiment $\mathsf{aExtrWit}_{\mathcal{A}}$ for a PPT adversary $\mathcal{A}$ with a keypair (sk_B, pk_B), meaning the attacker plays the role of Bob in the protocol.

$\mathsf{aExtrWit}_{\mathcal{A}}(n)$

1 : $\mathbb{S} := \emptyset$
2 : $((sk_A, pk_A, k_A, \Lambda), (sk_B, pk_B, k_B, \Lambda)) \leftarrow \mathsf{dKeyGen}\langle 1^n,\ 1^n\rangle$
3 : $(m,\ (x, X) \in R) \leftarrow \mathcal{A}^{\mathcal{O}_{ps}(\cdot,\cdot,\cdot,\cdot)}(pk_A)$
4 : $\tilde{\sigma_A} \leftarrow \mathsf{signPrt}(m,\ sk_A,\ k_A,\ \Lambda)$
5 : $\tilde{\sigma_B} \leftarrow \mathsf{signPrt}(m,\ sk_B,\ k_B,\ \Lambda)$
6 : $(\hat{\sigma_B},\ \sigma_{fin}) \leftarrow \mathcal{A}^{\mathcal{O}_{ps}(\cdot,\cdot,\cdot,\cdot)}(pk_B)$
7 : $x^* \leftarrow \mathsf{extWit}(\sigma_{fin},\ \tilde{\sigma_A},\ \hat{\sigma_B})$
8 : **return** $(m \notin \mathbb{S} \ \wedge\ (X,\ x^*) \notin R \ \wedge\ \mathsf{verf}(m,\ \sigma_{fin},\ \Lambda.pk))$

$\mathcal{O}_{ps}(m, pk_A, pk_B, \Lambda)$

1 : $\mathbb{S} := \mathbb{S} \cup m$
2 : $\tilde{\sigma_A} \leftarrow \mathsf{signPrt}(m,\ sk_A,\ k_A,\ \Lambda)$
3 : $\tilde{\sigma_B} \leftarrow \mathsf{signPrt}(m,\ sk_B,\ k_B,\ \Lambda)$
4 : $\sigma_{fin} \leftarrow \mathsf{finSig}(\tilde{\sigma_A},\ \tilde{\sigma_B})$
5 : **return** σ_{fin}

In order to satisfy witness extractability the following must hold:

$$\Pr[\mathsf{aExtrWit}_{\mathcal{A}}(n) \ =\ 1] \ \leq\ \mathsf{negl}(n)$$

4.2 Schnorr-based instantiation

We start by providing a general instantiation of a signature scheme (see definition 3.2): We assume we have a group $\mathbb{G}$ with prime p, H is a secure hash function as defined in definition 3.3 and $m \in \{0,1\}^*$ is a message.

- $(sk, pk) \leftarrow \mathsf{keyGen}(1^n)$: The keygen function creates a keypair (sk, pk), the public key can be distributed to the verifier(s) and the secret key has to be kept private.

- $\sigma \leftarrow \mathsf{sign}(m,\ sk)$: The signing function creates a signature consisting of a variable s and R which is a commitment to the secret nonce k used during the signing process. As an input it takes a message m and the secret key sk of the signer.

$\mathsf{keyGen}(1^n)$	$\mathsf{sign}(m,\ sk)$	$\mathsf{verf}(m,\ \sigma,\ pk)$
1 : $x \leftarrow\$\ \mathbb{Z}_p^*$	1 : $k \leftarrow\$\ \mathbb{Z}_p^*$	1 : $(s,\ R) \leftarrow \sigma$
2 : **return** $(sk := x,\ pk := g^x)$	2 : $R := g^k$	2 : $e := \mathsf{H}(m \mid\mid R \mid\mid pk)$
	3 : $e := \mathsf{H}(m \mid\mid R \mid\mid pk)$	3 : **return** $g^s \stackrel{?}{=} R \cdot pk^e$
	4 : $s := k + e \cdot sk$	
	5 : **return** $\sigma := (s, R)$	

Figure 4.1: Schnorr Signature Scheme as first defined in [Sch89]

- $\{1, 0\} \leftarrow \mathsf{verf}(m,\ \sigma,\ pk)$: The verification function allows a verifier knowing the signature σ, message m and the provers public key pk to verify the signatures validity.

A concrete implementation can be seen in figure 4.1. The signature scheme is called schnorr signature scheme, first defined in [Sch89] and is widely employed in many cryptography systems. Correctness of the scheme is easy to derive. As s is calculated as $k + e \cdot sk$, when generator g is raised to s, we get $g^{k\ +\ e\ \cdot\ sk}$ which we can transform into $g^k \cdot g^{sk\ \cdot\ e}$, and finally into $R \cdot pk^e$ which is the same as the right side of the equation.

From the regular schnorr signature we now provide an instantiation for the two-party case defined in definition 4.1. Note that this two-party variant of the scheme is what is currently implemented in the mimblewimble-based cryptocurrencies and will provide a basis from which we will build our adapted scheme.

First we define a auxiliary function $\mathsf{setupCtx}$ to use for the instantion:

$\mathsf{setupCtx}(\Lambda,\ pk_A,\ R_A)$
1 : $\Lambda.pk := \Lambda.pk \cdot pk_A$
2 : $\Lambda.R := \Lambda.R \cdot R_A$
3 : **return** Λ

This function helps the particpents to setup and update the signature context shared between them. In figure 4.2 we show a concrete instantiation of the protocol and functions. In $\mathsf{dKeyGen}$ Alice and Bob will each randomly chose their secret key and nonce. They further require to create a zero-knowledge proof attesting to the fact that they have generated their key before any message was exchanged. This is essential for the scheme to achieve EUF-CMA as described by Lindell et al. [**?**].

In dKeyGen Alice will initially setup the signature context and send it to Bob, together with her public and zk-proof. Bob verifies the proof (and exits if it is invalid). He will proceed by adding his parameters to the signature context and send it back to Alice, together with his public key and zk-proof, which Alice will verify.

signPrt and vrfPt are generally similiar to the instantiation of the normal schnorr signature scheme. Note however that for computing the schnorr challenge e the input into the hash function will be the combined public key pk and combined nonce commitment R, which the participants can read from the context object Λ. This has the effect that the partial signature itself are not yet a valid signature (neither under pk nor under pk_A or pk_B). This is because to be valid under pk the partial signatures are missing the s values from the other participants. They are also not valid under the partial public keys pk_A or pk_B because the schnorr challenge is computed already with the combined values. There we have to introduce the slightly adjusted vrfPt to be able to verify specifically the partial signatures.

We further show in figure 4.3 how Alice and Bob can cooperate to produce a final signature which fulfills Correctness as defined in definition 3.2.

The final signature is a valid signature to the message m with the composite public key $pk := pk_A \cdot pk_B$. A verifier knowing the signed message m, the final signature σ_{fin} and the composite public key pk can now verify the signature using the regular verf procedure.

In figure 4.4 we further provide a schnorr-based instantiation for the fixed witness adapted signature scheme as defined in definition 4.2.

adaptSig will add the secret witness x to the s value of the signature, this means we will not be able to verify the adapted signature using vrfPt anymore. Therefore we introduce verifyAptSig which takes as additional parameter the statement X which will be included in the verifiers equation. Now the function verifies not only validity of the portial signature, but also that it indeed has been adapted with the witness value x, being the discrete logarithm of X. After obtaining σ_{fin}, we can then cleverly unpack the secret x, which is shown in the $\mathsf{aExtrWit}_{\mathcal{A}}$ function.

Again in figure 4.5 we show another example interaction between Alice and Bob creating a signature σ_{fin} for the composite public key $pk := pk_A \cdot pk_B$ while Bob will hide his secret x which Alice can extract after the signing process has completed. One thing to note is that in this protocol only Bob is able to call finSig to create the final signature. This is because the function requires Bobs unadapted partial signature $\tilde{\sigma_B}$ as input, which Alice does not know. (She only knows Bobs adapted variant). Therefore one further interaction is needed to send the final signature to Alice.

4.2.1 Correctness & Security

We now prove that the outlined schnorr-based instantiation is correct, i.e. Adaptor Signature Correctness holds, as well as secure with regards to the definition 4.3.

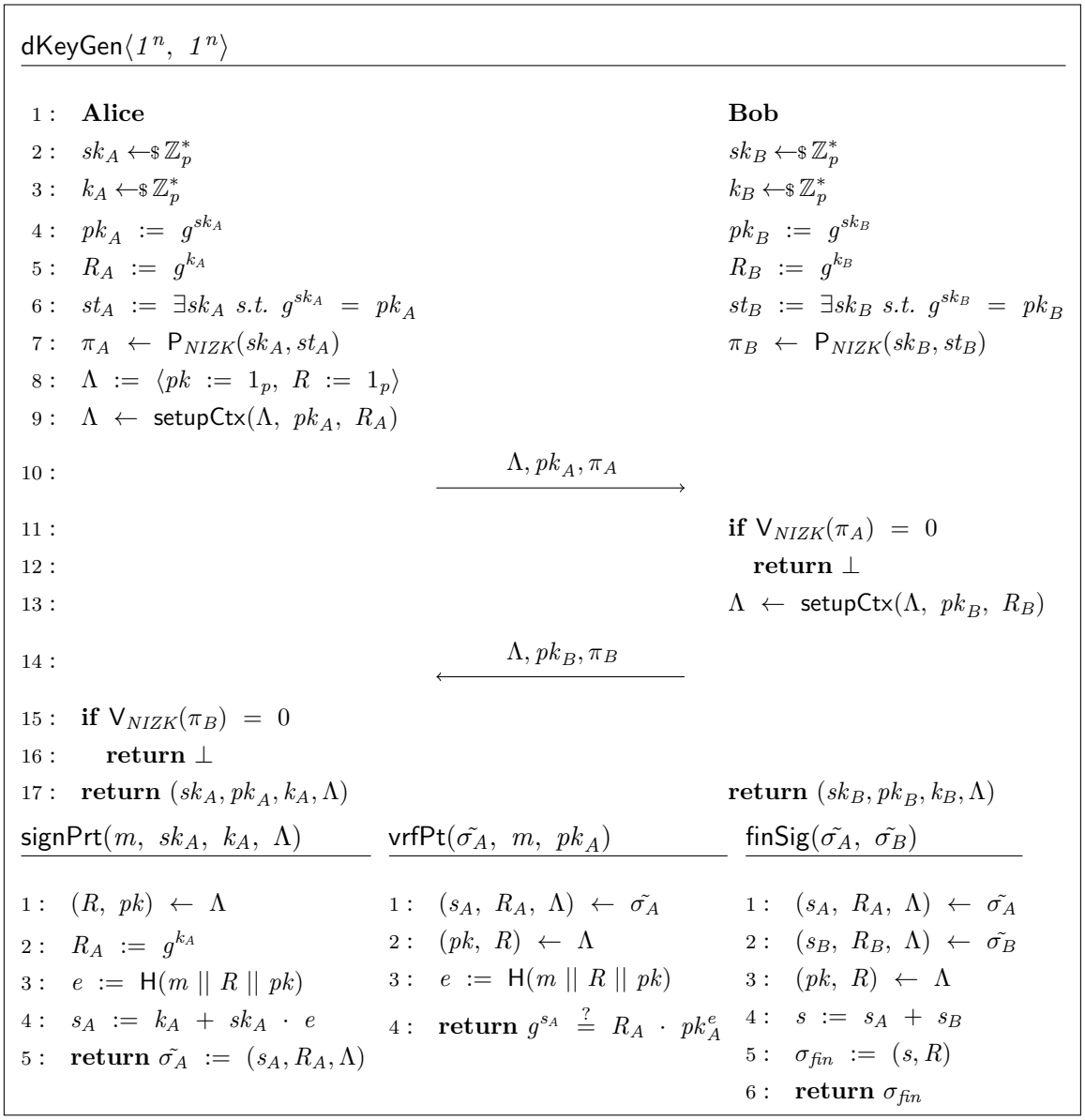

Figure 4.2: Two Party Schnorr Signature Scheme

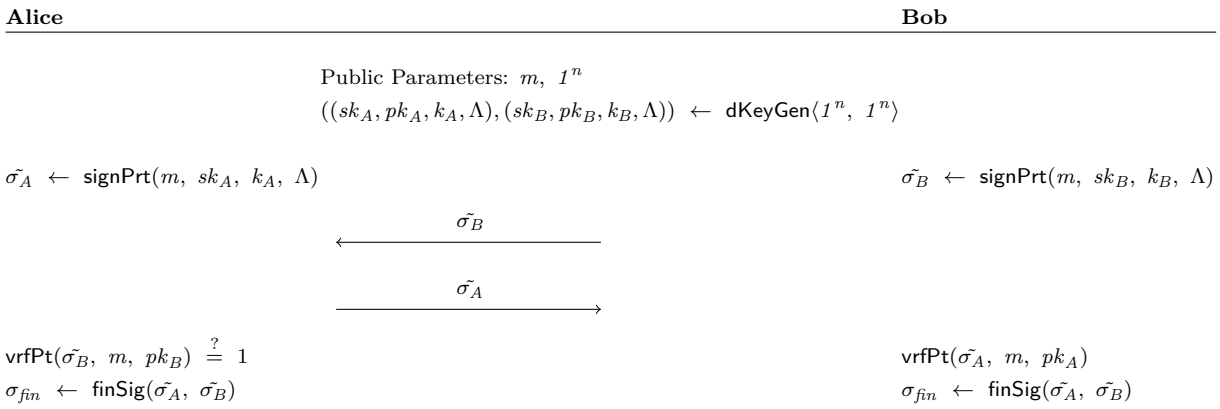

Figure 4.3: Two Party Schnorr Signature Scheme Interaction

$\mathsf{adaptSig}(\tilde{\sigma},\ x)$

1 : $(s,\ R_A,\ \Lambda) \leftarrow \tilde{\sigma}$
2 : $s^* := s + x$
3 : **return** $\hat{\sigma} := (s^*,\ R_A,\ \Lambda)$

$\mathsf{verifyAptSig}(\hat{\sigma_A},\ m,\ pk_A,\ X)$

1 : $(s_A,\ R_A,\ \Lambda) \leftarrow \hat{\sigma_A}$
2 : $(pk,\ R) \leftarrow \Lambda$
3 : $e := \mathsf{H}(m \mid\mid R \mid\mid pk)$
4 : **return** $g^{s_A} \stackrel{?}{=} R_A \cdot pk_A^e \cdot X$

$\mathsf{extWit}(\sigma_{fin},\ \tilde{\sigma_A},\ \hat{\sigma_B})$

1 : $(s,\ R) \leftarrow \sigma_{fin}$
2 : $(s_A,\ R_A,\ \Lambda) \leftarrow \tilde{\sigma_A}$
3 : $(\hat{s_B},\ R_B,\ \Lambda) \leftarrow \hat{\sigma_B}$
4 : $s_B := s - s_A$
5 : $x := \hat{s_B} - s_B$
6 : **return** (x)

Figure 4.4: Fixed Witness Adaptor Schnorr Signature Scheme

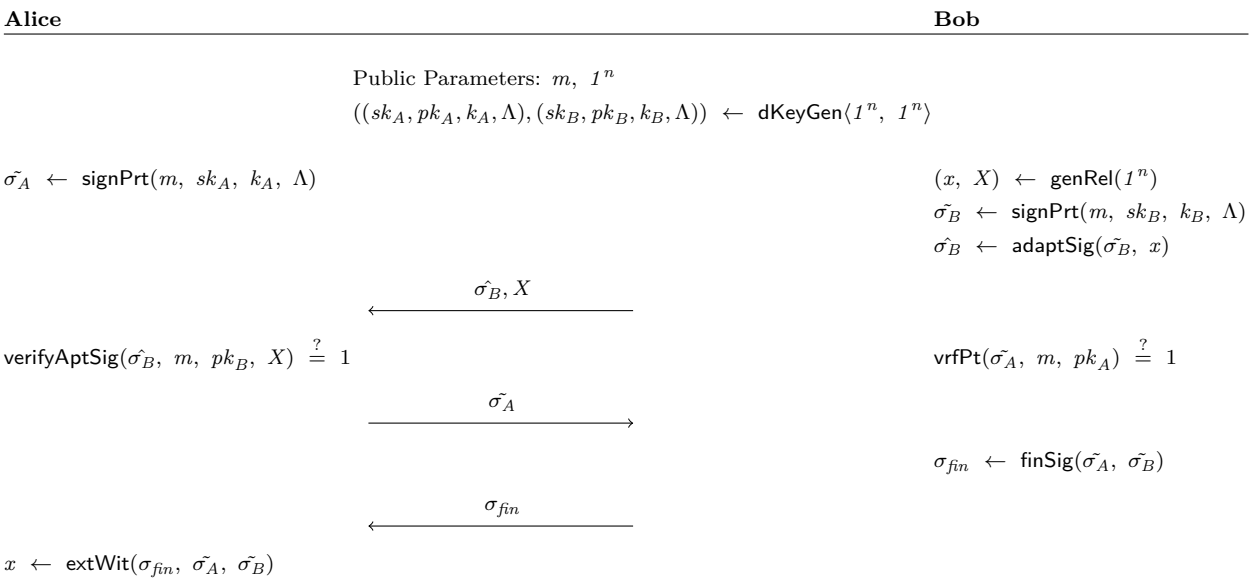

Figure 4.5: Fixed Witness Adaptor Schnorr Signature Interaction

4.2.2 Adaptor Signature Correctness

To prove that Adaptor Signature Correctness holds we have 3 statements to prove, first we prove that $\mathsf{verf}(m,\ \sigma_{fin},\ \Lambda.pk) \stackrel{?}{=} 1$ holds in our schnorr-based instantiation of the signature scheme, whereas Λ is setup such that $pk = pk_A \cdot pk_B$.

Proof. For this prove we assume the setup already specified in definition 4.4. The proof is by showing equality of the equation checked by the verifier of the final signature by continuous substitutions in the left side of equation:

$$g^s = R \cdot pk^e \tag{4.1}$$
$$g^{s_A} \cdot g^{s_B} \tag{4.2}$$
$$g^{k_A + e \cdot sk_A} \cdot g^{k_B + e \cdot sk_B} \tag{4.3}$$
$$g^{k_A} \cdot pk_A^e \cdot g^{k_B} \cdot pk_B^e \tag{4.4}$$
$$R_A \cdot pk_A^e \cdot R_B \cdot pk_B^e \tag{4.5}$$
$$R \cdot pk^e = R \cdot pk^e \tag{4.6}$$
$$1 = 1 \tag{4.7}$$

It remains to prove that with the same setup $\mathsf{verifyAptSig}(\hat{\sigma_B},\ m,\ pk_B,\ X) \stackrel{?}{=} 1$ and $(X,\ x^*) \in R$ hold.

$$\mathsf{verifyAptSig}(\hat{\sigma_B},\ m,\ pk_B,\ X) \stackrel{?}{=} 1$$

The proof is by continuous substitutions in the equation checked by the verifier:

$$g^{\hat{\sigma_B}} = R_B \cdot pk_B^e \cdot X \tag{4.8}$$
$$g^{\tilde{\sigma_B} + x} \tag{4.9}$$
$$g^{k_B + sk_B \cdot e + x} \tag{4.10}$$
$$g^{k_B} \cdot g^{sk_B \cdot e} + g^x \tag{4.11}$$
$$R_B \cdot pk_B^e \cdot X = R_B \cdot pk_B^e \cdot X \tag{4.12}$$
$$1 = 1 \tag{4.13}$$

We now continue to prove the last equation required:

$$((X,\ x^*) \in R)$$

We do this by showing that x is calculated correctly in extWit:

$$x := \hat{s} - (s - s_A) \tag{4.14}$$
$$\hat{s} - ((s_A + s_B) - s_A) \tag{4.15}$$
$$s_B + x - (s_B) \tag{4.16}$$
$$x := x \tag{4.17}$$
$$\tag{4.18}$$

□

4.2.3 Secure Adaptor Signature Scheme

In order to prove the security of the scheme we need to provide a proof that both $\mathsf{aEUF-CMA}$ and Witness Extractability hold in our instantiation. To perform the proof we must first recall the regular definition of $\mathsf{EUF-CMA}$ given for regular schnorr signatures. [Sch89] For that we define the game $\mathsf{forgeSig}_{\mathcal{A}}$:

$\mathsf{forgeSig}_{\mathcal{A}}(n)$	$\mathcal{O}_s(m, pk)$
1 : $\mathbb{S} \leftarrow \emptyset$	1 : $\sigma \leftarrow \mathsf{sign}(m,\ sk)$
2 : $(sk, pk) \leftarrow \mathsf{keyGen}(1^n)$	2 : $\mathbb{S} := \mathbb{S} \cup \{m\}$
3 : $(m,\ \sigma) \leftarrow \mathcal{A}^{\mathcal{O}_s(\cdot,\cdot)}$	3 : **return** σ
4 : **return** $((m) \notin \mathbb{S} \wedge \mathsf{verf}(m,\ \sigma,\ pk))$	

$\mathsf{EUF-CMA}$ holds if $\Pr[\mathsf{forgeSig}_{\mathcal{A}}(n) = 1] \leq \mathsf{negl}(n)$.

Proof. We proof $\mathsf{aEUF-CMA}$ holds by providing a black box reduction from $\mathsf{aEUF-CMA}$ to $\mathsf{EUF-CMA}$ of schnorr signatures. Intuitively if we suppose there exists a PPT adversary $\mathcal{A}$ that wins the $\mathsf{forgeAptSig}_{\mathcal{A}}$ game with probability 1, then $\mathcal{A}$ will also be able

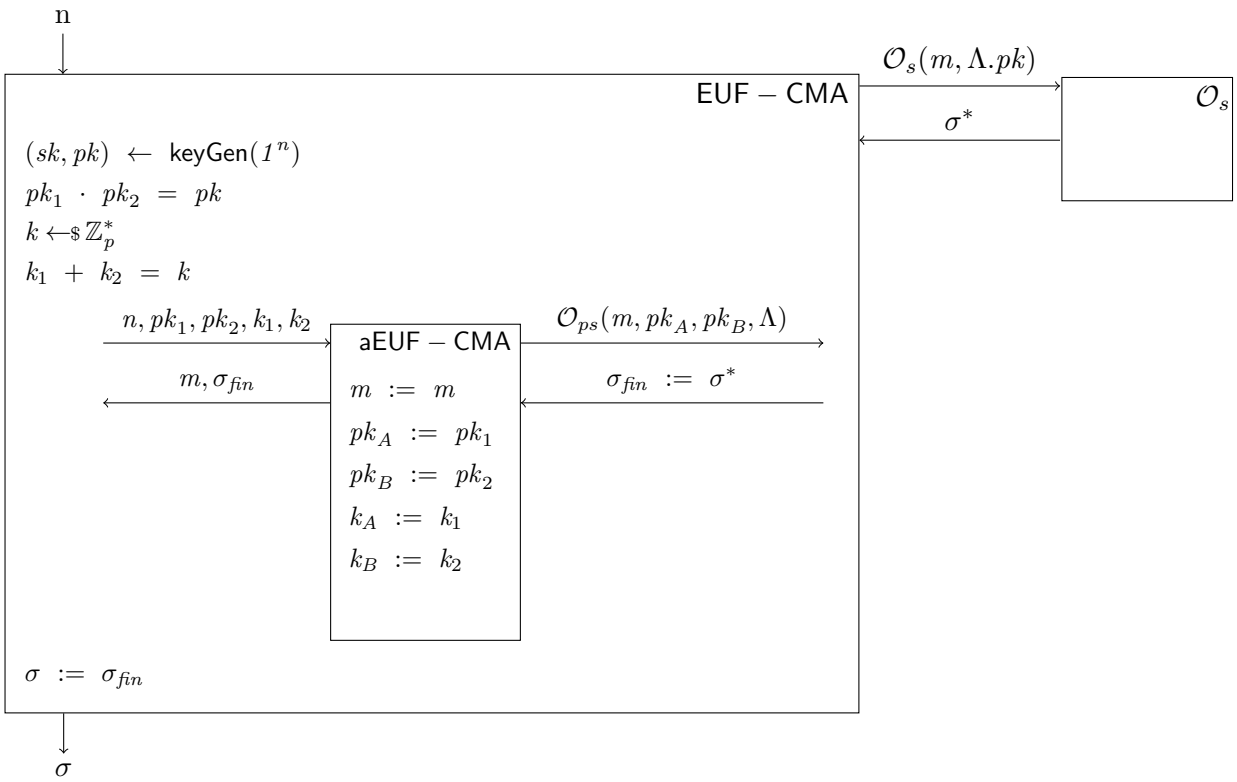

Figure 4.6: Reduction from aEUF − CMA to EUF − CMA

to win the $\mathsf{forgeSig}_{\mathcal{A}}$ game with the same probability, which leads us to a contradiction. This can be achieved by splitting up the public key chosen by the challenger in the $\mathsf{forgeSig}_{\mathcal{A}}$ game into pk_1 and pk_2 and then running the $\mathsf{forgeAptSig}_{\mathcal{A}}$ game using the two split keys. Since the signing oracle in aEUF − CMA and Witness Extractability both provide a signature valid unter the composite of the two public keys, we can simulate the oracle queried by the adversary $\mathcal{A}$ simply by forwarding the query to the EUF − CMA oracle with the original unsplit version of the public key. The output of the $\mathsf{forgeAptSig}_{\mathcal{A}}$ game will be a forged final signature valid under the combined public key of pk_1 and pk_2 which we can then use to win the $\mathsf{forgeSig}_{\mathcal{A}}$ game. See figure 4.6 for the black-box reduction.

In a very similiar way we can provide a reduction from Witness Extractability to EUF − CMA . Again if we suppose there exists a PPT adversary $\mathcal{A}$ able to win the $\mathsf{aExtrWit}_{\mathcal{A}}$ game with probability 1, then $\mathcal{A}$ will always be able to win the $\mathsf{forgeSig}_{\mathcal{A}}$, leading to a contradiction. Similiar to the previous proof the adversary $\mathcal{A}$ splits up the secret key pk computed during the $\mathsf{forgeSig}_{\mathcal{A}}$ game into pk_1 and pk_2 to use them in the $\mathsf{aExtrWit}_{\mathcal{A}}$. The forged final signature σ_{fin} can then be used to win the $\mathsf{forgeSig}_{\mathcal{A}}$ game. As the black-box reduction is the same as before we again refer to figure 4.6 to see the details.

□

CHAPTER 5

Adaptor Signature Based Atomic Swaps Between Bitcoin and a Mimblewimble Based Cryptocurrency

In this section, we will formalize Mimblewimble transactions and define security properties, similiar to those found by Fuchsbauer et al. in [FOS19]. We will only focus on the creation of new transfer transactions (transfering value from one or many parties to one or many parties), the notions of transaction aggregation, coin minting and transaction publishing discussed in [FOS19] will not be topic of this formalization. We then provide four different types of the transaction protocol, we separate between output coins which are controlled by a single party (denominated as $SoloCoin$) and a coin which is controlled by multiple parties owning a share of the blinding key, denominated as $SharedCoin$. These multiparty outputs can only spend by the two owners collaborating and are therefore semantically similiar to multisignature outputs in Bitcoin. [**?**] From this we can then derive the different types of transactions $Solo2SharedTx$, $Solo2SharedTx$, $Shared2SoloTx$ and $Shared2SharedTx$. Note that in our case we restrict ourselfs to $SharedCoin$ outputs which are controlled by two parties, as those are the ones needed to construct the Atomic Swap protocol. Proving the security for coins controlled by n participants would be a topic for future research. We conclude by providing proofs that all security definitions for Mimblewimble transactions hold for all four types of transactions. Finally, we define an Atomic Swap protocol from these building blocks, which allows us to securely swap funds from a Mimblewimble blockchain with those on another Blockchain, such as Bitcoin.

5.1 Definitions

As we have already discussed in section 3.3.3 for the creation of a transaction, it is immanent that both the sender and receiver collaborate and exchange messages via a secure channel. To construct the transaction protocol we assume that we have access to a two-party signature scheme Φ_{MP} as defined in definition 4.1, a zero-knowledge Range-proofs system Π such as Bulletproofs, as described in section 3.3.2 and a homomorphic commitment scheme COM as defined in definition 3.5 such as Pedersen Commitments 3.6.

Fuchsbauer et al. have defined three procedures Send, Rcv and Ldgr with regards to the creation of a transaction. Send called by the sender will create a pre-transaction, Rcv take the pretransaction and add the receivers output and Ldgr. As we already pointed out in this thesis we won't discuss the transaction publishing phase therefore we will not cover the functionality of the Ldgr procedure. Furthermore to better fit with out definitions of the multiparty signature scheme Φ_{MP} instead of having two seperate functions Send, Rcv we instead combine those two into a single distributed protocol spendOutput.

Definition 5.1 (Mimblewimble Transaction Scheme)**.** A Mimblewimble transaction scheme $MW[COM, \Phi_{MP}, \Pi]$ consist of the following procedures:

$$MW[COM, \Phi_{MP}, \Pi] \ := \ (\mathsf{spendCoins}, \mathsf{recvCoins}, \mathsf{finTx}, \mathsf{verfTx})$$

- $(ptx, r_A) \leftarrow \mathsf{spendCoins}([\mathcal{C}_{inp}], [r], p, v)$ is a procedure called by the sending party to initiate a transaction As input it takes a list of coins $[\mathcal{C}_{inp}]$ which should be spent, the respective keys $[x]$ to the input coins and a value p which should be transferred to the receiver as well as v which is the total value stored in the input coins. It outputs a pre-transaction ptx which can be sent to a receiver, as well as the senders signing key r_A.
- $ptx \leftarrow \mathsf{recvCoins}(ptx, p)$ is a procedure called by the receiver takes as input a pre-transaction ptx and a fund value p and will output a modified pre-transaction ptx.
- $tx \leftarrow \mathsf{finTx}(ptx, r_A)$ is a procedure that takes as input a pre-transaction ptx and a signing key r_A. The function will output a finalized transaction tx.
- $\langle 1, 0 \rangle \leftarrow \mathsf{verfTx}(tx)$ takes as input a transaction tx and verifies it and outputs either 1 on verification success or 0 otherwise.

An essential security property for a Mimblewimble Transaction Scheme MW is that a transaction can only transfer value from a sender to a receiver, but not create any new value, meaning the value of a transactions output coins must always be equal (or less accounting for transaction fees) then the sum of the value of its input coins.

Definition 5.2 (Inflation Resistence)**.**

Definition 5.3 (Theft-resistance)**.**

Definition 5.4 (Transaction indistinguishability)**.**

5.2 Grin instantiation

In this section we describe instantiations for the four transaction types defined in 5.1 specifically tailored for the Grin cryptocurrency. In section 5.3 we use this transaction types as building block for the Mimblewimble side of our Atomic Swap protocol.

5.2.1 *Solo2SoloTx*

The *Solo2SoloTx* is the simplest form of a transaction in which a sender wants to transfer some value p to a receiver. The spendOutput protocol is called by a sender providing the to be transferred value p, the input coin $\mathcal{C}_{inp}$ with a value v of at least p, its secret blinding factor r and a receiver just having to provide the same value p. Note that the sender could also provide a list of input coins and blinding factors. However, since the protocol works in the same way for both cases, for simplicity we here only describe the case of a single input coin.

For the execution of the protocol we assume to have access to a homomorphic commitment scheme such as Pedersen Commitment as defined in definition 3.6. Furthermore we require a Rangeproof system as defined in 3.3.2 and a two-party signature scheme as defined in 4.1. The instantiation of the transaction protocol can be found in figure 5.1.

The protocol begins by the sender creating his change coin of value $v - p$, this is the difference between the coin the sender spends and the value transferred to the receiver. (Again here it would be possible to create a list of change coins instead of one without changing the protocol) After creating the change coin a range proof is calculated. The sender then concludes by calculating his share of the secret to the final excess value $\mathcal{E}$ to which the participents have to sign to. Optionally the sender can choose a message m to be included in the transaction. This optional message, the change coin together with its range proof, input coin and a commitment to his share of the excess secret is then sent to the receiver. The receiver will create his output of value p together with a range proof and send back the coin, the proof and a commitment to his share of the excess secret. The participents then run in the signPt protocol to calculate a signature whereas $\mathcal{E}$ is the shared public key used in the signing process. After the signature is finalized the transaction can be published.

5.2.2 *Solo2SharedTx*

In this type of transaction we create an output coin which is controlled by two parties rather then just one. We achieve this by creating a coin which blinding factor is composed of two blinding factors each controlled by one of the parties. The coin then will only be spendable if both parties cooperate, which is generally referred to a multisignature

$\mathsf{createCoin}(v, r)$	$\mathsf{createTx}(m, [\mathcal{C}_{inp}], [\mathcal{C}_{out}], [\pi], \Lambda, [C], \sigma)$
1 : $\mathcal{C} \leftarrow \mathsf{commit}(v, r)$	1 : **return** (
2 : $\pi \leftarrow \mathsf{ranPrf}(\mathcal{C}, v, r)$	2 : $m := m$,
3 : **return** $(\mathcal{C}, \pi)$	3 : $inp := [\mathcal{C}_{inp}]$,
	4 : $out := [\mathcal{C}_{out}]$,
	5 : $\Pi := [\pi]$,
	6 : $\Lambda := \Lambda$,
	7 : $com := [C]$,
	8 : $\sigma := \sigma$
	9 :)

output as defined in [**?**]. The protocol is similiar to the standard *Solo2SoloTx*, however the sender initially will create multiple blinding factors, one for his change coin and one for the shared output coin. He needs to send one additional commitment to the receiver to his share of the to be created output coin, with which the receiver can build the final shared output coin. One further difference to the standard protocol is that the parties have to cooperate to create the rangeproof for the shared output coin, since nobody of them know the full blinding factor (which is needed to calculate the proof).

Again we assume that we have access to a homomorphic commitment scheme, a rangeproof protocol and two-party signature scheme. The rangeproof protocol needs to support creation of multiparty rangeproofs for this protocol to be executeable. The concrete instantiation can be found in figure 5.2.

5.2.3 *Shared2SoloTx*

In this instantiation we want to spend a input coin controlled by both parties (each of them holding a share of the blinding factor) and create a output coin controlled only the receiver and a change output controlled by the sender. The protocol is very similiar to the standard *Solo2SoloTx*, the only difference is that both participents need to provide their share of the input coin blinding factor to the protocol. As a result the receivers share of the final excess secret (which always calculates as output coin blinding factors minus input coins blinding factors) is calculated slightly different.

Again with the same assumptions as in *Solo2SoloTx* we provide the concrete instantiation in 5.3.

5.2.4 *Shared2SharedTx*

The final instantiation is a combination of *Solo2SharedTx* and *Shared2SoloTx* as we want to spend a shared coin and create a new shared coin. Note that for simplicity in

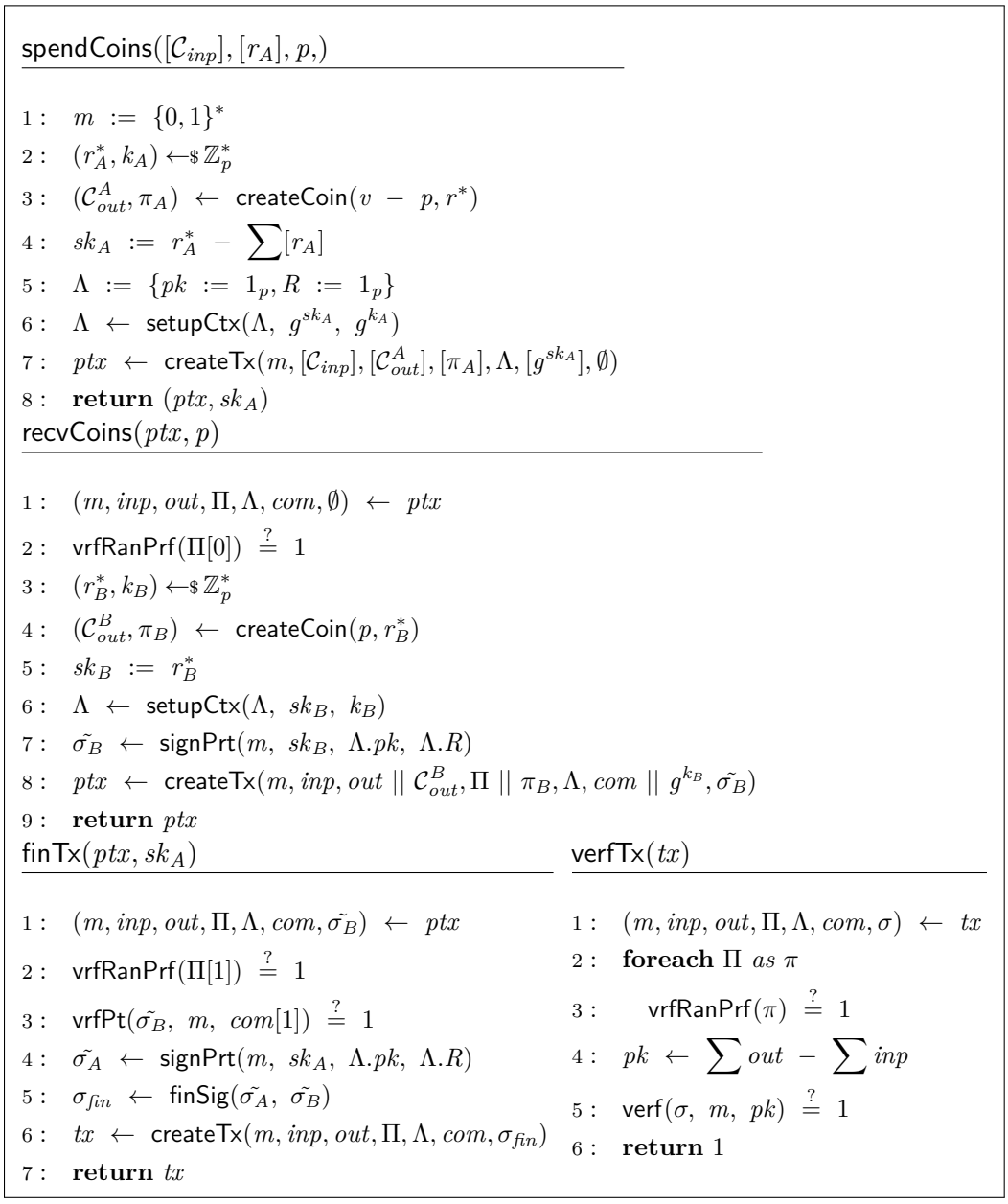

Figure 5.1: Instantiation of *Solo2SoloTx* transaction protocol.

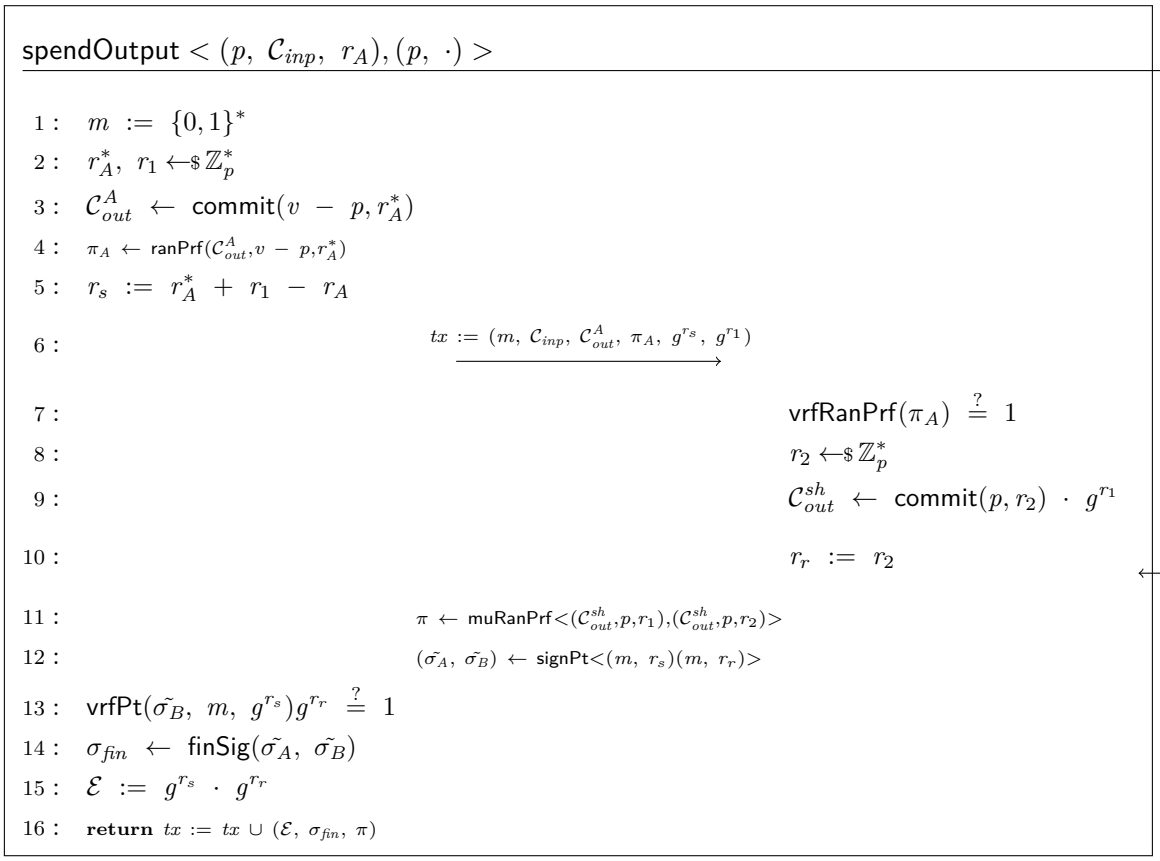

Figure 5.2: Instantiation of *Solo2SharedTx* transaction protocol.

explaining the protocol we assume the parties controlling the input coin are the same as the ones who should control the output coin, however, this can easily be adapted to arbitrary parties.

5.3 Atomic Swap protocol

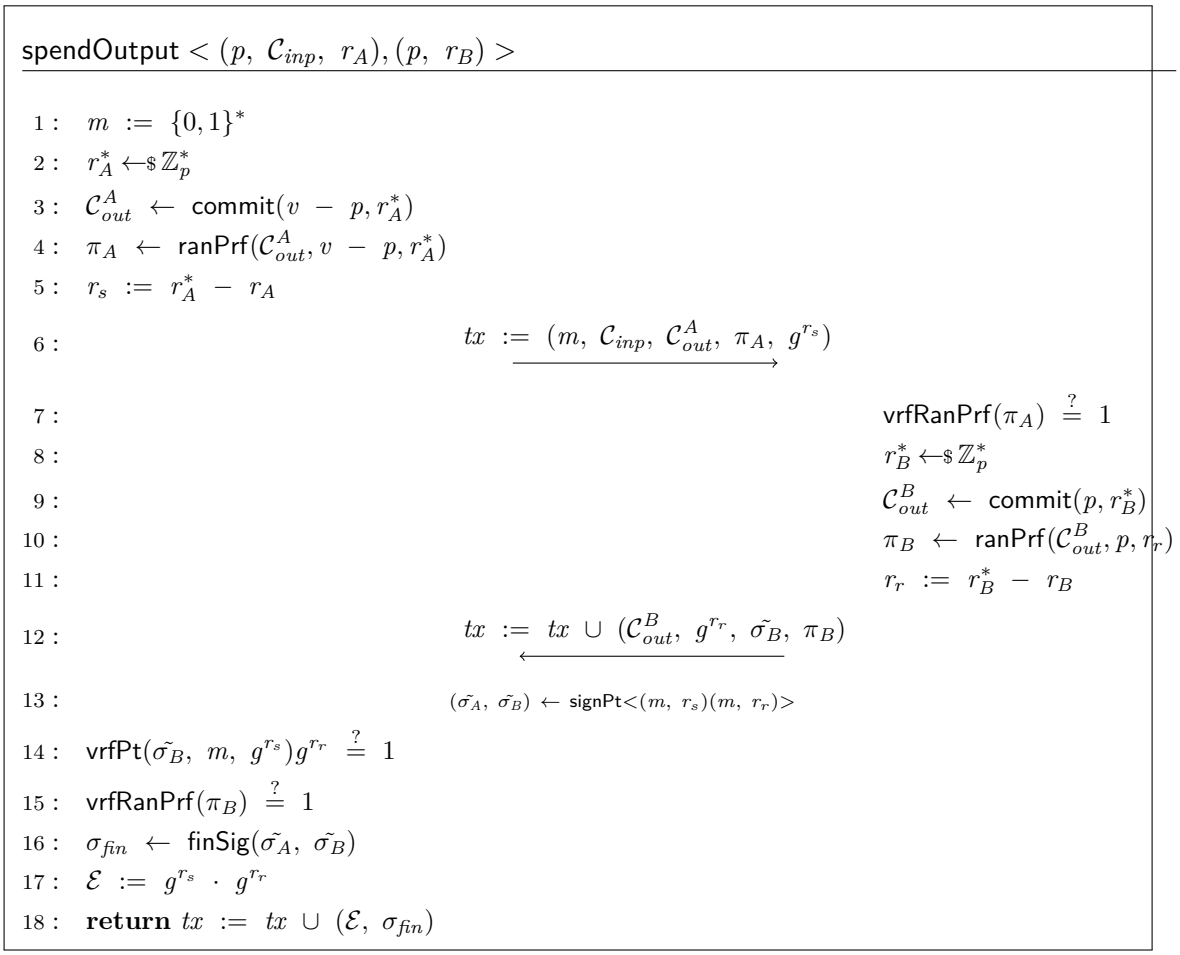

Figure 5.3: Instantiation of *Shared2SoloTx* transaction protocol.

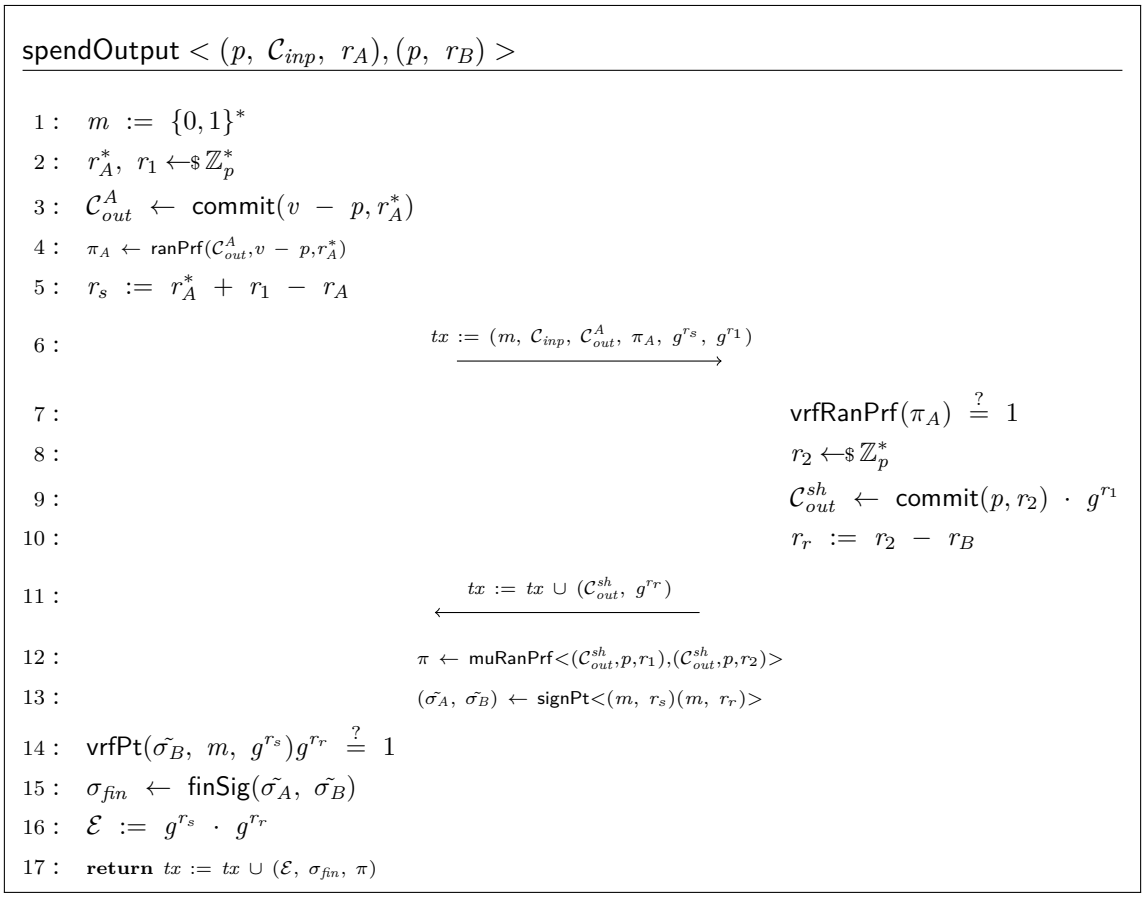

Figure 5.4: Instantiation of *Shared2SharedTx* transaction protocol.

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