

Consider $\nabla \epsilon \triangle u = f$, discretization by the spectral method and solved using multigrid.

Discretizing the domain by the Spectral Method and solving using multigrid yields a solution with a fourier representation that is accurate up to k coefficients. To get higher accuracy, one can refine the grid further.

Now consider the buffer zone introduced by segmental refinement, which can be represented as a new grid with two boundary conditions encoding the error. The boundary will thus be functions with fourier coefficients of k or greater since the coarser grid was accurate up to k coefficients. The solution to this grid with boundary values of frequency k will decay to discretization error by the time it leaves the buffer zone. On this finer grid, then, we will have error of frequency $2k$ or higher, combined with the error that is introduced by the buffer zone. Overall, this function will still be accurate up to coefficients of $2k$.

Inductively, we will see that at every level of refinement, we get accuracy up to coefficients of $(2^n)k$

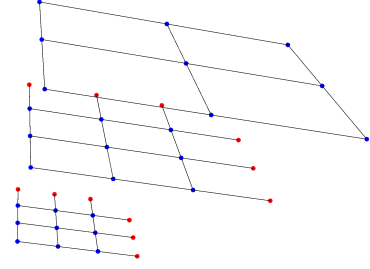


Figure 1: Blue:grid points to be calculated. Red:fixed boundary estimates