Time-Frequency Analysis with Python

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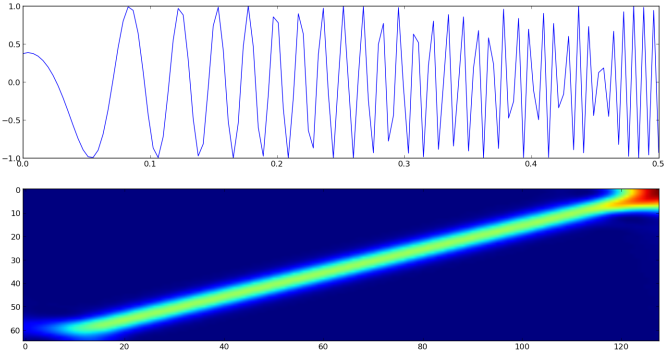
SciPy.In 2012,
IIT Bombay

International Year of Statistics 2013

Recap

- pyhht (`github.com/jaidevd/pyhht`) is a Python implementation of the Hilbert-Huang transform
- scikit-signal (github.com/scikit-signal) is a scikit for advanced signal processing
  12 forks, 6 active :(
- Developer Talks
  1. Filter Design
  2. Interpolation - Splines
  3. Periodograms/Spectrograms
  4. Wavelets
  5. **Time-Frequency Analysis**

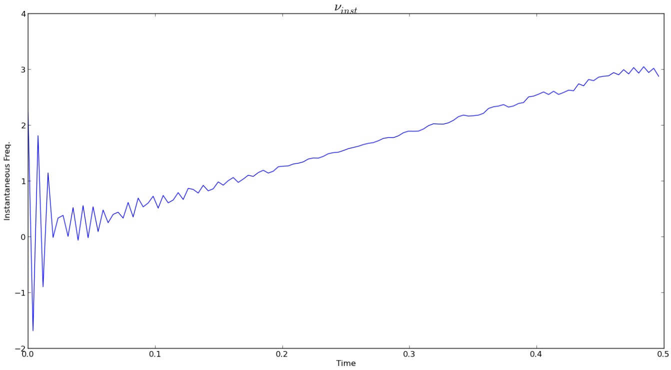
The Problem: Chirp



- Analytical signals via the Hilbert Transform:

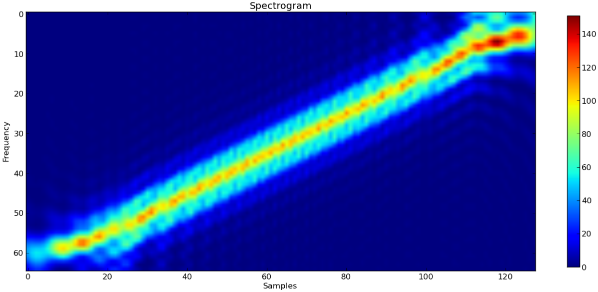
$$\nu_{inst} = \tan^{-1} \frac{Im(\hat{x})}{Re(\hat{x})} \tag{1}$$

where $\hat{x} = x + jH(x)$

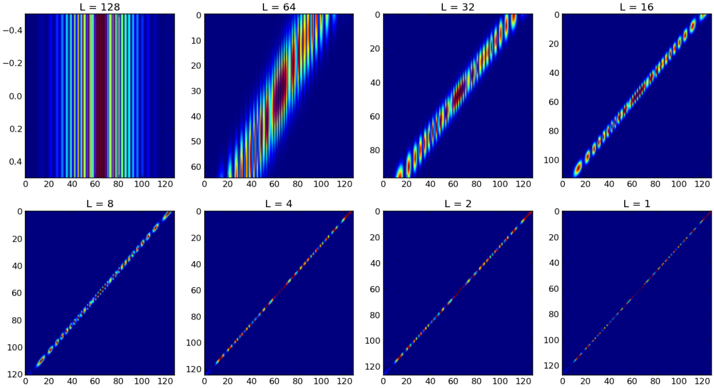


The Solutions

- Short-time Fourier Transform

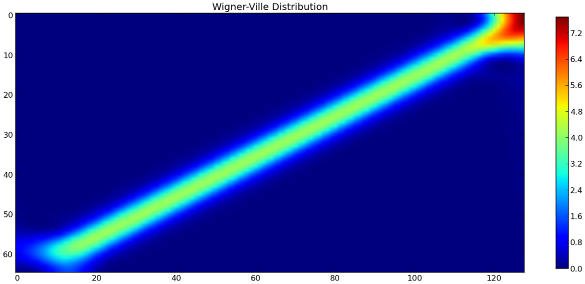


The Solutions



- The Wigner-Ville Class

$$W_x(t,\nu) = \int_{-\infty}^{\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})e^{-j2\pi\nu\tau}d\tau \qquad (2)$$



scikit.signal.timefreq

- Generators
  - Linear chirps
  - Gaussian modulations
  - Other AM/FM modulations
- Representations
  - Spectrograms
  - Instantaneous frequencies based on IMFs
  - Wigner/Cohen class of distributions
- Other
  - Group delay
  - Time-bandwidth products
  - Analytical signals
  - Windowed operations

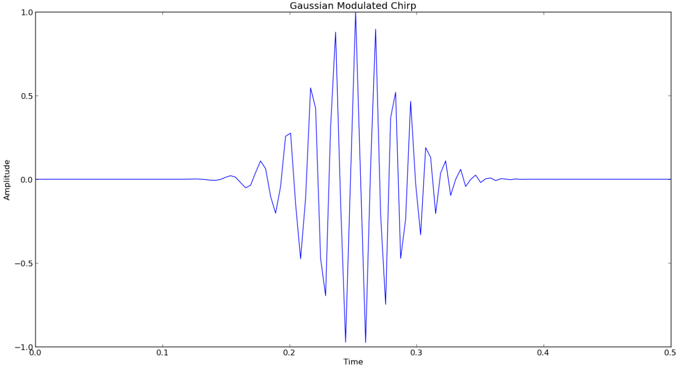
- Heisenberg-Gabor Inequality / The Uncertainty Principle
- Time-Bandwidth Product: $T \times B \geq 1$
- '(The uncertainty principle) is a statement about two variables whose associated operators do not commute.' - Leon Cohen
- For any two quantities *a* and *b* represented by the respective operators α and β

$$\Delta a \times \Delta b \geq \frac{1}{2} \times |\langle \alpha, \beta \rangle| \tag{3}$$

where Δ denotes the standard deviation

Heisenberg-Gabor Inequality / The Uncertainty Principle

- Ideal decompositions must have a large time-bandwidth product
- Gaussian signals show the lower bound
- Hence, work on gaussian decompositions with large $T \times B$



Future work - What the scikit needs

- Time-Frequency Analysis
  - Better spectrograms
  - Wavelet-based spectra
  - Time-frequency representations via the HHT
- Wavelets!
- Better filter design
- Better interpolation

Read the discussion here - `http://goo.gl/elRp3`
and the summary here - `http://goo.gl/YHo7G`
Find the repository at `github.com/scikit-signal`

Thank You!
Questions?