RRC Summer Sessions 2020 **(Due: 20/05/20)**

Assignment 1

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Note:

- Solve the questions on paper. Email the photos of your answer sheets in a zipped folder to the instructor.
- Use the **Ref:** links to quickly review the concepts before solving the questions.

Problem 1: SVD. Ref: SVD Example

Consider the matrix

$$A = \begin{pmatrix} 2 & 2 & 2 & 2 \\ 17/10 & 1/10 & -17/10 & -1/10 \\ 3/5 & 9/5 & -3/5 & -9/5 \end{pmatrix}$$

(a) Find the Singular Value Decompostion (SVD) of A.

(b) *SVD is the Swiss Army Knife of Linear Algebra.* Support this statement with atleast 5 uses of SVD. You may also use the solution of (a) in support of your answer in each case.

Problem 2: QR Decomposition. Ref: Gram-Schmidt Example

Consider the matrix

$$B = \begin{pmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 0 & 2 & 2 \end{pmatrix}$$

(a) Find the QR decomposition of B using Gram-Schmidt procedure.

(b) Write the pseudocode of a function that implements Gram-Schmidt procedure with a matrix as input and its Q and R matrices as output.

Problem 3: Quadratic Function and Definiteness

Consider the equation

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} S \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 4(x_1 - 2x_2 + x_3)^2$$

(a) Find the 3×3 matrix S that satisfies the above equation.
(b) Find its rank, eigen values and determinant.
(c) Is S, a positive-definite matrix ?

Problem 4: Gradient and Hessian

Find the gradient and hessian of the function $f(x, y, z) = 3x^2yz$.

Problem 5: Lagrange Multipliers. Ref: Paul's Online Notes and Practice Problems

Find the maximum and minimum values of $f(x, y) = 3x - 6y$ subject to the constraint $4x^2 + 2y^2 = 25$.