

$$\begin{aligned} KB &= \{(A \vee B) \rightarrow C, A\} \\ (A \vee B) \rightarrow C &= \neg(A \vee B) \vee C \\ &= (\neg A \wedge \neg B) \vee C \\ &= (\neg A \vee C) \wedge (\neg B \vee C) \end{aligned}$$

Hence, In CNF form:

$$KB = \{\neg A \vee C, \neg B \vee C, A\}$$

Applying Modes Ponens:

$$\frac{\neg A \vee C, A}{C} \text{ as it is equivalent to } \frac{A \rightarrow C, A}{C}$$

$$\text{Hence, } KB = \{\neg A \vee C, \neg B \vee C, A, C\}$$

$$\begin{aligned} KB &= \{A \vee B, B \rightarrow C, (A \vee C) \rightarrow D\} \\ B \rightarrow C &= \neg B \vee C \\ (A \vee C) \rightarrow D &= \neg(A \vee C) \vee D \\ &= (\neg A \wedge \neg C) \vee D \\ &= (\neg A \vee D) \wedge (\neg C \vee D) \end{aligned}$$

Hence, In CNF form:

$$KB = A \vee B, \neg B \vee C, \neg A \vee D, \neg C \vee D$$

Applying resolution:

$$\frac{A \vee B, \neg B \vee C}{A \vee C}$$

$$KB = A \vee B, \neg B \vee C, \neg A \vee D, \neg C \vee D, A \vee C$$

Applying resolution:

$$\frac{A \vee C, \neg C \vee D}{A \vee D}$$

$$KB = A \vee B, \neg B \vee C, \neg A \vee D, \neg C \vee D, A \vee C, A \vee D$$

Applying resolution:

$$\frac{\neg A \vee D, A \vee D}{D \vee D} = \frac{\neg A \vee D, A \vee D}{D}$$

$$KB = A \vee B, \neg B \vee C, \neg A \vee D, \neg C \vee D, A \vee C, A \vee D, D$$

## Question 5.b, Scheduling, CS221

---

The key fact to notice in this problem is that the first constraint says that every number has a unique successor. But since the set is finite, finding successors repeatedly of a number must form cycle and start repeating. But successor relationship also implies the "larger than" relationship and hence will repeat too. Also since "larger than" relationship is also transitive, we will end up with forcing "larger than" relationship to be reflexive as well.

More formally, let  $X$  be a finite set of numbers such that  $|X| = n$ . Here  $X = \{x_0, x_1, x_2, \dots, x_{n-1}\}$ . Let us start with element  $x_i$  and start evaluating successors relationship repeatedly  $n$  times to produce the sequence  $Y = \{y_0, y_1, \dots, y_n\}$ . Here:

$y_0 = x_i$   
 $y_1$  is the successor of  $y_0$ . i.e.,  $Successor(y_0, y_1)$  holds true.  
 $y_2$  is the successor of  $y_1$ . i.e.,  $Successor(y_1, y_2)$  holds true.  
*...andsoon*

Since  $|Y| = n + 1$ , there must be two elements in  $Y$  that are same. Also, since  $Successor(y_i, y_j) \rightarrow Larger(y_i, y_j)$ , and  $Successor(y_i, y_{i+1})$  holds true for all  $0 \leq i < n$ ,  $Larger(y_i, y_{i+1})$  also holds true for any consecutive pair of  $(y_i, y_{i+1})$ . Additionally,  $Larger$  relationship is transitive, so  $Larger(y_i, y_j)$  holds true for any pair  $(i, j)$  where  $0 \leq i < j \leq n + 1$ .

But we have already proved that there are two elements in  $Y$  that are same.  $Larger$  relationship holds true for these two elements too. This is in contradiction to the new rule that is being added in this question. Hence, no finite set can satisfy all 7 constraints