WHAT IS A "VALID" SCORE IN THE GAME OF RUGBY?

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Context

There are three ways you can get points in the game of rugby:

1. A "try" is worth five points;
2. A "conversion" is worth two points, but must be scored after a try;
3. A non-conversion kick (often called, but not limited to, a "penalty") is worth three points.

Let the set of individual scores be S, where

$$S = \{2, 3, 5\}. \tag{1}$$

Let us also say that the set of all valid scores in rugby S' is exactly $\{\mathbb{N} \cup 0\}$ (i.e., no negative numbers).

PROPOSITION 1

We want to prove that the only scores that are impossible to obtain in this game are scores of 1, 2, and 4.

Zero

We start with the base case: you can get zero points, as everyone starts with zero points at the start of the game. The lack of scoring anything will leave you with zero points.

One

We can also not score 1 because

$$1 < s \ \forall s \in S. \tag{2}$$

Two

We cannot obtain a score of 2, because we can only score a conversion under the condition that we score a try first.

Three

Scoring a 3 is as "simple" as kicking the ball over the H-looking thing at the correct end of the field.

Four

The only permutation of S that makes is two of the score 2. By the same logic as the score 2, we cannot obtain this because we can only score a conversion by first scoring a try.

Five

We can score 5 points by *committing* a try.

Six

The score 6 can be obtained by scoring 3 twice.

Seven

Score a try, and successfully convert the ball over the H thing.

Induction Step

By induction, we can score 8 by scoring 5 (defined above) and scoring another 3 (also defined above). We can score 9 by scoring 6 and scoring another 3. We can score 10 by scoring 7 and scoring another 3.

In general, we can write a recursive function s of the score n such that:

$$s(n) = \begin{cases} s(n-3)+3, & \text{if } n > 7 \\ a+b, & \text{if } n \leq 7, \text{ with } a, b \in \{S \cup 0\} \end{cases}. \tag{3}$$

Hence, the only scores we cannot obtain are 1, 2, and 4, and by induction every other score is possible:

$$i \notin S' \; \forall i \in \{1, 2, 4\}. \tag{4}$$