Maestría en Estadística

: Justo Andrés Manrique Urbina - 20091107

Pregunta (1)

→ Hallar $V_{(pe)} - V_{(neyman)}$:

→ Hallar $V_{(neyman)}$ - Paso 1

$$(1)\quad n_{h,neyman} = \frac{N_h S_h n}{\sum_{i=1}^{H} N_i S_i}$$

$$(2)\quad V(\bar{y}) = \sum_{h=1}^{H} \left(\frac{N_h^2}{N^2} \times \frac{S_h^2}{n_h} \times \left[1 - \frac{n_h}{N_h} \right] \right)$$

reemplazando (1) en (2)

$$(3)\quad \sum_{h=1}^{H} \left(\frac{N_h^2}{N^2} \times \frac{\frac{S_h^2}{1}}{\frac{N_h S_h n}{\sum_{i=1}^{H} N_i S_i}} \times \left[1 - \frac{\frac{N_h S_h n}{\sum_{i=1}^{H} N_i S_i}}{\frac{N_h}{1}} \right] \right)$$

$$(4)\quad \sum_{h=1}^{H} \left(\frac{N_h S_h \sum_{i=1}^{H} N_i S_i}{N^2 n} \times \left[1 - \frac{S_h n}{\sum_{i=1}^{H} N_i S_i} \right] \right)$$

$$(5)\quad \sum_{h=1}^{H} \left(\frac{N_h S_h \sum_{i=1}^{H} N_i S_i}{N^2 n} - \frac{N_h S_h^2}{N^2} \right)$$

$$(6)\quad \frac{1}{N^2 n} \left(\sum_{h=1}^{H} N_h S_h \right)^2 - \frac{1}{N^2} \sum_{h=1}^{H} N_h S_h^2$$

$$(7)\quad V_{(neyman)} = \frac{1}{N^2} \left[\frac{\left(\sum_{h=1}^{H} N_h S_h \right)^2}{n} - \sum_{h=1}^{H} N_h S_h^2 \right]$$

→ Hallar $n_{h(prop)}$, paso 2:

(1) $n_{h,prop} : \frac{N_h n}{N}$

(2) $V(\bar{y}) \quad \sum_{h=1}^{H} \left(\frac{N_h^2}{N^2} \times \frac{S_h^2}{n_h} \times \left[1 - \frac{n_h}{N_h} \right] \right)$

reemplazando (1) en (2)

(3) $\sum_{h=1}^{H} \left(\frac{N_h^2}{N^2} \times \frac{\frac{S_h^2}{1}}{\frac{N_h n}{\cancel{N}}} \times \left[1 - \frac{\frac{N_h n}{N}}{\frac{N_h}{1}} \right] \right)$

(4) $\sum_{h=1}^{H} \left(\frac{N_h}{N} \times \frac{S_h^2}{n} \times \left[1 - \frac{n}{N} \right] \right)$

(5) $\sum_{h=1}^{H} \left(\frac{N_h S_h^2}{N n} - \frac{N_h S_h^2}{N^2} \right)$

(6) $\sum_{h=1}^{H} \left(\frac{N_h S_h^2}{N n} \right) - \sum_{h=1}^{H} \left(\frac{N_h S_h^2}{N^2} \right) = V(\bar{y})_{prop}$

→ Hallar $V_{(prop)} - V_{(neyman)}$ paso 3:

(1) $\sum_{h=1}^{H} \frac{N_h S_h^2}{N n} - \cancel{\sum_{h=1}^{H} \frac{N_h S_h^2}{N^2}} - \frac{(\sum_{h=1}^{H} N_h S_h)^2}{N^2 n} + \cancel{\sum_{h=1}^{H} \frac{N_h S_h^2}{N^2}}$

(2) $\sum_{h=1}^{H} \frac{N_h S_h^2}{N n} - \frac{(\sum_{h=1}^{H} N_h S_h)^2}{N^2 n}$

(3) $\frac{1}{n} \left[\sum_{h=1}^{H} \frac{N_h}{N} \left(S_h^2 - S_h \frac{\sum_{j=1}^{H} N_j S_j}{N} \right) \right.$

(4) $\frac{1}{n} \left[\sum_{h=1}^{H} \frac{N_h}{N} \left[\left(S_h - \frac{\sum_{j=1}^{H} N_j S_j}{N} \right)^2 + S_h \frac{\sum_{j=1}^{H} N_j S_j}{N} - \left(\frac{\sum_{j=1}^{H} N_j S_j}{N} \right)^2 \right] \right.$

(5) $\frac{1}{n}\left[\sum_{h=1}^{H}\frac{N_h}{N}\left(S_h - \frac{\sum_{j=1}^{H} N_j S_j}{N}\right)^2 + \cancel{\sum_{h=1}^{H}\frac{N_h S_h}{N}} \times \frac{\sum_{j=1}^{H} N_j S_j}{N} - \underbrace{\sum_{h=1}^{H}\frac{N_h}{N}}_{1}\cancel{\left(\sum_{j=1}^{H}\frac{N_j S_j}{N}\right)^2}\right]$

(6) $\frac{1}{n}\left[\sum_{h=1}^{N}\frac{N_h}{N}\left(S_h - \sum_{j=1}^{H}\frac{N_j S_j}{N}\right)^2\right]$

b) Hallar cuando $H=2$ ...

$N_2 + N_1 = N \therefore N_2 = N - N_1$

$$\rightsquigarrow \frac{1}{n}\left[\frac{N_1}{N}\left(S_1 - \sum_{j=1}^{2}\frac{N_j S_j}{N}\right)^2 + \frac{N-N_1}{N}\left(S_2 - \sum_{j=1}^{2}\frac{N_j S_j}{N}\right)^2\right]$$

$$\rightsquigarrow \frac{1}{n}\left[\frac{N_1}{N}\left(S_1 - \frac{N_1S_1 + NS_2 - N_1S_2}{N}\right)^2 + \frac{N-N_1}{N}\left(S_2 - \frac{N_1S_1 + NS_2 - N_1S_2}{N}\right)^2\right]$$

$$\rightsquigarrow \frac{1}{n}\left[\frac{N_1}{N}\left(\frac{NS_1 - N_1S_1 - NS_2 + N_1S_2}{N}\right)^2 + \frac{N-N_1}{N}\left(\frac{NS_2 - N_1S_1 - NS_2 + N_1S_2}{N}\right)^2\right]$$

$$\rightsquigarrow \frac{1}{n}\left[\frac{N_1}{N}\left(\frac{(N-N_1)(S_1-S_2)}{N}\right)^2 + \frac{N-N_1}{N}\left[\frac{N_1^2\,(S_2-S_1)^{2*}}{N^2}\right]\right]$$

$$\rightsquigarrow \frac{1}{n}\left[\frac{N_1}{N}\left(\frac{N-N_1}{N}\right)(S_1-S_2)^2\left[\cancel{\frac{N-N_1}{N} + \frac{N_1}{N}}\right]^1\right]$$

$$\rightsquigarrow \frac{1}{n}\left[\frac{N_1}{N}\left(\frac{N-N_1}{N}\right)(S_1-S_2)^2\right]$$

* esto es igual a $(S_1 - S_2)^2$

→ Análisis de sensibilidad

a) Se observa que cuando N tiende al infinito, ambas varianzas serán iguales.

b) Se observa que si N_1 tiende a N, las varianzas serán iguales.

c) Si las varianzas dentro de cada estrato son iguales entre sí, ambas varianzas serán iguales.

La diferencia será mayor cuando:

b) $N_1 = N_2$; así aseguramos que la diferencia de varianzas es máxima.

c) La diferencia entre varianzas de cada estrato sean lo más dispares entre sí.

a) N no tienda al infinito.