Examen 2 - Estadística Computacional

14 de julio de 2019

a) Encuentre la función de distribución a posteriori de $p(\theta|Y, W)$ y las distribuciones condicionales de λ y γ.

1. Resolución

Primero hallamos la función de verosimilitud de $p(Y, W|\theta)$, la cual es:

$$\prod_{i=1}^{n} \frac{e^{-n_i\gamma\lambda} (n_i\gamma\lambda)^{y_i}}{y_i!} \prod_{j=1}^{m} \frac{e^{-m_j\lambda} \left(m_j\lambda\right)^{w_j}}{w_j!}$$

$$e^{-\lambda\gamma \sum_{i=1}^{n} n_i} (\lambda\gamma)^{\sum_{i=1}^{n} y_i} e^{-\lambda \sum_{j=1}^{m} m_j} \lambda^{\sum_{j=1}^{m} w_j} \prod_{i=1}^{n} \frac{n_i^{y_i}}{y_i!} \prod_{j=1}^{m} \frac{m_j^{w_j}}{w_j!}$$

$$p(Y, W|\theta) = L(.) = e^{-\lambda(\gamma \sum_{i=1}^{n} n_i + \sum_{j=1}^{m} m_j)} \lambda^{\sum_{i=1}^{n} y_i + \sum_{j=1}^{m} w_j} \gamma^{\sum_{i=1}^{n} y_i} \prod_{i=1}^{n} \frac{n_i^{y_i}}{y_i!} \prod_{j=1}^{m} \frac{m_j^{w_j}}{w_j!}$$

Posteriormente, hallamos la distribución a posteriori:

$$p(\theta|Y, W) = p(\theta)p(Y, W|\theta)$$

$$p(\theta|Y, W) = \frac{1}{\Gamma(a)} b^a \lambda^{a-1} e^{-b\lambda} * \frac{1}{\Gamma(c)} c^d \gamma^{c-1} e^{-d\gamma} * p(Y, W|\theta)$$

Finalmente, encontramos las distribuciones condicionales de *lambda* y *gamma*:

$$p(\lambda|\gamma, Y, W) \propto \lambda^{a-1} e^{-b\lambda} * e^{-\lambda(\gamma \sum_{i=1}^{n} n_i + \sum_{j=1}^{m} m_j)} \lambda^{\sum_{i=1}^{n} y_i + \sum_{j=1}^{m} w_j}$$

$$p(\lambda|\gamma, Y, W) \propto \lambda^{\left(a + \sum_{i=1}^{n} y_i + \sum_{j=1}^{m} w_j\right) - 1} e^{-\lambda\left(\gamma \sum_{i=1}^{n} n_i + \sum_{j=1}^{m} m_j + b\right)}$$

$$\lambda \sim gamma\left(a+\sum_{i=1}^{n} y_i+\sum_{j=1}^{m} w_j, \gamma \sum_{i=1}^{n} n_i+\sum_{j=1}^{m} m_j+b\right)$$

$$p(\gamma|\lambda, Y, W) \propto \gamma^{c-1} e^{-d\gamma} \gamma^{\sum_{i=1}^{n} y_i} e^{-\lambda(\gamma \sum_{i=1}^{n} n_i)}$$

$$p(\gamma|\lambda, Y, W) \propto \gamma^{(\sum_{i=1}^{n} y_i+c)-1} e^{-\gamma(d+\lambda \sum_{i=1}^{n} n_i)}$$

$$\gamma \sim gamma\left(\sum_{i=1}^{n} y_i+c, d+\lambda \sum_{i=1}^{n} n_i\right)$$