SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

50.017 Graphics and Visualization

Quiz 5

Date: 2022-July-21

Time: 6:00pm – 6:30pm

Duration: 30 mins

Student Name:

Student ID:

Instructions:

1. This quiz consists of 3 questions and 4 printed pages.
2. This is an Open Book quiz.
3. You may use calculators. Whether or not you choose to use a calculator, you should clearly and systematically write out all steps in your solutions.
4. Draft paper will be provided on request.

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Q1. Given two orientations represented as quaternions: $\mathbf{q}_0 = (1; (0,0,0))$ and $\mathbf{q}_1 = (\frac{1}{2}; (\frac{\sqrt{3}}{2}, 0,0))$. Interpolate between $\mathbf{q}_0$ and $\mathbf{q}_1$ with $t = 0.5$ using $\text{slerp}(\mathbf{q}_0,\ \mathbf{q}_1,\ t)$. [3 Points]

Solution:

The angle ω between the two quaternions is:

$$\omega = \text{acos}\frac{\mathbf{q}_0 \cdot \mathbf{q}_1}{\|\mathbf{q}_0\|\|\mathbf{q}_1\|} = \text{acos}\left(\frac{1}{2}\right) = 60^o$$

Interpolate between $\mathbf{q}_0$ and $\mathbf{q}_1$ with $t = 0.5$ using:

$$\text{slerp}(\mathbf{q}_0, \mathbf{q}_1, t) = \frac{\mathbf{q}_0 \sin\big((1-t)\omega\big) + \mathbf{q}_1 \sin(t\omega)}{\sin(\omega)}$$

$$= \frac{\mathbf{q}_0 \sin(30^o) + \mathbf{q}_1 \sin(30^o)}{\sin(60^o)}$$

$$= \frac{\mathbf{q}_0 \frac{1}{2} + \mathbf{q}_1 \frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\mathbf{q}_0 + \mathbf{q}_1}{\sqrt{3}}$$

$$= (\frac{\sqrt{3}}{2}; (\frac{1}{2}, 0,0))$$

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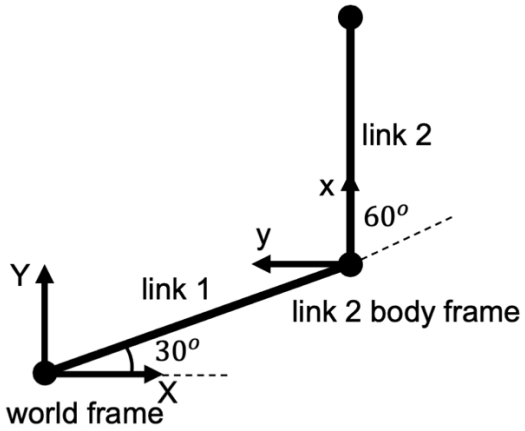
Q2. Given a 2-bar linkage shown below, where each link has length equal to 2. Calculate the transformation matrix of the link 2 body frame with respect to the world frame. [4 Points]



Solution:

The transformation matrix of the link 2 body frame with respect to the link 1 body frame is:

$$\mathbf{M}^{21} = \begin{bmatrix} \cos 60^o & -\sin 60^o & 2 \\ \sin 60^o & \cos 60^o & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 2 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The transformation matrix of the link 1 body frame with respect to the word frame is:

$$\mathbf{M}^{1w} = \begin{bmatrix} \cos 30^o & -\sin 30^o & 0 \\ \sin 30^o & \cos 30^o & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The transformation matrix of the link 2 body frame with respect to the word frame is:

$$\mathbf{M}^{2w} = \mathbf{M}^{1w}\ \mathbf{M}^{21}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 2 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & \sqrt{3} \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

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Q3. Explain three techniques that are needed to animate a character based on a set of key joint configurations of the character's skeleton, where a joint configuration is a vector that concatenates all the joint angles in the character's skeleton. [3 Points]

Solution:

Keyframing.

Interpolate the key joint configurations to obtain a joint configuration at any time t.

Forward kinematics of the skeleton.

The skeleton consists of multiple open chains. Compute forward kinematics for each open chain to animate the skeleton based on the interpolated joint configurations.

Skinning or skeletal subspace deformation.

Animate the skin mesh of the character based on the motion of the skeleton using the skinning technique.