SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

50.017 Graphics and Visualization

Quiz 4

Date: 2022-July-07

Time: 6:00pm – 6:30pm

Duration: 30 mins

Student Name:

Student ID:

Instructions:

1. This quiz consists of 3 questions and 4 printed pages.
2. This is an Open Book quiz.
3. You may use calculators. Whether or not you choose to use a calculator, you should clearly and systematically write out all steps in your solutions.
4. Draft paper will be provided on request.

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Q1. Given a ray with origin $\mathbf{o} = (0,0,0)^{\mathrm{T}}$ and direction $\mathbf{d} = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)^{\mathrm{T}}$. Given a plane with normal $\mathbf{n} = (0,1,0)^{\mathrm{T}}$ and point $\mathbf{c} = (1,1,1)^{\mathrm{T}}$. Compute the intersection point between the ray and the plane (if any). [4 Points]

Solution:

The plane equation is: $\mathbf{n}^{\mathrm{T}}\mathbf{x} - d = 0$, where $d = \mathbf{n}^{\mathrm{T}}\mathbf{c}$. Hence, $d = (0,1,0)(1,1,1)^{\mathrm{T}} = 1$.

Denote the distance between the intersection point and the ray origin as t, we have:

$$\mathbf{n}^{\mathrm{T}}(\mathbf{o} + t\mathbf{d}) - d = 0.$$

This gives us the following equation:

$$(0,1,0)\left((0,0,0)^{\mathrm{T}} + t\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)^{\mathrm{T}}\right) - 1 = 0.$$

Solving this equation gives us

$$t = \sqrt{2}.$$

Hence, the intersection point is: $\mathbf{o} + t\mathbf{d} = (1,1,0)^{\mathrm{T}}$

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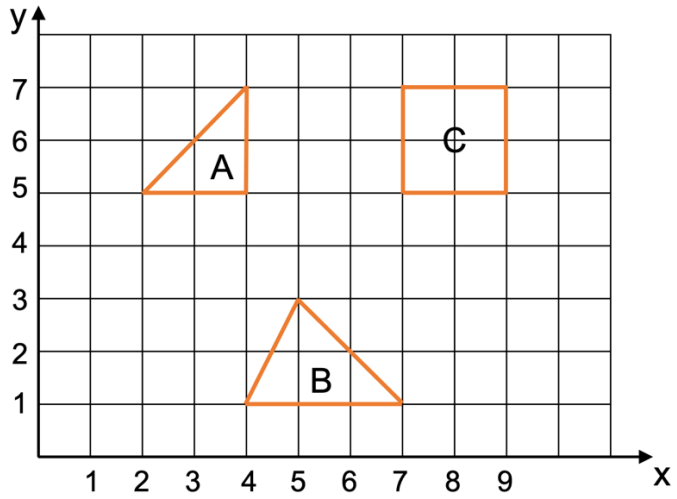
Q2. Given three 2D objects, A, B, and C, as shown in the following figure. Compute the smallest bounding circle (represented as center $\mathbf{c}$ and radius r) of each object, and the smallest axis-aligned bounding rectangle (represented as the minimum and maximum vertices: $\mathbf{V}_{\min}$ and $\mathbf{V}_{\max}$) of each object. [3 Points]



Solution:

Bounding circle of object A: $\mathbf{c} = (3,6)^{\mathrm{T}},\ r = \sqrt{2}$

Bounding rectangle of object A: $\mathbf{V}_{\min} = (2,5)^{\mathrm{T}},\ \mathbf{V}_{\max} = (4,7)^{\mathrm{T}}$

Bounding circle of object B:

The bounding circle should be the circumcircle of the triangle B. Denoted the circle center as (x, y), we have:

$$(x-4)^2 + (y-1)^2 =$$
$$(x-7)^2 + (y-1)^2 =$$
$$(x-5)^2 + (y-3)^2$$

By solving the above equations, we obtain $(x, y) = (5.5, 1.5)$, $r = \sqrt{\frac{5}{2}}$

Bounding rectangle of object B: $\mathbf{V}_{\min} = (4,1)^{\mathrm{T}},\ \mathbf{V}_{\max} = (7,3)^{\mathrm{T}}$

Bounding circle of object C: $\mathbf{c} = (8,6)^{\mathrm{T}},\ r = \sqrt{2}$

Bounding rectangle of object C: $\mathbf{V}_{\min} = (7,5)^{\mathrm{T}},\ \mathbf{V}_{\max} = (9,7)^{\mathrm{T}}$

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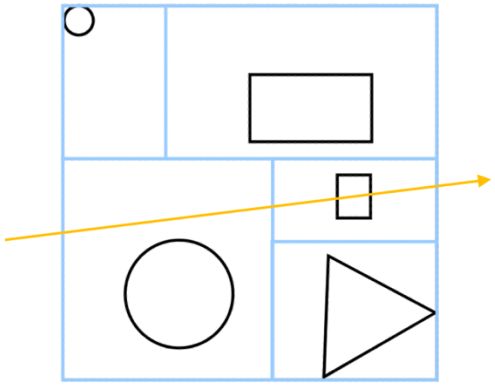
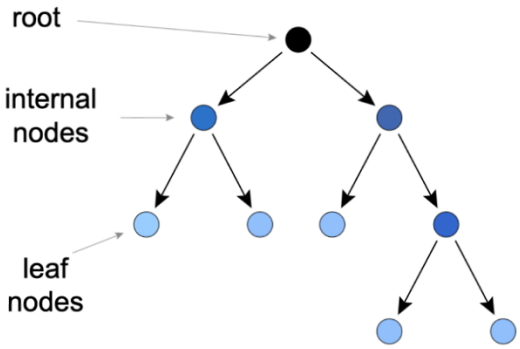
Q3. Given a 3D scene, we build a KD-tree as shown in the following figure. Now, we want to use the KD-tree to speed up the intersection test between a ray (in orange) and objects in the scene. Circle objects in the scene for which a ray-object intersection test is actually performed. Please justify your answer. [3 Points]





Solution:

Objects for which a ray-object intersection test is actually performed are:

1) large circle at the lower left of the scene;

2) small rectangle at the middle right of the scene.

The reason is that the ray intersects with the bounding cell of the object. Hence, we need to perform a ray-object intersection test to see whether the ray really intersects with the object.