SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

50.017 Graphics and Visualization

Quiz 1

Date: 2022-May-26

Time: 6:00pm – 6:30pm

Duration: 30 mins

Student Name:

Student ID:

Instructions:

1. This quiz consists of 4 questions and 5 printed pages.
2. This is an Open Book quiz.
3. You may use calculators. Whether or not you choose to use a calculator, you should clearly and systematically write out all steps in your solutions.
4. Draft paper will be provided on request.

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Q1. Given a 2D closed curve represented by the following implicit function:

$$\{(x, y) \in \mathbb{R}^2 | F(x, y) = \left(y - \sqrt[3]{x^2}\right)^2 + x^2 - 1 = 0\}.$$

Check if each of the two points $\boldsymbol{p}_1 = (1, 1)$ and $\boldsymbol{p}_2 = (0, 0)$ is on, in, or out of the closed curve. Please provide your justification. [2 points]

Solution:

Point $\boldsymbol{p}_1 = (1, 1)$ is on the curve since $F(1, 1) = 0$.

Point $\boldsymbol{p}_2 = (0, 0)$ is in the curve since $F(0, 0) < 0$.

Q2. Given a 2D Bezier curve with the following four control points:

$$\boldsymbol{b}_0 = (0, 0), \quad \boldsymbol{b}_1 = (0, 1), \quad \boldsymbol{b}_2 = (2, 2), \quad \boldsymbol{b}_3 = (3, 0).$$

Compute the point on the Bezier curve at $t = 0.5$ using the De Casteljau's algorithm. [3 points]

Solution:

Below are the steps to compute the point on the Bezier curve at t = 0.5, denoted as $\boldsymbol{b}_0^3$, using the De Casteljau's algorithm.

Initialization:

$\boldsymbol{b}_0^0 = \boldsymbol{b}_0 = (0, 0),$

$\boldsymbol{b}_1^0 = \boldsymbol{b}_1 = (0, 1),$

$\boldsymbol{b}_2^0 = \boldsymbol{b}_2 = (2, 2),$

$\boldsymbol{b}_3^0 = \boldsymbol{b}_3 = (3, 0)$

Recursion 1:

$\boldsymbol{b}_0^1 = (1 - 0.5)\boldsymbol{b}_0^0 + 0.5\boldsymbol{b}_1^0 = (0, 0.5),$

$\boldsymbol{b}_1^1 = (1 - 0.5)\boldsymbol{b}_1^0 + 0.5\boldsymbol{b}_2^0 = (1, 1.5),$

$\boldsymbol{b}_2^1 = (1 - 0.5)\boldsymbol{b}_2^0 + 0.5\boldsymbol{b}_3^0 = (2.5, 1)$

Recursion 2:

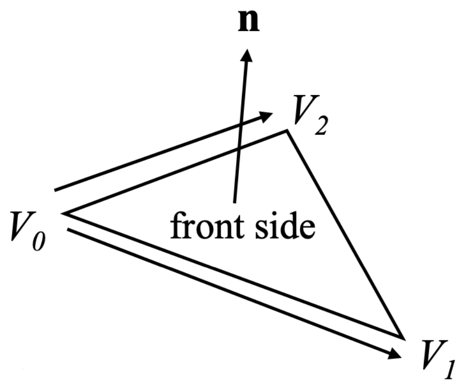
$\boldsymbol{b}_0^2 = (1 - 0.5)\boldsymbol{b}_0^1 + 0.5\boldsymbol{b}_1^1 = (0.5, 1),$

$\boldsymbol{b}_1^2 = (1 - 0.5)\boldsymbol{b}_1^1 + 0.5\boldsymbol{b}_2^1 = (1.75, 1.25)$

Recursion 3:

$\boldsymbol{b}_0^3 = (1 - 0.5)\boldsymbol{b}_0^2 + 0.5\boldsymbol{b}_1^2 = (1.125, 1.125),$

Q3. Given a 3D triangle with vertices $\mathbf{V}_0, \mathbf{V}_1, \text{and } \mathbf{V}_2$ that are in anti-clockwise order using right hand rule and have coordinates $\mathbf{V}_0 = (1, 0, 1)$, $\mathbf{V}_1 = (1, 2, 3)$, and $\mathbf{V}_2 = (2, 2, 2)$. Compute the normal (front side) $\mathbf{n}$ and area A of the triangle. [2 Points]



Solution:

Edge vector $\mathbf{E}_0 = \mathbf{V}_1 - \mathbf{V}_0 = (1,2,3,) - (1,0,1) = (0, 2, 2)$

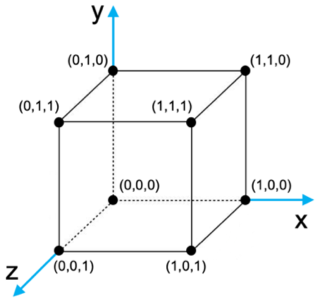
Edge vector $\mathbf{E}_1 = \mathbf{V}_2 - \mathbf{V}_0 = (2,2,2) - (1,0,1) = (1, 2,1)$

Triangle normal: $\mathbf{n} = \mathbf{E}_0 \times \mathbf{E}_1 = (0, 2, 2) \times (1, 2,1) = (-2, 2, -2)$

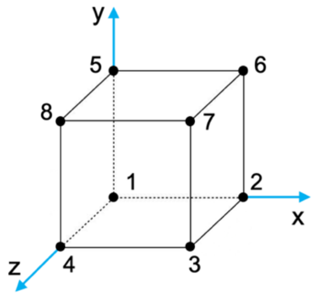
Normalize normal $\mathbf{n} = \frac{\mathbf{n}}{|\mathbf{n}|} = (\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}})$ (note: normal is preferred to be with unit length)

Triangle area: $\mathrm{A} = \frac{1}{2}|\mathbf{E}_0 \times \mathbf{E}_1|=\sqrt{3}$

Q4. Consider the following unit cube as a polygonal mesh with 8 vertices and 6 faces. Represent this mesh using the Indexed Face Set data structure by writing down the *Vertices* table and the *Polygons* table (hint: Polygons table can be easily extended from the Triangles table introduced in Lecture 4). [3 Points]



Solution:



This question has many solutions. Below is one example solution (corresponding to vertex numbering in the above figure). The important thing is that the each polygon in the Polygons table should be represented by corresponding vertex indices in the Vertices table.

Vertices table, where the left row is vertex ID and the right row is vertex coordinates. The table is correct even without the left row.

1	0 0 0
2	1 0 0
3	1 0 1
4	0 0 1
5	0 1 0
6	1 1 0
7	1 1 1
8	0 1 1

Polygons table, where the left row is face ID and the right row is vertex indices for each face (polygon). The table is correct even without the left row. The vertex indices of each polygon in organized in *counter-clockwise* such that the face normal always points outward for rendering purpose; compare the order of vertex indices in face 1 and 2.

1	1 2 3 4
2	5 8 7 6
3	2 6 7 3
4	1 4 8 5
5	1 5 6 2
6	3 7 8 4