SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

50.017 Graphics and Visualization

Quiz 3

Date: 2022-June-23

Time: 6:00pm – 6:30pm

Duration: 30 mins

Student Name:

Student ID:

Instructions:

1. This quiz consists of 4 questions and 5 printed pages.
2. This is an Open Book quiz.
3. You may use calculators. Whether or not you choose to use a calculator, you should clearly and systematically write out all steps in your solutions.
4. Draft paper will be provided on request.

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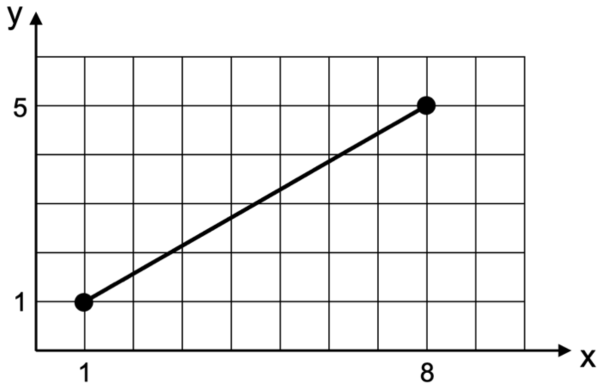
Q1. Discretize the following line from (1,1) to (8, 5) to the pixel grid using the Bresenham Algorithm. Write down your steps to derive position of each pixel on the line and draw these pixel positions in the grid. Note that the intersection points in the grid (rather than the grid cells) represent pixel positions. [3 Points]



Solution:

Here is the procedure to find pixels on the line by running the Bresenham Algorithm

$\Delta x = 7$

$\Delta y = 4$

$d = 2 * \Delta y - \Delta x = 1$

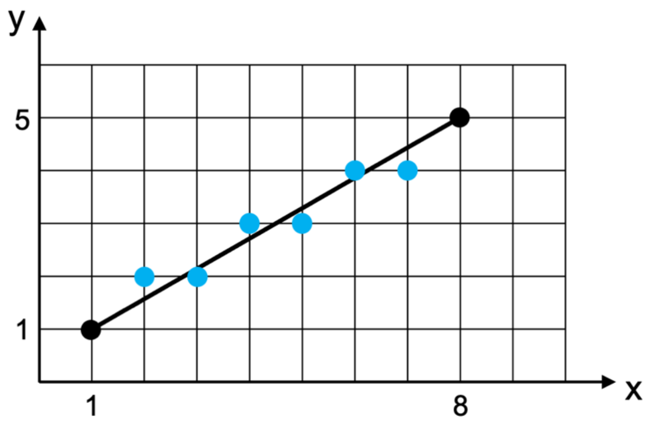
$\Delta E = 2 * \Delta y = 8$

$\Delta NE = 2 * (\Delta y - \Delta x) = -6$

d = 1 > 0, choose north east **(2, 2)**, d = d + ΔNE = -5
d = -5 <=0, choose east **(3, 2)**, d = d + ΔE = 3
d = 3 > 0, choose north east **(4, 3)**, d = d + ΔNE = -3
d = -3 <=0, choose east **(5, 3)**, d = d + ΔE = 5
d = 5 > 0, choose north east **(6, 4)**, d = d + ΔNE = -1
d = -1 <=0, choose east **(7, 4)**, d = d + ΔE = 7
d = 7 > 0, choose north east **(8, 5)**, d = d + ΔNE = 1

The pixels on the line (excluding endpoints) are:

(2, 2), (3, 2), (4, 3), (5, 3), (6, 4), (7, 4), (8, 5)



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Q2. Given a virtual camera with perspective projection and viewing volume [-2, 2] x [-2, 2] x [-1, -11] that defines the left/right, bottom/top, and near/far boundaries. Given a viewport with left bottom pixel position (20, 20) and dimension (800, 800). Project a 3D point with position $(1, 1, -5)^{\mathrm{T}}$ in the camera's viewing volume to the given viewport. Compute the 2D window pixel coordinate of the projected point. [3 Points]

Solution:

Perspective projection matrix is:

$$M_{\mathrm{P}} = \begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & -6/5 & -11/5 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Viewport transformation matrix is:

$$M_{\mathrm{v}} = \begin{bmatrix} 400 & 0 & 0 & 420 \\ 0 & 400 & 0 & 420 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The normalized device coordinate of the 3D point is:

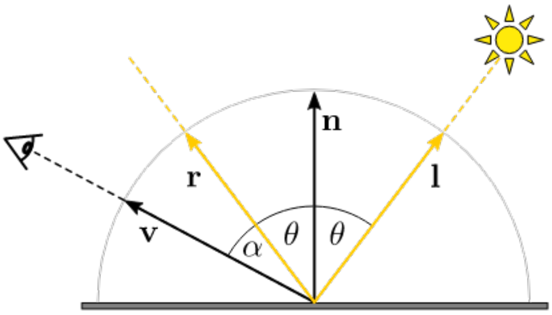
$$P_{\mathrm{D}} = M_{\mathrm{p}} \cdot \begin{bmatrix} 1 \\ 1 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ 19/5 \\ 5 \end{bmatrix} = \begin{bmatrix} 1/10 \\ 1/10 \\ 19/25 \\ 1 \end{bmatrix}$$

The 2D window pixel coordinate of the 3D point is:

$$P_{\mathrm{W}} = M_{\mathrm{v}} \cdot \begin{bmatrix} 1/10 \\ 1/10 \\ 19/25 \\ 1 \end{bmatrix} = \begin{bmatrix} 460 \\ 460 \\ 22/25 \\ 1 \end{bmatrix}$$

Hence, the pixel coordinate should be (460, 460).

Q3. Given a light source with intensity $I_l = 10$ and direction $\mathbf{l} = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)^{\mathrm{T}}$, and a viewing direction $\mathbf{v} = (-\frac{\sqrt{3}}{2}, \frac{1}{2}, 0)^{\mathrm{T}}$. Given a point with surface normal $\mathbf{n} = (0,1,0)^{\mathrm{T}}$, material's specular reflection coefficient $m_s = 0.2$, and material's shininess coefficient $s = 1$. Compute the intensity I of specular reflection light observed by the viewpoint. [2 Points]



Solution:

The dot product between $\mathbf{n}$ and $\mathbf{l}$ is:

$$\mathbf{n} \cdot \mathbf{l} = (0,1,0)^{\mathrm{T}} \cdot \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)^{\mathrm{T}} = \frac{\sqrt{2}}{2}$$

The reflected ray is:

$$\mathbf{r} = 2\mathbf{n}(\mathbf{n} \cdot \mathbf{l}) - \mathbf{l} = 2(0,1,0)^{\mathrm{T}} \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)^{\mathrm{T}} = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)^{\mathrm{T}}$$

The dot product between $\mathbf{r}$ and $\mathbf{v}$ is:

$$\mathbf{r} \cdot \mathbf{v} = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)^{\mathrm{T}} \cdot \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right)^{\mathrm{T}} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

The intensity I of specular reflection light is:

$$I = I_l m_s (\mathbf{r} \cdot \mathbf{v})^s = 10 \times 0.2 \times \frac{\sqrt{6}+\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{2}$$

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Q4. Given a triangle with vertices $\mathbf{A} = (-2,0,1)^{\mathrm{T}}$, $\mathbf{B} = (2,0,1)^{\mathrm{T}}$, and $\mathbf{C} = (0,3,1)^{\mathrm{T}}$. Compute the Barycentric coordinates of a point $\mathbf{P} = (1,1,1)^{\mathrm{T}}$ that is inside the triangle ΔABC. [2 Points]

Solution:

The signed area of the following four triangles are:

$$\mathrm{area}(\mathbf{A},\mathbf{B},\mathbf{C}) = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 2 & 0 & 1 \\ 0 & 3 & 1 \end{vmatrix} = 6$$

$$\mathrm{area}(\mathbf{P},\mathbf{B},\mathbf{C}) = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 3 & 1 \end{vmatrix} = 1/2$$

$$\mathrm{area}(\mathbf{A},\mathbf{P},\mathbf{C}) = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{vmatrix} = 7/2$$

$$\mathrm{area}(\mathbf{A},\mathbf{B},\mathbf{P}) = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2$$

Denote the Barycentric Coordinates of $\mathbf{P}$ as (α,β,γ). Then,

$$\alpha = \frac{\mathrm{area}(\mathbf{P},\mathbf{B},\mathbf{C})}{\mathrm{area}(\mathbf{A},\mathbf{B},\mathbf{C})} = \frac{1}{12}$$

$$\beta = \frac{\mathrm{area}(\mathbf{A},\mathbf{P},\mathbf{C})}{\mathrm{area}(\mathbf{A},\mathbf{B},\mathbf{C})} = \frac{7}{12}$$

$$\gamma = \frac{\mathrm{area}(\mathbf{A},\mathbf{B},\mathbf{P})}{\mathrm{area}(\mathbf{A},\mathbf{B},\mathbf{C})} = \frac{4}{12}$$

Hence, the Barycentric Coordinates of $\mathbf{P}$ is $(\frac{1}{12},\frac{7}{12},\frac{4}{12})$.