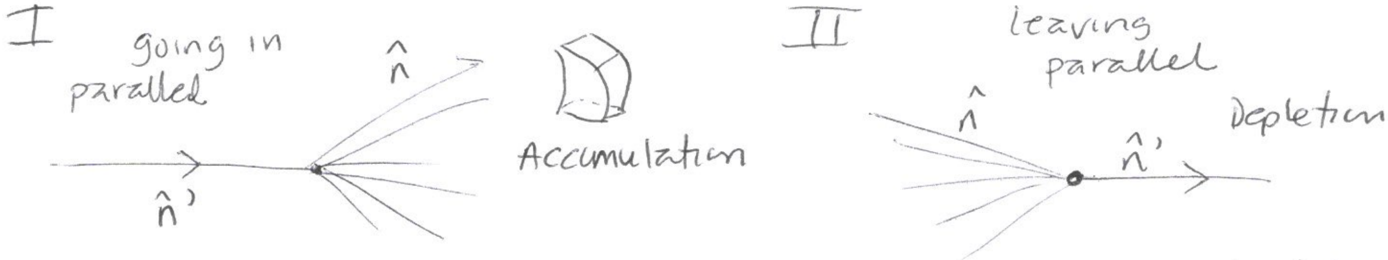
Scattering

Very similar to absorpition, but with two situations



So need to integrate over infinitesimal area $d^2\hat{n}'$

$$\frac{\partial}{\partial t} \ell(\hat{n}, \vec{x}, \nu, t) = c\rho(\vec{x}, t) \int d^2\hat{n} \Big[-K_s(\hat{n} \rightarrow \hat{n}'; \vec{x}, \nu, t)\ell(\hat{n}, \vec{x}, \nu, t)$$

$$+ K_s(\hat{n}' \rightarrow \hat{n}; \vec{x}, \nu, t)\ell(\hat{n}', \vec{x}, \nu, t) \Big]$$

Here, assume $\frac{1}{K_s \rho} \ll \ell$ different enough

Emission

How does thermal emission work? Nuclear?

Emission can be emitted at any time and in any direction; so

$$\frac{\partial}{\partial t} \ell(\hat{n}, \vec{x}, \nu, t) = \frac{j(\vec{x}, \nu, t)\rho(\vec{x}, t)}{4\pi}$$

solid angle.

Now assume l does not change with time, which means body is in equilibrium, and add all the contributions

W1.2.6

$$\frac{\partial l(\hat{n},\vec{x},\nu,t)}{\partial t} = 0 = -c\hat{n}\cdot\nabla l(\hat{n},\vec{x},\nu)$$
$$- cK_{abs}(\vec{x},\nu)\rho(\vec{x})l(\hat{n},\vec{x},\nu)$$
$$+ c\rho(\vec{x})\int d^2\hat{n}'\Big[-K_s(\hat{n}\rightarrow\hat{n}';\vec{x},\nu)l(\hat{n},\vec{x},\nu)$$
$$+K_s(\hat{n}'\rightarrow\hat{n};\vec{x},\nu)l(\hat{n}',\vec{x},\nu)\Big]$$
$$+ j(\vec{x},\nu)\rho(\vec{x})/4\pi$$

Now integrate W1.2.6 over ~~all~~ the directions of $\hat{n}$, with definitions

$$\mathcal{E}_{rad}(\vec{x},\nu) \equiv \int d^2\hat{n}\, l(\hat{n},\vec{x},\nu)$$
$$\Phi_i(\vec{x},\nu) \equiv c\int d^2\hat{n}\,\hat{n}_i\, l(\hat{n},\vec{x},\nu)$$

The scattering terms cancel each other and

$$\nabla\cdot\Phi(\vec{x},\nu) = -cK_{abs}(\vec{x},\nu)\rho(\vec{x})\mathcal{E}_{rad}(\vec{x},\nu) + j(\vec{x},\nu)\rho(\vec{x})$$

Notice that this quantities depend only on ν and $\vec{x}$. The differences at different $\vec{x}$ come from the density $\rho(\vec{x})$, the temperature $T(\vec{x})$, and the chemical composition at $\vec{x}$.

☆ What happens when $cK_{abs}(\vec{x},\nu)\mathcal{E}_{rad}(\vec{x},\nu) = j(\vec{x},\nu)$?

Black body.

In this case all the emission is absorbed, nothing escapes and this becomes a black body cavity, and $j = c\kappa_{abs}\,\mathcal{E}_{rad}$. In a real star, radiation <u>is</u> emitted that is not re-trapped, so

$$j = c\kappa(\vec{x},\nu)\,\mathcal{E}_{rad}(\vec{x},\nu) + \epsilon(\vec{x},\nu)$$

$\epsilon(\vec{x},\nu)$ is the rate per unit mass and frequency interval of energy generated by ☆ Nuclear reactions

so

~~Divergence of flux~~

$$\nabla\cdot\Phi(\vec{x},\nu) = \epsilon(\vec{x},\nu)\,\rho(\vec{x})$$

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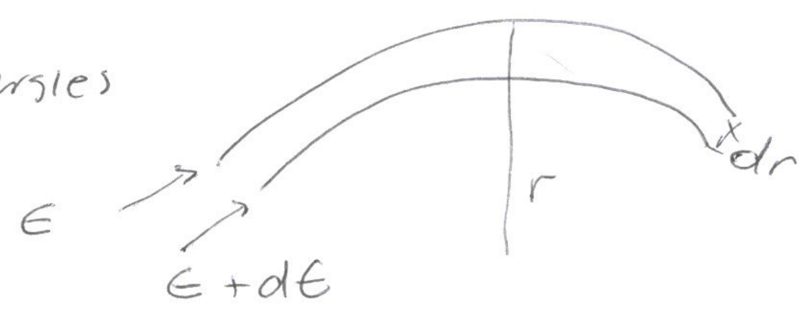
if there are two surfaces separated by dr with different energies



Net flow of radiation energy through shell at radius r per unit time is the <u>luminosity</u> $L(r)$. This should be the excess energy compared to a shell of larger radius divided by the time it takes the excess energy to flow the photons can be scattered, so they do a random walk, the expectation value of the displacement is $D = \sqrt{N}\,\underbrace{\frac{1}{K\rho}}_{\text{step size}}$

Number of steps required to ~~esc~~ move dr

$$(dr)^2 (K\rho)^2 = N$$

each step takes $dt = \dfrac{1}{K\rho \cdot c}$, so the total time is $t = \dfrac{(dr)^2 (K\rho)^{\cancel{2}}}{\cancel{K\rho} \cdot c}$

The excess energy density is $4\pi r^2\, dr\, dE$

$$L(r) \approx -\frac{4\pi r^2 \cancel{dr}\, dE}{(dr)^{\cancel{2}} K\rho / c} = -\frac{4\pi r^2 c}{K\rho} \cdot \frac{dE}{dr}$$

A more rigurous derivation includes an extra $\frac{1}{3}$ from the integration of a $\cos^2\theta$ solid angle.

$$\frac{L(r)}{4\pi r^2} = -\frac{c}{3K\rho}\,\frac{dE}{dr}$$

← energy densty gradient

diffusion coefficient

This is a diffusion equation

energy flux

☆ what happens if opacity is low? opacity high?

The energy density ϵ is very close to Blackbody radiation, so $\epsilon = aT^4$

where $a = \frac{8\pi^5 k_B^4}{15 h^3 c^3} = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$ Stefan-Boltzmann constant

$$\frac{d\epsilon}{dr} = \frac{d\epsilon}{dT} \cdot \frac{dT}{dr}$$

$$\frac{d\epsilon}{dT} = 4aT^3, \text{ so } \frac{d\epsilon}{dr} = 4aT^3 \frac{dT(r)}{dr}$$

$$\frac{L(r)}{4\pi r^2} = -\frac{c}{3\kappa\rho} 4aT^3 \frac{dT(r)}{dr}$$

$$\Rightarrow \boxed{\frac{dT(r)}{dr} = \frac{3\rho(r)\,\kappa(r)\,L(r)}{4ac\,T^3(r)\,4\pi r^2}}$$

W1.2.30

Equation of radiative energy transport

Finally, $dL = \epsilon\, dm = \epsilon \rho 4\pi r^2 dr$

$$\boxed{\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\,\epsilon(r)}$$

W1.2.28

conservation of energy

Along with $\frac{dP(r)}{dr}$ and $\frac{dM(r)}{dr}$, these differential equations govern stellar structure and the boundary conditions are

$$M(0) = 0, \quad L(0) = 0, \quad P(R) = 0, \quad M(r) = M$$

Hertzprung-Rusell (HR) relation.

fixed chemical composition

$L(r)$ - radiant energy per second

$M(r)$ mass

r - radius

$\rho(r)$ - density

$T(r)$ - temperature

P - pressure — fixed

K - opacity — fixed

ϵ - nuclear energy production per mass — fixed

Rewrite

$$\frac{dr(M)}{dM} = \frac{1}{4\pi r^2(M)\,\rho(M)}$$

$$\frac{dp(M)}{dM} = -\frac{GM}{4\pi r^4(M)}$$

$$\frac{dL(M)}{dM} = \epsilon(M)$$

$$\frac{dT(M)}{dM} = -\frac{3K(M)\,L(M)}{4caT^3(M)\,(4\pi r^2(M))^2}$$

$r(M) = L(M) = 0$ for $M = 0$

$\rho(M) = T(M) = 0$ at $M = M$

written this way, the mass is the only parameter that matters!