Star formation

At the beginning of the semester we saw that an isolated spherical mass obeys the virial theorem

$$\Pi + \Omega = 0 \qquad 2\Upsilon + \Omega = 0$$

Π is a measure of the thermal energy $\Pi \equiv 3\int_0^R 4\pi r^2 p(r)\,dr$

Ω is the gravitational self-energy $\Omega = -G\int_0^R 4\pi \rho(r) M(r) \, dr$

with $M(r) \equiv \int_0^R 4\pi r'^2 \rho(r')\,dr'$

The virial theorem is the boundary between non-equilibrium configurations

if $\Pi > -\Omega$, pressure makes the system expand

if $\Pi < -\Omega$, gravitational condensation occurs.

The interstellar medium is not homogeneous in its mass distribution, but consider some region with average density $\bar{\rho}$, $M = \int_0^R 4\pi r^2 \bar{\rho}\, dr = \frac{4\pi R^3 \bar{\rho}}{3}$, $p \sim c_s^2 \bar{\rho}$ HW6

$$-\Omega = +G \frac{4\pi}{3} R^3 \bar{\rho} \int_0^R \bar{\rho}\, 4\pi r\, dr = G \frac{(4\pi)^2}{6} \bar{\rho}^2 R^5$$

$$\Pi = 3 \cdot 4\pi c_s^2 \bar{\rho} \int_0^R r^2\,dr = \cancel{3} \cdot \frac{4\pi}{\cancel{3}} c_s^2 \bar{\rho} R^3$$

This is not an exact calculation, but the order of magnitude is correct.

$$\Pi \approx 4\pi c_s^2 \bar{\rho} R^3 \qquad -\Omega \approx (4\pi)^2 G \bar{\rho}^2 R^5 \qquad \underline{\underline{W3.4.5}}$$

There will be gravitational condensation if

$$\Pi < -\Omega$$

$$\cancel{4\pi} c_s^2 \cancel{\bar{\rho}} \cancel{R^3} < (4\pi)^{\cancel{2}} G \bar{\rho}^{\cancel{2}} R^{\cancel{5}\,2}$$

$$c_s^2 < 4\pi G \bar{\rho} R^2$$

$$\frac{c_s^2}{4\pi G\bar{\rho}} < R^2 = R_J^2 \quad \text{(Jeans lenght)} \qquad \underline{\underline{W3.4.7}}$$

gravitational condensation if $R < R_J$

The adiavatic speed of sound is $c_s^2 = \dfrac{\Gamma k_B T}{m}$

where Γ is the adiabatic index, m is the particle mass.

For a monoatoomic (ideal) gas, $\Gamma = 5/3$

For a diatomic (ideal) gas, $\Gamma = 7/5$

Γ is the ratio between ~~Cv an~~ the heat capacity at constant volume and pressure C_V/C_P

So the speed of sound increases with temp, decreases with particle mass and increasing degrees of freedom

Gas		Speed of sound	
~~Helium~~			
Hydrogen	H_2	1270 m/s	$A=2$
Helium	He	1007 m/s	$A=4$
Nitrogen	N_2	349 m/s	$A=14$
Oxygen	O_2	326 m/s	$A=16$
Xenon	Xe	178 m/s	$A=131.3$

(particle twice as massive, but monoatomic) → Helium

voice sounds cool.

$$R_J^2 \equiv \frac{\Gamma k_B T}{4\pi G A m_p \bar{\rho}}$$

where m_p is the proton mass and A is the number of nucleons

This gives an order of magnitude for when gravitational condensation occurs, but it is a bit problematic because both T and $\bar{\rho}$ will increase upon contraction.

The <u>Jeans mass</u> is defined as the mass within a sphere of diameter $2\pi R_J$, so $r = \pi R_J$

$$M(\pi R_J) = \int_0^{\pi R_J} 4\pi r^2 \bar{\rho}\, dr = \frac{4\pi}{3}\bar{\rho}(\pi R_J)^3 = \frac{4\pi^4}{3}\bar{\rho} R_J^3$$

$$M_J = \frac{4\pi^4}{3}\bar{\rho} \frac{C_s^3}{(4\pi)^{3/2} G^{3/2} \bar{\rho}^{3/2}} = \frac{4^{1/2}}{3}\frac{\pi^{5/2} C_s^3}{G^{3/2}\bar{\rho}^{1/2}}$$

$$M_J = \frac{\pi^{5/2} C_s^3 \cancel{t^{1/3}}}{6\, G^{3/2} \bar{\rho}^{1/2}}$$

$$\bar{\rho} = \frac{m}{V} = \frac{Am_p}{V} \Rightarrow V = \frac{Am_p}{\bar{\rho}} \Rightarrow \frac{1}{V} = n = \frac{\bar{\rho}}{Am_p}$$

$$M_J = \frac{\pi^{5/2} C_s^3}{6\, G^{3/2} n^{1/2} A^{1/2} m_p^{1/2}}$$

W3.4.9

$$M_J = \underbrace{\frac{\pi^{5/2}}{6G^{3/2} m_p^{1/2}}}_{\text{constant}} \cdot \underbrace{\frac{C_s^3}{(nA)^{1/2}}}_{\text{physics}} = \frac{\pi^{5/2} \cdot C_s^3/(nA)^{1/2}}{6\cdot(6.67\times10^{-11}\, m^3/\cancel{Kg}\, s^2)^{3/2}(1.67\times10^{-27}\, \cancel{Kg})^{1/2}}$$

$$M_J = \frac{\pi^{5/2}}{1.33\times10^{-28}\, \frac{m^{9/2}}{Kg\, s^3}} \frac{C_s^3}{(nA)^{1/2}} = 1.31\times10^{29}\, \frac{Kg\, s^3}{m^{9/2}} \frac{C_s^3}{(nA)^{1/2}}$$

$$= 6.55\times10^{-2}\, M_\odot\, \frac{s^3}{m^{9/2}}$$

$$\frac{\frac{m^3}{s^3}}{\frac{1}{m^{3/2}}} = \frac{m^{9/2}}{s^3}$$

~~with $C_s^3 = \left(\frac{\Gamma k_B T}{m_p A}\right)^{3/2}$~~

~~$$M_J = \frac{\pi^{5/2}}{6G^{3/2} m_p^{1/2}} \frac{\Gamma^{3/2} k_B^{3/2} T^{3/2}}{n^{1/2} A^{1/2} m_p^{3/2} A^{3/2}}$$~~

Some typical values:

$C_s = 600\, m/s$

$n = 10^{10}\, /m^3$

$A = 1$

$$\frac{C_s^3}{(nA)^{1/2}} = \frac{(600 m/s)^3}{(10^{10}/m^3)^{1/2}} = \frac{2160\, m^{9/2}}{s^3}$$

$$M_J = 6.55\times10^{-2} M_\odot \cdot 2160 = \underline{141\, M_\odot}$$

So a cloud with mass $141\, M_\odot$ mostly H would collapse with $$C_s^3 = \left(\frac{\Gamma k_B T}{m_p A}\right)^{3/2},$$

$$M_J = \frac{\pi^{5/2}}{6 G^{3/2} m_p^{1/2}} \frac{(\Gamma k_B T)^{3/2}}{(nA)^{1/2} (A m_p)^{3/2}} = \frac{\pi^{5/2} (\Gamma k_B T)^{3/2}}{6 G^{3/2} A^2 m_p^2 n^{1/2}}$$

$$M_J = \frac{\pi^{5/2} \left(1.38\times10^{-23}\ J/K\right)^{3/2}}{6\cdot\left(6.67\times10^{-11}\ m^3/kg\,s^2\right)^{3/2} \left(1.67\times10^{-27}\ kg\right)^2} \quad \frac{(\Gamma T)^{3/2}}{A^2 n^{1/2}}$$

$$M_J = \frac{\cancel{2.44\times10^{-22}}\ 8.97\times10^{-34}\ \frac{kg^{3/2}\, m^3}{s^3\, K^{3/2}}}{\cancel{8.46\times10^{-42}}\ 9.11\times10^{-69}\ \frac{m^{9/2}}{kg^{1/2}\, s^3}\ \cancel{kg}\cdot kg} \quad \frac{(\Gamma T)^{3/2}}{A^2 n^{1/2}}$$

$$M_J = \frac{\cancel{1.64\times10^{8}}\ kg^{\cancel{2}}\ \cancel{m^3}\ \cancel{s^3}}{9.8\times10^{34}\ m^{3/2}\ \cancel{s^3}\ K^{3/2}} \quad \frac{(\Gamma T)^{3/2}}{A^2 n^{1/2}} = 4.9\times10^4 \frac{M_\odot}{m^{3/2} K^{3/2}} \frac{(\Gamma T)^{3/2}}{A^2 n^{1/2}}$$

Mass needed to collapse decreases with decreasing temperature, decreases with increasing density and molecular weight, decreases if diatomic molecules present.

For 25% He and 75% H at 15000K

(conditions similar to those right after the Big Bang)

$\Gamma = 5/3$, $A = 4(0.25) + 1(0.75) = 1.75$, $n = 10^{10}/m^3$

$$M_J = \frac{4.9\times10^4 M_\odot}{m^{3/2}K^{3/2}} \cdot \frac{(5/3 \cdot 15{,}000\,K)^{3/2}}{(1.25)^2 (10^{10}/m^3)^{1/2}}$$

fix

$$M_J = 4.9\times10^4 \frac{M_\odot}{m^{3/2}K^{3/2}} \cdot \frac{3.95\times10^6\,K^{3/2}}{1.56\times10^5\,/m^{3/2}}$$

$$M_J = 4.9\times10^4 \cdot 25.3\,M_\odot = 1.24\times10^6\,M_\odot$$

$\approx 1\times10^6\,M_\odot$

Nowadays, $A \simeq 1.25$ but $\Gamma \approx 7/5$, $T \simeq 20K$,

$n = 10^{12}/m^3$, so

$$M_J = 4.9\times10^4 \frac{M_\odot}{K^{3/2}m^{3/2}} \frac{(7/5 \cdot 20K)^{3/2}}{(1.25)^2 (10^{12})^{1/2}} = 4.9\times10^4 \frac{M_\odot}{K^{3/2}m^{3/2}} \frac{148\,K^{3/2}}{1.56\times10^6/m^{3/2}}$$

$$M_J = 4.9\times10^4 \cdot 9.5\times10^{-5}\,M_\odot = 4.7\,M_\odot$$

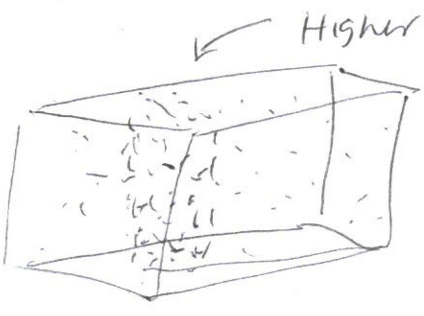
So stars can form now much more readely than right after the Big Bang, and stars are much smaller than they used to be, e.g., in Pop III because they were massive, Pop III ~~died~~ died very quickly, but the metals they manufactured helped create smaller stars in Pop II.

The Jeans mass gives the necessary condition for gravitational condensation, but not sufficient.

We can observe clouds with masses ~ $1000 M_\odot$ that are not collapsing and probably have been around for a very long time. It is believed that turbulence and magnetic fields contribute an aditional "pressure" in addition to thermal pressure.

HOW DOES A CLOUD COLLAPSE?

Consider a homogeneous cloud but disturbed by a shock wave and assume that this change in density allowed this region to reach its M_J and starts condensating.



$M_J \propto \frac{T^{3/2}}{n^{1/2}}$, when the density increases, the temparature increases. This might make $M_J > M$, so the condensation stops. As the cloud cools down, the now distinct region again has $M_J < M$ and gravitational condensation is reignited. The contraction will continue slowly. contracting - heating up - cooling down - contracting, it will contract at the rate it can radiate energy away.

At some point, the temperature will be high enough that molecules will start dissociating, e.g. $H_2 \rightarrow 2H$, using 4.5eV per molecule. At this point the gravitational potential energy is not being converted into kinetic energy that must be radiated away. Instead, it is going into breaking Coulomb interactions (molecular bonds), so $T \sim$ constant and $n \sim$ increases, so M_J decreases and the initial cloud might fracture and create several stars.

At some other point, the temperature will be high enough that hydrogen atoms will start to ionize, e.g. $H \rightarrow e^- + p$ using 13.6 eV per atom. At this point the gravitational potential energy goes into breaking further Coulomb interactions, so $T \sim$ constant and $n \sim$ increases.

Eventually, the temperature and density will be high enough that nuclear reactions will start at the core. A star is born!

This last step, the temperature increase happens quickly as there is nothing else dissociate or ionize.

Consider a mass δm at the "surface" of the Jeans sphere.

$$dU = -\frac{GM(R_0)\,\delta m}{R_0^2} \qquad R_0^{\text{mass inside}}$$

As it "free-falls", the velocity of δm will be

$$\frac{1}{2}\delta m v^2 \doteq \frac{1}{2}\delta m\left(\frac{dr}{dt}\right)^2 = \frac{GM(R_0)\delta m}{r} - \frac{GM(R_0)\,\delta m}{R_0}$$

mass inside R_0 also free-falling

$$\left(\frac{dr}{dt}\right) = \left\{2GM(R_0)\left[\frac{1}{r} - \frac{1}{R_0}\right]\right\}^{1/2}$$

$$\int_0^{R_0}\left\{2GM(R_0)\left[\frac{1}{r} - \frac{1}{R_0}\right]\right\}^{-1/2} dr = \int_0^{\tau_{eff}} dt$$

$$\Rightarrow \tau_{eff} = \left(\frac{3\pi}{32G\bar{\rho}}\right)^{1/2}$$

$$\text{Since } M_J = \frac{\pi^{5/2} C_s^3}{6G^{3/2}\bar{\rho}^{1/2}} \Rightarrow \bar{\rho} = \left(\frac{\pi^{5/2} C_s^3}{6G^{3/2}M_J}\right)^2$$

For $M_J = 1\,M_\odot$ and $C_s = 600\,m/s$

$$\bar{\rho} = \left[\frac{\pi^{5/2}\cdot(300\,m/s)^3}{6\,(6.67\times10^{-11}\,m^3/kg\,s^2)^{3/2}(2\times10^{30}\,kg)}\right]^2 = \left[\frac{4.77\times10^{8}\ m^3/s^3}{6.54\times10^{15}\ \frac{m^{3/2}}{kg^{1/2}s^3}\,kg}\right]^2 = \cancel{3.3\times10^{-13}}\ 5.2\times10^{-15}\ \frac{kg}{m^3}$$

$$\tau_{eff} = \left(\frac{3\pi}{32\,(6.67\times10^{-11}\ \mathrm{m^3/kg\,s^2})(\cancel{3.}5.\cancel{2}2\times10^{-1\cancel{5}3}\ \mathrm{kg/m^3})} \right)^{1/2}$$

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$$\tau_{eff} = \left(\frac{3\pi}{\cancel{7\times10^{-22}}\ 1.11\times10^{-23}/s^2} \right)^{1/2} = (\cancel{1.35}\ 8.48\times10^{2\cancel{2}3}\ s^2)^{1/2} = 9.15\times10^{11}\ s$$

$$1.15\times10^{11}\ \cancel{s} \left(\frac{1\ \cancel{day}}{86\,400\ \cancel{s}} \right) \left(\frac{1\ year}{365.25\ \cancel{days}} \right) = \cancel{3{,}600}\ years$$

29,000 years.