

THE INTERSTELLAR MEDIUM (ISM) ^{Oct-22-20} (114)

• The space between stars is filled with matter. The density is much lower than the best vacuum achievable on earth, but the distances between stars is huge, so there is a lot of mass. This mass is critical for the structure, dynamics, and evolution of galaxies.

• Most of the mass is in the form of hydrogen and helium:

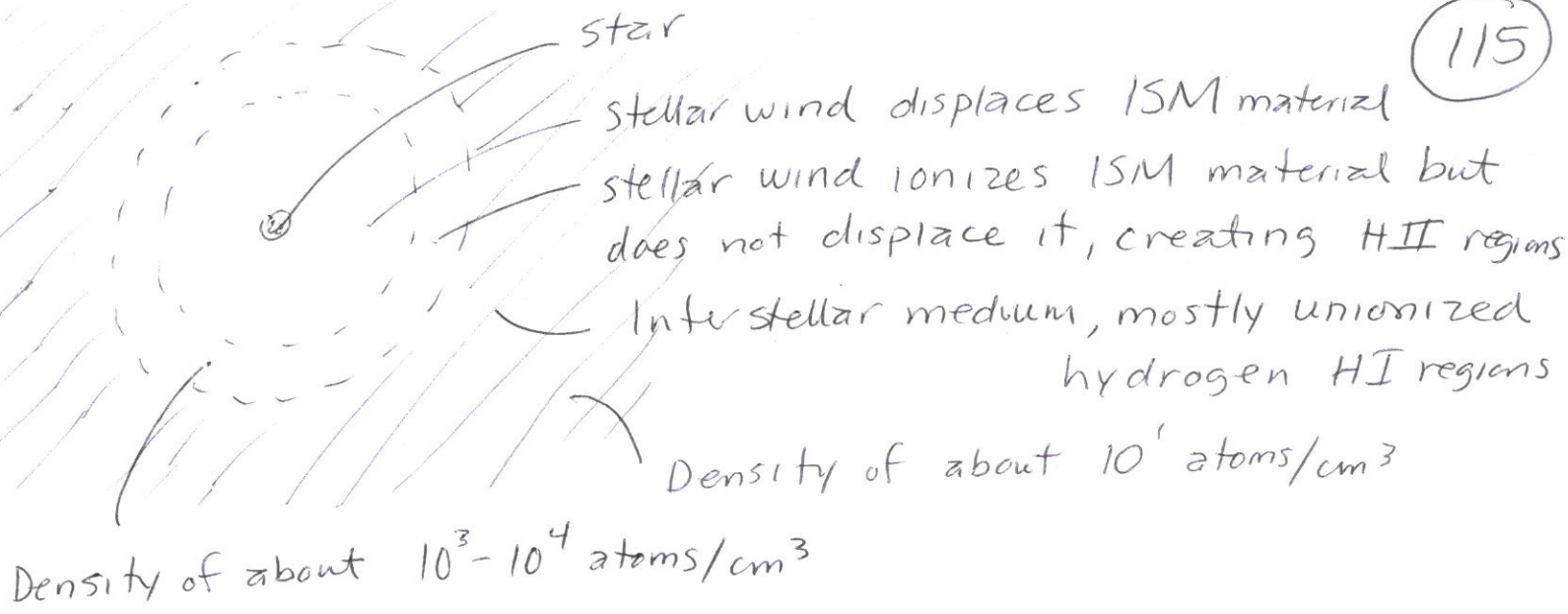
neutral hydrogen (HI)	one proton and one electron
ionized hydrogen (HII)	essentially just p^+ a proton
molecular hydrogen (H_2)	two protons, two electrons, two nuclei
neutral helium ^3He , ^4He	two protons in single nucleus, two e^-
ionized helium	no e^-

By mass, about 70% hydrogen, about 30% helium
About 2-3% everything else, enriched by the "cyclical" process of star formation (birth and death).

Ionized corona (or halo) extends far beyond galactic plane, more or less spherically, with temperatures 10^6 K
 $10^{-2} - 10^{-4}$ atoms per cm^3 ,
gravitationally bound, but energy comes from supernova, etc. events



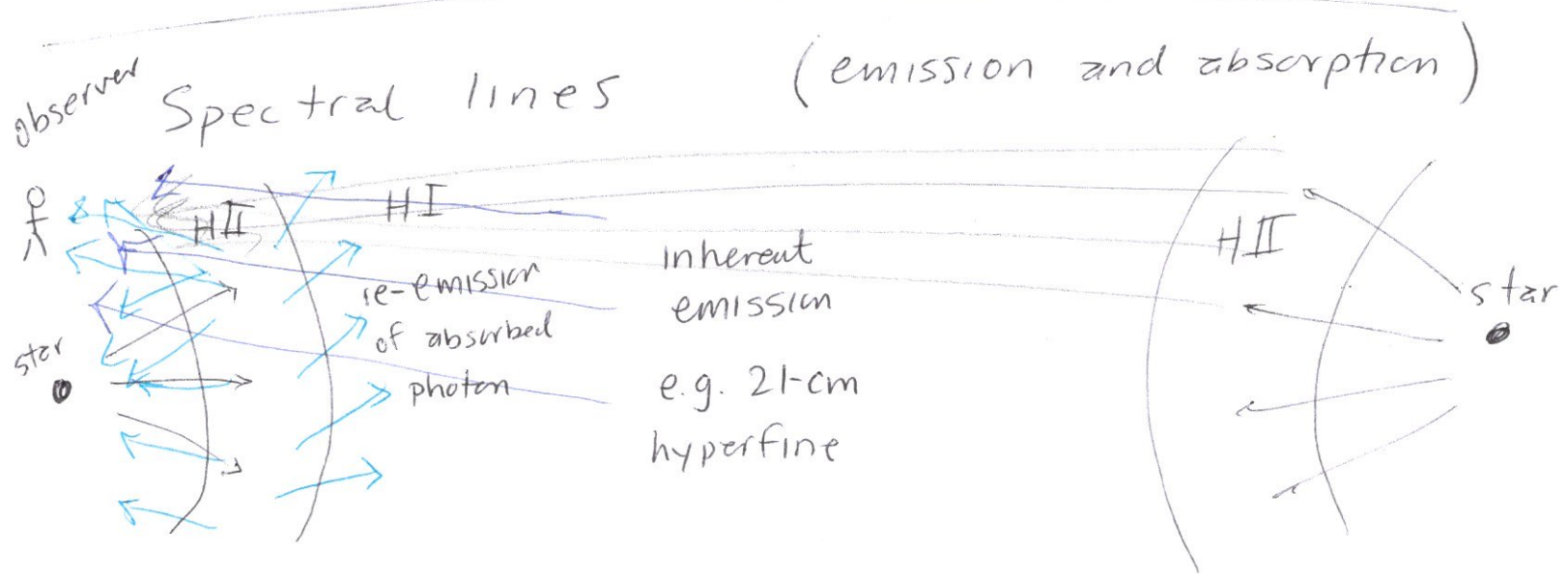
Cold molecular clouds in the galactic plane, $50 \text{ K} - 150 \text{ K}$, star formation
 ~ 10 atoms/ cm^3



Other ingredients of ISM

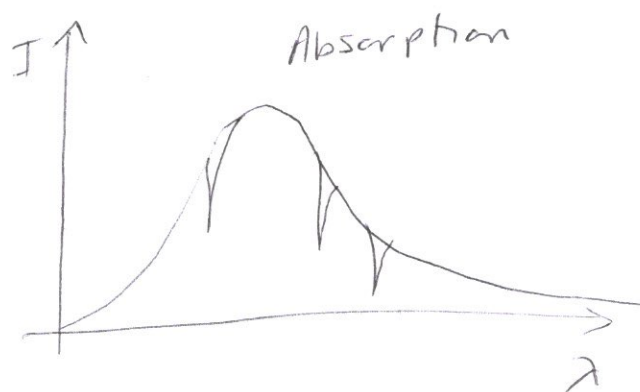
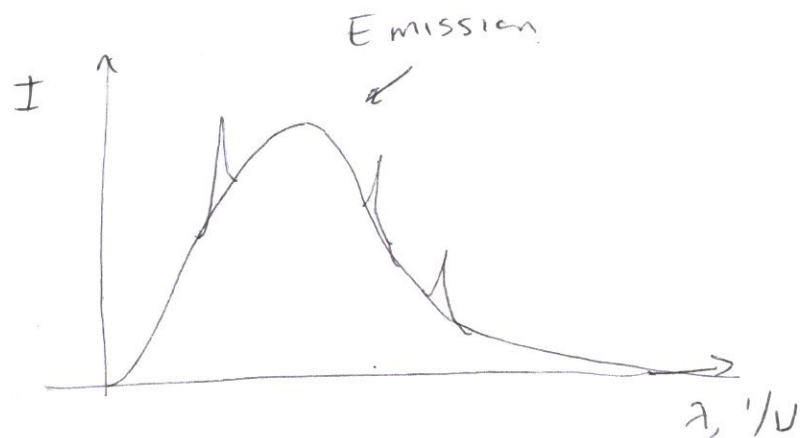
- "Metals" Carbon, nitrogen, oxygen } most abundant
- other atoms lithium, sodium, magnesium, aluminum, silicon, phosphorus, sulfur, calcium, iron } measurable

- Dust Grains of solid matter, ~~mostly~~ made out of metals, 10^{-4} cm³ in size, or less

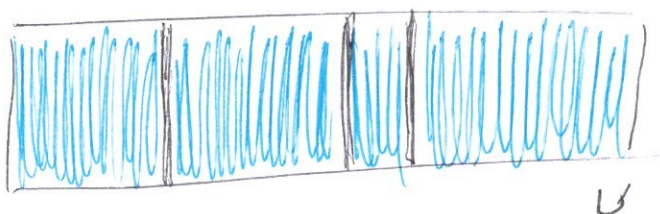


"reflection" "inherent" "absorption"

The emission/absorption is on top of the black body radiation spectrum, although these clouds are sparse and tend to be cold, so the intensity of black-body radiation can be low.

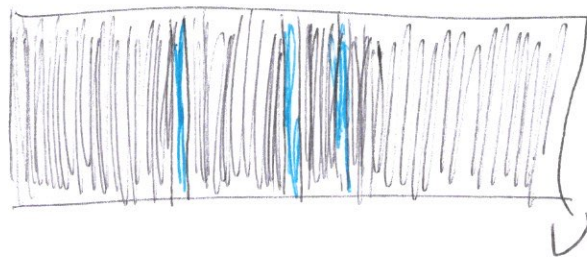


Correcting for the background, spectroscopy usually looks



Low intensity

High intensity



★ WHAT PRODUCES THESE SEEMINGLY DISCRETE EMISSION/ABSORPTION LINES?

The Romans knew that a prism can separate ^{white} light into a rainbow, Newton was the first to develop a theory of optics, In 1802, William Hyde Wollaston combined a lens with a prism, focused the light from Opticks published in 1704

the sun and discovered absorption lines.

(117)

He thought that they were boundaries between colors.

Joseph von Fraunhofer replaced the prism with a diffraction grating, eventually with thousands of slits.

This allowed experiments and measurements to be quantitatively reproducible. He independently re-discovered the absorption lines in the sun's spectra and they are now known as Fraunhofer lines.

In 1756 Thomas Melvill discovered emission lines when adding salts to alcohol flames. In 1835 Charles Wheatstone discovered that elements could be identified by their emission lines and Foucault demonstrated experimentally that emission and absorption lines for a given material are the same, depending on the temperature which one you get (1849). Independently, Angstrom theorized the same thing in 1853.

In the 1860's Bunsen and Kirchhoff established spectrochemical analysis and discovered the elements Caesium and Rubidium. Also in this decade husband and wife William and Margaret Huggins showed that stars are composed of the same elements found on earth and used Doppler shift in 1868 to measure the axial speed of Sirius.

Also in 1868, Jules Janssen discovered helium in the spectrum of the chromosphere of the sun during a solar eclipse.

The Balmer lines are emission lines ^{in hydrogen} known since the 1860s, but it was Johan Balmer who in 1885 came up with the following empirical equation to predict them.

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \quad n = 3, 4, 5, \dots$$

R is the Rydberg constant $1.097 \times 10^7 \text{ m}^{-1}$
 0.01097 /nm

H_{α} is for $n=3$

$$\frac{1}{\lambda} = \frac{0.01097}{1\text{nm}} \left(\frac{1}{4} - \frac{1}{9} \right) = \frac{0.01097}{1\text{nm}} \left(\frac{5}{36} \right)$$

$$\frac{1}{\lambda} = \frac{0.001524}{1\text{nm}} \Rightarrow \lambda = \frac{1\text{nm}}{0.001524} = 656.3\text{nm}$$

H_{β} is for $n=4$

$$\frac{1}{\lambda} = \frac{0.01097}{1\text{nm}} \left(\frac{12}{64} \right) = \frac{0.002057}{1\text{nm}}$$

$$\Rightarrow \lambda = \frac{1\text{nm}}{0.002057} = 486.2 \text{ nm}$$

H_{γ} , $n=5$

$$\frac{1}{\lambda} = \frac{0.01097}{1\text{nm}} \left(\frac{21}{100} \right) = \frac{0.002304}{1\text{nm}} \Rightarrow \lambda = 434.0 \text{ nm}$$

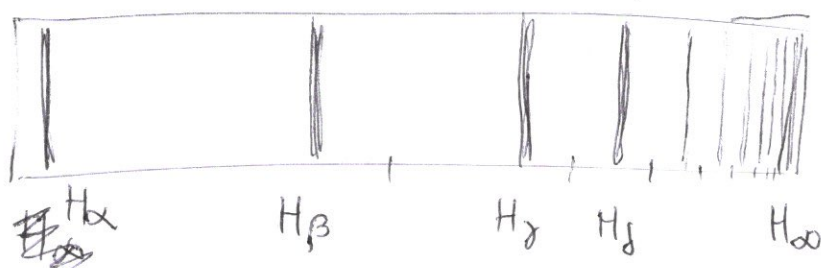
H_{δ} , $n=6$

$$\frac{1}{\lambda} = \frac{0.01097}{1\text{nm}} \left(\frac{32}{144} \right) = \frac{0.002438}{1\text{nm}} \Rightarrow \lambda = 410.2 \text{ nm}$$

$$H_{\infty}, n=\infty, \quad \frac{1}{\lambda} = \frac{0.01097}{1 \text{ nm}} \left(\frac{1}{4} \right) = \frac{0.002743}{1 \text{ nm}}$$

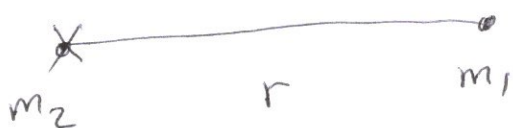
(119)

$$\Rightarrow \lambda = 364.6 \text{ nm}$$



*WHAT IS THE ORIGIN OF THIS RELATIONSHIP?

Consider the model for binary compact objects, but with $m_2 \gg m_1$, and $R_1 \ll r$. Also, origin of force is Coulomb interaction



$$F_{\text{cent}} = \frac{m_1 v^2}{r}$$

$$F_{\text{elec}} = \frac{1}{4\pi\epsilon_0} \frac{Z_1 Z_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

If the orbit is stable, then $F_{\text{cent}} = F_{\text{elec}}$, let's call m_1 and m_2 what they are, m_e & m_p .

$$\frac{m_e v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \Rightarrow v = \left(\frac{e^2}{4\pi\epsilon_0 m_e r} \right)^{1/2} = \frac{e}{\sqrt{4\pi\epsilon_0 m_e r}}$$

$$KE = \frac{1}{2} m_e v^2 = \frac{1}{2} m_e \frac{e^2}{4\pi\epsilon_0 m_e r} = \frac{e^2}{8\pi\epsilon_0 r}$$

$$\text{Total} = \frac{e^2}{8\pi\epsilon_0 r} - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \left(\frac{1}{2} - 1 \right) = - \frac{e^2}{8\pi\epsilon_0 r}$$

↑
Expected?

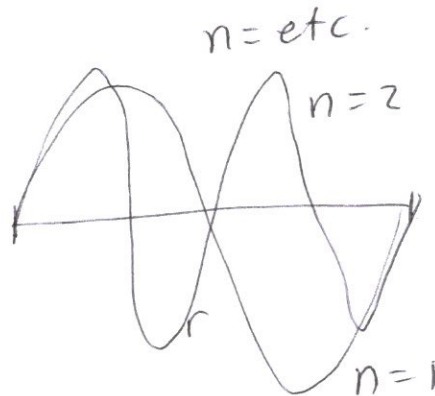
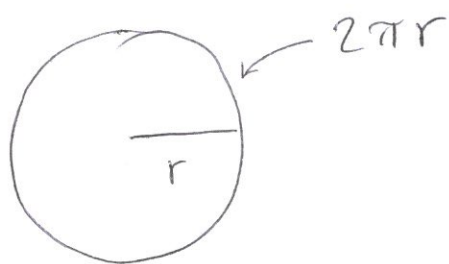
120

The de Broglie wavelength of such electron is

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{\frac{h}{1}}{\frac{m_e e}{\sqrt{4\pi\epsilon_0 m_e r}}} = \frac{h \sqrt{4\pi\epsilon_0 m_e r}}{e m_e}$$

$$\lambda = \frac{h}{e} \sqrt{\frac{4\pi\epsilon_0 m_e r}{m_e^2}} = \frac{h}{e} \sqrt{\frac{4\pi\epsilon_0 r}{m_e}}$$

This wavelength has to wrap around in a loop, otherwise it interferes with itself.



For $n=1$

$$2\pi r = \frac{h}{e} \sqrt{\frac{4\pi\epsilon_0 r}{m_e}} \Rightarrow \cancel{4\pi r^2} = \frac{h^2}{e^2} \frac{\cancel{4\pi\epsilon_0 r}}{m_e}$$

$$\Rightarrow \frac{h^2 \epsilon_0}{\pi e^2 m_e} = r = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s} \right)^2 \left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2 \right)}{\pi \left(1.602 \times 10^{-19} \text{ C} \right)^2 \left(9.11 \times 10^{-31} \text{ kg} \right)}$$

- permittivity of free space

$$r = \frac{(4.39 \times 10^{-67} \text{ N m}^2 \text{ s}^2) \left(\frac{8.85 \times 10^{-12} \text{ C}^2}{\text{N m}} \right)}{(8.06 \times 10^{-38} \text{ C}^2) (9.11 \times 10^{-31} \text{ kg})}$$

(121)

$$r = \frac{(3.88 \times 10^{-78} \frac{\text{kg m s}^2}{\text{s}^2})}{7.34 \times 10^{-68} \text{ kg}} = 5.3 \times 10^{-11} \text{ m}$$

This is known as the Bohr radius, usually a_0

remember that the fine structure constant is $\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c}$,

$$\text{so } \epsilon_0 = \frac{1}{4\pi\alpha} \frac{e^2}{\hbar c}$$

$$a_0 = \frac{(2\pi\hbar)^2}{\pi e^2 m_e} \frac{1}{4\pi\alpha} \frac{e^2}{\hbar c} = \frac{4\pi^2 \hbar^2}{4\pi^2 m_e c \alpha} = \frac{\hbar}{m_e c \alpha}$$

The Bohr radius is the most probable distance between the electron and proton in a hydrogen atom in its ground state. It was derived by Niels Bohr in 1913 using quantization of the angular momentum (de Broglie had not yet discovered the de Broglie wavelength). It made everybody uncomfortable, but it works.

More generally, $n\lambda = 2\pi r_n$, where n is called the quantum number of the orbit.

$$2\pi r_n = n \frac{h}{e} \sqrt{\frac{4\pi \epsilon_0 r}{m_e}} \Rightarrow \cancel{4\pi}^2 r_n^2 = \frac{n^2 h^2}{e^2} \frac{\cancel{4\pi} \epsilon_0 \cancel{r}}{m_e}$$

$$\Rightarrow r_n = \frac{n^2 h^2 \epsilon_0}{\pi m_e e^2}$$

$$\text{Since } a_0 = \frac{h^2 \epsilon_0}{\pi m_e e^2}, \quad r_n = n^2 a_0$$

Before we had derived $E = -\frac{e^2}{8\pi \epsilon_0 r}$, so

$$E_n = -\frac{e^2}{8\pi \epsilon_0 r_n} = -\frac{e^2 / 1}{8\pi \epsilon_0 \frac{n^2 h^2 \epsilon_0}{\pi m_e e^2}}$$

$$E_n = -\frac{m_e e^4}{8 \epsilon_0^2 h^2} \left(\frac{1}{n^2} \right)$$

$$\text{Since } E_1 = -\frac{m_e e^4}{8 \epsilon_0^2 h^2}, \quad E_n = \frac{E_1}{n^2}$$

$$E_1 = -\frac{(9.11 \times 10^{-31} \text{ kg})(1.602 \times 10^{-19} \text{ C})^4}{8 \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right)^2 \left(6.626 \times 10^{-34} \text{ J} \cdot \text{s} \right)^2}$$

$$E_1 = - \frac{6.0 \times 10^{-106} \text{ Kg } \cancel{\text{C}^4}}{\left(6.26 \times 10^{-22} \frac{\cancel{\text{C}^4}}{\text{N}^2 \text{ m}^2}\right) \left(4.39 \times 10^{-67} \cancel{\text{N}^2} \text{ m}^2 \text{ s}^2\right)}$$

$$E_1 = - \frac{6.0 \times 10^{-106} \text{ Kg}}{2.75 \times 10^{-88} \frac{\text{s}^2}{\text{m}^2}} = -2.18 \times 10^{-18} \frac{\text{Kg m}^2}{\text{s}^2}$$

$$E_1 = -2.18 \times 10^{-18} \text{ J} = -13.6 \text{ eV}$$

$$\text{Let } \Delta E = h \Delta \nu = E_i - E_f$$

E_i is the initial (higher energy) state and
 E_f is the final (lower energy) state.

$$E_i - E_f = \frac{E_1}{n_i^2} - \frac{E_1}{n_f^2} = E_1 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = -E_1 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\nu = \frac{E_i - E_f}{h} = - \frac{E_1}{h} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\text{with } \lambda = \frac{c}{\nu}, \quad \frac{1}{\lambda} = \frac{\nu}{c}$$

$$\frac{1}{\lambda} = - \frac{E_1}{hc} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = - \frac{(-2.18 \times 10^{-18} \text{ J})}{(6.626 \times 10^{-34} \text{ J} \cdot \text{s}) (3 \times 10^8 \frac{\text{m}}{\text{s}})} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\frac{1}{\lambda} = \frac{2.18 \times 10^{-18}}{1.99 \times 10^{-25} \text{ m}} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = 1.097 \times 10^7 \text{ m}^{-1} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

This was
 $\frac{1}{2^2}$ for the Balmer
 series. What does
 it mean?