Accretion disks

11-17-20

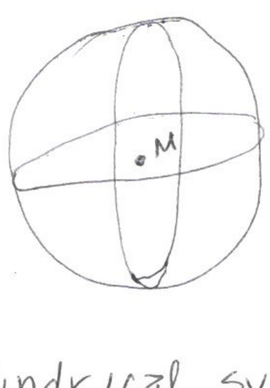
Disk vs Sphere: If the interstellar material is at rest, it would accrete as a sphere, nevertheless, it is typically moving, so once gravitational condensation starts, the material acquires angular momentum.

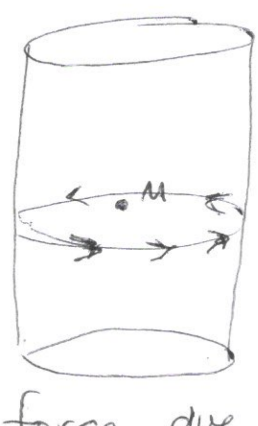
Ubiquitous: Star creation process, planet creation process, quasars, etc.

Luminous: Not subject to the Eddington limit.

$$H(R_{eff})\, L_* < 4\pi G c M_*$$

In spherical symmetry, the force produced by thermal radiation can't be stronger than gravitational attraction, otherwise the material is dispersed (the star or protostar has a radius that balances this).





In cylindrical symmetry, the repulsive force due to radiation pressure (thermal energy) ~~can't be stronger~~ plus centrifugal force can't be stronger than gravitational attraction.

Notice the following about the cylindrical symmetry:

- Even though initially there will be pandemonium, the system has angular momentum that must be conserved.

- If we consider a slice, we get an accretion disk. Initially it is not a disk, but matter not on the disk plane (where the center of mass is located) will experience a force towards the disk. Due to friction and collisions, eventually the configuration of mass will be very much disk-shaped.

~~[illegible]~~ i) If $\Omega(R)$ is the angular frequency of bodies in an orbit of radius R, then, using Kepler's third law

$$\Omega^2 = \frac{GM}{r^3} \Rightarrow \Omega^2(R)\,\vec{R} = \frac{GM}{R^2} = \frac{F_{cent}}{\delta m}$$ centrifugal force per mass

ii) Let $\kappa_T \vec{\ell}/c$ be the force per mass due to radiation, with κ_T the opacity due to Thomson scattering (scattering of photons due to charged particles), c the speed of light and $\vec{\ell}$ is a vector in the direction of the radiation with magnitude of energy per time per area.

$$\frac{F_{rad}}{\delta m} = \frac{K_T \vec{l}}{c}$$

K units of m^2/kg, c is m/s $\quad \dfrac{\dfrac{m^{\cancel{2}}}{kg}}{\dfrac{\cancel{m}}{s}} = \dfrac{m \cdot s}{kg}$

$\vec{l}$ is $\dfrac{J}{s \cdot m^2}$ $\quad \dfrac{\dfrac{kg\,\cancel{m^2}}{s^2}}{\dfrac{\cancel{m^2} \cdot s}{1}} = \dfrac{kg}{s^3}$, so $\dfrac{K_T \vec{l}}{c}$ units $\dfrac{m \cdot \cancel{s}}{\cancel{kg}} \dfrac{\cancel{kg}}{s^{\cancel{3}\,2}}$

For a sphere; $\vec{l} = \dfrac{L}{4\pi r^2}$, although we are not assuming spherical symmetry.

acceleration as expected $\underline{\underline{m/s^2}}$

iii) The gravitational potential is $\phi = \dfrac{GM}{R} = \dfrac{U_{grav}}{\delta m}$,

so $\dfrac{F_{grav}}{\delta m} = -\nabla \phi$

The maximum luminosity in this case is

$$\underbrace{\Omega^2(R)\vec{R} + K_T \vec{l}/c}_{\text{vector outward}} \leq \underbrace{\nabla \phi}_{\text{vector inward}}$$

(Applies to each vector component)

<u>W3.5.1</u>

Notice that if $\Omega^2(R) = 0$ so there is no angular momentum nor centrifugal force, we get $\dfrac{K_T l}{c} \leq \dfrac{d}{dR}\dfrac{GM}{R} = \dfrac{GM}{R^2}$

$$\frac{K_T L}{c\, 4\pi \cancel{R^2}} \quad \frac{GM}{\cancel{R^2}}$$

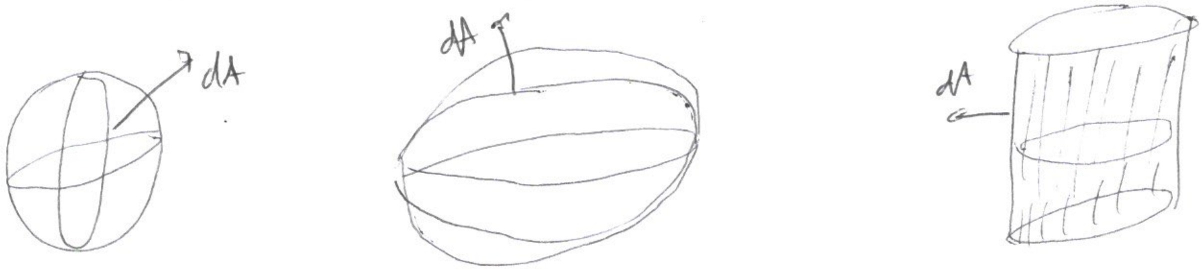
$K_T L < 4\pi GcM$ Eddington

More generally, $\vec{l} \le \frac{c}{K_T}\left[-\Omega^2(R)\vec{R} + \nabla\phi\right]$

W3.5.2

$$L = \oint_A \vec{l}\cdot d\vec{A} \le \frac{c}{K_T}\oint_A \left[-\Omega^2(R)\vec{R} + \nabla\phi\right]\cdot d\vec{A}$$

$d\vec{A}$ is a vector normal to the volume enclosing the area of accretion



Using the divergence theorem $\int_V (\nabla\cdot\vec{F})\,dV = \int_A F\cdot d\vec{A}$

$$L \le \frac{c}{K_T}\int_V \nabla\cdot\left[-\Omega^2(R)\vec{R} + \nabla\phi\right]dV$$

$$L \le \frac{c}{K_T}\int_V \left[-\nabla\cdot\left[\Omega^2(R)\vec{R}\right] + \nabla^2\phi\right]dV$$

$$4\pi\frac{\partial^2}{\partial x^2}\phi = \frac{\partial}{\partial x^2}\frac{GM}{x} = 4\pi\frac{GM}{x^3} = \underline{\underline{4\pi G\rho}}$$

Poisson's equation for gravity

$$\nabla\cdot\left[\Omega^2(R)\vec{R}\right] = \Omega^2(R)\nabla\cdot\vec{R} + \vec{R}\,\nabla\cdot\Omega^2(R)$$

Since $\vec{R}$ is a vector in the plane of the orbit, $\vec{R} = (x, y, 0)$

$$\nabla \cdot \vec{R} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial}{\partial z} 0 = 1 + 1 = 2$$

$$\nabla \cdot \left[\Omega^2(R) \vec{R} \right] = 2\Omega^2(R) + R \frac{d}{dR} \Omega^2(R)$$

so

$$L_{max} = \frac{c}{K_T} \left[\int_V 4\pi G \underbrace{\rho \, dV}_{M} - \int_V \left[2\Omega^2(R) + R \frac{d}{dR} \Omega^2(R) \right] dV \right]$$

$$L_{max} = \underbrace{\frac{c}{K_T} 4\pi G M}_{\text{Eddington luminosity} \; - \; L_E} - \frac{c}{K_T} \int_V \overline{\left[2\Omega^2(R) + R \frac{d}{dR} \Omega^2(R) \right]}^{W3.5.4} dV$$

The maximum luminosity produced by an accretion disk is larger than the Eddington luminosity if the integral is negative

For a solid disk the angular frequency Ω is independent of R, so $\frac{d}{dR} \Omega^2(R) = 0$ and

$$L_{max} = L_E - \frac{c}{K_T} \left(2\Omega^2 \int_V dV \right)$$

L_{max} is less than the Eddington luminosity

Nevertheless, for Keplerian orbits, $\Omega(R) = K R^{-3/2}$, so

$$\frac{d}{dR} \Omega^2(R) = K^2 \frac{d}{dR} R^{-3} = \frac{-3K^2}{R^4} \qquad \Omega^2(R) = \frac{K^2}{R^3}$$

$$\int_V \left[2\Omega^2(R) - \frac{3RK^2}{R^{*3}} \right] dV$$

$$\int_V \left[\frac{2K^2}{R^3} - \frac{3K^2}{R^3} \right] dV = -\int_V \frac{K^2}{R^3}\, dV = -\int_V \Omega^2(R)\, dV$$

so

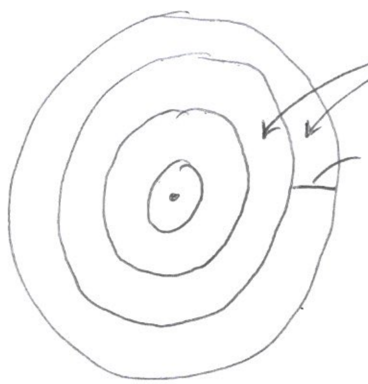
$$L_{max} = L_E + \frac{c}{K_T} \int_V \Omega^2(R)\, dV$$ which is greater than the Eddington limit

☆ WHY ARE THE PLANETS NOT ACCRETED TOWARDS THE SUN?

L_{max} is a <u>cool</u> equation. The faster a body is rotating and the more compact it is, the higher its luminosity.

The accretion disks of protostars or young stars radiate with maximum intensity in the infrared, whereas accretion disks around neutron stars and black holes radiate with maximum intensity in the x-ray range. Notice that this is "hotter" than the most massive stars. Supermassive black holes with substantial accretion disks are called quasars, have luminosities several thousand times that of the whole Milky Way with its 200 billion stars, and are the most powerful phenomena we have discovered.

Planets interact via collisionless interactions, they never physically touch. This is not the case for accretion disks. Their thermal energy comes from the collisions between the particles, from friction, or viscosity.



- Ultimately described by the Navier-Stokes equation (fluid dynamics)
- Particles lose energy and angular momentum due to viscous interaction with other particles at adjecent radii.
- The exact process by which viscous friction operates is <u>not</u> known.

If a particle moves from ~~r+~~ r+dr to r, half of its potential energy is converted to kinetic energy, but the other 1/2 can be converted to thermal energy (disordered kinetic energy).

$$d\mathcal{E} = \frac{1}{2}\left(\frac{GM\,dm}{r} - \frac{GM\,dm}{r+dr}\right)$$

$$\Rightarrow dL = \frac{d\mathcal{E}}{dt} = \frac{1}{2} GM \frac{dm}{dt}\left(\frac{1}{r} - \frac{1}{r+dr}\right) = \frac{1}{2} GM\dot{M}\frac{dr}{r^2}$$