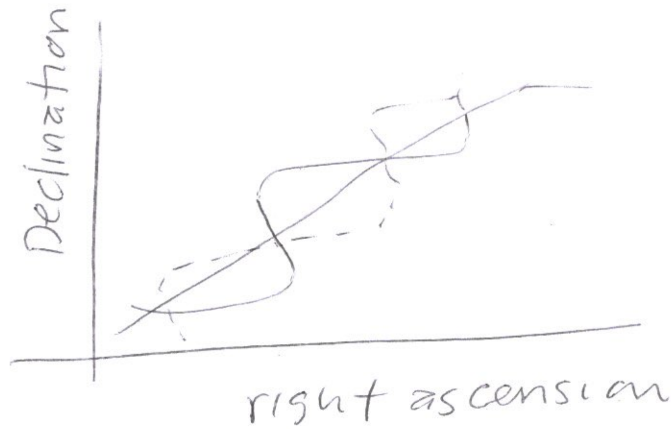
From definition of center of mass,

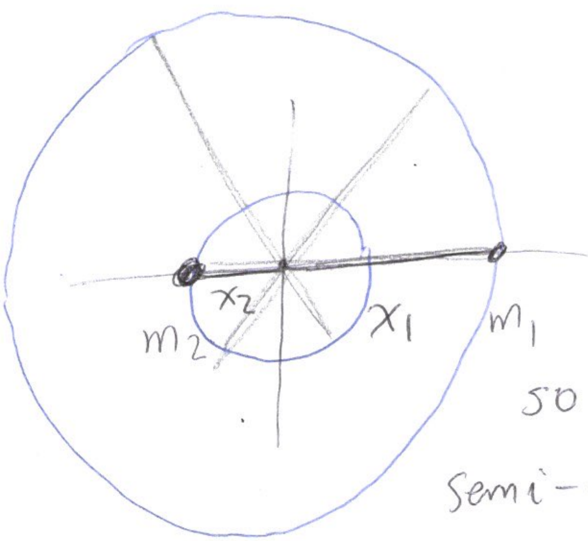
$$m_1 \vec{x}_1 + m_2 \vec{x}_2 = 0$$

$$|m_1 \vec{x}_1| = |-m_2 \vec{x}_2|$$

$$\frac{m_1}{m_2} = \frac{x_2}{x_1}$$

Visual binaries





so x_1 and x_2 are semi-major axes for m_1 and m_2

(each one moves in an elliptical orbit about the center of mass

$$\frac{m_1}{m_2} = \frac{x_2}{x_1} = \frac{a_2}{a_1}$$

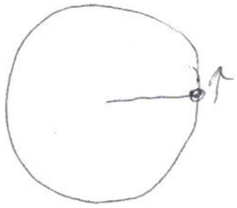
$$\alpha_2 = \frac{a_2}{d}$$

$$\alpha_1 = \frac{a_1}{d}$$

d ← distance to system (from observer)

α_1 ← angle in radians ← angular separation

Consider the case for low eccentricity $e \approx 0$



$$F_c = m_1 \frac{v^2}{x_1} = \frac{G m_1 m_2}{r^2} = \frac{\Omega^2 x_1^2}{\cancel{x_1}}$$

with $x_1 = \frac{m_2}{M} r$,

$$\Omega^2 \left(\frac{m_2}{M}\right) r = \frac{G m_2}{r^2}$$

$$\Rightarrow \Omega^2 = \frac{GM}{r^3}$$ This is Kepler's third law

$$m_1 + m_2 = \frac{\Omega^2 (x_1 + x_2)^3}{G}$$

x_1 and x_2 are known if the distance to the system is known, Since we have two equations m_1/m_2 and m_1+m_2, and two unknowns, m_1 and m_2.

Since $\Omega = \frac{2\pi}{T}$, $\frac{(2\pi)^2}{T^2} = \frac{GM}{r^3}$

$$\Rightarrow \sqrt{\frac{4\pi^2 r^3}{GM}} = T$$

$$\Rightarrow M = \frac{(2\pi)^2 r^3}{G T^2}$$

Consider the case of the earth-sun system.

$$T = 1 \text{ year} = 365 \text{ days} \left(\frac{24\text{h}}{1\text{ day}}\right)\left(\frac{3600\text{s}}{1\text{ h}}\right) = 3.15 \times 10^7 \text{ s}$$

distance is 1 A.U. $= 1.5 \times 10^{11}$ m

$$M = \frac{(2\pi)^2}{(3.15 \times 10^7 \text{ s})^2} \cdot \frac{(1.5 \times 10^{11} \text{ m})^3}{6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2} = \frac{\cancel{3.38 \times 10^{33} \text{ m}^3}}{\cancel{6.61 \times 10^{-3} \text{ m}^3/\text{kg}}}$$

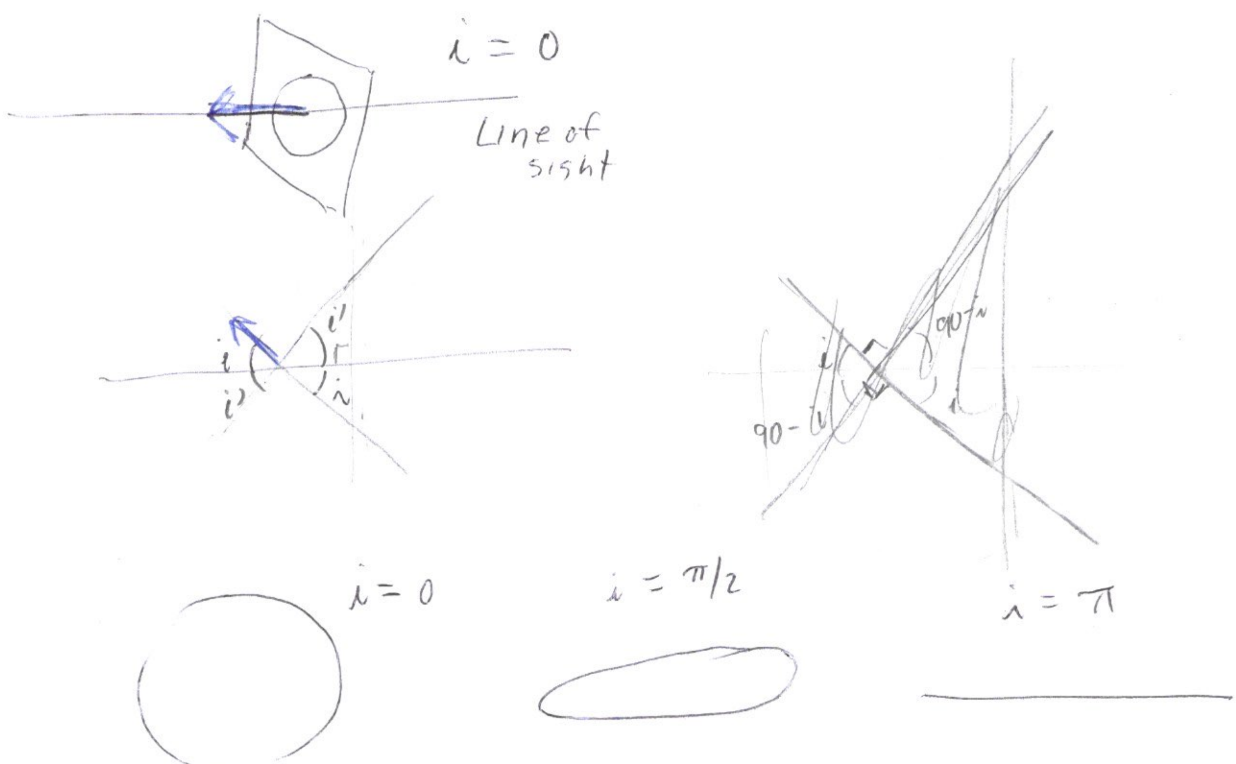
~~$M = 4.33 \times 10^{35}$~~

$$M = \frac{4\pi^2 \cdot 3.38 \times 10^{33}\ \cancel{m^3}}{9.92 \times 10^{14}\ \cancel{s^2} \cdot 6.67 \times 10^{-11}\ \cancel{m^3}/kg\,\cancel{s^2}} = \frac{1.33 \times 10^{35}\ m^3}{6.61 \times 10^4\ m^3/kg}$$

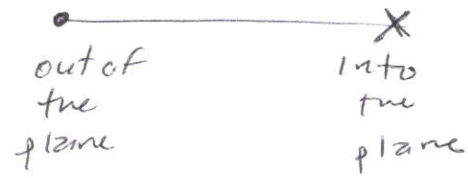
$M = 2.0 \times 10^{30}$ kg // as expected

☆ How would you measure it? (Although it is not as easy to measure everything...)

In the general case the system will be at an angle i, called the angle of inclination



$i = \pi$



$$|v_{1,obs}| = |v_1| \sin i$$

$$|v_{2,obs}| = |v_2| \sin i$$

$$|v_1| = \frac{2\pi\, x_1}{T} \qquad |v_2| = \frac{2\pi\, x_2}{T}$$

so $$\frac{|v_{1,obs}|}{|v_{2,obs}|} = \frac{|v_1| \cancel{\sin i}}{|v_2| \cancel{\sin i}} = \frac{\cancel{2\pi} x_1}{\cancel{T}} \cdot \frac{\cancel{T}}{\cancel{2\pi} x_2} = \frac{x_1}{x_2} = \frac{m_2}{m_1}$$

$$x_1 = \frac{|v_1|\, T}{2\pi} = \frac{|v_{1,obs}|}{\sin i} \frac{T}{2\pi}$$

$$x_2 = \frac{|v_2|\, T}{2\pi} = \frac{|v_{2,obs}|}{\sin i} \frac{T}{2\pi}$$

$$|\vec{r}| = |\vec{x}_1| + |\vec{x}_2| = \frac{T}{2\pi \sin i}\left[|v_{1,obs}| + |v_{2,obs}| \right]$$

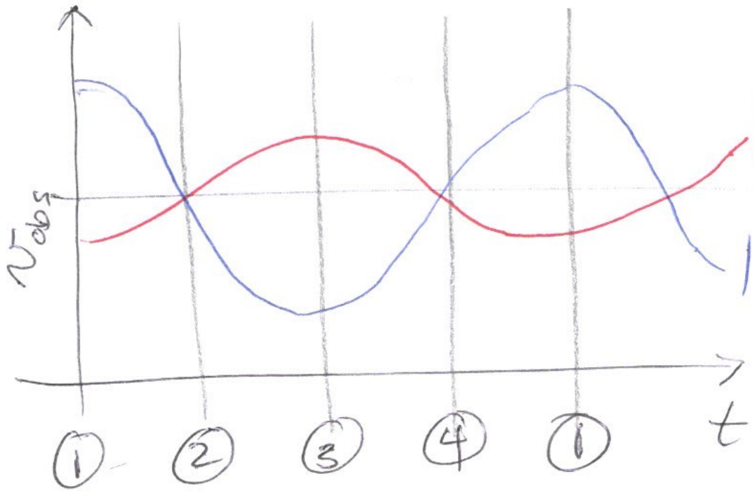
using Kepler's law

$$M = \frac{\cancel{(2\pi)^2}}{G \cancel{T^2}} \cdot \frac{T^{\cancel{3}}}{(2\pi)^{\cancel{3}} \sin^3 i} \left(|v_{1,obs}| + |v_{2,obs}| \right)^3$$

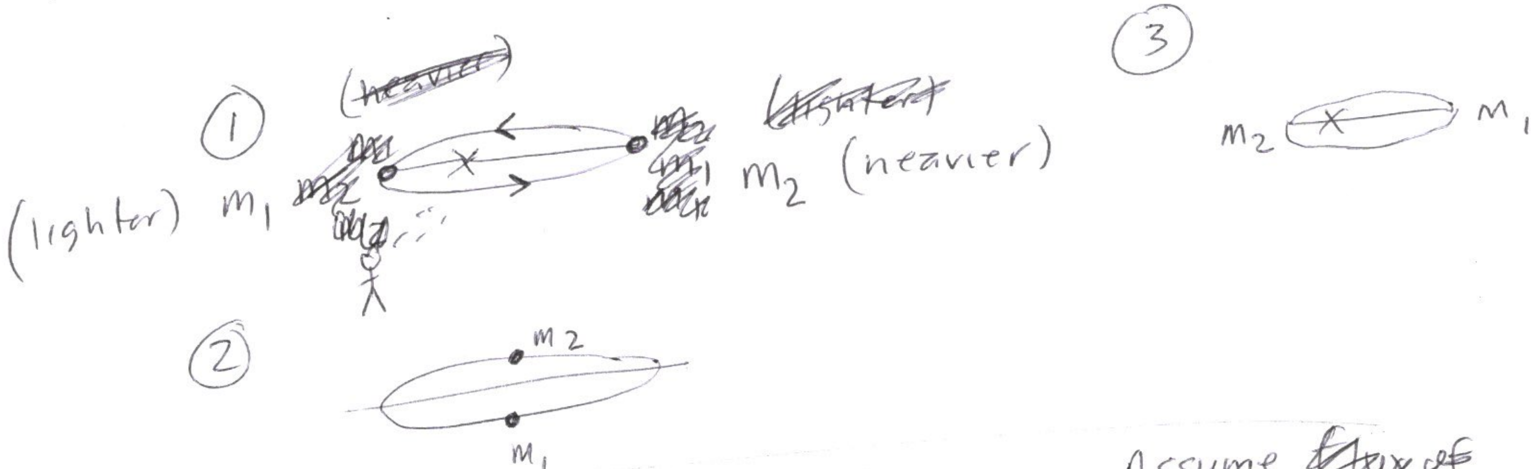
$$M \sin^3 i = \frac{T \left(|v_{1,obs}| + |v_{2,obs}| \right)^3}{2\pi G}$$

only up to a factor of $\sin^3 i$

if $i \approx \pi$, then it is eclipsing and the masses can be determined but NOT otherwise if the stars or star/planet <u>is</u> eclipsing though,

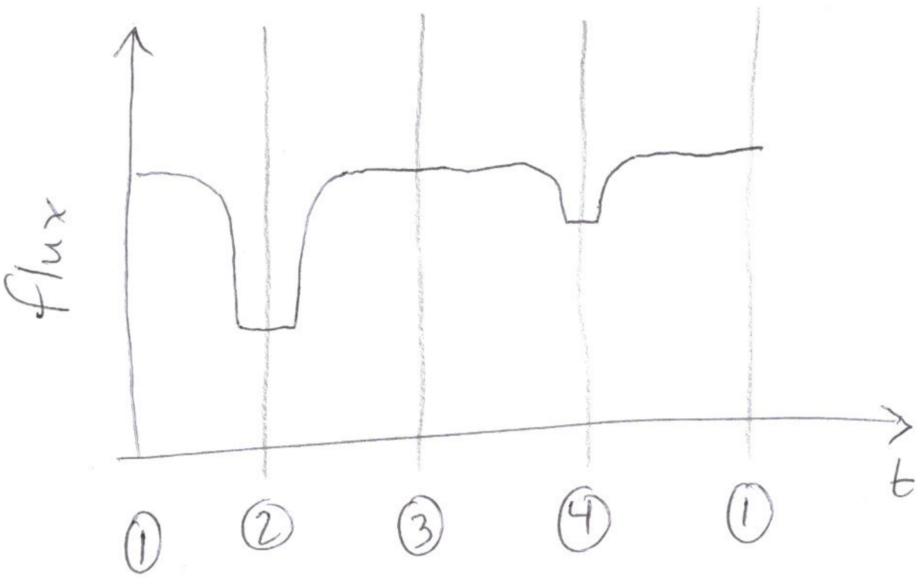


Radial velocity curve



light curve

Assume ~~flux of~~ luminosity of 2 is greater



since we know the velocities, we can also find the radius of bodies

If only one velocity is observed, such in the case of an exoplanet

$$|v_{2,obs}| = \frac{m_1}{m_2}|v_{1,obs}|$$

$$M\sin^3 i = \frac{T\left[|v_{1,obs}| + (m_1/m_2)|v_{1,obs}|\right]^3}{2\pi G}$$

$$M\sin^3 i = \frac{T|v_{1,obs}|^3(1+m_1/m_2)^3}{2\pi G}$$

Mass function

$$\frac{\cancel{(m_1+m_2)}\sin^3 i}{\frac{(m_1+m_2)^{\cancel{3}}}{m_2^3}} = \frac{m_2^3}{M^2}\sin^3 i = \boxed{\frac{T|v_{1,obs}|^3}{2\pi G}}$$

So now there are 3 unknowns: m_1, m_2, i and only one equation. But if $i \approx \pi$ and $m_2 << m_1$

$M \approx m_1$

$$\frac{m_2^3}{M^2}\sin^3 i \approx \frac{m_2^3}{m_1^2}\sin^3 i$$

~~$m_2^3 \,/\, m_1^2 + 2m_1m_2 + m_2^2$~~

$$\Rightarrow m_2^3\sin^3 i \approx \frac{T|v_{1,obs}|^3 m_1^2}{2\pi G}$$

$v_{1,obs}$ can be measured to about $3\frac{m}{s}$

$$m_2\sin^3 i \approx \left(\frac{T}{2\pi G}\right)^{1/3}|v_{1,obs}|\,m_1^{2/3}$$