9/1/2020

Quick round of introductions post short intro
Teams? Bb?

Quick review:

- The only force we have explicitely considered is gravity. In order for a sphere to be in hydrostatic equilibrium with gravitational attraction,

$$\frac{dp(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

Fundamental equation of hydrostatic equilibrium

  - A pressure gradient is needed, but it can be provided by any force.
  - From boundary conditions,

$$p(0) \geq \frac{GM^2}{8\pi R^4}$$

Humble but powerful equation

★ What happens if the pressure drops, or suddenly drops? For example, because the nuclear fuel was exhausted in a star? Explotion?

- The pressure can be thought of as being produced by collisions of particles against a wall, they bounce off and so there's a change in momentum ⇒ force. Temperature is the average energy of the particles. The higher the temperature, the higher the pressure (larger change of momentum and collisions more often). So thermal energy density $\mathcal{E}$ and pressure p are proportional. Let $\frac{1}{\Gamma - 1}$ be the proportionality constant. $\Gamma = 5/3$ for an ideal gas and $\Gamma = 4/3$ for pure radiation.

- Let $E = \Omega + \Upsilon$ be the total energy, Ω be the gravitational potential energy and Υ be the kinetic (thermal) energy. Then

$$\Upsilon = -\frac{E}{3\Gamma - 4} \quad ; \quad \Omega = \frac{E(\Gamma - 1)}{\Gamma - 4/3}$$

which blows up for pure radiation and gives you the virial theorem for the ideal gas $2\Upsilon = \Omega$

- The system will be gravitationally bound when $E < 0$. A cloud of gas/dust radiates if it is at finite temperature, so it is "bound" to collapse.
When $E < 0$, Υ is positive, so the thermal energy increases
- Losing energy heats up the system, so negative heat capacity

☆ What happens with the most energetic stars/particles?
They evaporate.

- All this physics is valid for stars, clusters of stars, galaxies. ☆ Universe?

Hydrostatic equilibrium depends on the pressure, which is a function of energy / temperature. How the temperature varies depends on how energy is transported.

Three mechanisms:

- Radiation
- Convection
- Conduction

★ What is each one?

negligible in stars due to short mean free path

We will study radiative energy transport. Consider

$$\ell(\hat{n}, \vec{x}, \nu, t)\, d^2\hat{n}\, d\nu$$

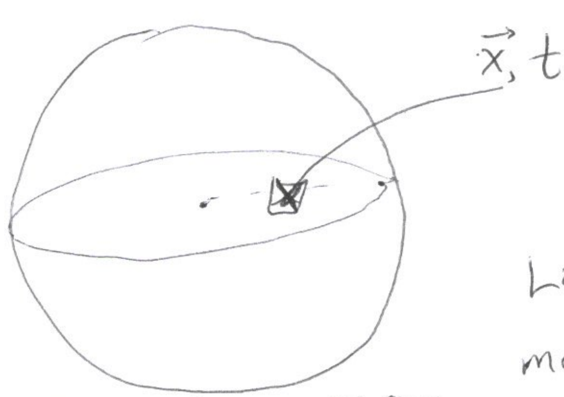
energy per volume (energy density)

at position $\vec{x}$ and time t

of photons with directions within solid angle $d^2\hat{n}$

around unit vector $\hat{n}$

and frequencies between ν and $\nu + d\nu$



Since it is at a given ~~t~~ ν, the energy density depends on how many photons there are in the volume

Large solid angle

means that photons can be traveling in many directions, but as $d^2\hat{n}$ becomes infinitesimally small, only photons that are traveling "straight" are considered

In the same volume you can have photons of different energies

depends on the physics

Looks intimidating, but is not so bad. We will look at the contributions and use $\frac{d\ell}{}=0$ to rewrite the hydrostatic equation in terms of luminocity, opacity, etc. that we can actually measure for stars.

- Transport (nothing happens to photons)
- Absorption (uncorrelated photon motion)
- Scattering (correlated) ☆What is each one?
- Emission (thermal & nuclear)

Transport

everything that goes in — comes out

$$\frac{\partial}{\partial t}\ell(\hat{n},\vec{x},\nu,t)+\nabla\cdot\left(c\hat{n}\cdot\ell(\hat{n},\vec{x},\nu,t)\right)=0.$$

volume density

$$\frac{\partial}{\partial t}\int \ell\, d^3r = -\oint \ell c\cdot dn$$

volume

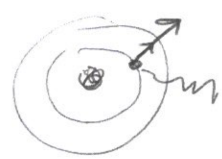
continuity equation

☆ $$\frac{\partial\rho}{\partial t}+\nabla\cdot\rho\vec{v}=0$$

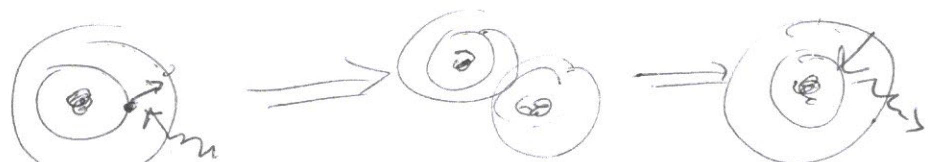
$$\frac{M}{V}\qquad \frac{M}{\frac{\partial V}{\partial t}}=\frac{M}{vA}$$

Absorption

bound-free



bound-bound



free-free

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \cancel{(\rho \vec{v})} = \text{~~constant~~ something}$$

0 goes to zero

(since it is being accumulated)

the fraction of radiation absorbed ~~per unit time~~ in a given time should be proportional to the number of absorbers and how efficient the absorbers are for radiation of a particular frequency.

$\#$ absorbers $\sim \rho(\vec{x}, t)$

efficiency $\sim K_{abs}(\vec{x}, \nu, t) \sim c K_{abs}(\vec{x}, \nu, t)$

fraction ~~per~~ in unit time $\quad c K_{abs}(\vec{x}, \nu, t) \rho(\vec{x}, t)\, dt$

so $\dfrac{d\, \ell(\hat{n}, \vec{x}, \nu, t)}{dt} = -c K_{abs}(\vec{x}, \nu, t) \rho(\vec{x}, t)\, \ell(\hat{n}, \vec{x}, \nu, t) \cancel{dt}$

fraction should be unitless, so

$$\frac{\cancel{m}}{\cancel{s}} K_{abs} \frac{kg}{m^{\cancel{3}2}} \cancel{s} = 1 \Rightarrow K_{abs} = \frac{m^2}{kg}$$

☆ what is this? cross-section! is it fundamental or phenomenological?

Since K_{abs} is a proportionality constant, it does not matter if we add the speed of light c, but it has advantages.

Also, $\dfrac{1}{K_{abs}\rho} = \dfrac{1}{\frac{\cancel{m^2}}{\cancel{kg}} \frac{\cancel{kg}}{\cancel{m^3}} m} = m$ what is this?