9/8/2020 check BOB edition. Send email

If the chemical composition is fixed and uniform, then $P(r)$ (pressure), $K(r)$ (opacity) and $\epsilon(r)$ (nuclear energy production per mass) are fixed functions of the density $\rho(r)$ and temperature $T(r)$.

unknown functions

The structure of the star is described by

- $M(r)$ mass contained within sphere of radius $\underline{r}$
  determines gravitational potential energy
  (and kinetic)
- $L(r)$ luminosity is radiant energy per second flowing outward through a spherical surface of radius $\underline{r}$
- $\rho(r)$ density at $\underline{r}$
- $T(r)$ temperature at $\underline{r}$

They are related

depend only on mass distribution

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dL(r)}{dr} = 4\pi r^2 \epsilon(r)\rho(r)$$

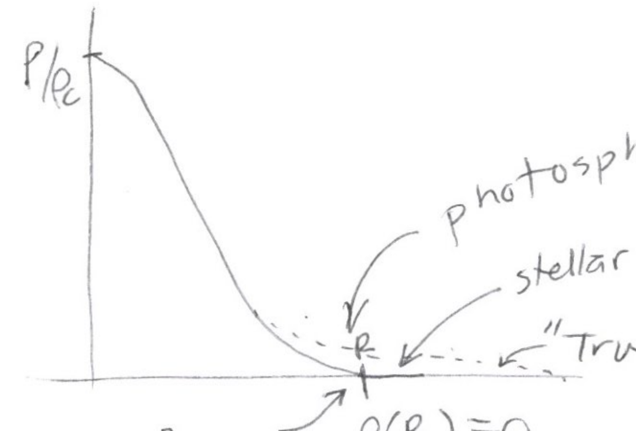
$$\frac{dT(r)}{dr} = -\frac{3K(r)\rho(r)L(r)}{4caT^3(r)\,4\pi r^2}$$

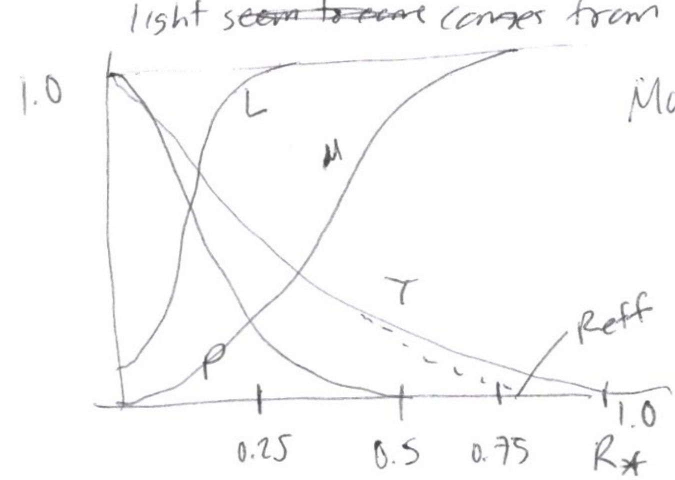
$M(0) = L(0) = 0$

$\rho(R) = T(R) = 0$

How can the temperature be zero at R? (25)

well it can't. Our condition that $\frac{1}{\kappa\rho} \ll R-r$ is not valid close to the surface of the star





Assume all the ~~net~~ luminosity comes from sphere of radius Reff, then

$$\sigma T^4 R_{eff} \times 4\pi R_{eff}^2 = L$$

$$\sigma = \frac{ac}{4} = \frac{2\pi^5 R_{eff}^4}{15 h^3 c^2 N_A^4}$$

The optical depth of the stellar atmosphere is $\int_{R_{eff}}^{R_{true}} \kappa(r)\rho(r)\,dr = \tau_{eff}$

what this means is that

$$\rho(R) << \rho(0) \quad \text{and} \quad T(R) << T(0)$$

For the sun $\rho_c = 98 \pm 15 \frac{g}{cm^3}$ (water is $1 g/cm^3$)

The Teff effective temperature is obtained by fitting the black body radiation spectrum at Teff to observations.

$\rho_{\tau_{eff}=10} = 5 \times 10^{-7} \frac{g}{cm^3}$ — 9 orders of magnitude

$T_c = 13.6 \pm 1.2 \times 10^6$ K

$T_{\tau_{eff}=10} = 9700$ K — 4 orders of magnitude

Vogt-Russell theorem

4 first-order differential equations, 4 boundary conditions of single parameter R

if composition is unique, there is a unique solution to the system

BUT: The solution is not necessarily unique: there might be other combinations of parameters that produce the same solution!

Any given star is going to ~~have~~ experience pressure due to both radiation and matter (ideal gas)

$$P = P_{gas} + P_{rad}$$

we know that for black body radiation, $P_{rad} = \frac{a}{3} T^4$

and for the ideal gas, $P_{gas} = \frac{\rho k_B T}{m_1 \mu}$ where μ is the molecular weight and m_1 is the mass of the nucleon (mass of unit atomic weight)

Remember

$$\frac{dT}{dr} = - \frac{3 K \rho L}{4 c a T^3 4\pi r^2}$$

$$\frac{dP_{rad}}{dT} = \frac{4}{3} a T^3 \Rightarrow dT = \frac{dP_{rad}}{\frac{4}{3} a T^3} = \frac{3}{4} \frac{dP_{rad}}{a T^3}$$

so

$$\frac{\cancel{3}}{\cancel{4aT^3}} \frac{dP_{rad}}{dr} = - \frac{\cancel{3} H \rho L}{\cancel{4aT^3} c 4\pi r^2}$$

$$\frac{dP_{rad}}{dr} = - \frac{K(r) \rho(r) L(r)}{4\pi c r^2}$$

Now, remember $$\frac{dP(r)}{dr} = - \frac{G M(r) \rho(r)}{r^2}$$

$$P(r) - P_{rad}(r) = P_{gas}(r)$$

$$\overset{P_{rad}}{-\frac{K(r)\rho(r)L(r)}{4\pi c r^2}} + \overset{\rho(r)}{\frac{GM(r)\rho(r)}{r^2}} \geq 0$$

otherwise it would be pure radiation and star would explode

$$\cancel{-\frac{dP(r)}{dr}} = -\frac{dP_{gas}(r)}{dr}$$

$$\Rightarrow \frac{GM(r)\cancel{\rho(r)}}{\cancel{r^2}} > \frac{K(r)\cancel{\rho(r)}L(r)}{4\pi c \cancel{r^2}}$$

$$\Rightarrow K(r)L(r) < 4\pi G c M(r)$$

At $r = R_{eff}$, $\boxed{K(R_{eff})L_* < 4\pi G c M_*}$

Eddington Limit

If this limit is violated, the star disperses material until it is valid (reaches hydrostatic equilibrium). How far away it is from ~~equilibrium~~ being equal depends on the mass of the star (or galactic nucleus). Very massive star have almost exclusively P_{rad} due to particles moving close to the speed of light. If you know the mass of the star and can estimate $K(R_{eff})$, you get L_* from which distance can be obtained.