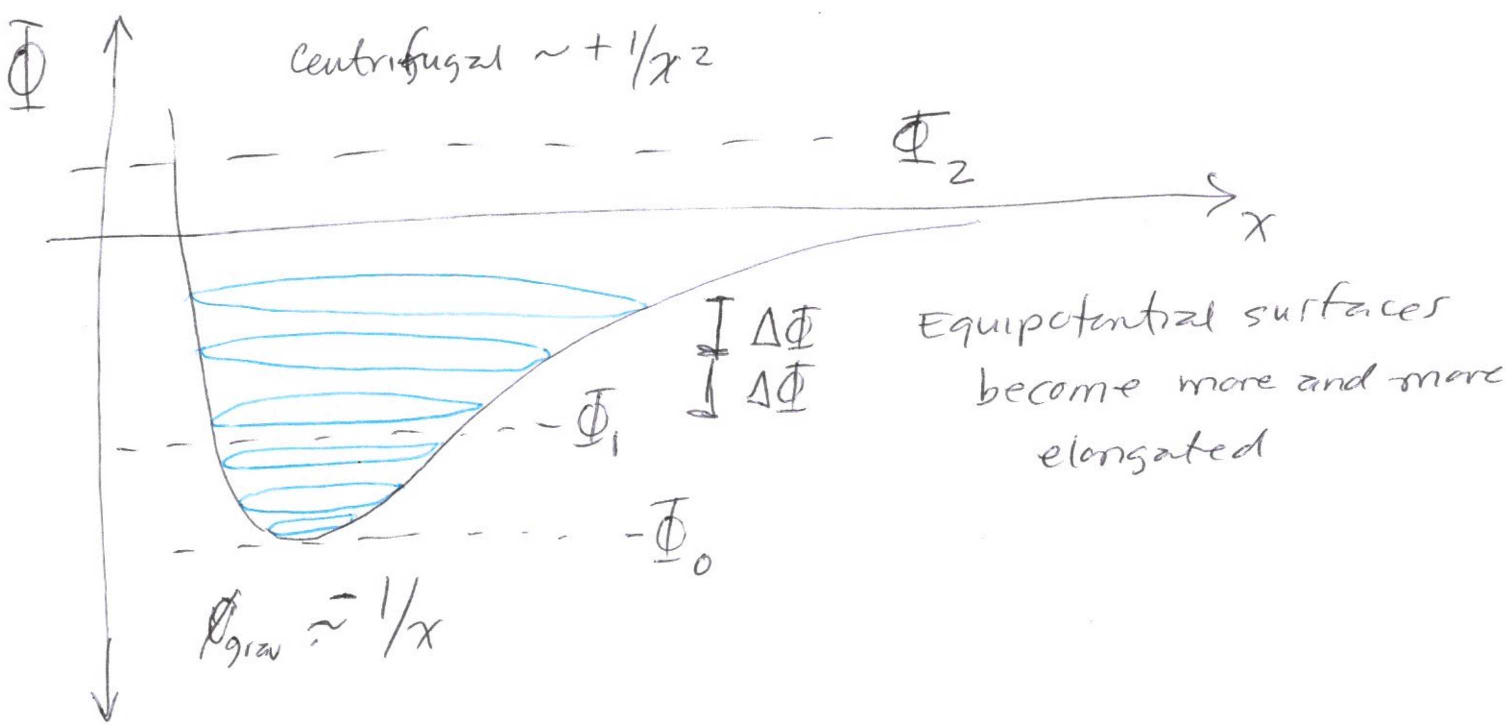
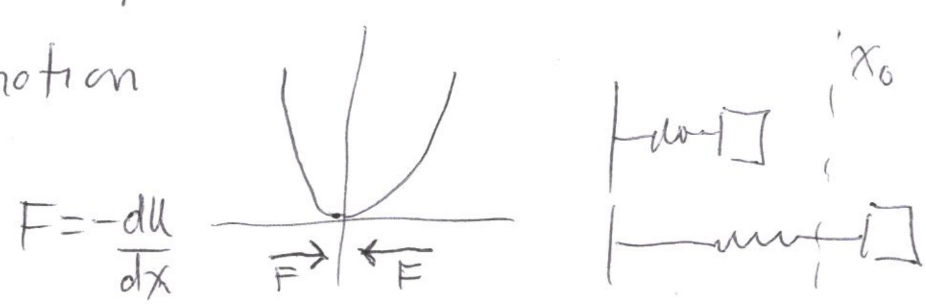
Effective gravitational potentials & corrections

Consider the effective potential



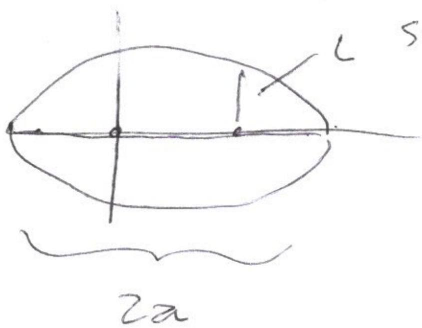
There is kinetic energy, but some is "trapped" in the rotational motion, so if you have $-\Phi_0$, the $|F_{grav}| = |F_{cent}|$ gravitational pull is perfectly cancelled by the centripetal push and the orbit is perfectly circular.

If there is more energy in the orbital motion, say $-\Phi_1$, then that extra energy is accomodated by making the orbit elliptical. Consider the case of simple harmonic motion



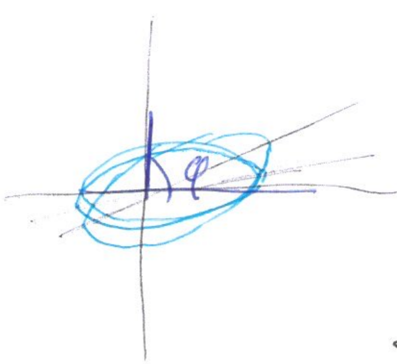
More energy would be accomodated by a larger radius if we had a harmonic potential, but since it has an asymmetry the "turning points" occur at different x.

In the elliptical orbit, these "turning points" are the apastron $r_{max} = a(1+e)$ and periastron $r_{min} = a(1-e)$



semi latus rectum $L = a(1-e^2)$

Since the 2-body system is itself rotating, in general r_{max} and r_{min} will not occur at the same spot.

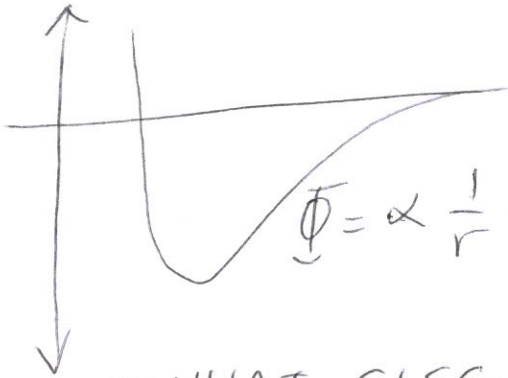


The angle φ changes with time, this is called the "precesion" of the elliptical orbit.

★ HAVE YOU HEARD ABOUT THE PRECESION OF MERCURY?

★ DOES THE EARTH PRECEDE?

Multipole expansion



$$\Phi = \alpha \frac{1}{r} + \beta \frac{1}{r^2} + \gamma \frac{1}{r^4} + \delta \frac{1}{r^8} + \ldots$$

*WHAT ELSE COULD AFFECT THE SHAPE OF THE POTENTIAL? General relativity, mass distributions

The sun is elongated at ~~the~~ its equator, since it is not perfectly spherical, it will have small dipole, quadrupole, etc. corrections.

Since mass itself affects the "shape" of space-time, we can find an ~~Newtonian~~ effective potential that encapsulates the GR effects.

The precession of the orbit of mercury was used to demonstrate that GR ~~was~~ is correct.

$$\Delta\varphi = \frac{6\pi MG}{Lc^2} = \frac{6\pi MG}{ac^2(1-e^2)} \quad \text{per revolution}$$

with $M \simeq M_\odot = 2\times10^{30}$ kg $\quad e = 0.20$

$a = 5.79\times10^{10}$ m

$$\Delta\varphi = \frac{6\pi(2\times10^{30}\,\text{kg})(6.67\times10^{-11}\,\text{m}^3/\text{kg s}^2)}{5.79\times10^{10}\,\text{m}\,(3\times10^8\,\text{m/s})^2(1-0.2^2)} = \frac{2.51\times10^{21}\,\text{m}^3/\text{s}^2}{4.92\times10^{27}\,\text{m}^3/\text{s}^2} = 5.1\times10^{-7}\,\text{rad}$$

The orbital period of Mercury is 88 days,

so in 1 century. $\frac{100 \text{ years}}{1 \text{ century}} \cdot \frac{365.25 \text{ days}}{1 \text{ year}} = 36525$ days

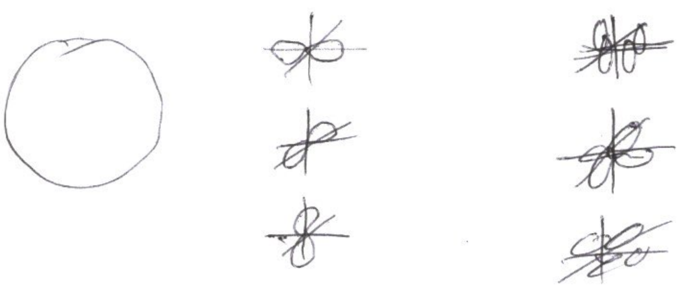
it completes $\frac{36525 \text{ days}}{88 \text{ days}} = 415$ orbits

so it precedes $\left(2.1 \times 10^{-4} \text{ radians}\right)\left(\frac{2.06 \times 10^{5} \text{ seconds}}{\text{radian}}\right)$ ~~206264 arcsec~~

<u>43</u> arcseconds per century   3600 seconds in 1°

Famous number.

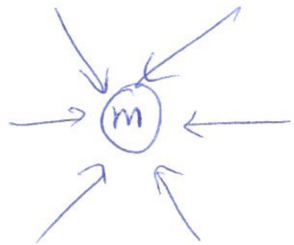
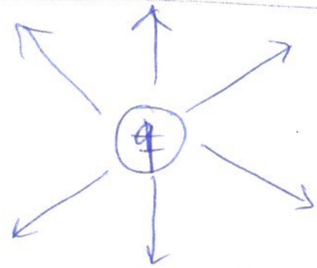
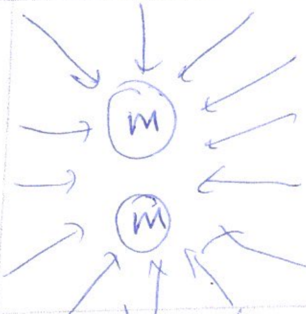
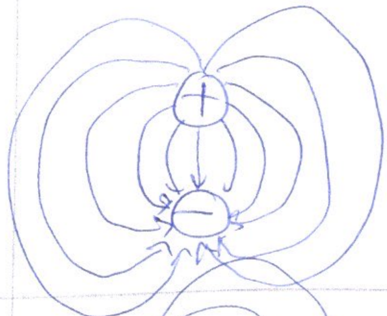
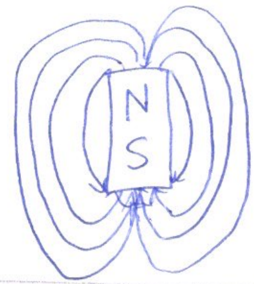
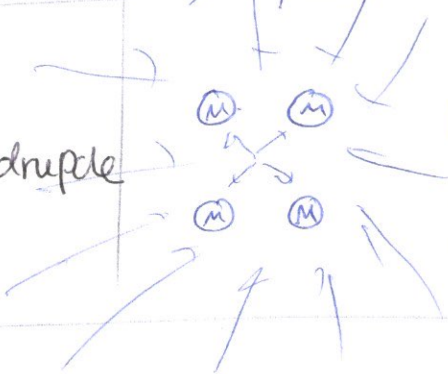
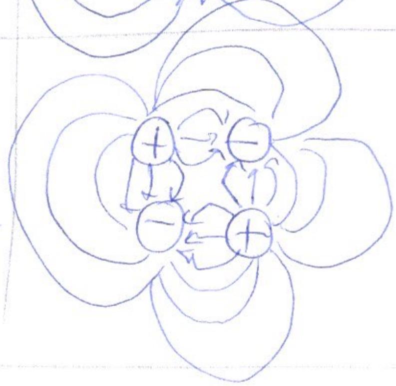
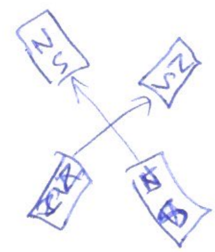
The different multipole corrections are actually the spherical harmonics, so they look like electronic orbitals



etc

s   p   d   f

(1 pole) (2 poles) (4 poles) (8 poles)

	Gravity	Electricity	Magnetism	
monopole			N/A	$U \sim 1/r$
dipole				$U \sim 1/r^2$
quadrupole				$U \sim 1/r^3$

☆DOES MASS IN MONOPOLE CONFIGURATION RADIATES? It can't due to conservation of mass-energy

IN DIPOLE? It can't due to conservation of momentum

IN QUADRUPOLE? Yes.

If you rotate an electric dipole you produce light radiation, if you rotate a gravitational quadrupole you produce gravitational radiation (gravitational waves).

Something important to notice is that as the system radiates energy, it goes down in the potential, so eventually the orbit becomes circular.

Consider the Hulse-Taylor ~~pa~~ binary pulsar PSR 1913+16 discovered in 1974 (Nobel prize 1993)

$m_1 = 1.44\, M_\odot$

$m_2 = 1.39\, M_\odot$

(smaller than the orbit of Mercury)

$e = 0.617$

$T = 0.32 \text{ days} \Rightarrow a^3 = \frac{GMT^2}{2\pi} = 1.95 \times 10^9 \text{ m}$

$\dot{T} = -2.29 \times 10^{-12}$