Before we looked at radiative energy transport in the context of stars. The energy per volume (energy density) is $\ell(\hat{n}, \vec{x}, \nu, t)\, d^2\hat{n}\, d\nu$. There were 4 processes that can change the energy density:

- Transport (but nothing happens to photons)
- Absorption (uncorrelated photon motion)
- Scattering (correlated photon motion)
- Emission (thermal and nuclear)

Scattered photons do not travel in a fixed direction and transport does not affect photons. We will focus then on ~~scattering~~ absorption and emission, the phenomena that will affect what an observer looking in a straight line would observe.

Remember that the fraction of absorvers is $H_{abs}(\nu, \vec{x})\rho(\vec{x})$. We will call this $K(\nu, \vec{x})$. The rate of emission $j(\nu, \vec{x})\rho(\vec{x})$ will be $J(\nu, \vec{x})$. The time-independent radiation transport equation is then

$$\hat{n} \cdot \nabla \ell(\nu, \hat{n}, \vec{x}) = -K(\nu, \vec{x})\ell(\nu, \hat{n}, \vec{x}) + J(\nu, \vec{x})/4\pi c$$

If we take only a straight line, with fixed direction $\hat{n}$ towards the observer, we can drop $\hat{n}\cdot$. Let $\vec{x} = s\hat{n}$, then

$$\frac{d}{ds}\ell(\nu, s) = -K(\nu, s)\ell(\nu, s) + J(\nu, s)/4\pi c$$

If the absorption is negligible, $K(\nu,s) \approx 0$,

$$\frac{d}{ds} \ell(\nu,s) = J(\nu,s)/4\pi c$$

$$\int_{s_1}^{s} d\ell(\nu,s) = \frac{1}{4\pi c}\int_{s_1}^{s} J(\nu,s)\,ds$$

$$\ell(\nu,s) = \ell(\nu,s_1) + \frac{1}{4\pi c}\int_{s_1}^{s} J(\nu,s)\,ds$$

s_1 is a convenient referencepoint. It could be the location of a light source, or the back of an interstellar cloud.

If the absorption is not negligible, define the optical depth

$$\tau(\nu,s) = \int_{s_1}^{s} K(\nu,s')\,ds' \qquad \underline{\underline{W3.1.4}} \qquad \tau \approx 0 \text{ if } s \approx s_1$$

$$d\tau(\nu,s) = K(\nu,s')\,ds' \Rightarrow K(\nu,s) = \frac{d\tau(\nu,s)}{ds}$$

$$\frac{d\ell(\nu,s)}{ds} = -K(\nu,s)\,\ell(\nu,s) + J(\nu,s)/4\pi c$$

$$\frac{d}{ds}\ell(\nu,s) = -\frac{d\tau(\nu,s)}{ds}\,\ell(\nu,s) + J(\nu,s)/4\pi c$$

$$\frac{d}{ds}\ell(\nu,s) = d\tau(\nu,s)\left[-\frac{\ell(\nu,s)}{ds} + \frac{J(\nu,s)/4\pi c}{d\tau(\nu,s)}\right]$$

$$\frac{d}{d\tau}\ell(\nu,s) = ds\left[-\frac{\ell(\nu,s)}{ds} + \frac{J(\nu,s)/4\pi c}{d\tau(\nu,s)}\right]$$

$$\frac{d}{d\tau} \ell(\nu,s) = \cancel{ds}\left[-\frac{\ell(\nu,s)}{\cancel{ds}} + \frac{J(\nu,s)/4\pi c}{K(\nu,s)\,\cancel{ds}}\right]$$

$$e^{-\tau(\nu,s)}\frac{d\ell(\nu,s)}{d\tau(\nu,s)} = -e^{-\tau(\nu,s)}\ell(\nu,s) + \frac{J(\nu,s)/4\pi c}{K(\nu,s)}\,e^{-\tau(\nu,s)}$$

But

$$\frac{d}{d\tau(\nu,s)}\left[e^{-\tau(\nu,s)}\ell(\nu,s)\right] = \ell(\nu,s)\frac{d}{d\tau(\nu,s)}e^{-\tau(\nu,s)} + e^{-\tau(\nu,s)}\frac{d\,\ell(\nu,s)}{d\tau(\nu,s)}$$

$$= -e^{-\tau(\nu,s)}\ell(\nu,s) + e^{-\tau(\nu,s)}\frac{d}{d\tau(\nu,s)}\ell(\nu,s)$$

$$\Rightarrow e^{-\tau(\nu,s)}\frac{d}{d\tau(\nu,s)}\ell(\nu,s) = \frac{d}{d\tau(\nu,s)}\left[e^{-\tau(\nu,s)}\ell(\nu,s)\right] + e^{-\tau(\nu,s)}\ell(\nu,s)$$

so

$$\frac{d}{d\tau(\nu,s)}\left[e^{-\tau(\nu,s)}\ell(\nu,s)\right] = \frac{J(\nu,s)/4\pi c}{K(\nu,s)}\,e^{-\tau(\nu,s)}$$

$$\int_{s_1}^{s} d\left[e^{-\tau(\nu,s)}\ell(\nu,s)\right] = \int_{s_1}^{s}\frac{J(\nu,s)/4\pi c}{K(\nu,s)}\,e^{-\tau(\nu,s)}\,d\tau(\nu,s)$$

$$\cancel{\ell(\nu,s)\,de^{-\tau(\nu,s)} + e^{-\tau(\nu,s)}\,d\ell(\nu,s)}$$

$$\ell(\nu,s)\,e^{-\tau(\nu,s)}\,d\tau + e^{-\tau(\nu,s)}\,d\ell(\nu,s)$$

Assume that, even though $J(\nu,s)$ and $K(\nu,s)$ depend explicitely on s, their ratio does not,

so $\dfrac{J(\nu,s)/4\pi c}{K(\nu,s)} = \dfrac{J(\nu)}{4\pi c K(\nu)}$ can be taken out of the integral

$$I(\nu,s) = I(\nu,s_1)\, e^{-\tau(\nu,s)} + \frac{J(\nu)}{4\pi c K(\nu)} \int_{s_1}^{s} e^{-\tau(\nu,s)}\, d\tau(\nu,s')$$

Eventually,

$$I(\nu,s) = e^{-\tau(\nu,s)} I(\nu,s) + \frac{J(\nu)}{4\pi c K(\nu)} \left[1 - e^{-\tau(\nu,s)}\right] \qquad \text{W3.1.6}$$

In practice, $\dfrac{J(\nu)}{K(\nu)}$ is an average of $\dfrac{J(\nu,s)}{K(\nu,s)}$

☆ HOW ARE THEY CALCULATED?

In 1917, Einstein defined a quantity A_a^b as the rate at which an atom spontaneously makes a transition from energy level E_a to lower energy level E_b emitting a photon frequency $\nu_{ab} = \dfrac{E_a - E_b}{h}$. The quantities are called Einstein coefficients.

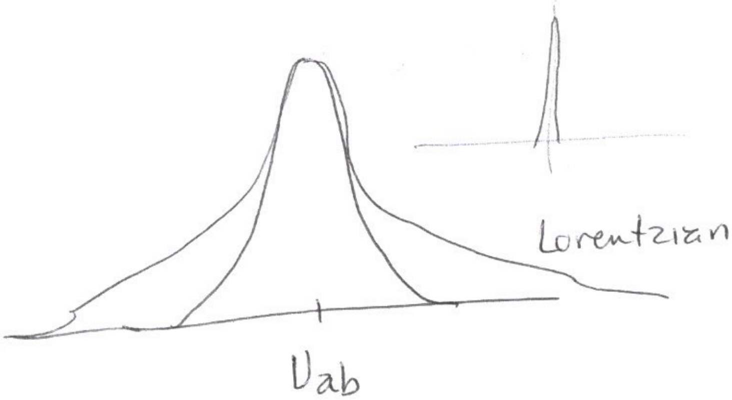
The frequencyes are not perfectly sharp. The uncertainty principle states that $\Delta E \Delta t \geq \hbar/2$

$$\Rightarrow \Delta E \geq \frac{\hbar}{2\Delta t}$$

In order for the frequency to be perfectly sharp, $\Delta t \to \infty$, but photons are not produced continuosly, the transition occurs over some finite Δt and hence $\Delta E \neq 0$.

The atoms or molecules producing the radiation are moving in random directions, so they will have a small Dopler shift that also contributes to the linewidth of the emission.

If the probability that the phothon is emitted with frequency in the range $[\nu, \nu + d\nu]$ is $\phi(\nu)d\nu$, sharply peaked at $\nu = \nu_{ab}$ with $\int \phi(\nu) d\nu = 1$, the rate of emission per volume, per solid angle, and per frequency interval is $\phi(\nu) A_a^b n_a / 4\pi$ where n_a is the number density of atoms in energy level E_a



$\phi(\nu) > 0$ otherwise forbiden

A_a^b probability usually depends on energy difference between states

n_a self explanatory, each atom has same probability

It takes a photon a time $t = \frac{ds}{c}$ to travel a distance ds, so the amount of additional radiant energy per distance is $\frac{J(\nu)}{4\pi c}$ per solid angle per frequency is $h\nu\phi(\nu)A_a^b n_a/4\pi c$, so

$$J(\nu) = h\nu\phi(\nu)A_a^b n_a$$

Applying Dopler shift: $\nu = \nu_{ab}\left[1 - v/c\right]$ where v is the velocity in the line of sight

☆WHAT IS THE DISTRIBUTION OF THE VELOCITIES WILL BE? MAXWELL-BOLTZMAN

in 1-D $P(v) = \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(-m(v-\bar{v})^2/2k_B T\right)$

so $\phi(\nu) = \frac{c}{\nu_{ab}} P\left(c\left(1 - \nu/\nu_{ab}\right)\right)$

it is the same for absorption, althoug that one includes stimulated emission, so

$$K(\nu) = h\nu\phi(\nu)\left(\underbrace{B_b^a n_b}_{\text{absorb}} - \underbrace{B_a^b n_a}_{\text{emitt}}\right)/c$$

To work out the coefficients $\frac{n_a}{n_b} = \frac{g_a e^{-E_a/k_B T}}{g_b e^{-E_b/k_B T}}$ Boltzman distribution