HEATING AND COOLING IN ISM

Consider an HII region of space that is produced due to starlight. Consider also the definition of heat : energy in flux. (Heating and cooling are the same thing, although the direction of the energy flux is inverted).

The HII region is heated up by the starlight, which means that energy is transferred from the photons to the ISM. ✱ WHAT ARE THE POTENTIAL MECHANISMS FOR ENERGY TRANSFER? REMEMBER THAT THE MAIN COMPONENTS ARE PROTONS & ELECTRONS

?) Absorption of photons by free electrons

$$E^2 = (mc^2)^2 + p^2c^2$$ ✱ Relativistic energy and momentum

$$E_f^2 = E_0^2$$ ✱ must be conserved

$$E_{e0}^2 = m_e^2 c^4 + c^2 p_{e0}^2 \; ; \; E_{ef}^2 = m_e^2 c^4 + c^2 p_{ef}^2$$

$$E_{p0} = p_{p0} c \; , \; E_{pf} = p_{pf} c$$

since m_e and c are constants, $m_e^2 c^4$ is constant, so the only way the electron can change its energy is by changing its momentum. (Makes sense)

Let q be the photon momentum

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$$p_0 + q = p_f \quad \text{if absorbed}$$

Conservation of energy $\quad E = \left[(mc^2)^2 + p^2c^2\right]^{1/2}$

$$\left\{\left[(m_ec^2)^2 + p_0^2c^2\right]^{1/2} + qc\right\}^2 = \left\{\left[(m_ec^2)^2 + p_f^2c^2\right]^{1/2}\right\}^2$$

$$\cancel{(m_ec^2)^2} + p_0^2c^2 + 2qc\left[(m_ec)^2 + p_0^2c^2\right]^{1/2} + q^2c^2 = \cancel{(m_ec^2)^2} + p_f^2c^2$$

$q = p_f - p_0$, so

$$p_0^2c^2 + 2(p_f - p_0)c\sqrt{m_e^2c^4 + p_0^2c^2} + q^2c^2 = p_f^2c^2$$

$$\cancel{c^2}\left[p_0^2 + 2(p_f - p_0)\sqrt{m_e^2c^2 + p_0^2} + q^2\right] = \cancel{c^2}\,p_f^2$$

$$p_0^2 - p_f^2 + (p_f - p_0)^2 + 2(p_f - p_0)\sqrt{m_e^2c^2 + p_0^2} = 0$$

$$p_0^2 - \cancel{p_f^2} + \cancel{p_f^2} - 2p_fp_0 + p_0^2 + 2(p_f - p_0)\sqrt{m_e^2c^2 + p_0^2} = 0$$

$$\cancel{2}p_0^2 - \cancel{2}p_fp_0 = -\cancel{2}(p_f - p_0)\sqrt{m_e^2c^2 + p_0^2}$$

$$p_fp_0 - p_0^2 = (p_f - p_0)\sqrt{m_e^2c^2 + p_0^2} = p_0(p_f - p_0)$$

One solution is $p_f = p_0$ ... but this implies that the momentum of the electron can't change!

if the momentum of the electron can't change, then the photon can't be absorbed! So a free electron can't absorb a photon, so this is not a possible way to heat up the ISM. Even if we divide

$$p_0 \cancel{(p_f - p_0)} = \cancel{(p_f - p_0)} \sqrt{m_e^2 c^2 + p_0^2}$$

$$p_0 = \sqrt{m_e^2 c^2 + p_0^2}$$

In the frame of reference in which the electron is initially at rest, $p_0 = 0$, so $m_e^2 c^2 = 0$. The only way for this to happen is for the mass of the electron to be zero, ~~but it In that cas~~ which can't happen.

★ CAN BOUND ELECTRONS ABSORB PHOTONS?

?) Compton Scattering

In this situation the photon is not absorbed, instead, the ~~photon is~~ momentum of the photon changes, transfering energy to the electron.

$$\Delta E = \frac{E^2 (1 + \cos^2 \theta)}{m_e c^2 - E(1 + \cos^2 \theta)}$$

where θ is the angle between the original and final direction

This is maximum for $m_e c^2 - E(1+\cos^2\theta) \to 0$

$$m_e c^2 = (9.11 \times 10^{-31}\,\text{kg})(3 \times 10^8 \tfrac{m}{s})^2 = 8.2 \times 10^{-14}\,J$$

$$\max\{1+\cos^2\theta\} = 2$$

$$h\nu = E = \frac{1}{2}\, 8.2 \times 10^{-14}\,J = 4.1 \times 10^{-14}\,J$$

$$\nu = \frac{4.1 \times 10^{-14}\,J}{6.626 \times 10^{-34}\,J \cdot s} = 6.2 \times 10^{19}\,Hz = 62\ EHz$$

This is in the range of Hard x-rays or gamma rays while not impossible, quite infrequent not in supernovas.

In the UV, $(6.626 \times 10^{-34}\,J\text{-}s)(\cancel{6.626}\,60 \times 10^{15}/s) = 4 \times 10^{-17}\,J$ ← PHz
ultraviolet

$$\frac{4 \times 10^{-17}\,J}{8 \times 10^{-14}\,J} \approx 0.0005\, m_e c^2,$$

so $\Delta E \approx 2 \times 10^{-20}$, so it is negligible.

As long as there is an appreciable amount of neutral hydrogen, the dominant mechanism for heating will be <u>photoionization</u>. The photon will knock-off the electron from the hydrogen atom, the energy of the electron will be the original energy of the photon minus the binding energy $E_I = 13.6\ eV = 2.18 \times 10^{-18}\,J$

What is the weighted average energy of the photoelectrons? 150

$$\overline{\Delta E} = \frac{\int_{E_I}^{\infty} \overbrace{(E - E_I)}^{\text{electron energy}} L(E)\, dE}{\int_{E_I}^{\infty} L(E)\, dE}$$

How many photons of each energy E

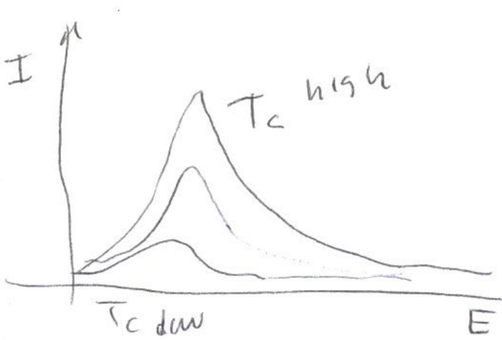
All photons emitted per unit time.

from the definition of the mean, where $L(E)$ is the number of photons per unit time of energy E. To a good approximation, since the photons are emitted by a star, $L(E) \propto \frac{E^2}{\exp(E/k_B T_c) - 1}$ which is the black-body radiation at "color temperature" T_c

In the limit $k_B T_c \ll E_I \Rightarrow T_c \ll \frac{13.6\,eV}{8.62 \times 10^{-5}\, eV/K}$

$$T_c \ll 1.6 \times 10^5\, K$$

the integrals are dominated by energies just above the Ionization. (so even though they go to infinite, there is no much intensity there)



with $E = E_I + w \quad dE = dw \quad$ ~~but~~ $E_I = E$

$$\overline{\Delta E} = \frac{\int_0^\infty (\cancel{E_I} + w - \cancel{E_I})(E_I + w)^2 / \exp\left((E_I + w)/k_B T_c\right) dw}{\int_0^\infty (E_I + w)^2 / \exp\left((E_I + w)/k_B T_c\right) dW}$$

$$E_I >> w, \text{ so } (E_I + w)^2 \approx E_I^2$$

$$= \frac{\int_0^\infty w \cancel{E_I^2} \exp\left[-(E_I + w)/k_B T_c\right] dw}{\int_0^\infty \cancel{E_I^2} \exp\left[-(E_I + w)/k_B T_c\right] dw}$$

Let $aw = -\frac{E_I}{k_B T_c} - \frac{w}{k_B T_c} \qquad a = -\frac{1}{k_B T_c}$

$$= \frac{\int_0^\infty w \exp\left(-w/k_B T_c\right) dw}{\int_0^\infty \exp\left(-w/k_B T_c\right) dw} = \frac{\cancel{e^{-w/k_B T_c}} \left[\frac{-w k_B T_c}{1} - \frac{1}{k_B^2 T_c^2}\right]_0^\infty}{\cancel{e^{-w/k_B T_c}} \left(-\frac{1}{k_B T_c}\right)\Big|_0^\infty}$$

$$= k_B T_c$$

For the case $k_B T >> E_I$, we can ignore the lower bound, so

$$\overline{\Delta E} \approx \frac{\int_0^\infty E^3 / [\exp(E/k_B T) - 1] \, dE}{\int_0^\infty E^2 / [\exp(E/k_B T) - 1] \, dE}$$

This is a well known integral in thermodynamics, but pretty long.

$$\overline{\Delta E} \approx \frac{\pi^4}{30 \zeta(3)} k_B T_c$$, where $\zeta(3)$ is the Riemann zeta function.

so ΔE is between $k_B T_c$ and $2.7 \, k_B T_c$

In equilibrium, the rate of ionization ~~as~~ has to be equal to the rate of recombinations, $\alpha n_e n_p$. The heating function is $\Gamma = \alpha n_e n_p \overline{\Delta E}$