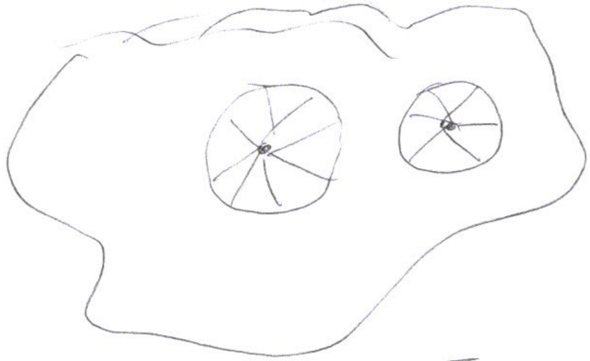
HII regions 11/3/2020 (138)



O, B stars inside HI cloud. Their radiation is energetic enough that it ionizes the hydrogen, separating protons and electrons. The protons and electrons will recombine, but this recombination might be overwhelmed by the radiation from the star. The energy density of the radiation decreases since it is a sphere (with distance from the star), so at some point (distance r) the ionization and recombination processes will be in equilibrium. There will be only ionized hydrogen (HII) inside the sphere and neutral hydrogen (HI) outside the sphere.

☆ CAN THE SUN CREATE SUCH A BUBBLE??

Because HI is opaque to this radiation, the surface of the sphere is distinct. The protons and electrons will recombine initially in excited states, and as they move to their ground state, they emitt radiation in other wavelengths, particularly Lyman α. These bubbles are called <u>Strömgren spheres</u>.

Consider a ~~uka~~ uniform medium of neutral hydrogen but with a Strömgren sphere inside. The <u>number density</u> is n and the fraction of neutral hydrogen at distance r from the star is $\xi(r)$ (xi). Hence, the number density of hydrogen is $\xi(r)n$

and the number densities of protons and electrons is $[1-\xi(r)]n$. if the star emitts L photons per time isotropically in all directions, the flux at r is

$$\Phi(r) = \frac{L}{4\pi r^2}\exp(-\tau(r)) \qquad \underline{\underline{W3.2.1}}$$

This look like the typicall flux ~~pas~~ from a point source with spherical symmetry, <u>but</u> with an attenuation factor <u>except</u> that the attenuation is ~~not~~ a functional of the position and not just a function.

$$\tau(r) = \int_{r_0}^{r} K(\nu,s')\,ds' = \int_{r_0}^{r} n\sigma\xi(r')\,dr$$

number density of hydrogen

fraction of hydrogen

where σ is an absorption (in this case due to photo~~ionization~~ -ionization) cross-section.

The number of ionizations per volume per time at r is

$$\underbrace{\xi(r)n}_{\text{How many hydrogens}}\;\underbrace{\sigma}_{\text{How efficent "area" covered}}\;\Phi(r) \qquad \frac{\#_h}{V}\,A\,\frac{\#_p}{A\cdot t}$$

← How many photons per area per time

Number of recombinations is proportional to the number density of protons and electrons

$$\alpha\left[n(1-\xi(r))\right]^2 = \alpha n^2(1-\xi(r))^2$$

α is an effectiveness parameter with units V/t

$$\frac{V}{t}\,\frac{\#^2}{V^2} = \frac{\#^2}{t}$$

← pairs per time

The condition for equilibrium is thus:

$$\xi(r) n \sigma \Phi(r) = \alpha n^2 (1 - \xi(r))^2$$

so $\Phi(r) = \dfrac{\alpha n^2 (1-\xi(r))^2}{\xi(r) n \sigma}$ ~~using W3.2.1~~

using W 3.2.1

$$\frac{\alpha n^2 (1-\xi(r))^2}{\xi(r) n \sigma} = \frac{L}{4\pi r^2} \exp(-\tau(r))$$

$$\Rightarrow \exp(\tau(r)) = \frac{L\sigma}{4\pi r^2 \alpha n} \frac{\xi(r)}{[1-\xi(r)]^2}$$ W 3.2.4

☆ HOW DOES THIS QUANTITY DEPEND ON R?

On the left hand side,

$$\frac{de^{\tau(r)}}{dr} = e^{\tau(r)} \frac{d\tau(r)}{dr} = e^{\tau(r)} \frac{d}{dr} \int_{r_0}^{r} n\sigma \xi(r') dr' = e^{\tau(r)} n \sigma \xi(r)$$

with the right hand side,

$$e^{-\tau(r)} \frac{L\sigma}{4\pi\alpha n} \frac{d}{dr}\left(\frac{\xi(r)}{r^2 [1-\xi(r)]^2} \right) = \cancel{e^{\tau(r)}} n\sigma\xi(r) \cancel{e^{-\tau(r)}}$$

since $e^{-\tau(r)} = \dfrac{4\pi r^2 \Phi(r)}{L}$,

$$\frac{\cancel{4\pi}\,\Phi(r)}{\cancel{\Delta}}\;\frac{\cancel{\Delta}\,\sigma}{\cancel{4\pi}\,\alpha n}\;r^2\frac{d}{dr}\left(\frac{\xi(r)}{r^2[1-\xi(r)]^2}\right)=n\sigma\,\xi(r)$$

$$r^2\frac{d}{dr}\left(\frac{\xi(r)}{r^2[1-\xi(r)]^2}\right)=\frac{n\sigma\,\xi(r)\,\alpha n}{\Phi(r)\,\sigma}$$

Remember that $\xi(r)\,n\sigma\,\Phi(r)=\alpha n^2(1-\xi(r))^2$, W3.2.3

so

$$\frac{\xi(r)}{[1-\xi(r)]^2}=\frac{\alpha n^{\cancel{2}}}{\cancel{n}\sigma\,\Phi(r)}$$

and

$$r^2\frac{d}{dr}\left(\frac{\xi(r)}{r^2[1-\xi(r)]^2}\right)=n\sigma\,\frac{\xi^2(r)}{[1-\xi(r)]^2}\qquad\text{W3.2.5}$$

In equation W3.2.5, $n\sigma$ are astrophysical parameters. The other ones enter through the intial condition of the differential equation. From W3.2.4,

~~[illegible]~~ for $\tau(r_0)=0$ the beginning of the bubble

$$\exp(\tau(r_0))=1=\frac{L\sigma}{4\pi r_0^2\,\alpha n}\,\frac{\xi(r_0)}{[1-\xi(r_0)]^2}$$

$$\Rightarrow\quad\frac{\xi(r_0)}{[1-\xi(r_0)]^2}=\frac{4\pi r_0^2\,\alpha n}{L\sigma}$$

W 3.2.5 has analytic solutions in two cases. The first one is inside the sphere. Since by definition the inside of the bubble has almost no neutral hydrogen, its fraction $\xi(r) \approx 0$, so $[1-\xi(r)]^2 \approx 1$.

In this case,

$$r^2 \frac{d}{dr}\left(\frac{\xi(r)}{r^2}\right) = n\sigma \xi^2(r) \qquad \underline{\underline{W\ 3.2.7}}$$

~~$d(\xi(r)/r^2) = n\sigma$~~ Let $y = \frac{\xi(r)}{r^2}$, then

$$\frac{\xi(r)}{y}\frac{dy}{dr} = n\sigma\xi^2(r) \Rightarrow \frac{1}{y}\frac{dy}{dr} = n\sigma y r^2$$

so $$\int \frac{dy}{y^2} = n\sigma \int_{r_0}^{r} r^2 dr$$

Integrating on both sides we get

$$-\frac{1}{y} + K = \frac{n\sigma}{3}(r^3 - r_0^3) \;\overline{\overline{\cancel{\Rightarrow}}}\; -\frac{r^2}{\xi(r)} + K$$

~~at~~ @ $r = r_0$,

$$-\frac{r_0^2}{\xi(r_0)} + K = \frac{n\sigma}{3}(r_0^3 - r_0^3) = 0 \Rightarrow K = \frac{r_0^2}{\xi(r_0)}$$

Hence,

$$-\frac{r^2}{\xi(r)} + \frac{r_0^2}{\xi(r_0)} = \frac{n\sigma}{3}(r^3 - r_0^3)$$

$$+\frac{r^2}{\xi(r)} = -\frac{n\sigma}{3}\left(r^3 - r_0^3\right) + \frac{r_0^2}{\xi(r_0)}$$

$$\frac{\xi(r)}{r^2} = \left[r_0^2/\xi(r_0) - n\sigma\,(r^3 - r_0^3)/3\right]^{-1} \qquad \text{W3.2.8}$$

Let's analyze this eq. Remember that a cross-section times a density gives you a meanfree path to the -1

$$n\sigma \rightarrow \frac{\#}{m^{\cancel{3}}}\,\cancel{m^2} = \#/m$$

$d \equiv 1/n\sigma$ with units of meters per collision
length.

L is the number of photons emitted in all directions per unit time. If we consider the radius R_s of the ~~co~~ bubble of completely ionized gas, right after R_s there is one recombination for every photon emitted by the star. Hence, the number of recombinations per volume per time is $\alpha n^2\left(1-\xi(R_s)\right)^2 \approx \alpha n^2$, so number of recombinations per time is

$$\alpha n^2\left(\frac{4}{3}\pi R_s^3\right) = L \qquad \text{W3.2.10}$$

using W3.2.6, $L = \dfrac{4\pi r_0^2 \alpha n}{\sigma \xi(r)}$

$$\frac{4\pi r_0^2 \alpha n}{\sigma \xi(r)} = \alpha n^2 \frac{4\pi}{3} R_s^3 \Rightarrow R_s^3 = \frac{3 r_0^2}{\sigma n \xi(r_0)} = \frac{3 d r_0^2}{\xi(r_0)}$$

$$\Rightarrow \quad \frac{\xi(r_0)}{r_0^2} = \frac{3d}{R_s^3} \quad \Rightarrow \quad \frac{r_0^2}{\xi(r_0)} = \frac{R_s^3}{3d}$$

we can now combine this with W3.2.8 to get

$$\frac{\xi(r)}{r^2} = \left[R_s^3/3d - (r^3 - r_0^3)/3d \right]^{-1} = \frac{3d}{R_s^3 - r^3 + r_0^3}$$

$$\xi(r) = \frac{3dr^2}{R_s^3 - r^3 + r_0^3}$$

$3 \times 10^{16} - 3 \times 10^{17}$ m

For typical strömgren spheres, R_s is 10–100 pc, d is 0.1 pc, whereas r_0, the radius of the star might be 5×10^{-6} pc

3×10^{15} m $\qquad$ 5×10^{10} m.

we can thus leave r_0^3 out.

$$\xi(r) = \frac{3dr^2}{R_s^3 - r^3} \qquad \underline{\underline{W3.2.11}}$$

if $r < R_s$, then $\xi(r) \sim d/R_s \ll 1$

The other situation in which we can get an analytical solution via an approximation is at the surface of the sphere. This is close to R_s, and is a very narrow region compared R_s, so

$$r^2 \frac{d}{dr}\left(\frac{\xi(r)}{r^2(1-\xi(r))^2}\right) = n\sigma \frac{\xi^2(r)}{[1-\xi(r)]^2}$$

$$\approx \frac{R_s^2}{R_s^2} \frac{d}{dr}\left(\frac{\xi(r)}{[1-\xi(r)]^2}\right) = n\sigma \frac{\xi^2(r)}{[1-\xi(r)]^2} = \frac{1}{d} \frac{\xi^2(r)}{[1-\xi(r)]}$$

$$\frac{r_b - r_a}{d} = \int_{\xi_a}^{\xi_b} \frac{(1-\xi)^2}{\xi^2} \frac{d}{d\xi}\left(\frac{\xi}{(1-\xi)^2}\right) d\xi = -\frac{1}{\xi} + 2\ln\left(\frac{\xi}{1-\xi}\right)\Bigg|_{\xi_a}^{\xi_b}$$

Now, remember that $J(\nu) = h\nu\phi(\nu) A_a^b n_a$

The Einstein coefficient is probability per unit time, n_a is ~~vol~~ number density. For a Strömgren sphere,

$$J(\nu,s)\,d\nu = \alpha n^2(s)(1-\xi(s))^2 h\nu\phi(\nu)\,d\nu$$

where s is along the line of sight.