Consider a cavity in a perfectly conductive material. The cavity is a cube of edge L. There is an infinite number of electromagnetic modes within this cavity, although of course, not all will be occupied.

The charge in the cavity is zero, since it is a cavity.

From Gauss' Law, $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 = 0$

charge permitivity

electric flux surface integral

In integral form (PHYS 2421) you saw it as $\phi_E = \oiint \vec{E}\cdot d\vec{A}$

electric field area element

In differential form (PHYS 4341) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$

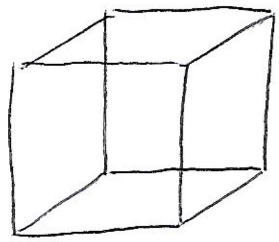
In case you didn't know, "Del" or "Nabla" is shorthand for

$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$, so the dot product

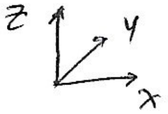
$$\vec{\nabla}\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$ KK Eq. 4.10

is the divergence of the electric field.

Since the walls are conductive, charge is free to move. If there is an electric field, charge will move until it neutralizes that ~~magnetic~~ electric field, and this will happen pretty much instantaneously, so $E_{\parallel} = 0$ (the electric field parallel to the wall is zero.



$x=0$ $\quad$ $x=L$



There are planes at $y=0,\ y=L,\ z=0,\ z=L$

$$\underbrace{(x,0,z)\ (x,L,z)}_{xz\text{ plane}}\ \underbrace{(x,y,0)\ (x,y,L)}_{xy\text{ plane}}$$

$E_{\parallel}=0$, so $E_x = E_z = 0$ $\qquad$ $E_x = E_y = 0$

We ~~are~~ are not going to derive KK Eq. 4.9, but it is a solution to the electromagnetic wave equation. Using $k_x = n_x\pi/L$, $k_y = n_y\pi/L$, $k_z = n_z\pi/L$, we can rewrite

$$E_x = \boxed{E_{x0}}\,\boxed{\sin(\omega t)}\,\boxed{\cos(k_x x)\sin(k_y y)\sin(k_z z)}$$
$$E_y = \boxed{E_{y0}}\,\boxed{\sin(\omega t)}\,\boxed{\sin(k_x x)\cos(k_y y)\sin(k_z z)}$$
$$E_z = \boxed{E_{z0}}\,\boxed{\sin(\omega t)}\,\boxed{\sin(k_x x)\sin(k_y y)\cos(k_z z)}$$

KK Eq. 4.9

Amplitude $\qquad$ temporal part $\qquad$ spatial part

k is called the "wavenumber."

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\frac{\partial E_x}{\partial x} = \frac{\partial}{\partial x}\left[E_{x_0} \overset{u}{\cos(k_x x)}\, \overset{v}{\sin(k_y y)}\, \overset{w}{\sin(k_z z)}\right] =$$

$$E_{x_0}\Big[\cos(k_x x)\sin(k_y y)\cancel{\frac{\partial}{\partial x}\sin(k_z z)}^{\,0}$$

$$+ \sin(k_y y)\sin(k_z z)\frac{\partial}{\partial x}\cos(k_x x)$$

$$+ \sin(k_z z)\cos(k_x x)\cancel{\frac{\partial}{\partial x}\sin(k_y y)}^{\,0}\Big]$$

product rule for three terms

$$\frac{\partial E_x}{\partial x} = -E_{x_0} k_x \sin(k_x x)\sin(k_y y)\sin(k_z z)$$

Similarly,

$$\frac{\partial E_y}{\partial y} = -E_{y_0} k_y \sin(k_x x)\sin(k_y y)\sin(k_z z)$$

$$\frac{\partial E_z}{\partial z} = -E_{z_0} k_z \sin(k_x x)\sin(k_y y)\sin(k_z z)$$

so

$$\vec{\nabla} \cdot \vec{E} = -\left[E_{x_0} k_x + E_{y_0} k_y + E_{z_0} k_z\right]\sin(k_x x)\sin(k_y y)\sin(k_z z) = 0$$

$$\Rightarrow E_{x_0}\frac{n_x \pi}{L} + E_{y_0}\frac{n_y \pi}{L} + E_{z_0}\frac{n_z \pi}{L} = 0$$

$$\Rightarrow E_{x_0} n_x + E_{y_0} n_y + E_{z_0} n_z = \vec{E}_0 \cdot \vec{n} = 0$$ KK Eq. 4.11

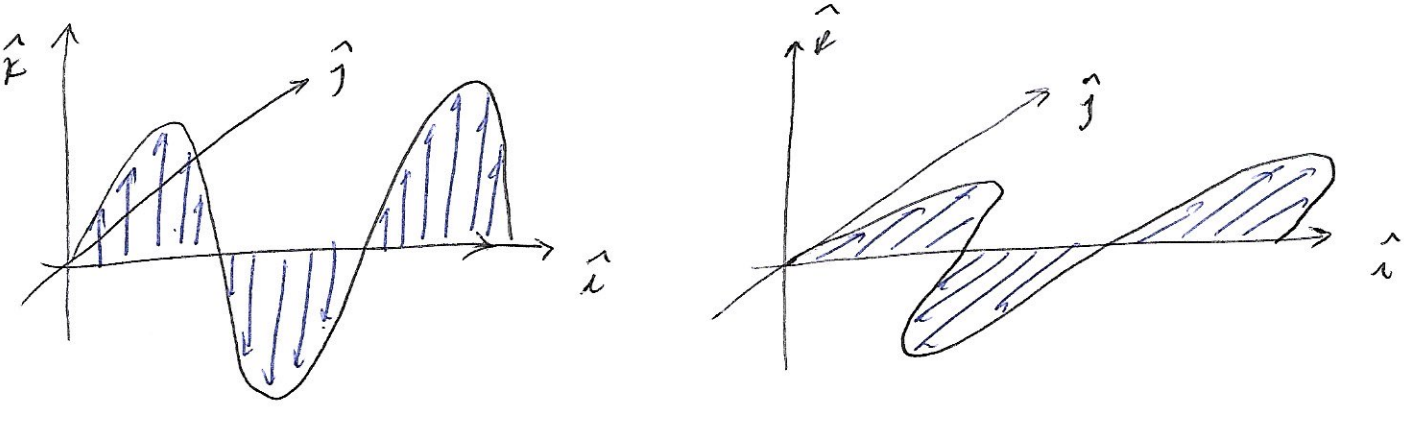
with $\vec{E}_0 = E_{x_0}\hat{\imath} + E_{y_0}\hat{\jmath} + E_{z_0}\hat{k}$ and $\vec{n} = n_x \hat{\imath} + n_y \hat{\jmath} + n_z \hat{k}$

Eq. 4.11 is interesting. One case that satisfies

$\vec{E}_0 \cdot \vec{n} = 0$ is $n_x \hat{i} + 0\hat{j} + 0\hat{k}$ and $E_{x0} = 0$

also $0\hat{i} + n_y \hat{j} + 0\hat{k}$ and $E_{y0} = 0$

$0\hat{i} + 0\hat{j} + n_z \hat{k}$ and $E_{z0} = 0$



standing wave in the direction $\hat{i}$ "rattles" the electric field in the transverse directions.

One way to write the wave equation is $\left(\underset{\text{Laplacian}}{\nabla^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = 0$

In the case of E_z,

$$c^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)E_z = \frac{\partial^2 E_z}{\partial t^2}$$ KK Eq. 4.12

Looking at the spatial part,

$$c^2 \frac{\partial}{\partial z}\frac{\partial E}{\partial z^2} = c^2 \frac{\partial}{\partial z}\left[-E_{z0} k_z \sin(k_x x)\sin(k_y y)\sin(k_z z)\right]$$ ← got it before.

$$= -c^2\left[\underbrace{E_{z0} k_z^2 \sin(k_x x)\sin(k_y y)\cos(k_z z)}_{E_z}\right]$$

$$= -k_z^2 c^2 E_z$$

If we repeat for x, y, we get

$$-c^2\left[E_x k_x^2 + E_y k_y^2 + E_z k_z^2\right] = -c^2 \vec{E} \cdot \vec{k}^2$$

For the temporal part

$$\frac{\partial^2}{\partial t^2} E_z = \frac{\partial^2}{\partial t^2} = -\omega^2 \sin(\omega t) E_z \text{ (stuff)}$$

for the three parts it will be $-\omega^2 \vec{E} \sin(\omega t)$

with boundary conditions, eventually

$$-c^2 \vec{E} \cdot \vec{k}^2 = -\omega^2 \vec{E} \Rightarrow c^2 \vec{k}^2 = \omega^2$$

$$c^2 \frac{\pi^2}{L^2}(n_x^2 + n_y^2 + n_z^2) = \omega^2$$

KK Eq. 4.13 $c^2\pi^2(n_x^2 + n_y^2 + n_z^2) = \omega^2 L^2$

with $n \equiv (n_x^2 + n_y^2 + n_z^2)^{1/2}$, $\omega_n^2 = \frac{c^2\pi^2 n^2}{L^2} \Rightarrow \omega_n = \frac{cn\pi}{L}$ KK Eq. 4.15

KK Eq. 4.14

c is the speed of wave propagation, in this case the speed of light.

Before, we calculated the expectation value of the number of photons of a given angular frequency ω at temperature τ. This was the Plank distribution. By multiplying by $\hbar\omega$, we got the expectation value of the energy for state "s." The total energy, hence, will be $U = 2\sum_n \langle \varepsilon_n \rangle = 2\sum_n \frac{\hbar\omega_n}{\exp(\hbar\omega_n/\tau) - 1}$

there are 2 polarization modes

KK Eq 4.16