Temperature

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There is something else that happens in thermal equilibrium: the temperatures of the two systems becomes the same, so $T_1 = T_2$

Indeed, we will define the fundamental temperature τ by $\boxed{\frac{1}{\tau} = \left(\frac{\partial \sigma}{\partial U}\right)_N}$ KK Eq. 2.26

what are the units? σ is unitless, U is energy, so τ has units of energy. Humans were able to measure temperature before they knew it was just an energy, so there is a proportionality constant between τ and T, $\boxed{\tau = k_B T}$ KK Eq. 2.27

k_B is the Boltzmann constant

$$k_B = 1.380 \times 10^{-23}\ J/K$$ KK Eq. 2.25

$$k_B = 8.617 \times 10^{-5}\ eV/K$$

$$\frac{1}{T} = k_B \left(\frac{\partial \sigma}{\partial U}\right)_N$$ KK Eq. 2.24

S is called the conventional entropy

Finally, in classical thermodynamics, $\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_N = \frac{k_B}{\tau}$

$$\frac{k_B}{\tau} = k_B\left(\frac{\partial \sigma}{\partial U}\right)_N \Rightarrow k_B \left(\frac{\partial \sigma}{\cancel{\partial U}}\right)_N = \left(\frac{\partial S}{\cancel{\partial U}}\right)_N \Rightarrow \boxed{k_B \sigma = S}$$ KK Eq. 2.30

Consider two systems in thermal contact but still with different temperatures (so the ~~sys~~ whole system is still not in thermal equilibrium).

If $\tau_2 > \tau_1$, energy will flow from 2 to 1

$$\frac{1}{\tau_1} > \frac{1}{\tau_2} \Rightarrow \frac{1}{\tau_1} - \frac{1}{\tau_2} > 0 \Rightarrow \left(\frac{\partial \sigma_1}{\partial U_1}\right)_{N_1} - \left(\frac{\partial \sigma_2}{\partial U_2}\right)_{N_2} > 0$$

Alternatively, energy will flow from 1 to 2 if $\tau_2 < \tau_1$

$$\frac{1}{\tau_1} < \frac{1}{\tau_2} \Rightarrow \frac{1}{\tau_1} - \frac{1}{\tau_2} < 0 \Rightarrow \left(\frac{\partial \sigma_1}{\partial U_1}\right)_{N_1} - \left(\frac{\partial \sigma_2}{\partial U_2}\right)_{N_2} < 0$$

$$-\frac{1}{\tau_1} + \frac{1}{\tau_2} > 0 \Rightarrow \left(\frac{\partial \sigma_1}{\partial U_1}\right)_{N_1} + \left(\frac{\partial \sigma_2}{\partial U_2}\right)_{N_2} > 0$$

If energy is removed from 1 and transferred to 2

$$\Delta\sigma = \left(\frac{\partial \sigma_1}{\partial U_1}\right)_{N_1} (-\Delta U) + \left(\frac{\partial \sigma_2}{\partial U_2}\right)_{N_2} (\Delta U)$$

$$= \left(-\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)\Delta U$$

KK Eq. 2.31

When the whole system reaches thermal equilibrium,

$$\Delta\sigma = 0 \iff \tau_1 = \tau_2$$

which one produces which.

Before that, $\Delta\sigma > 0$, which only holds if ~~the~~ the body with higher temperature loses energy and vice versa.

☆ WHY DOES ENTROPY ALWAYS INCREASE WHEN BODIES AT DIFFERENT TEMPERATURES ARE BROUGHT INTO THERMAL CONTACT AND ALLOWED TO REACH THERMAL EQUILIBRIUM?

Essentially, due to the properties of combinations

Consider an Einstein Solid. At this moment it does not matter what the details are, we will spend plenty of time latter in the course. The particles in the Einstein solid are distinguishable

Red Green particles: $U_1 = 5$, $N_1 = 2$, $\mathcal{S}_1$

Microstates available to the system: R, G

5, 0	0, 5
4, 1	1, 4
3, 2	2, 3

Blue white: $U_2 = 1$, $N = 2$, $\mathcal{S}_2$

B, W

1, 0
0, 1

Before thermal contact, $g = g_1 g_2 = 6 \cdot 2 = 12$

$$\sigma = \ln g = \ln g_1 + \ln g_2$$

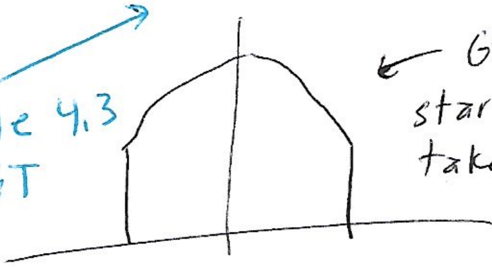
$$= \ln 6 + \ln 2 = \sigma = 2.48$$

After thermal contact $U_1 + U_2 = 5+1 = 6$

U_1	Microstates	g_1	U_2	Microstates	g_2	$g_1 g_2$
6	6,0 0,6 5,1 1,5 4,2 2,4 3,3	7	0	0,0	1	7
5	5,0 0,5 4,1 1,4 3,2 2,3	6	1	1,0 0,1	2	12
4	4,0 0,4 3,1 1,3 2,2	5	2	2,0 0,2 1,1	3	15
3	3,0 0,3 2,1 1,2	4	3	3,0 0,3 2,1 1,2	4	16
2	2,0 0,2 1,1	3	4	4,0 0,4 3,1 1,3 2,2	5	15
1	1,0 0,1	2	5	5,0 0,5 4,1 1,4 3,2 2,3	6	12
0	0,0	1	6	6,0 0,6 5,1 1,5 4,2 2,4 3,3	7	7

Notice that original g is included in the equilibrium g

Table 4.3 in GT



Gaussian starting to take shape

total $g = 84$

$\sigma = \ln g = \ln 84 = 4.43$

$$\Delta\sigma = \ln 84 - \ln 12 = \ln\left(\frac{84}{12}\right) = \ln 7 = 1.945$$

Entropy is maximum if temperatures are the same

If the total energy is conserved, the multiplicity after the bodies come in contact is

$$g(u) = \sum_{u_1} g_1(u_1)\, g_2(u - u_1)$$ KK Eq. 2.33

Notice that the initial multiplicity $g_1(u_{10})\, g_2(u - u_{10})$ is but one of many terms in $g(u)$ and it is always included. Therefore, entropy always increases.

Note: the Tsallis entropy discussed in the literature review paper "might" be valid while the system is in the process of reaching thermal equilibrium, the Boltzmann entropy is without a doubt correct in thermal equilibrium.

Physical processes at the microscopic level are time-symmetric, the laws of mechanics, for example, work the same to predict the future or the past. This is (obviously) not true at the macroscopic level. Since entropy is always higher in the future than in the past, thermodynamics provides a direction to time.
IS IT "THE" DIRECTION? Difficult to say for sure.

WAYS OF INCREASING ENTROPY THAT WE WILL STUDY IN THE COURSE: Add particles, add energy, increase volume, decompose molecules, let a linear polymer curl up. (Fig. 2.9 KK)

Laws of thermodynamics

<u>Zeroth law.</u> If two systems are in thermal equilibrium with a third, they must be in thermal equilibrium with each other. If $\tau_1 = \tau_3$ and $\tau_2 = \tau_3$, then $\tau_1 = \tau_2$

<u>First law.</u> Heat is a form of energy and energy is conserved $dU_1 + dU_2 = 0$

<u>Second law.</u> If a system in thermal contact is not in thermal equilibrium, entropy increases in successive instants of time until it reaches equilibrium, at which point the entropy does not change. $d(\sigma_1 + \sigma_2) \geq 0$

<u>Third law.</u> The entropy of a system approaches a constant as the temperature approaches zero. This is a consequence of quantum mechanics. $\lim_{\tau \to 0} \sigma = \text{constant}.$