Several examples of heat engines in KK, but going with a different one. 4/19/21

Lipscombe and Mungan, Phys. Teach. 58, 150 (2020)

Breathtaking Physics: Human respiration as a heat engine.

Air is a diatomic gas : N_2, O_2 (mostly) so it has 5 degrees of freedom (you derived the rotational in the exam)

3 translational, 2 rotational

$$U = \frac{5}{2} N\tau$$

Assume ideal gas, $U = \frac{5}{2} PV$, let V be volume of lungs and P the gauge pressure.

using a body plethysmograph

esophageal catheter

Empirical model of the volume of the lungs: (measured, fit fitting parameters to the data)

$$V = A\left(1-e^{-KP}\right)$$

$\frac{V}{A} = 1 - e^{-Kp}$

$\frac{V}{A} - 1 = -e^{-Kp}$

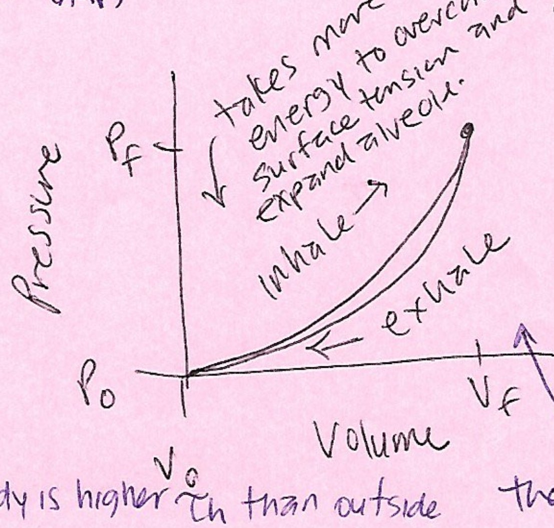
$\frac{1}{1} - \frac{V}{A} = e^{-Kp} = \frac{A-V}{A}$

$-Kp = \ln\left(\frac{A-V}{A}\right) \Rightarrow \ln\left(\frac{A}{A-V}\right) = Kp$

$p = \frac{1}{K}\ln\left(\frac{A}{A-V}\right)$

A_I, K_I during inhalation; A_E, K_E during exhalation

Inverting $$P = \frac{1}{K}\ln\left(\frac{A}{A-V}\right)$$ K is coefficient of compressibility



$$V_f = A_I\left(1-e^{-K_I P_f}\right) = A_E\left(1-e^{-K_E P_f}\right)$$

$K_E > K_I$ since the alveoli are easier to compress when they are "inflated"

Refrigerators are heat engines in reverse: they consume work to move heat from low τ_ℓ to high τ_h

Body is higher τ_h than outside

In case you are interested $A_I = 3.57\times10^{-3}\ m^3$

$K_I = 0.964\times10^{-3}\ 1/Pa$

$A_E = 3.35\times10^{-3}\ m^3$

$K_E = 1.644\times10^{-3}\ 1/Pa$

so $V_f = 3.31\times10^{-3}\ m^3$

$P_f = 2730\ Pa$ ← gauge pressure, so relative to atmospheric pressure

$dW = PdV$; $V = A(1-e^{-KP}) = A - Ae^{-KP}$

$$\Rightarrow dV = -Ae^{-KP}(-KdP) = AKe^{-KP}dP$$

$$W = \int P\cdot AKe^{-KP}dP = AK\int_0^{P_f} Pe^{-KP}dP = AKe^{-KP}\left[\frac{P}{-K} - \frac{1}{K^2}\right]_0^{P_f}$$

$$W = -\frac{AKe^{-KP}P}{K} - \frac{AKe^{-KP}}{K^2} = -\frac{A}{K}\left[KPe^{-KP} + e^{-KP}\right]$$

$$W = \frac{A}{K}\left[-(1+KP)e^{-KP}\right]_0^{P_f}$$ ~~$= -\frac{A}{K}\left[(1+KP)\left(1-\frac{V}{A}\right)\right]$~~

~~$= -\frac{A}{K}$~~ $$W = -\frac{A}{K}\left[(1+KP_f)e^{-KP_f}\right] + \frac{A}{K}\left[(1)(1)\right]$$

$$W = \frac{A}{K}\left[1 - (1+KP_f)e^{-KP_f}\right]$$

functional form

with $e^{-KP_f} = 1 - \frac{V_f}{A}$,

$$W = \frac{A}{K}\left[1-(1+KP_f)\left(1-\frac{V_f}{A}\right)\right] = \frac{A}{K}\left[\cancel{1} - \left[\cancel{1} + KP_f + \frac{V_f}{A} + \frac{KP_fV_f}{A}\right]\right]$$

149.5

$$W = \int \rho A K e^{-K\rho} d\rho = AK \int_0^{\rho_f} \rho e^{-K\rho} d\rho$$

Let $u = \rho$ $\quad dv = e^{-K\rho} d\rho \Rightarrow v = \int e^{-K\rho} d\rho$

$du = d\rho$

$$a = -K\rho$$

$$da = -K\, d\rho \Rightarrow d\rho = -\frac{da}{K}$$

$$v = \frac{1}{-K} \int e^{a} da = -\frac{1}{K} e^{a} = \frac{1}{-K} e^{-K\rho}$$

$$\int u\, dv = uv - \int v\, du$$

$$= -\frac{\rho}{K} e^{-K\rho} + \int \frac{1}{K} e^{-K\rho} d\rho = -\frac{\rho}{K} e^{-K\rho} - \frac{1}{K^2} e^{-K\rho}$$

$$W = AKe^{-K\rho} \left[-\frac{\rho}{K} - \frac{1}{K^2} \right]_0^{\rho_f}$$

$$W = -\frac{A\cancel{K}P_f}{\cancel{K}} + \frac{AV_f}{\cancel{A}K} + \frac{A\cancel{K}P_fV_f}{A\cancel{K}} = \frac{V_f}{K} - AP_f + P_fV_f$$

$$\text{so } \boxed{W_{net}} = W_I - W_E = \frac{V_f}{K_I} - A_IP_f + \cancel{P_fV_f} - \frac{V_f}{K_E} + A_EP_f - \cancel{P_fV_f}$$

Area under the curve

Inhale curve

exhale curve

$$W_{net} = V_f\left[\frac{1}{K_I} - \frac{1}{K_E}\right] + P_f\left[A_E - A_I\right]$$

$$W_{net} = V_f\left[\frac{1}{K_I} - \frac{1}{K_E}\right] - P_f\left[A_I - A_E\right] \quad \text{Eq. 10}$$

using fitted values, $W_{net} = 0.820\ J$.

$$dU + pdV = \frac{5}{2}d(pV) + pdV = đQ$$

$$Q_I = \Delta U_I + W_I = \frac{5}{2}P_fV_f + \frac{V_f}{K_I} - A_IP_f + P_fV_f$$

$$Q_I = \frac{V_f}{K_I} - A_IP_f + \frac{7}{2}P_FV_f = 25.3\ J$$

$$\text{Coefficent of performance} = \frac{Q_I}{W_{net}} = \frac{25.3J}{0.820J} = 31$$

Each individual breath requres little energy, but we do breath 20,000/day, so breathing is about 6% of the 9×10^6 J we spend every day

Laws of thermodynamics — revisited

You must play the game

Zeroth law. If two thermodynamic systems are in contact with a third one, they are in thermal equilibrium with each other.

$$\left(\frac{\partial \ln g_1}{\partial U_1}\right)_{N_1} = \left(\frac{\partial \ln g_3}{\partial U}\right)_{N_3} ; \left(\frac{\partial \ln g_2}{\partial U_2}\right)_{N_2} = \left(\frac{\partial \ln g_3}{\partial U_3}\right)_{N_3}$$

~~First law~~

$$\tau_2 = \tau_3 \ \& \ \tau_1 = \tau_3 \Rightarrow \tau_1 = \tau_2$$

First law. $dW = dU - \overset{\tau d\sigma}{dQ}$ Conservation of energy

You can't win

You can't do more work than the amount of heat you take in

Second law. $\eta = \frac{\tau_h - \tau_l}{\tau_h} = 1 - \frac{\tau_l}{\tau_h}$

You can't break even unless you quit

Your efficiency is always less than 100%, unless τ_l is ~~ze~~ absolute zero

Third law. $C_V = \tau \frac{\partial \sigma}{\partial \tau}$; For $C_{el} = \gamma \tau$

You can't quit

$$\Rightarrow d\sigma = \frac{\gamma \cancel{\tau} d\tau}{\cancel{\tau}}$$

$$\boxed{\tau = \frac{\sigma}{\gamma} + c} \quad \sigma = \int d\sigma = \gamma \int d\tau = \gamma\tau + c$$

As $\tau \to 0$, you still have c

can $c = 0$??

You can't reach absolute zero. Also from QM

Just like PHYS 3331

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