Homework 7

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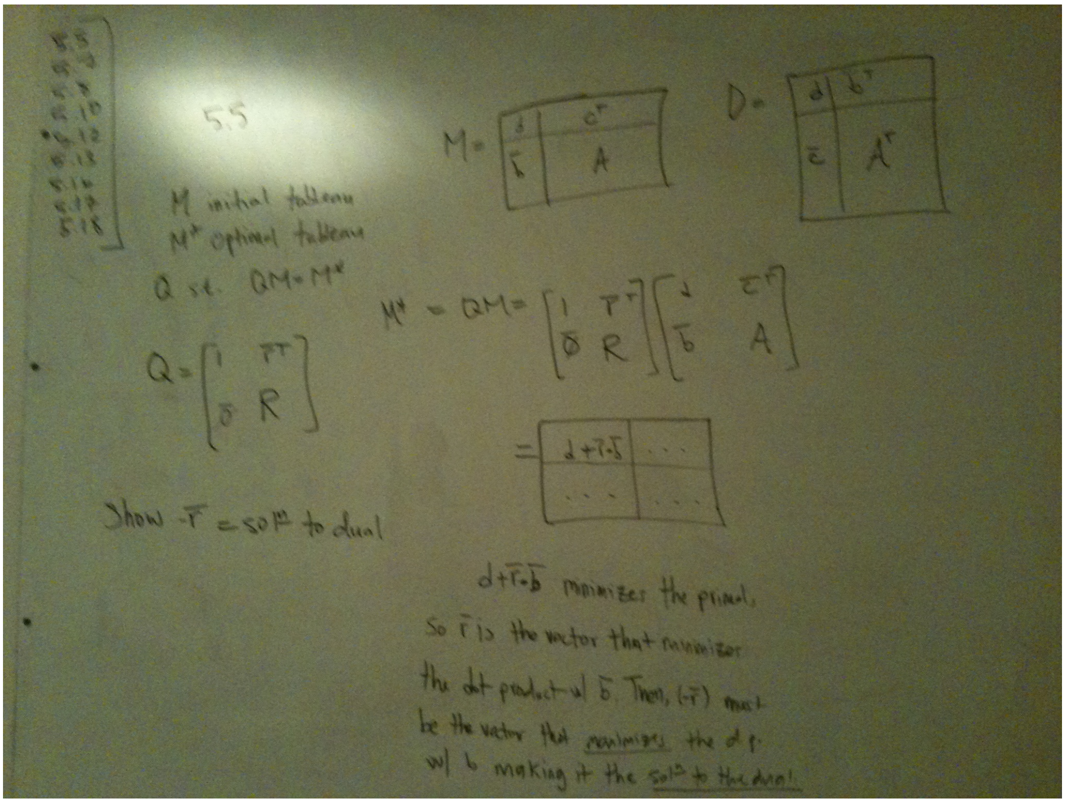
5.5



5.7

$\boxed{5.7}$

a) \$2

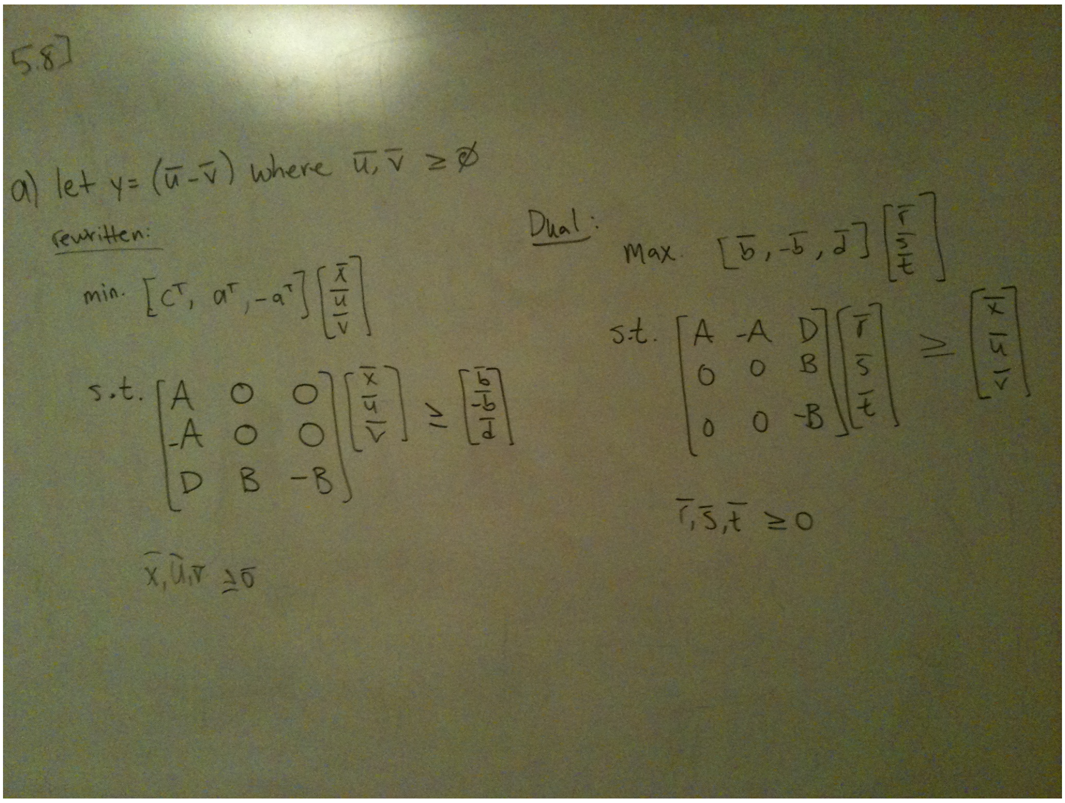
b) R2: \$0

R3: \$3

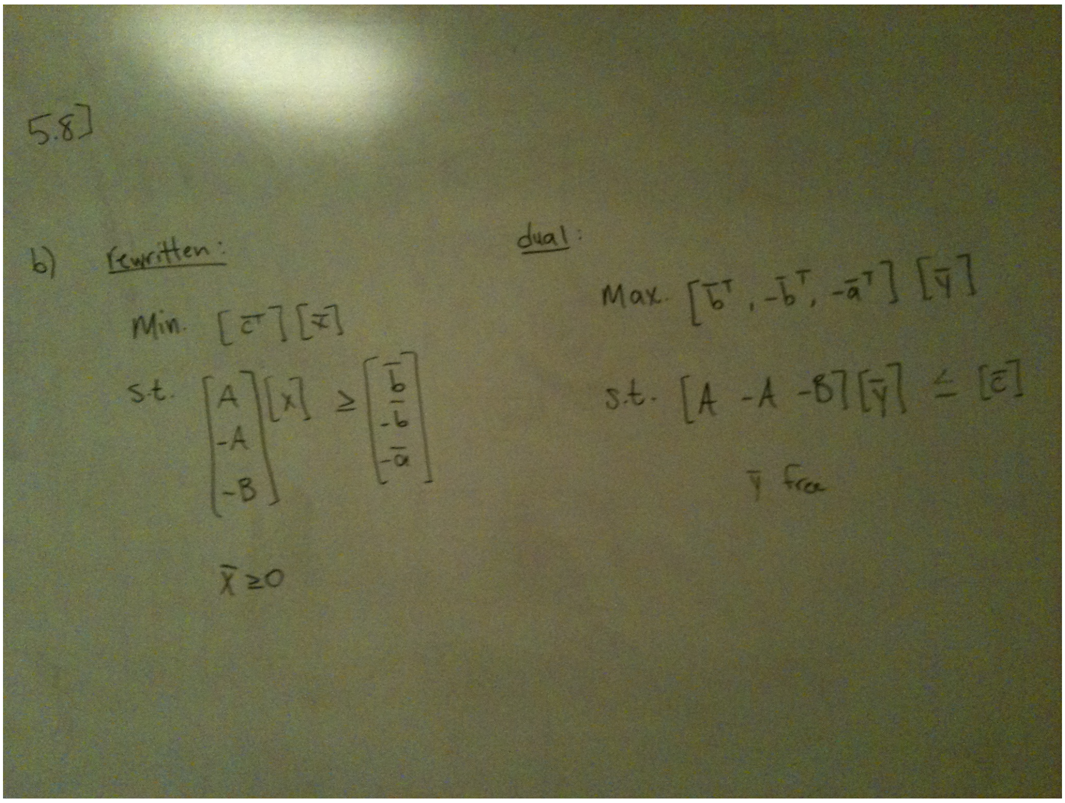
c) 40 units

5.8

a


5.8]
a) let $y = (\bar{u} - \bar{v})$ where $\bar{u}, \bar{v} \geq \emptyset$
rewritten:
min. $[c^T, a^T, -a^T] \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix}$
s.t. $\begin{bmatrix} A & 0 & 0 \\ -A & 0 & 0 \\ D & B & -B \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix} \geq \begin{bmatrix} \bar{b} \\ -\bar{b} \\ \bar{d} \end{bmatrix}$
$\bar{x}, \bar{u}, \bar{v} \geq \bar{0}$
Dual:
Max. $[\bar{b}, -\bar{b}, \bar{d}] \begin{bmatrix} \bar{r} \\ \bar{s} \\ \bar{t} \end{bmatrix}$
s.t. $\begin{bmatrix} A & -A & D \\ 0 & 0 & B \\ 0 & 0 & -B \end{bmatrix} \begin{bmatrix} \bar{r} \\ \bar{s} \\ \bar{t} \end{bmatrix} \geq \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix}$
$\bar{r}, \bar{s}, \bar{t} \geq 0$

b



5.8]

b) rewritten:

Min. $[\bar{c}^T][\bar{x}]$

s.t. $\begin{bmatrix} A \\ -A \\ -B \end{bmatrix}[\bar{x}] \geq \begin{bmatrix} \bar{b} \\ -\bar{b} \\ -\bar{a} \end{bmatrix}$

$\bar{x} \geq 0$

dual:

Max. $[\bar{b}^T, -\bar{b}^T, -\bar{a}^T][\bar{y}]$

s.t. $[A \;\; -A \;\; -B][\bar{y}] \leq [\bar{c}]$

$\bar{y}$ free

c

rewritten:

c) $\text{Max.} \begin{bmatrix} c^T, & a^T, & -a^T \end{bmatrix} \begin{bmatrix} \bar{w} \\ \bar{r} \\ \bar{s} \end{bmatrix}$

$$\text{s.t.} \begin{bmatrix} A & B & -B \\ -A & -B & B \end{bmatrix} \begin{bmatrix} \bar{w} \\ \bar{r} \\ \bar{s} \end{bmatrix} \leq \begin{bmatrix} \bar{b} \\ -\bar{b} \end{bmatrix}$$

$\bar{w}, \bar{r}, \bar{s} \geq \bar{0}$

and $\bar{v} = (\bar{r} - \bar{s})$

dual:

$$\text{min.} \begin{bmatrix} b^T, & -b^T \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix}$$

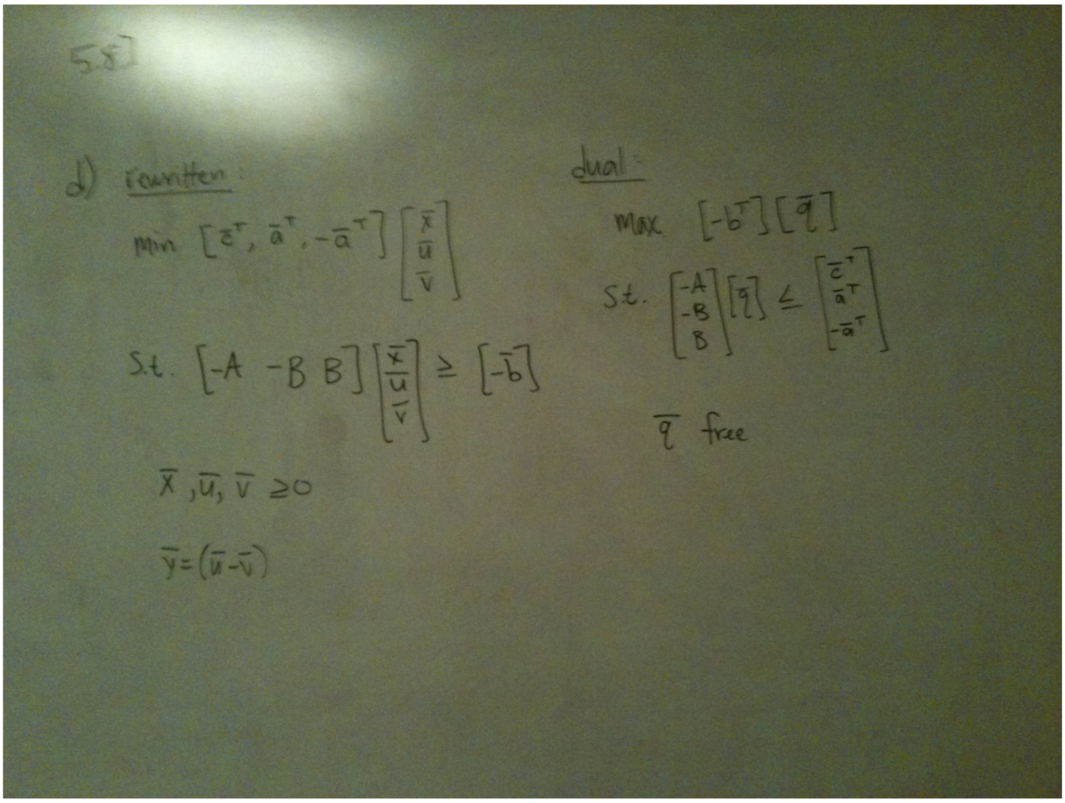
$$\text{s.t.} \begin{bmatrix} A & -A \\ B & -B \\ -B & B \end{bmatrix} \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} \geq \begin{bmatrix} \bar{c} \\ \bar{a} \\ -\bar{a} \end{bmatrix}$$

$\bar{y}, \bar{z} \geq 0$

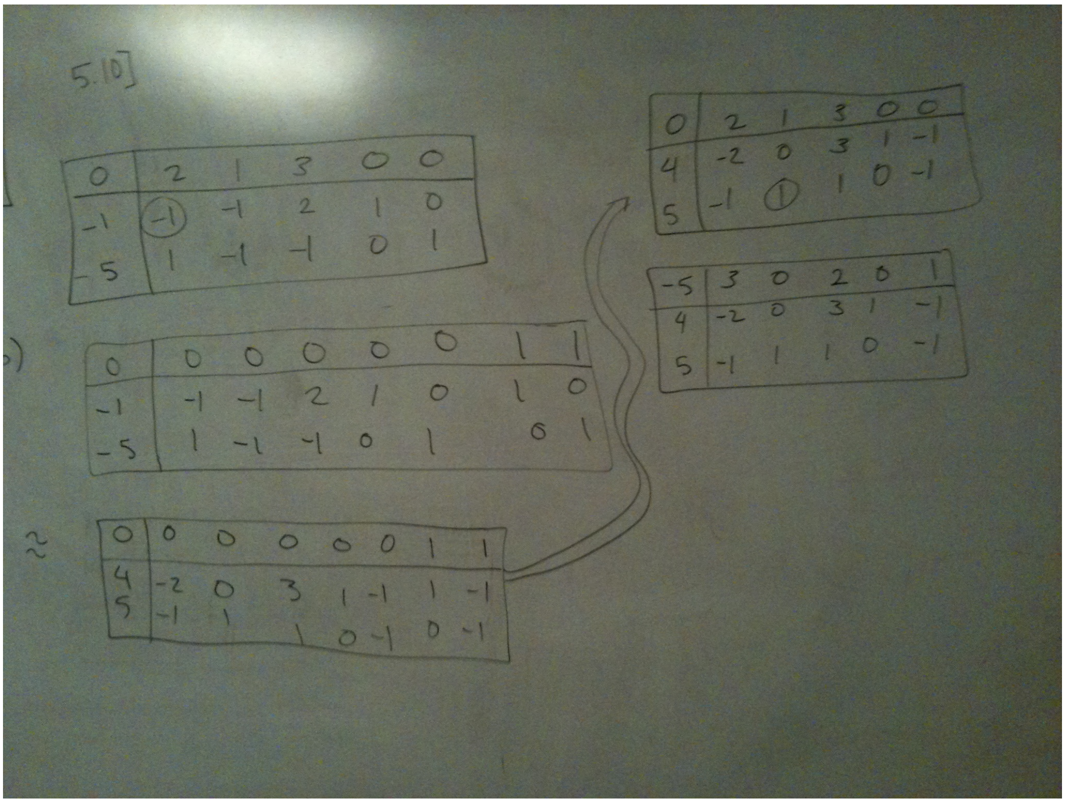
$$\Rightarrow \text{min. } b^T(\bar{y} - \bar{z})$$
$$\text{s.t. } A(\bar{y} - \bar{z}) \geq \bar{c}$$
$$B(\bar{y} - \bar{z}) \geq \bar{a}$$
$$-B(\bar{y} - \bar{z}) \geq -a$$
$$\bar{y}, \bar{z} \geq 0$$

$$\boxed{\begin{array}{l} \Rightarrow \text{min } \bar{b}^T \bar{x} \\ \text{s.t. } A\bar{x} \geq \bar{c} \\ \quad B\bar{x} = \bar{a} \\ \bar{x} \text{ free} \\ \bar{x} = (\bar{y} - \bar{z}) \end{array}}$$

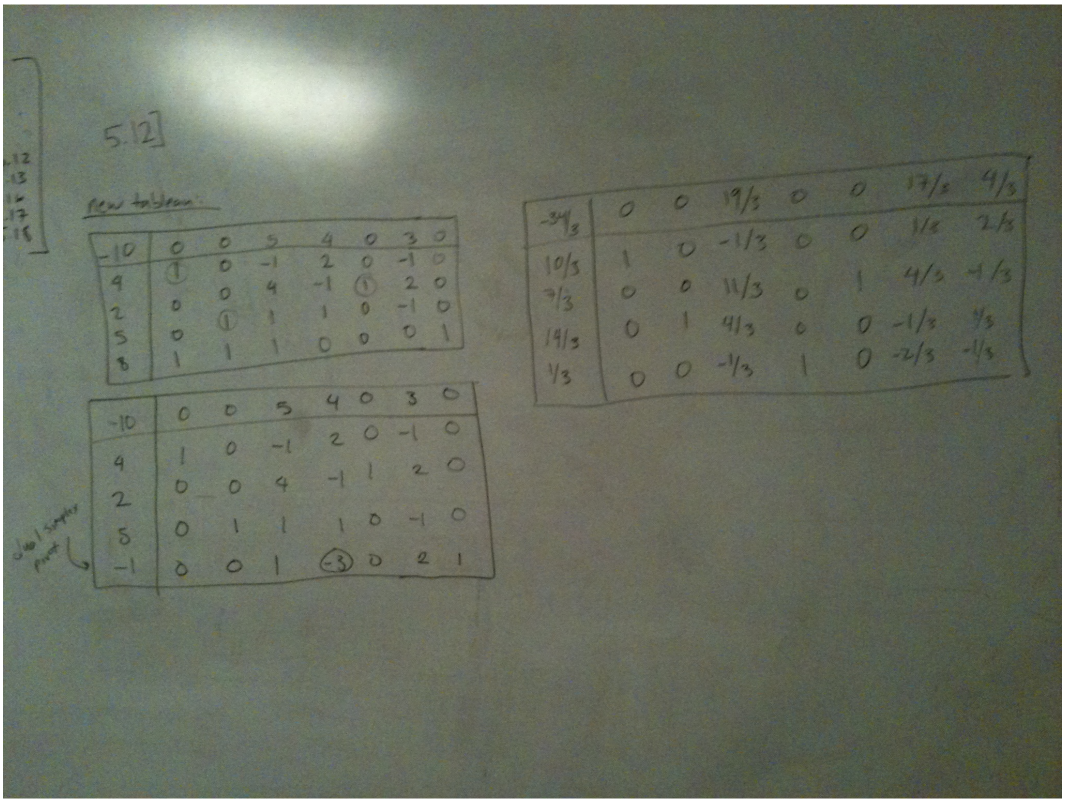
d


5.8]
d) rewritten:
min $[\bar{c}^T, \bar{a}^T, -\bar{a}^T] \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix}$
s.t. $[-A \ -B \ B] \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix} \geq [-\bar{b}]$
$\bar{x}, \bar{u}, \bar{v} \geq 0$
$\bar{y} = (\bar{u} - \bar{v})$
dual:
max $[-\bar{b}^T][\bar{q}]$
s.t. $\begin{bmatrix} -A \\ -B \\ B \end{bmatrix} [\bar{q}] \leq \begin{bmatrix} \bar{c}^T \\ \bar{a}^T \\ -\bar{a}^T \end{bmatrix}$
$\bar{q}$ free

5.10



5.12



5.16

5.16]

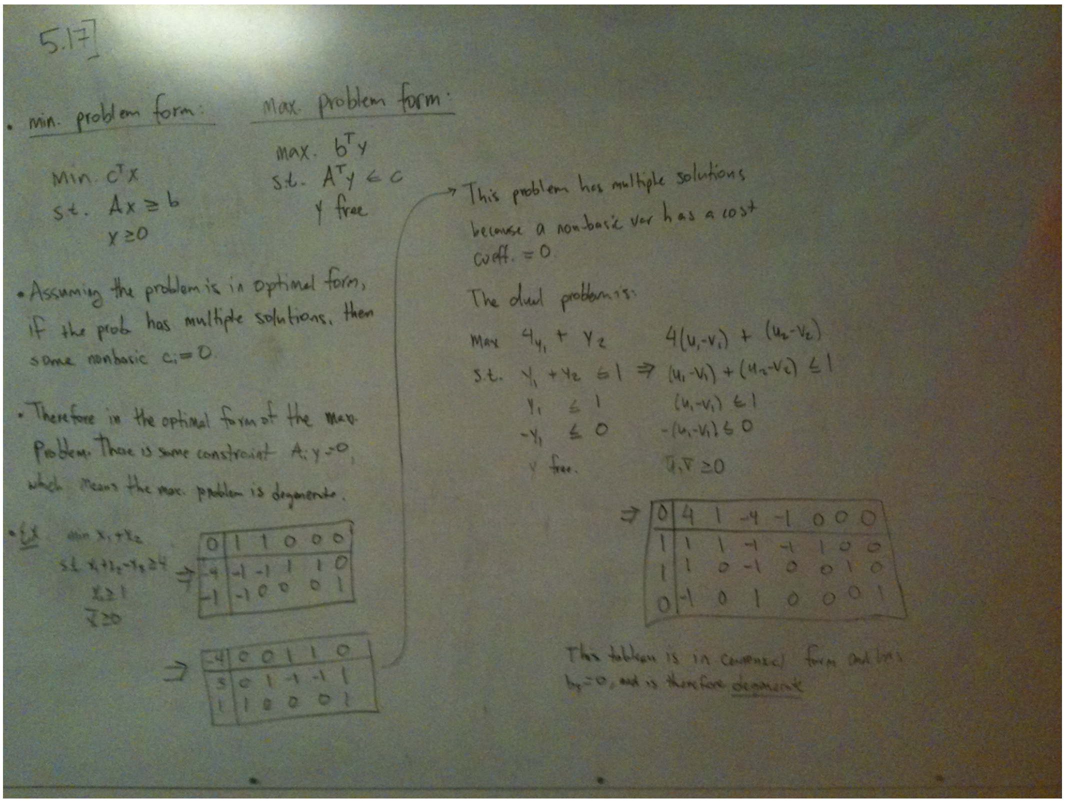
a) False - both problems may be infeasible

b) False - non-basic variables can have cost coefficients of 0, which implies a shadow price of zero if it is this slack variable.

c) True - Duality relations say if one problem is feasible, then its dual is also feasible.

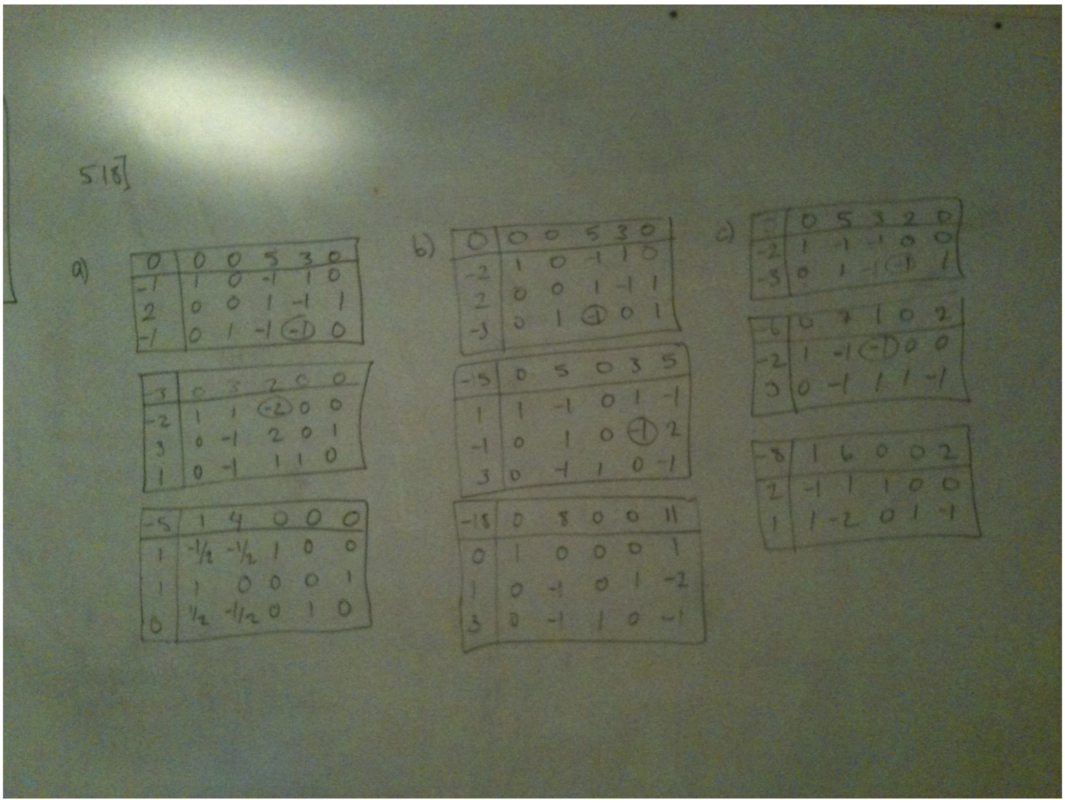
d) True. If the first component of the optimal vector is true. then in optimal form, $b_0 = 0$. Then in the dual problem, the first cost coeff. $= 0$, which implies that the first constraint is active

5.17



5.17
• min. problem form:
Min. $c^T x$
s.t. $Ax \geq b$
$x \geq 0$
Max. problem form:
max. $b^T y$
s.t. $A^T y \leq c$
y free
• Assuming the problem is in optimal form, if the prob. has multiple solutions, then some nonbasic $c_i = 0$.
• Therefore in the optimal form of the max. problem there is some constraint $A_i y = 0$, which means the max. problem is degenerate.
• Ex. min $x_1 + x_2$
s.t. $x_1 + x_2 - x_3 \geq 4$
$x_1 \geq 1$
$\bar{x} \geq 0$
This problem has multiple solutions because a non-basic var has a cost coeff. = 0
The dual problem is:
Max $4y_1 + y_2$
s.t. $y_1 + y_2 \leq 1$
$y_1 \leq 1$
$-y_1 \leq 0$
y free.
$4(u_1 - v_1) + (u_2 - v_2)$
$(u_1 - v_1) + (u_2 - v_2) \leq 1$
$(u_1 - v_1) \leq 1$
$-(u_1 - v_1) \leq 0$
$\bar{u}, \bar{v} \geq 0$
This tableau is in canonical form and has $b_3 = 0$, and is therefore degenerate

5.18



5.18

a)

0	0	0	5	3	0
-1	1	0	-1	1	0
2	0	0	1	-1	1
-1	0	1	-1	-1	0

-3	0	3	2	0	0
-2	1	1	-2	0	0
3	0	-1	2	0	1
1	0	-1	1	1	0

-5	1	4	0	0	0
1	-1/2	-1/2	1	0	0
1	1	0	0	0	1
0	1/2	-1/2	0	1	0

b)

0	0	0	5	3	0
-2	1	0	-1	1	0
2	0	0	1	-1	1
-3	0	1	-1	0	1

-15	0	5	0	3	5
1	1	-1	0	1	-1
-1	0	1	0	-1	2
3	0	-1	1	0	-1

-18	0	8	0	0	11
0	1	0	0	0	1
1	0	-1	0	1	-2
3	0	-1	1	0	-1

c)

[illegible]	0	5	3	2	0
-2	1	-1	-1	0	0
-5	0	1	-1	-1	1

-6	0	7	1	0	2
-2	1	-1	-1	0	0
3	0	-1	1	1	-1

-8	1	6	0	0	2
2	-1	1	1	0	0
1	1	-2	0	1	-1