Homework 3

Matt Forbes

October 19, 2010

Problem 3.8

a)

0	2	-1	1	1
5	1	2	0	-1
10	-1	1	-2	1

Pivots to:

15	0	0	$-\frac{7}{3}$	3
(-5)	1	0	$\frac{4}{3}$	-1
5	0	1	$-\frac{2}{3}$	0

Subproblem on row 2 gives:

0	3	0	$\frac{5}{3}$	0
5	-1	0	$-\frac{4}{3}$	1
5	0	1	$-\frac{2}{3}$	0

Which is in optimal form giving $\mathbf{x} = (0 \quad 5 \quad 0 \quad 5)^T$

b)

0	2	-1	1	1
5	1	2	0	-1
10	-1	1	-2	1

In artificial problem form:

0	0	0	0	0	1	1
5	1	2	0	-1	1	0
10	-1	1	-2	1	0	1

Which pivots to:

0	0	0	0	0	$\frac{1}{2}$	1
5	0	1	$-\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$
5	-1	0	$-\frac{4}{3}$	1	$-\frac{1}{3}$	$\frac{2}{3}$

This artificial problem's minimum is 0, so the original is feasible: Reformulate:

0	2	-1	1	1
5	0	1	$-\frac{2}{3}$	0
5	-1	0	$-\frac{4}{3}$	1

Pivots to:

0	3	0	$\frac{5}{3}$	0
5	-1	0	$-\frac{4}{3}$	1
5	0	1	$-\frac{2}{3}$	0

Which is in optimal form giving $\mathbf{x} = (0 \quad 5 \quad 0 \quad 5)^T$

Problem 3.15

This problem formulated using $|D_i| = D_i^+ + D_i^-$ is:

$$\begin{aligned}
\min \quad & D_1^+ + D_1^- + D_2^+ + D_2^- + D_3^+ + D_3^- + D_4^+ + D_4^- \\
\text{s. t.} \quad 5 &= a + b + c - D_1^+ + D_1^- \\
13 &= 4a + 2b + c - D_2^+ + D_2^- \\
30 &= 16a + 4b + c - D_3^+ + D_3^- \\
45 &= 25a + 5b + c - D_4^+ - D_4^-
\end{aligned}$$

Which has the simplex tableau of:

0	0	0	0	1	1	1	1	1	1	1	1
5	1	1	1	-1	1	0	0	0	0	0	0
13	4	2	1	0	0	-1	1	0	0	0	0
30	16	4	1	0	0	0	0	-1	1	0	0
45	25	5	1	0	0	0	0	0	0	-1	1

Pivots to:

-93	-46	-12	-4	2	0	2	0	2	0	2	0
5	1	1	1	-1	1	0	0	0	0	0	0
13	4	2	1	0	0	-1	1	0	0	0	0
30	16	4	1	0	0	0	0	-1	1	0	0
45	25	5	1	0	0	0	0	0	0	-1	1

Which is now in canonical form. Now using successive ratio rules for pivoting:

-10.2	0	-2.8	-2.16	2	0	2	0	2	0	0.16	1.84
3.2	0	0.8	0.96	-1	1	0	0	0	0	0.04	-0.04
5.8	0	1.2	0.84	0	0	-1	1	0	0	0.16	-0.16
1.2	0	0.8	0.36	0	0	0	0	-1	1	0.64	-0.64
1.8	1	0.2	0.04	0	0	0	0	0	0	-0.04	0.04

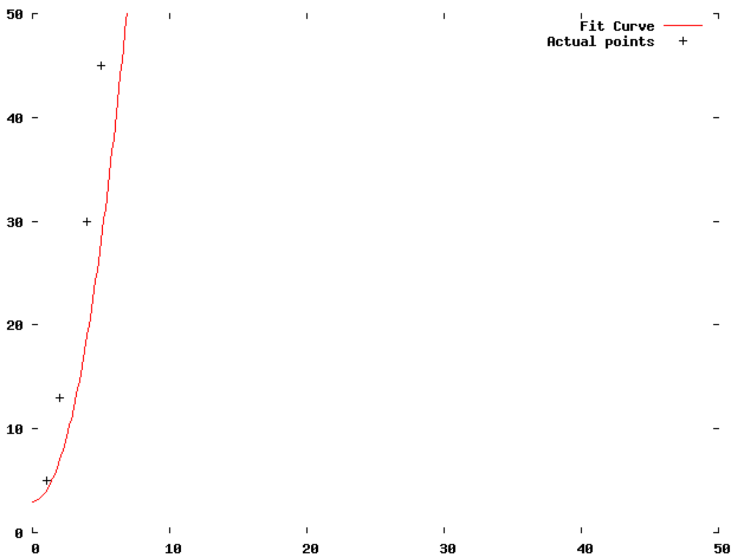
Again:

-6	0	0	-0.9	2	0	2	0	-1.5	3.5	2.4	-0.4
2	0	0	0.6	-1	1	0	0	1	-1	-0.6	0.6
4	0	0	0.3	0	0	-1	1	1.5	-1.5	-0.8	0.8
1.5	0	1	0.45	0	0	0	0	-1.25	1.25	0.8	-0.8
1.5	1	0	-0.05	0	0	0	0	0.25	-0.25	-0.2	0.2

And finally:

-3	0	0	0	0.5	1.5	2	0	0	2	1.5	0.5
3.33333	0	0	1	-1.66667	1.66667	0	0	1.66667	-1.66667	-1	1
3	0	0	0	0.5	-0.5	-1	1	1	-1	-0.5	0.5
0	0	1	0	0.75	-0.75	0	0	-2	2	1.25	-1.25
1.66667	1	0	0	-0.0833333	0.0833333	0	0	0.333333	-0.333333	-0.25	0.25

This gives values: $a = \frac{5}{3}, b = 0, c = \frac{10}{3}$
Here is a graph of the fit curve vs actual points:



Problem 3.16

a)

3	1	0	0	1	0
-1	1	1	0	-1	0
-4	-1	0	1	-1	0
1	1	0	0	0	1

Pivots to:

2	0	0	0	1	-1
-2	0	1	0	-1	-1
-3	0	0	1	-1	1
1	1	0	0	0	1

And in optimal form:

-1	0	0	1	0	0
1	0	1	-1	0	-2
3	0	0	-1	1	-1
1	1	0	0	0	1

With solution $\mathbf{x} = (1 \quad 1 \quad 0 \quad 3 \quad 0)^T$

b)

-1	0	0	-1	0	1
-1	1	0	0	2	-1
-1	0	0	1	1	-1
-2	-3	1	5	0	-2

Pivots to:

-2	1	0	-1	2	0
1	-1	0	0	-2	1
0	-1	0	1	-1	0
0	-5	1	5	-4	0

And in optimal form:

-2	0	0	0	1	0
1	-1	0	0	-2	1
0	-1	0	1	-1	0
0	0	1	0	1	0

With solution $\mathbf{x} = (0 \quad 0 \quad 0 \quad 0 \quad 1)^T$

c)

This tableau is already in optimal form:

0	2	5	3	0	0	0
5	1	1	0	1	0	0
15	2	1	2	0	1	0
8	1	1	1	0	0	1

With solution $\mathbf{x} = (0 \quad 0 \quad 0 \quad 0 \quad 5 \quad 15 \quad 8)^T$

Problem 3.17

Original Problem:

$$\begin{aligned} \min \quad & x_1 + 2x_2 \; -x_3 \\ \text{s. t.} \quad & x_1 - x_2 \quad +x_3 \leq 1 \\ & x_1 + x_2 \; -2x_3 \leq 4 \\ & x_1 \geq 0, x_2, x_3 \; \text{ free} \end{aligned}$$

Reformulated with slack variables, and: $x_2 = (x_2' - x_2''), x_3 = (x_3' - x_3'')$:

$$\begin{aligned} \min \quad & x_1 + 2x_2' - 2x_2'' - x_3' + x_3'' \\ \text{s.t.} \quad & x_1 - x_2' + x_2'' + x_3' - x_3'' + x_4 = 1 \\ & x_1 + x_2' - x_2'' - 2x_3' + 2x_3'' + x_5 = 4 \end{aligned}$$

Has the simplex tableau:

0	1	2	-2	-1	1	0	0
1	1	-1	1	1	-1	1	0
4	1	1	-1	-2	2	0	1

Pivots to:

2	3	0	0	1	-1	2	0
1	1	-1	1	1	-1	1	0
5	2	0	0	-1	1	1	1

And in optimal form:

7	5	0	0	0	0	3	1
6	3	-1	1	0	0	2	1
5	2	0	0	-1	1	1	1

With solution $\mathbf{x} = (0 \quad 0 \quad 6 \quad 0 \quad 5 \quad 0 \quad 0)^T$

Resubstituting $x_2 = x_2' - x_2''$ and $x_3 = x_3' - x_3''$ gives this solution to the original problem: $(0 \quad -6 \quad -5)^T$

Problem 3.19

Reformulating the system of equations into an artificial problem will show whether or not it has any feasible solution.

0	0	0	0	0	0	0	0	1	1	1
-1	1	1	0	-1	1	0	0	1	0	0
1	-1	0	1	1	0	1	0	0	1	0
-1	0	1	-1	0	0	0	1	0	0	1

Pivoting using subproblem (obj function row 1):

0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	0	1	1	0	1	1	0
1	-1	0	1	1	0	1	0	0	1	0
-1	0	1	-1	0	0	0	1	0	0	1

Again pivoting with a subproblem (obj function row3)

0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	0	1	1	0	1	1	0
1	-1	-1	0	1	-1	0	0	-1	0	0
-1	0	2	0	0	1	1	1	1	1	1

This tableau is in infeasible form 2. The system has no solution.

Problem 3.21

This problem calls for minimizing the max (minimax) of two expressions. This is not a linear objective function, but can be reformulated as such. If we create some free variable w and constrain it to being greater than both expressions, then finding the minimum value of w will also be the smallest max of the two expressions.

w is a free variable so we can write it as $w = w' - w''$ and then constrain $w', w'' \geq 0$.

In standard form (with slack variables):

$$\begin{aligned} \min \quad & w' - w'' \\ \text{s.t.} - w' + w'' & + 2x_1 - x_2 + x_3 = 0 \\ -w' + w'' & - 3x_1 + 2x_2 + x_4 = 0 \\ & 4x_1 + x_2 + x_5 = 5 \end{aligned}$$

The simplex tableau:

0	1	-1	0	0	0	0	0
0	-1	1	2	-1	1	0	0
0	-1	1	-3	2	0	1	0
5	0	0	4	1	0	0	1

Which is already in canonical form, pivoting:

0	0	0	2	-1	1	0	0
0	-1	1	2	-1	1	0	0
0	0	0	-5	3	-1	1	0
5	0	0	4	1	0	0	1

Pivot again for optimal form:

0	0	0	0.333333	0	0.666667	0.333333	0
0	-1	1	0.333333	0	0.666667	0.333333	0
0	0	0	-1.66667	1	-0.333333	0.333333	0
5	0	0	5.66667	0	0.333333	-0.333333	1

This gives a solution of $(0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 5)^T$
which means that $x_1 = 0, x_2 = 0$, which is within the constraints and does produce the smallest value of either expression in the objective function.

Problem 3.23

a)True
b)True
c)False
d)True
e)True
f)True
g)False
i)True