

# Homework 7

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5.5

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$M$ : initial tableau  
 $M^*$ : optimal tableau

$Q$  s.t.  $QM = M^*$

$Q = \begin{bmatrix} I & \bar{r} \\ \bar{0} & R \end{bmatrix}$

$M^* = QM = \begin{bmatrix} I & \bar{r} \\ \bar{0} & R \end{bmatrix} \begin{bmatrix} I & \bar{z} \\ \bar{b} & A \end{bmatrix}$

$= \begin{bmatrix} I + \bar{r}\bar{z} & \dots \\ \dots & \dots \end{bmatrix}$

Show  $\bar{r} = \text{sol}^2$  to dual

$d + \bar{r}^T \bar{b}$  minimizes the primal.  
So  $\bar{r}$  is the vector that minimizes  
the dot product w/  $\bar{b}$ . Then,  $(-\bar{r})$  must  
be the vector that maximizes the d.p.  
w/  $b$  making it the sol<sup>2</sup> to the dual!

5.7

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- a) \$2
- b) R2: \$0  
R3: #3
- c) 40 unit

## 5.8

a

5.8)

a) let  $y = (\bar{u} - \bar{v})$  where  $\bar{u}, \bar{v} \geq 0$

rewritten:

$$\text{min. } [C^T, a^T, -a^T] \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix}$$

$$\text{s.t. } \begin{bmatrix} A & O & O \\ -A & O & O \\ D & B & -B \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix} \geq \begin{bmatrix} b \\ -b \\ d \end{bmatrix}$$

$$\bar{x}, \bar{u}, \bar{v} \geq 0$$

Dual:

$$\text{Max. } [\bar{b}, -\bar{b}, \bar{d}] \begin{bmatrix} \bar{r} \\ \bar{s} \\ \bar{t} \end{bmatrix}$$

$$\text{s.t. } \begin{bmatrix} A & -A & D \\ O & O & B \\ O & O & -B \end{bmatrix} \begin{bmatrix} \bar{r} \\ \bar{s} \\ \bar{t} \end{bmatrix} \geq \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix}$$

$$\bar{r}, \bar{s}, \bar{t} \geq 0$$

b

5.8]

b) rewritten:

$$\text{Min. } [c^T] [x]$$

$$\text{s.t. } \begin{bmatrix} A \\ -A \\ -B \end{bmatrix} \begin{bmatrix} x \end{bmatrix} \geq \begin{bmatrix} \bar{b} \\ -\bar{b} \\ -\bar{a} \end{bmatrix}$$

$$x \geq 0$$

dual:

$$\text{Max. } [\bar{b}^T, -\bar{b}^T, -\bar{a}^T] [\bar{y}]$$

$$\text{s.t. } [A \ -A \ -B] [\bar{y}] \leq [c]$$

$\bar{y}$  free

c

rewritten:

c) Max.  $\begin{bmatrix} c^T, \alpha^T, -\alpha^T \end{bmatrix} \begin{bmatrix} \bar{w} \\ \bar{r} \\ \bar{s} \end{bmatrix}$

dual:

min.  $\begin{bmatrix} b^T, -b^T \end{bmatrix} \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix}$

s.t.  $\begin{bmatrix} A & B - B \\ A & -B & B \end{bmatrix} \begin{bmatrix} \bar{w} \\ \bar{r} \\ \bar{s} \end{bmatrix} \leq \begin{bmatrix} \bar{b} \\ -\bar{b} \end{bmatrix}$

s.t.  $\begin{bmatrix} A & -A \\ B & -B \\ -B & B \end{bmatrix} \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} \geq \begin{bmatrix} \bar{c} \\ \bar{\alpha} \\ -\bar{\alpha} \end{bmatrix}$

$\bar{y}, \bar{z} \geq 0$

$\bar{w}, \bar{r}, \bar{s} \geq 0$

and  $\bar{v} = (\bar{r} - \bar{s})$

$\Rightarrow \min. b^T(\bar{y} - \bar{z})$

s.t.  $A(\bar{y} - \bar{z}) \geq \bar{c}$

$B(\bar{y} - \bar{z}) \geq \bar{\alpha}$

$-B(\bar{y} - \bar{z}) \leq -\bar{\alpha}$

$\bar{y}, \bar{z} \geq 0$

$\Rightarrow \min \bar{b}^T \bar{x}$   
 s.t.  $A \bar{x} \geq \bar{c}$   
 $B \bar{x} = \bar{\alpha}$   
 $\bar{x}$  free  
 $\bar{x} = (\bar{y} - \bar{z})$

d

58)

d) rewritten:

$$\min \begin{bmatrix} \bar{c}^T, \bar{a}^T, -\bar{a}^T \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix}$$

s.t.  $\begin{bmatrix} -A & -B & B \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix} \geq \begin{bmatrix} -\bar{b} \end{bmatrix}$

dual:

$$\max \begin{bmatrix} -\bar{b}^T \end{bmatrix} \begin{bmatrix} \bar{q} \end{bmatrix}$$

s.t.  $\begin{bmatrix} -A \\ -B \\ B \end{bmatrix} \begin{bmatrix} \bar{q} \end{bmatrix} \leq \begin{bmatrix} \bar{c}^T \\ \bar{a}^T \\ -\bar{a}^T \end{bmatrix}$

$\bar{q}$  free

$\bar{x}, \bar{u}, \bar{v} \geq 0$

$\bar{y} = (\bar{u} - \bar{v})$

## 5.10

5.10]

0	2	1	3	0	0
-1	(-1)	-1	2	1	0
-5	1	-1	-1	0	1

  

)

0	0	0	0	0	1	1
-1	-1	-1	2	1	0	1
-5	1	-1	-1	0	1	0

  

$\approx$

0	0	0	0	0	1	1
4	-2	0	3	1	-1	1
5	-1	1	1	0	-1	0

## 5.12

5.12]

*new tableau:*

-10	0	0	5	4	0	3	0
4	1	0	-1	2	0	-1	0
2	0	0	4	-1	1	2	0
5	0	1	1	1	0	-1	0
8	1	1	1	0	0	0	1

  

-10	0	0	5	4	0	3	0
4	1	0	-1	2	0	-1	0
2	0	0	4	-1	1	2	0
5	0	1	1	1	0	-1	0
-1	0	0	1	(3)	0	2	1

*Use 1/3rd pivot*

  

-34/3	0	0	19/3	0	0	27/3	4/3
10/3	1	0	-1/3	0	0	1/3	2/3
7/3	0	0	11/3	0	1	4/3	-1/3
14/3	0	1	4/3	0	0	-1/3	4/3
4/3	0	0	-1/3	1	0	-2/3	-1/3

## 5.16

5.16]

- a) False - both problems may be infeasible
- b) False - non-basic variables can have cost coefficients of 0, which implies a shadow price of zero if it is this slack variable.
- c) True - Duality relations say if one problem is feasible, then its dual is also feasible.

d) True. If the first component of the optimal vector is true, then in optimal form,  $b_0 = 0$ . Then in the dual problem, the first cost coeff. = 0, which implies that the first constraint is active.

## 5.17

5.17

<p><u>min. problem form:</u></p> $\begin{array}{l} \text{Min. } c^T x \\ \text{s.t. } Ax \geq b \\ x \geq 0 \end{array}$	<p><u>max. problem form:</u></p> $\begin{array}{l} \text{Max. } b^T y \\ \text{s.t. } A^T y \leq c \\ y \text{ free} \end{array}$
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Assuming the problem is in optimal form, if the prob has multiple solutions, then some nonbasic  $c_i = 0$ .

Therefore in the optimal form of the max. Problem, there is some constraint  $A^T y \leq c$ , which means the max. problem is degenerate.

Ex:

$$\begin{array}{l} \text{Min. } x_1 + x_2 \\ \text{s.t. } x_1 + x_2 - x_3 \geq 4 \\ x_2 \geq 1 \\ x_3 \geq 0 \end{array}$$

$$\Rightarrow \begin{array}{c|ccccc} & 0 & 1 & 1 & 0 & 0 & 0 \\ \hline x_1 & -1 & -1 & 1 & 1 & 0 & 0 \\ x_2 & 1 & 0 & 0 & 0 & 1 & 0 \\ \hline & -1 & 1 & 0 & 0 & 0 & 1 \end{array}$$

$$\Rightarrow \begin{array}{c|ccccc} & 4 & 1 & -4 & -1 & 0 & 0 \\ \hline x_1 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ x_2 & 1 & 1 & 0 & -1 & 0 & 0 & 1 \\ x_3 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ \hline & 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{array}$$

This problem has multiple solutions because a nonbasic var has a const coeff. = 0.

The dual problem:

$$\begin{array}{ll} \text{Max. } 4y_1 + y_2 & 4(u_1 - v_1) + (u_2 - v_2) \\ \text{s.t. } y_1 + y_2 \leq 1 & \Rightarrow (u_1 - v_1) + (u_2 - v_2) \leq 1 \\ y_1 \leq 1 & (u_1 - v_1) \leq 1 \\ -y_1 \leq 0 & -(u_1 - v_1) \leq 0 \\ y \text{ free.} & u, v \geq 0 \end{array}$$

$\Rightarrow$  This tableau is in canonical form. And  $b_2 = 0$ , and is therefore degenerate.

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5.18]									
a)	$\begin{array}{ c c c c c } \hline 0 & 0 & 0 & 5 & 3 & 0 \\ \hline -1 & 1 & 0 & -1 & 1 & 0 \\ \hline 2 & 0 & 0 & 1 & -1 & 1 \\ \hline -1 & 0 & 1 & -1 & (-1) & 0 \\ \hline \end{array}$			$\begin{array}{ c c c c c } \hline 0 & 0 & 0 & 5 & 3 & 0 \\ \hline -2 & 1 & 0 & -1 & 1 & 0 \\ \hline 2 & 0 & 0 & 1 & -1 & 1 \\ \hline -3 & 0 & 1 & \oplus 0 & 1 & \\ \hline \end{array}$		$\begin{array}{ c c c c c } \hline 0 & 5 & 3 & 2 & 0 & 0 \\ \hline -2 & 1 & -1 & 0 & 0 & 0 \\ \hline 3 & 0 & 1 & (-1) & 1 & \\ \hline \end{array}$			
b)	$\begin{array}{ c c c c c } \hline 0 & 0 & 0 & 5 & 3 & 0 \\ \hline -2 & 1 & 0 & -1 & 1 & 0 \\ \hline 2 & 0 & 0 & 1 & -1 & 1 \\ \hline -3 & 0 & 1 & \oplus 0 & 1 & \\ \hline \end{array}$			$\begin{array}{ c c c c c } \hline -5 & 0 & 5 & 0 & 3 & 5 \\ \hline 1 & 1 & -1 & 0 & 1 & -1 \\ \hline -1 & 0 & 1 & 0 & \oplus 1 & 2 \\ \hline 3 & 0 & -1 & 1 & 0 & -1 \\ \hline \end{array}$			$\begin{array}{ c c c c c } \hline 6 & 0 & 7 & 1 & 0 & 2 \\ \hline -2 & 1 & -1 & (-1) & 0 & 0 \\ \hline 3 & 0 & -1 & 1 & 1 & -1 \\ \hline \end{array}$		
c)	$\begin{array}{ c c c c c } \hline -5 & 1 & 4 & 0 & 0 & 0 \\ \hline 1 & -\frac{1}{2} & -\frac{1}{2} & 1 & 0 & 0 \\ \hline 1 & 1 & 0 & 0 & 0 & 1 \\ \hline 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 1 & 0 \\ \hline \end{array}$			$\begin{array}{ c c c c c } \hline -18 & 0 & 8 & 0 & 0 & 11 \\ \hline 0 & 1 & 0 & 0 & 0 & 1 \\ \hline 1 & 0 & -1 & 0 & 1 & -2 \\ \hline 3 & 0 & -1 & 1 & 0 & -1 \\ \hline \end{array}$			$\begin{array}{ c c c c c } \hline -8 & 1 & 6 & 0 & 0 & 2 \\ \hline 2 & -1 & 1 & 1 & 0 & 0 \\ \hline 1 & 1 & -2 & 0 & 1 & -1 \\ \hline \end{array}$		