Matt Forbes
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Homework 1

1 Problem One

1.1

The minimum number of socks that need to be drawn to guarantee a pair of any color is 4. You could possibly have 3 socks that do not make a pair, but on the next draw, one of the socks will become a match.

1.2

Likewise, the minimum number of socks from a drawer of n different color socks (assuming there is at least 2 of each color) is n+1. For the same reason that you can have no match on the nth draw, but on the n+1th draw, one sock will match.

2 Problem Two

Prove by induction that $F_{n+k} = F_k F_{n+1} + F_{k-1} F_n$

2.1 Basis

$$\begin{aligned} \text{let n=0, k >= 1} \\ F_{0+k} = F_k F_1 + F_{k-1} F_0 \\ F_k = F_k(1) + F_{k-1}(0) \\ F_k = F_k \end{aligned}$$

2.2 Inductive Hypothesis

For all $n >= 1, c >= 0$, assuming the equation is true for $n-1$ then it is also true for n.

2.3 Proof

$$\begin{aligned}
F_{n+c} &= F_c F_{n+1} + F_{c-1} F_n \\
F_{(n-1)+(c+1)} &= F_c F_{n+1} + F_{c-1} F_n \\
F_{c+1} F_n + F_c F_{n-1} &= F_c(F_{n-1} + F_n) + F_{c-1} F_n \\
(F_{c-1} + F_c) F_n + F_c F_{n-1} &= F_c F_{n-1} + F_c F_n + F_{c-1} F_n \\
F_{c-1} F_n + F_c F_n + F_c F_{n-1} &= F_{c-1} F_n + F_c F_n + F_c F_{n-1}
\end{aligned}$$

3 Problem Three

Prove $\sum_{i=0}^{n} \binom{n}{i} = 2^n$ with induction and given identity.

3.1 Basis

$$\begin{aligned}
\sum_{i=0}^{1} \binom{1}{i} &= \binom{1}{0} + \binom{1}{1} \\
&= 1 + 1 \\
&= 2^1
\end{aligned}$$

3.2 Inductive Hypothesis

For all $n > 1$, assuming the sum is true for $n - 1$, it is also true for n.

3.3 Induction

$$
\begin{aligned}
\sum_{i=0}^{n} \binom{n}{i} &= \binom{n}{0} + \sum_{i=1}^{n} \binom{n}{i} \text{[pull first value from sum]} \\
&= 1 + \sum_{i=1}^{n} \binom{n-1}{i} + \binom{n-1}{i-1} \text{[given identity]} \\
&= 1 + \sum_{i=1}^{n} \binom{n-1}{i} + \sum_{i=1}^{n} \binom{n-1}{i-1} \text{[break up sum]} \\
&= 1 + [(\sum_{i=0}^{n} \binom{n-1}{i}) - \binom{n}{0}] + \sum_{i=0}^{n} \binom{n-1}{i} \text{[change sum limits]} \\
&= 1 + [2^{n-1} - 1] + 2^{n-1} \text{[inductive hypthosis]} \\
&= 2 * 2^{n-1} \\
&= 2^{n}
\end{aligned}
$$

4 Problem Four