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Math 312 Assignment 1
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1 Problem 1

2 Problem 2

3 Problem 3

Goal: Prove $\forall a \in \mathbb{R}\ s.t.\ -1 < a < 1, \forall n \in \mathbb{N} : |a|^n \leq \dfrac{|a|}{n(1-|a|)+|a|}$.

3.1 Lemma $\forall x \in \mathbb{R}, n \in \mathbb{N} :\ 0 \leq x \leq 1 \Rightarrow x^n \leq 1$

Proof by induction:

Let $x \in \mathbb{R}$ be such that $0 \leq x \leq 1$.
Define $P(n) : x^n \leq 1$.

Basis: $P(1) = x^1 \leq 1$, which is trivially true.

Inductive Hypothesis (I.H.): let $k \in \mathbb{N}$ be arbitrary; Assume $P(k)$ is true.
$x^k \leq 1$ by I.H.
$xx^k \leq x$ by OM.
$x^{k+1} \leq x$ by def. of powers.
$x^{k+1} \leq 1$ by transitiviy.
$P(k+1)$ is true, thus $P(k) \Rightarrow P(k+1)$.

$\forall x \in \mathbb{R}, n \in \mathbb{N}, 0 \leq x \leq 1 \Rightarrow x^n \leq 1$.

3.2 Lemma $\forall x \in \mathbb{R}\ s.t.\ 0 \leq x \leq 1 :\ x \leq \dfrac{1}{x}$

Let $x \in \mathbb{R}$ be such that $0 \leq x \leq 1$.
$x \leq 1$, so $\dfrac{1}{x} \geq 1$ by 312 Notes 2.2.2(g).
By transitivity, $x \leq \dfrac{1}{x}$.

3.3 Proof

Let $n \in \mathbb{N}, a \in \mathbb{R}$ s.t. $-1 < a < 1$
Set $b = -1 + |a|$.
$0 < |a| < 1$ so $-1 < b < 0$.
By Bernouilli's inequality, $(1 + b)^{n+1} \geq 1 + b(n + 1)$.
Substituting $b = -1 + |a|$: $(1 + -1 + |a|)^{n+1} \geq 1 + (-1 + |a|)(n + 1)$.
$|a|^{n+1} \geq 1 - n(1 - |a|) - 1 + |a|$.

4 Problem 4

5 Problem 5

6 Problem 6