Homework 2

Matt Forbes

October 12, 2010

Problem 3.1

a)

$$\begin{aligned} \min \quad & -2x_1 - 3x_2 \\ \text{s. t.} \quad & x_1 + x_2 + x_3 = 4 \\ & x_1 - 2x_2 + x_4 = 1 \\ & \mathbf{x} \geq 0 \end{aligned}$$

b)

$$\begin{array}{|c|cccc|} \hline 0 & -2 & -3 & 0 & 0 \\ \hline 4 & 1 & 1 & 1 & 0 \\ 1 & 1 & -2 & 0 & 1 \\ \hline \end{array}$$

c)

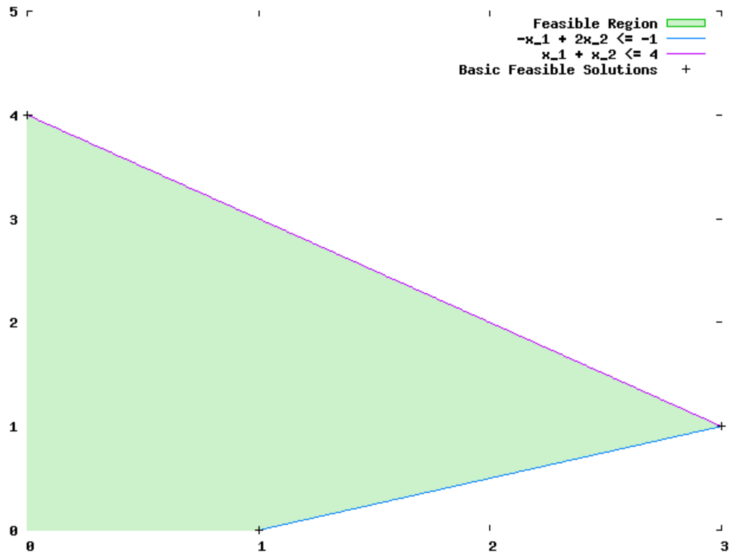
$$\begin{array}{|c|cccc|} \hline 2 & 0 & -7 & 0 & 2 \\ \hline 3 & 0 & 3 & 1 & -1 \\ 1 & 1 & -2 & 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|cccc|} \hline 9 & 0 & 0 & \frac{7}{3} & -\frac{1}{3} \\ \hline 1 & 0 & 1 & \frac{1}{3} & -\frac{1}{3} \\ 3 & 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{|c|cccc|} \hline 12 & 1 & 0 & 3 & 0 \\ \hline 4 & 1 & 1 & 1 & 0 \\ 9 & 3 & 0 & 2 & 1 \\ \hline \end{array}$$

optimal form: $\mathbf{x} = (0, 4, 0, 9)^T$

d)



Problem 3.2

a)

$$
\begin{aligned}
\min \quad & -2x_1 + x_2 \\
\text{s.t} \quad & x_1 + 2x_2 + x_3 \qquad = 10 \\
& -x_1 + x_2 \qquad - x_4 = 5 \\
& \mathbf{x} \geq 0
\end{aligned}
$$

b)

If moving from original problem to standard form:

- The feasible point is of the form $(x_1 \quad x_2)^T$
- A point for the standard form needs an x_3 and x_4, which are equal to $10 - x_1 - 2x_2$ and $-x_1 + x_2 - 5$ respectively.
- So this solution in the standard form would be:

$$\begin{pmatrix} x_1 \\ x_2 \\ 10 - x_1 - 2x_2 \\ -x_1 + x_2 - 5 \end{pmatrix}$$

If moving from standard form to original problem:

- The feasible point is $(x_1 \quad x_2 \quad x_3 \quad x_4)^T$
- Only x_1 and x_2 are needed for original form, so the solution would be:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

c)

Both programs are optimizing the same function with the same constraints. Therefore, if a minimum is found for one program, it is the minimum for the other.

d)

Same reasoning as part c. They are the same problem, just formatted differently. There cannot be different solutions.

e)

You're kidding right? They are the same problem.

Problem 3.3

	a	b	c	d	e
i	≥ 0	≥ 0	c	d	≥ 0
ii	≥ 0	< 0	< 0	< 0	e
iii	< 0	b	≥ 0	d	e
iv	≥ 0	$= 0$	c	d	$= 0$

Problem 3.6

$$\begin{array}{rl} \min & 2x_1 + x_2 - 4x_3 \\ \text{s.t.} & 3x_1 - 2x_2 + 2x_3 + x_4 = 25 \\ & -x_1 - x_2 + 2x_3 + x_5 = 20 \\ & -x_1 - x_2 + x_3 + x_6 = 5 \\ & \mathbf{x} \geq 0 \end{array}$$

0	2	1	-4	0	0	0
25	3	-1	2	1	0	0
20	-1	-1	2	0	1	0
5	-1	-1	1	0	0	1

20	-2	-3	0	0	0	4
15	5	1	0	1	0	4
10	1	1	0	0	1	-2
5	-1	-1	1	0	0	1

50	7	0	0	0	3	-2
5	2	0	0	1	-1	0
10	3	1	0	0	1	-2
15	2	0	1	0	1	-1

This problem is unbounded and therefore has no finite optimal solution.

Problem 3.7

a)

0	2	-1	0	3	0
10	1	1	0	1	1
6	3	-1	1	-2	0

b)

Basic sequence $S = \{5, 1\}$

c)

Basic feasible solution $\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 6 \\ 0 \\ 10 \end{pmatrix}$

d)

Solving 1st constraint for x_2: $x_2 = 10 - x_1 - x_4 - x_5$
Replacing x_2 in objective function: $-10 + 3x_1 + 4x_4 + x_5$
Replacing x_2 in 2nd constraint: $4x_1 + x_3 - x_4 + x_5 = 16$
Which produces a new LP problem:

$$\begin{array}{lllll} \min & 3x_1 & & + 4x_4 + x_5 - 10 & \\ \text{s.t} & x_1 + 2x_2 + x_3 + x_4 & & & = 10 \\ & x_1 & + x_3 - x_4 & + x_5 & = 16 \end{array}$$

Which has a simplex tableau of:

-10	3	0	0	4	1
10	1	2	1	1	0
16	4	0	1	-1	1

Likewise,
Pivoting in the 2nd column of the tableau from a) results in the same form as above.

-10	3	0	0	4	1
10	1	2	1	1	0
16	4	0	1	-1	1

Problem 3.10

a) Row 3.
b) Row 2.
c) Column 2, Row 1.
d) You can rearrange the columns so they have the identiy columns first, making it lexicographically positive. A possible new ordering could be $x_4, x_7, x_5, x_1, x_2, x_3, x_6$.

Problem 3.11

a) If you were to pivot on that column, then all the rows that tie for minimum ratio (besides pivot row) would have their b-value equal to 0.
b) Yes, pivoting on a column with it's c-value greater than 0 would be counter-productive, but after the pivot the rows that tied for minimum ratio would have b-value 0.

Problem 3.22

Both $\mathbf{c}$ and $\mathbf{x}$ are nonnegative, so the only way to minimize the function is to decrease the values of $\mathbf{x}$ towards 0. So the ideal value of $\mathbf{x}$ would be $\mathbf{0}$, which turns out to be a valid solution because $\mathbf{Ax} \leq \mathbf{b}$, where $\mathbf{b}$ is nonnegative. The minimum value must be 0.