

Homework 4

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3.20

a) Using subproblem

Original problem:

0	4	0	-1
-1	1	1	-1
1	-1	1	0

Phase 0:

4	0	0	-1
-1	1	0	-0.5
0	0	1	-0.5

Now form subproblem with obj. function as row 1, and pivot:

6	-2	0	0
2	-2	0	1
1	-1	1	0

b) Using artificial problem

Initial artificial problem:

0	0	0	0	1	1
-1	1	1	-1	1	0
1	-1	1	0	0	1

Pivoted to optimal form (feasible because d = 0):

0	0	0	0	1	1
2	-2	0	1	-1	1
1	-1	1	0	0	1

Revert to original problem's objective function:

0	4	0	-1
2	-2	0	1
1	-1	1	0

3.24

Make one pivot on the original matrix such that it decreases the objective function by the smallest amount.

Original Tableau:

-9	0	4	5	0	2	0
4	1	2	4	0	1	0
5	0	3	1	1	-1	0

After the pivot:

-14	-1.25	1.5	0	0	0.75	0
1	0.25	0.5	1	0	0.25	0
4	-0.25	2.5	0	1	-1.25	0

So the next best feasible solution is $\mathbf{x} = (0, 0, 1, 4, 0, 0)^T$

3.26

$\begin{matrix} 3.26 \\ 3.30 \\ 4.5 \end{matrix}$	$\left \begin{array}{ccc ccccc} 1 & 0 & 0 & -9 & 0 & -8 & 10 & -1 & 0 \\ 0 & 1 & 0 & 5 & 0 & -1 & -5 & 3 & 1 \\ 0 & 0 & 1 & 2 & 1 & 1 & -1 & 0 & 0 \end{array} \right $
Q	$\left \begin{array}{ccc ccccc} 1 & 0 & 8 & 7 & 8 & 0 & 18 & -9 & 0 \\ 0 & 1 & 1 & 7 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & 1 & 2 & 1 & 1 & 1 & -1 & 0 \end{array} \right $
Q_{-1}	$\left \begin{array}{ccc ccccc} 1 & \frac{9}{2} & 0 & \frac{7}{2} & \frac{25}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & -2 & 1 & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 & 0 & \frac{1}{2} & \frac{3}{2} & 1 & -1 & 0 \end{array} \right $

$$Q = \begin{bmatrix} 1 & \frac{9}{2} & \frac{25}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

3.30

3.30

$$\begin{array}{ll} \text{Min} & -\bar{x}_1 + \bar{x}_2 - 2\bar{x}_3 + \hat{x}_1 - \hat{x}_2 + 2\hat{x}_3 \\ \text{s.t.} & \bar{x}_1 - 2\bar{x}_2 + 5\bar{x}_3 + \bar{s}_1 = 25 \\ & \bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{s}_2 = 10 \\ & -2\bar{x}_1 + 5\bar{x}_2 - 7\bar{x}_3 + \bar{s}_3 = 20 \\ & \text{and similar for } \hat{x}. \end{array}$$

0	-1	1	-2	1	-1	2	0	0	0	0	0	0
25	1	-2	5	0	0	0	1	0	0	0	0	0
10	1	1	1	0	0	0	0	1	0	0	0	0
20	-2	3	-7	0	0	0	0	0	1	0	0	0
25	0	0	0	1	-2	5	0	0	0	1	0	0
10	0	0	0	1	1	1	0	0	0	0	1	0
20	0	0	0	-2	3	-7	0	0	0	0	0	1

Initial Tableau:

0	-1	1	-2	1	-1	2	0	0	0	0	0	0
25	1	-2	5	0	0	0	1	0	0	0	0	0
10	1	1	1	0	0	0	0	1	0	0	0	0
20	-2	3	-7	0	0	0	0	0	1	0	0	0
25	0	0	0	1	-2	5	0	0	0	1	0	0
10	0	0	0	1	1	1	0	0	0	0	1	0
20	0	0	0	-2	3	-7	0	0	0	0	0	1

Pivoting...

10	0	2	-1	1	-1	2	0	1	0	0	0	0
15	0	-3	4	0	0	0	1	-1	0	0	0	0
10	1	1	1	0	0	0	0	1	0	0	0	0
40	0	5	-5	0	0	0	0	2	1	0	0	0
25	0	0	0	1	-2	5	0	0	0	1	0	0
10	0	0	0	1	1	1	0	0	0	0	1	0
20	0	0	0	-2	3	-7	0	0	0	0	0	1

Pivoting...

13.75	0	1.25	0	1	-1	2	0.25	0.75	0	0	0	0
3.75	0	-0.75	1	0	0	0	0.25	-0.25	0	0	0	0
6.25	1	1.75	0	0	0	0	-0.25	1.25	0	0	0	0
58.75	0	1.25	0	0	0	0	1.25	0.75	1	0	0	0
25	0	0	0	1	-2	5	0	0	0	1	0	0
10	0	0	0	1	1	1	0	0	0	0	1	0
20	0	0	0	-2	3	-7	0	0	0	0	0	1

Pivoting...

20.4	0	1.25	0	0.333	0	-0.333	0.25	0.75	0	0	0	0.333
3.75	0	-0.75	1	0	0	0	0.25	-0.25	0	0	0	0
6.25	1	1.75	0	0	0	0	-0.25	1.25	0	0	0	0
58.8	0	1.25	0	0	0	0	1.25	0.75	1	0	0	0
38.3	0	0	0	-0.333	0	0.333	0	0	0	1	0	0.667
3.33	0	0	0	1.67	0	3.33	0	0	0	0	1	-0.333
6.67	0	0	0	-0.667	1	-2.33	0	0	0	0	0	0.333

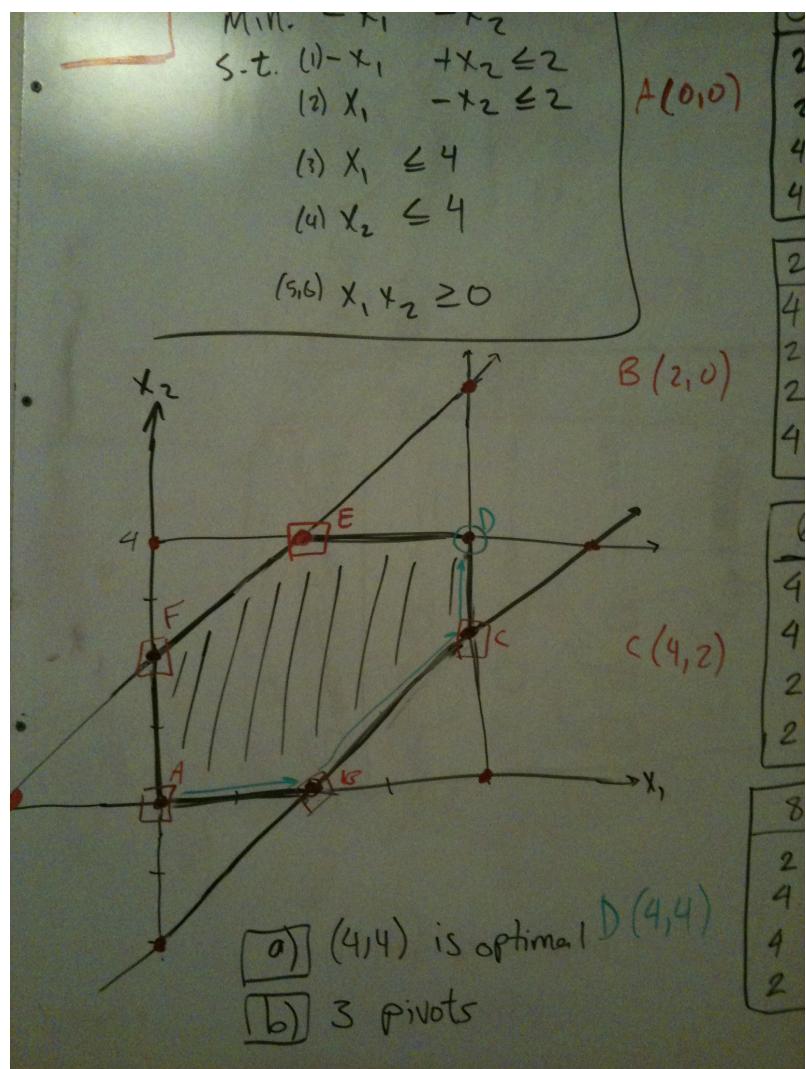
And in optimal form:

20.8	0	1.25	0	0.5	0	0	0.25	0.75	0	0	0.1	0.3
3.75	0	-0.75	1	0	0	0	0.25	-0.25	0	0	0	0
6.25	1	1.75	0	0	0	0	-0.25	1.25	0	0	0	0
58.8	0	1.25	0	0	0	0	1.25	0.75	1	0	0	0
38	0	0	0	-0.5	0	0	0	0	0	1	-0.1	0.7
1	0	0	0	0.5	0	1	0	0	0	0	0.3	-0.1
9	0	0	0	0.5	1	0	0	0	0	0	0.7	0.1

Has solution $\bar{x} = (6.25, 0, 3.75)^T$ and $\hat{x} = (0, 9, 1)^T$

4.5

This answer is a series of two shots of my whiteboard. Answers a-h are labeled in the picture.



[c]

0	-1	-1	0	0	0	0
2	-1	1	1	0	0	0
2	1	-1	0	1	0	0
4	1	0	0	0	1	0
4	0	1	0	0	0	1

(0,0)

(2,0)

c(4,2)

D(4,4)

[d]

- (1) : [F,E]
- (2) : [B,C]
- (3) : [C,D]
- (4) : [E,D]
- (5) : [A,B]
- (6) : [A,F]

[e]

$$\begin{aligned} (0,0) &= (1) \times (5) \\ (2,0) &= (5) \times (2) \\ (4,2) &= (2) \times (3) \\ (4,4) &= (3) \times (4) \end{aligned}$$

[f]

Red points on Graph

[g]

- Pivot on row 3, col 1
- [h]
- Pivot on row 3, col 1
- Pivot in row 4 col 1