

Homework 2

Matt Forbes

October 21, 2010

1 Problem One

a)

found is true. For this case to occur, we had to be within some iteration of the loop. In order to be in an iteration, the expression $(low \leq high)$ AND **!found** must be true. Therefore, low does not equal $high$, because low must be less than $high$. The first if statement within the loop body checks if

$A[low] + A[high] == x$ is true, and will set **found** to true if that is the case. Therefore, if **found** is true, then low and $high$ are indices of **A** whose elements' sum is x .

b)

low \geq high. I will prove the loop invariant provided to show that if $low \geq high$, there does not exist two distinct elements in **A** that sum to x .

Basis

At the start of the first iteration,

$low = 0, high = n - 1,$

$S = \{A[0] \dots A[low - 1]\} \cup \{A[high] \dots A[n - 1]\} = \{\}$

So there are no distinct pairs in S sum to x . The L.I. holds before we enter the loop.

Maintenance

Assuming the L.I. held for all iterations up to this iteration j , then:

- Right before this loop started, there was no distinct pair of elements in the set $S = \{A[0] \dots A[low - 1]\} \cup \{A[high + 1] \dots A[n - 1]\}$ whose sum = x .
- During this iteration, $A[low] + A[high]$ could be:
equal to x: We found a distinct pair that sums x , done.

less than x: low is incremented, and thus $A[low]$ is 'added' to S in the next iteration, in which case the L.I. would still hold for these reasons: $A[low] + A[i], i = 0 \dots low - 1$ will always be less than x , and $A[low] + A[j], j =$

$high + 1 \dots n - 1$ will always be greater than x . Therefore there will be no pair in S whose sum is exactly x .

greater than x : $high$ is decremented, and this $A[high]$ is 'added' to S in the next iteration, in which case the L.I. would still hold for these reasons: $A[high] + A[j], j = high + 1 \dots n - 1$ will always be greater than x , and $A[i] + A[high], i = 0 \dots low$ will always be less than x . Therefore there will be no pair in S whose sum is exactly x .

- The L.I. will always hold after this iteration, granted that it held up to this point for the reasons listed above.

Termination

- After each iteration, either low is incremented or $high$ is decremented, so they have to converge at one point as long as **found** is never set to true, so the loop is guaranteed to terminate.
- According to the L.I. at the end of the last iteration, there is no distinct pair of elements in $S = \{A[0] \dots A[low - 1]\} \cup \{A[high + 1] \dots A[n - 1]\}$ whose sum is x . Well at the end of the loop, S is equal to all of the elements in **A**. Therefore, there is no distinct pair of elements in **A** whose sum is equal to x .

2 Problem Three

Algorithm

median(L, R, n):

 return median-helper(L, R, 0, 0, 0, n)

median-helper(L, R, lbound, rbound, cuts, n):

 if L is empty or R is empty:

 'remove' (n-cuts-1) cards from the non-empty hand
 return first remaining card in the non-empty hand (+1) }