Homework 7

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5.5

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$$M = \left[\begin{array}{c|c} 0 & \bar{c} \\ \hline b & A \end{array}\right]$$

$$M^* = \left[\begin{array}{c|c} -d & \bar{c}^{*T} \\ \hline b^* & A^* \end{array}\right]$$

$$Q = \left[\begin{array}{c|c} 1 & \bar{r}^T \\ \hline \bar{0} & R \end{array}\right]$$

$$M^* = QM = \left[\begin{array}{c|c} 1 & \bar{r}^T \\ \hline \bar{0} & R \end{array}\right] \left[\begin{array}{c|c} 0 & \bar{c}^T \\ \hline \bar{b} & A \end{array}\right] = \left[\begin{array}{c|c} \bar{r}^T\bar{b} & \bar{c}^T + \bar{r}^T A \\ \hline R\bar{b} & RA \end{array}\right]$$

- $-d = \bar{r}^T\bar{b}$
- $\bar{c}^{*T} = \bar{c}^T + \bar{r}^T A \geq 0$

So, $-\bar{r}^T A \leq \bar{c}^T \implies -\bar{r}$ is feasible for dual ✓

and $d = -\bar{r}^T\bar{b} \implies -\bar{r}$ is optimal for dual ✓

Therefore $-\bar{r}$ is the optimal soln for the dual problem. ✓

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$\boxed{5.7}$ a) \$2

b) R2: \$0

R3: \$3

c) 40 units

5.8

a

5.8

a) let $y = (\bar{u} - \bar{v})$ where $\bar{u}, \bar{v} \geq \emptyset$

rewritten:

$$\min. \begin{bmatrix} c^T, & a^T, & -a^T \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix}$$

$$\text{s.t.} \begin{bmatrix} A & 0 & 0 \\ -A & 0 & 0 \\ D & B & -B \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix} \geq \begin{bmatrix} \bar{b} \\ -\bar{b} \\ \bar{d} \end{bmatrix}$$

$$\bar{x}, \bar{u}, \bar{v} \geq \bar{0}$$

Dual:

$$\text{Max.} \begin{bmatrix} \bar{b}, & -\bar{b}, & \bar{d} \end{bmatrix} \begin{bmatrix} \bar{r} \\ \bar{s} \\ \bar{t} \end{bmatrix}$$

$$\text{s.t.} \begin{bmatrix} A & -A & D \\ 0 & 0 & B \\ 0 & 0 & -B \end{bmatrix} \begin{bmatrix} \bar{r} \\ \bar{s} \\ \bar{t} \end{bmatrix} \geq \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix}$$

$$\bar{r}, \bar{s}, \bar{t} \geq 0$$

b

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b) <u>rewritten:</u>

$$\text{Min. } \begin{bmatrix} \bar{c}^T \end{bmatrix} \begin{bmatrix} \bar{x} \end{bmatrix}$$

$$\text{s.t. } \begin{bmatrix} A \\ -A \\ -B \end{bmatrix} \begin{bmatrix} \bar{x} \end{bmatrix} \geq \begin{bmatrix} \bar{b} \\ -\bar{b} \\ -\bar{a} \end{bmatrix}$$

$$\bar{x} \geq 0$$

<u>dual:</u>

$$\text{Max. } \begin{bmatrix} \bar{b}^T, -\bar{b}^T, -\bar{a}^T \end{bmatrix} \begin{bmatrix} \bar{y} \end{bmatrix}$$

$$\text{s.t. } \begin{bmatrix} A & -A & -B \end{bmatrix} \begin{bmatrix} \bar{y} \end{bmatrix} \leq \begin{bmatrix} \bar{c} \end{bmatrix}$$

$\bar{y}$ free

c

rewritten:

c) $$\text{Max. } [c^T, a^T, -a^T] \begin{bmatrix} \bar{w} \\ \bar{r} \\ \bar{s} \end{bmatrix}$$

$$\text{s.t. } \begin{bmatrix} A & B & -B \\ -A & -B & B \end{bmatrix} \begin{bmatrix} \bar{w} \\ \bar{r} \\ \bar{s} \end{bmatrix} \leq \begin{bmatrix} \bar{b} \\ -\bar{b} \end{bmatrix}$$

$$\bar{w}, \bar{r}, \bar{s} \geq \bar{0}$$

and $\bar{v} = (\bar{r} - \bar{s})$

dual:

$$\min. \ [b^T, -b^T] \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix}$$

$$\text{s.t. } \begin{bmatrix} A & -A \\ B & -B \\ -B & B \end{bmatrix} \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} \geq \begin{bmatrix} \bar{c} \\ \bar{a} \\ -\bar{a} \end{bmatrix}$$

$$\bar{y}, \bar{z} \geq 0$$

$$\begin{aligned} \Rightarrow \ & \min. \ b^T(\bar{y} - \bar{z}) \\ \text{s.t. } & A(\bar{y} - \bar{z}) \geq \bar{c} \\ & B(\bar{y} - \bar{z}) \geq \bar{a} \\ & -B(\bar{y} - \bar{z}) \geq -a \\ & \bar{y}, \bar{z} \geq 0 \end{aligned}$$

$$\boxed{\begin{aligned} \Rightarrow \ & \min \ \bar{b}^T \bar{x} \\ \text{s.t. } & A\bar{x} \geq \bar{c} \\ & B\bar{x} = \bar{a} \\ & \bar{x} \text{ free} \\ & \bar{x} = (\bar{y} - \bar{z}) \end{aligned}}$$

d

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d) rewritten:

$$\min \begin{bmatrix} \bar{c}^T, & \bar{a}^T, & -\bar{a}^T \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix}$$

$$\text{s.t.} \begin{bmatrix} -A & -B & B \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \\ \bar{v} \end{bmatrix} \geq \begin{bmatrix} -\bar{b} \end{bmatrix}$$

$$\bar{x}, \bar{u}, \bar{v} \geq 0$$

$$\bar{y} = (\bar{u} - \bar{v})$$

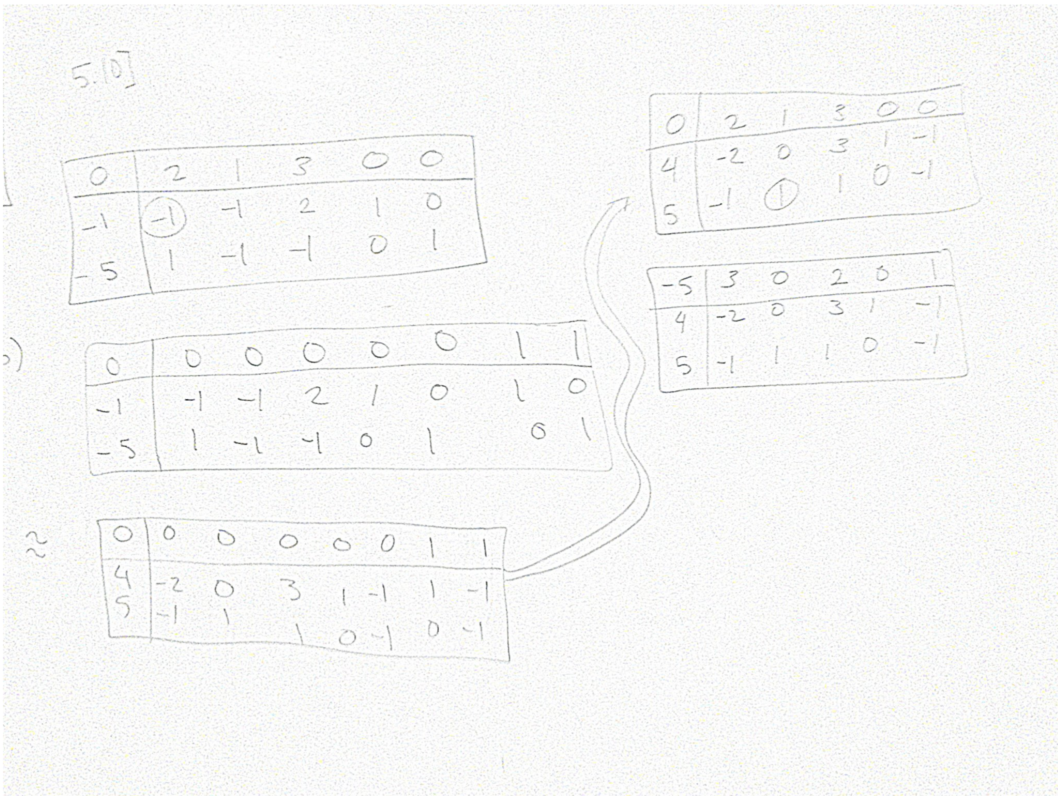
dual:

$$\max \begin{bmatrix} -\bar{b}^T \end{bmatrix} \begin{bmatrix} \bar{q} \end{bmatrix}$$

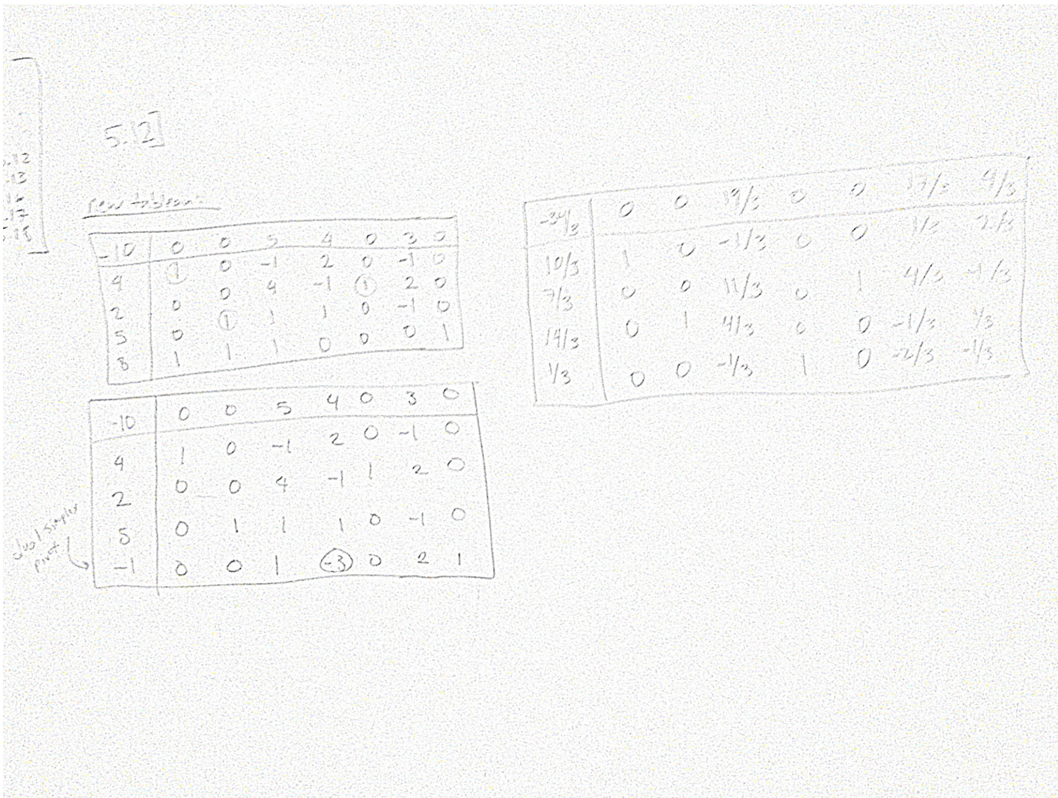
$$\text{s.t.} \begin{bmatrix} -A \\ -B \\ B \end{bmatrix} \begin{bmatrix} \bar{q} \end{bmatrix} \leq \begin{bmatrix} \bar{c}^T \\ \bar{a}^T \\ -\bar{a}^T \end{bmatrix}$$

$\bar{q}$ free

5.10


5.10
0 | 2 1 3 0 0
-1 | -1 -1 2 1 0
-5 | 1 -1 -1 0 1
0 | 0 0 0 0 0 1 1
-1 | -1 -1 2 1 0 1 0
-5 | 1 -1 -1 0 1 0 1
0 | 0 0 0 0 0 1 1
4 | -2 0 3 1 -1 1 -1
5 | -1 1 1 0 -1 0 -1
0 | 2 1 3 0 0
4 | -2 0 3 1 -1
5 | -1 1 1 0 -1
-5 | 3 0 2 0 1
4 | -2 0 3 1 -1
5 | -1 1 1 0 -1

5.12



5.12

new tableau:

-10	0	0	5	4	0	3	0
4	1	0	-1	2	0	-1	0
2	0	0	4	-1	1	2	0
5	0	1	1	1	0	-1	0
8	1	1	1	0	0	0	1

-10	0	0	5	4	0	3	0
4	1	0	-1	2	0	-1	0
2	0	0	4	-1	1	2	0
5	0	1	1	1	0	-1	0
-1	0	0	1	-3	0	2	1

dual simplex pivot

-34/3	0	0	19/3	0	0	17/3	9/3
10/3	1	0	-1/3	0	0	1/3	2/3
7/3	0	0	11/3	0	1	4/3	-1/3
14/3	0	1	4/3	0	0	-1/3	1/3
1/3	0	0	-1/3	1	0	-2/3	-1/3

5.13

$\boxed{5.13}$

a) min. $\sum_i^n -y_i$

s.t. $\begin{bmatrix} A^T \\ -A^T \end{bmatrix} \bar{y} \geq 0$

$\bar{y} \geq 0$

... System has [illegible] when min. [illegible] > 0

b) max. $\bar{0}^T \bar{x}$

s.t. $[A \;\; -A]\,\bar{x} \leq \begin{bmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}$

$\bar{x}$ free

c) If one system has an optimal solution then the other must as well. So if a sol'n exists for $A^T\bar{y} = 0$, then so must $Ax = \begin{bmatrix} -1 \\ \vdots \\ -1 \end{bmatrix}$ which is [illegible] which makes the other system infeasible, and vice versa.

5.16

5.16]

a) False - both problems may be infeasible

b) False - non-basic variables can have cost coefficients of 0, which implies a shadow price of zero if it is this slack variable.

c) True - Duality relations say if one problem is feasible, then its dual is also feasible.

d) True. If the first component of the optimal vector is true, then in optimal form, $b_0 = 0$. Then in the dual problem, the first cost coeff. $= 0$, which implies that the first constraint is active.

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5.17

- min. problem form:

$$\begin{aligned} \text{Min. } & c^T x \\ \text{s.t. } & Ax \geq b \\ & x \geq 0 \end{aligned}$$

Max. problem form:

$$\begin{aligned} \text{max. } & b^T y \\ \text{s.t. } & A^T y \leq c \\ & y \text{ free} \end{aligned}$$

- Assuming the problem is in optimal form, if the prob has multiple solutions, then some nonbasic $c_i = 0$.

- Therefore in the optimal form of the max. Problem. There is some constraint $A_i y = 0$, which means the max. problem is degenerate.

- Ex

$$\begin{aligned} \min\ & x_1 + x_2 \\ \text{s.t. } & x_1 + x_2 - x_3 \geq 4 \\ & x_1 \geq 1 \\ & x \geq 0 \end{aligned}$$

$\Rightarrow$

0	1	1	0	0	0
-4	-1	-1	1	1	0
-1	-1	0	0	0	1

$\Rightarrow$

-4	0	0	1	1	0
3	0	1	-1	-1	1
1	1	0	0	0	1

$\rightarrow$ This problem has multiple solutions because a non-basic var has a cost coeff. $= 0$.

The dual problem is:

$$\begin{aligned} \text{Max } & 4y_1 + y_2 & & 4(u_1 - v_1) + (u_2 - v_2) \\ \text{s.t. } & y_1 + y_2 \leq 1 & \Rightarrow \quad & (u_1 - v_1) + (u_2 - v_2) \leq 1 \\ & y_1 \leq 1 & & (u_1 - v_1) \leq 1 \\ & -y_1 \leq 0 & & -(u_1 - v_1) \leq 0 \\ & y \text{ free.} & & \bar{u}, \bar{v} \geq 0 \end{aligned}$$

$\Rightarrow$

0	4	1	-4	-1	0	0	0
1	1	1	-1	-1	1	0	0
1	1	0	-1	0	0	1	0
0	-1	0	1	0	0	0	1

This tableau is in canonical form and has $b_3 = 0$, and is therefore degenerate

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5.18

a)

0	0	0	5	3	0
-1	1	0	-1	1	0
2	0	0	1	-1	1
-1	0	1	-1	-1	0

-3	0	3	2	0	0
-2	1	1	-2	0	0
3	0	-1	2	0	1
1	0	-1	1	1	0

-5	1	4	0	0	0
1	$-1/2$	$-1/2$	1	0	0
1	1	0	0	0	1
0	$1/2$	$-1/2$	0	1	0

b)

0	0	0	5	3	0
-2	1	0	-1	1	0
2	0	0	1	-1	1
-3	0	1	-1	0	1

-15	0	5	0	3	5
1	1	-1	0	1	-1
-1	0	1	0	-1	2
3	0	-1	1	0	-1

-15	0	5	0	0	11
0	1	0	0	0	1
1	0	-1	0	1	-2
3	0	-1	1	0	-1

c)

[illegible]	0	5	3	2	0
-2	1	1	1	0	0
-3	0	1	-1	-1	1

-6	0	7	1	0	2
-2	1	-1	-1	0	0
3	0	-1	1	1	-1

-8	1	6	0	0	2
2	-1	1	1	0	0
1	1	-2	0	1	-1