Vibrations

small angles: $\cos\theta = 1$
$\sin\theta = \theta$
rads!

1. Equilibrium (sometimes)
2. Dynamic: defl = $\delta + \Delta y$
3. ① into ②... form $\ddot{x} + \omega_n^2 x = 0$
4. ω_n
5. $y = \ldots$
6. solve A,B / C

Free undamped Vibrations.

x: $\Sigma F = ma$

Performing for a single weight, single spring:

$$\ddot{x} + \omega_n^2 x = 0$$

y: Equilibrium:

$$-k\delta_{st} + mg = 0$$

$$\delta_{st} \text{ (static deflection)} = \frac{mg}{k}$$

Dynamics:

$$\Sigma F = ma \Rightarrow$$

$$-k\delta_{st} - ky + mg = m\ddot{y}$$

$$\ddot{y} + \omega_n^2 y = 0$$

- Get into this form with methods above.

DE Solutions

① $x = A\sin\omega_n t + B\cos\omega_n t$

from initial conditions.
derive for $\dot{x}, \ddot{x}$ and plug in to find A, B.

② $x = C\sin(\omega_n t + \phi)$

Amplitude → from initial conditions.
phase angle → from initial conditions.

natural, circular freq: $\omega_n = \sqrt{\frac{k}{m}}$

natural freq.: $f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{\tau}$ ~ period

1. equilibrium (y_{st})

4. x_c ; x_p

$\Sigma F_y / \Sigma F_x / \Sigma M$ 2. Dynamic

Disp = $(y_{st} + y)$

3. form $\ddot{y} + \omega_n^2 y = \frac{F_0}{m}\cos\omega t$

Undamped forced vibrations:

(force acting: $F = F_0 \sin \omega_0 t$)

$$\ddot{x} + \omega_n^2 x = \frac{F_0}{m}\sin(\omega_0 t)$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

DE Solution: $x = x_c + x_p$

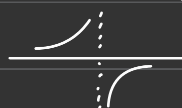
Transient solution: $x_c = C\sin(\omega_n t + \phi)$

Steady state solution: $x_p = \left(\frac{F_0/k}{1-(\omega_0/\omega_n)^2}\right)\sin\omega_0 t$

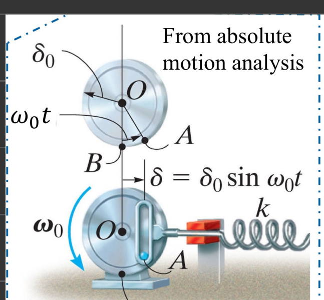
$$\tau = \frac{2\pi}{\omega_0} \qquad \text{Amp } |x| = \left|\frac{F_0/k}{1-(\omega_0/\omega_n)^2}\right|$$

Magnification factor: $M = \frac{x}{F_0/k} = \frac{1}{1-(\omega_0/\omega_n)^2}$

Resonance will occur when $\omega_0 \to \omega_n$



Support displacement:



$$\ddot{x} + \omega_n^2 x = \frac{\overbrace{k\delta_0}^{F_0}}{m}\sin\omega_0 t \quad F_0/k$$

$$x = C\sin(\omega_n t + \phi) + \left[\frac{\delta_0}{1-(\omega_0/\omega_n)^2}\right]\sin\omega_0 t$$

if we have multiple spring/damper this will change

1. Static (if there is static deflection)
2. Dynamic ($y_{st} + y$)
3. Form $m\ddot{y} + c\dot{y} + ky$?
4. Find λ from $m\lambda^2 + c\lambda + k = 0$
5. C_c

Free, damped vibrations.

$$m\ddot{x} + c\dot{x} + kx = 0 \rightarrow m\lambda^2 + c\lambda + k = 0$$

where c is damping coefficient.

$$\lambda_{1,2} = \frac{-c}{2m} \pm \sqrt{\left[\left(\frac{c}{2m}\right)^2 - \frac{k}{m}\right]} = \Delta$$

If more than 1 spring / damper: set $\Delta = 0$

Critical damping coefficient : $C_c = 2m\omega_n = 2m\sqrt{\frac{k}{m}}$

$\Rightarrow$ damping factor $\zeta = \frac{c}{C_c}$

Overdamp : $c > C_c$ $(\zeta > 1)$

$$x = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

(λ_1, λ_2 real)

no oscillations

Critically damped : $c = C_c$ $(\zeta = 1)$

$$x = (A + Bt)e^{-\omega_n t}$$

Underdamped : $c < C_c$ $(\zeta < 1)$

$$x = De^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)$$

(λ_1, λ_2 complex)

Damped natural freq : $\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{C_c}\right)^2}$ period of damped vibration : $\tau_d = \frac{2\pi}{\omega_d}$

A general approach : ① if there is static deflection:

↳ Equilibrium

② Dynamics

↳ $\Sigma F_x / \Sigma F_y / \Sigma M$

③ using ① and ② find function in std. form.

④ Derive / Use solutions.