Abgewandelter Davies and Harte method

$$Z(\omega) = K_\alpha \, \Gamma(1+\alpha)\,(+iz)^{(1-\alpha)}$$

↑ velocity auto-correlation function (VACF) im frequency domain for $\omega \to 0$, $0 < \alpha < 1$

with $$Z(\omega) = \int_0^\infty e^{-i\omega t} Z(t)\, dt$$

$$Z(t) = \frac{1}{d} \langle V(t) \cdot V(0) \rangle$$

Normal diffusion

$$\eta(t) = (\eta_1(t), \dots \eta_d(t))$$ ← independend Gaussian noise mean = 0, variance = 2Dt

η_j — $d \to$ amount of increments

$$\Delta R(t) = R(t) - R(0) = \int_0^t dt' \, \eta(t')$$ ← trajectory

$$\delta r^2(t) = \langle \Delta R(t)^2 \rangle = 2dDt$$

Anomal diffusion

$$\eta(\omega) = \int_0^\infty e^{-i\omega t} \eta(t)\, dt$$ ← independend velocities in the frequency domain

$$\eta_A(\omega) = \eta(\omega) \sqrt{\cancel{\text{Re}}[Z(\omega)] \cdot 2}$$ ← anomal diffusion velocities in frequency domain

$$\eta_A(t) = \int_{-\infty}^{\infty} e^{i\omega t} \eta_A(\omega)\, d\omega$$ ← anomal diffusion velocities increments

$$\Delta R_A(t) = R(t) - R(0) = \int_0^t dt' \, \eta_A(t')$$ ← trajectory of anomal diffusion

$$\delta r_A^2(t) = \langle \Delta R_A(t)^2 \rangle = 2d\underbrace{D}_{K_\alpha} t^\alpha$$

1.D. Discretisation

$\eta_i(t) = \mathcal{N}(0,1)$ ← Normal distribution with mean = 0, variance = 1

$$\eta(t) \to \eta_j(t)$$

$j = 0, \dots, d$

$$\eta_j(\omega) = \sum_{j=0}^{d-1} \eta_j(t)\, e^{\frac{-2\pi i j \omega}{d}}$$

$$\eta_0(\omega) = \sum_{j=0}^{d-1} \eta_j(t)\, e^0 = \Delta R(t)$$ ← das wollen wir nicht also

$$\to \eta_0(\omega) = \mathcal{N}\left(0, \sqrt{2 \cdot K_\alpha \cdot d^{(\alpha)}}\right)$$

$\cancel{\eta_{\frac{d}{2}}(\omega) =}$

1. Dimensionale Diskretisierung mit numpy fft

$d = 2n$

$\eta(t) = \eta_j(t)$ $\quad j = 0, 1, \dots, d$

$$\eta_J(\omega) = \sum_{j=0}^{d-1} \eta_j(t)\, e^{\frac{-2\pi i j J}{d}} \qquad J = 0, 1, \dots, d$$

$n \to$ Anzahl der resultierenden Inkremente

$$\eta_{AJ}(\omega) = \eta_J(\omega)\sqrt{kZ_i(\omega) \cdot 2}$$

$\eta_{A_0}(\omega) = \Delta R(t) \cdot 2$ das wollen wir nicht, also

$\to$ $\eta_{A_0}(\omega) = \mathcal{N}(0, \sqrt{2 \cdot k_d \cdot d^{\alpha}})$.

$\eta_{A_n}(\omega) = \sqrt{kZ_n(\omega) \cdot 2 \cdot n} \cdot \mathcal{N}(0,1)$ $\leftarrow$ Davies and Harte Warum?

$$\eta_{A_j}(t) = \frac{1}{d}\sum_{J=0}^{d-1} \eta_{AJ}(\omega)\, e^{\frac{2\pi i j J}{d}}$$

man halbiert die Länge von $\eta_{A_j}(t)$ sodass es nur bis n und nicht bis d geht

$\eta_{A_j}(t)$ $\quad j = 0, \dots, n$

$$\Delta R_{A_m}(t) = \sum_{j=0}^{m} \eta_{A_j}(t) \qquad m = 1 \dots j$$