A APPENDIX - PROPERTIES ON A TOY DATASET

Here we experimentally test the capability of T-SIRGN and its competitors to adhere to specific desired properties (including those discussed in **Section 4.2**). To this end, we created a toy temporal graph (**Figure 9-(a)**), and generated 6-dimensional embeddings from each method. Note that only the unsupervised versions of the related works are examined, as this dataset has no node labels. The embeddings for the lettered nodes in red (those of interest to demonstrate the properties) were then normalized and reduced to 2 dimensions using a PCA, in order to plot them in visual space. These plots are illustrated in **Figures 9-(b)–(j)**.

Aspects of interest. Specific characteristics of the toy dataset make it ideal to demonstrate our desired properties:

- **(C1)** Nodes N and P have neighborhoods with identical structure (i.e., those neighborhoods are isomorphic), in every timestamp. However, they have neighboring nodes with different node identity: this removes the possibility of generating similar embeddings based upon node connectivity.
- **(C2)** Nodes A and F have identical structures, different timestamps, but identical "time deltas", i.e., identical change in time over the timestamps ($t_2 - t_1 = t_4 - t_3 = 4$).
- **(C3)** Node J has the same time delta as A and F, along with the same degree, but a slightly different structure. Node Q has a different structure and degree than A, but identical timestamps. Both J and Q have different structure than all other lettered nodes and one another.
- **(C4)** Nodes K, M, and N have identical structure as A and F, but have time deltas of 5, 6, and 50, respectively, to test the effect of increasingly larger time deltas.

Findings. (**C1**) Our T-SIRGN (run with $\alpha = 10$) creates representations in which N and P are directly overlapping. This demonstrates that it generates identical embeddings for two nodes with identical structure and timestamps, regardless of the their connectivity properties (**Theorem 4.3**). Moreover, the embeddings of N and P are correctly far away from the other nodes, due to their vastly different time delta. Importantly, DynGem, U-CTGCN-C (using a connectivity-based loss), U-GCRN , and TIMERS all generate different representations for N and P, due to their connections with neighbor nodes with different node identities. U-CTGCN-S (using a structural loss) and TGAT create identical embeddings for N and P (but both have other issues, see next).

(**C2**) A and F are directly overlapping one another for T-SIRGN. This demonstrates the time invariance property of T-SIRGN (**Theorem 4.3**, again): two nodes with identical structure and time delta will also have identical embeddings. U-CTGCN-C does not capture this property, nor does U-GCRN , both in fact show no overlap between any nodes. TGAT does show some overlap between nodes, but oddly an overlap exists between A and Q, which have identical timestamps, but different structures and even node degree. U-CTGCN-S and TGN appear to capture the overlap between A and F, but closer inspection shows that U-CTGCN-S generates the same representation for A, F, J, K, M and Q, which includes nodes with differing timestamps, time deltas, and structures. TGN generates identical representations for A, F, J, P and K,M,N,Q. This is odd,

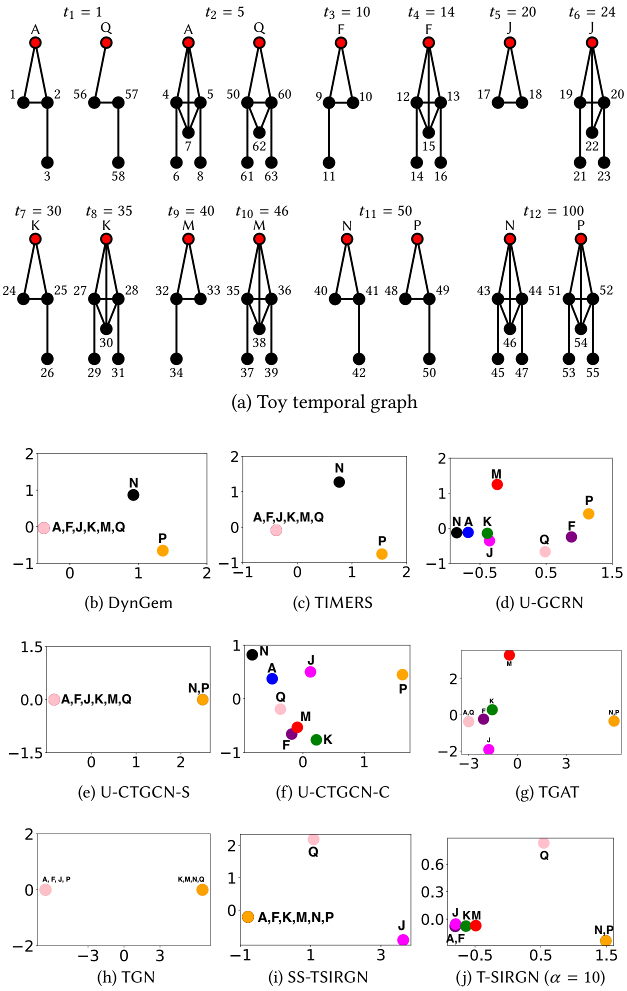

Figure 9: Results of the experiment to check the capability of the proposed T-SIRGN and its competitors of fulfilling a number of properties of interest. (a) Toy temporal graph, composed of twelve graph snapshots, from timestamp t_1 to timestamp t_{12}. Every snapshot is plotted right below its corresponding timestamp. Lettered nodes (in red) are the ones of interest. Black nodes have node identities all different than each other. (b)–(j) 2D projection of the embeddings generated on the toy graph by the various methods.

as A and Q have identical timestamps but different structure, and are placed far apart. Similarly, N and P have identical timestamps and structure but different node identities, and are placed apart, as well as A and F have identical structure but differing time deltas, and showed overlapping embeddings. No pattern of logic for TGN can be defined using these results. Thus, the overlap between A and F recognized by U-CTGCN-S and TGN results from an inability to discriminate between nodes, rather than an ability to capture structural or time delta similarities. The same observation holds for DynGem and TIMERS, as well.

(**C3**) T-SIRGN generates an embedding for J that is near those for A and F. This is desirable, as these nodes have structures that differ by only a single edge in each timestamp, and share identical time deltas. When the temporal aspect is given lower weight (with α close to 0, in the SS-TSIRGN variant), the structure differences become more impactful, and J is (incorrectly) separated farther from the others. Importantly, J and Q have different structures from A, F, K, M, N, and P and from each other. For a method to claim the ability to capture structural roles, J and Q should be separated in the embedding space from one another and from the others.

(**C4**) {A, F}, K and M have embeddings for T-SIRGN that sit close to one another, while remaining different. This is desirable as those nodes have identical structure, and only a small difference (i.e., 1) in their time deltas. Conversely, N, P is correctly placed far away from all those nodes: although they share the same structure with the others, their time deltas are much higher (i.e., 50 vs. 4,5, and 6). For TGAT, M is projected far from the other nodes, though it has identical structure to A, but with a time delta that differs by only 1. SS-TSIRGN, DynGem, U-CTGCN-S, and TIMERS create identical embeddings for (at least part of) these nodes, thus failing to recognize the temporal differences altogether.