Exercises for the lecture

Fundamentals of Simulation Methods

WS 2015/16

Lecturer: Volker Springel

Exercise sheet 7 *(due date: Dec 11, 2015, 11am)*

Fluid instability and blast waves

1) Trinity shock wave

Consider the image sequence in Figure 1, showing the blast wave of the first atomic bomb triggered in 1945. We expect the radius R_s of the blast wave (which is an extremely strong shock) to evolve as $R_s \sim (Et^2/\rho)^{1/5}$ in three dimensions (according to the so-called Sedov-Taylor solution), with a proportionality coefficient close to unity (and which we assume is exactly one). Here E is the energy released and ρ is the background density.

(a) Confirm by dimensional analysis that R_s is the only length scale that one can construct from an energy E, a density ρ, and a time t.

(b) Estimate the released explosion energy E based on the images, and convert to the equivalent in kilotons of TNT ($1\,\mathrm{kton\,TNT} \simeq 5.0 \times 10^{12}\,\mathrm{J}$).

(c) Estimate the post-shock temperature at time $t = 1\,\mathrm{ms}$.

2) Kelvin-Helmholtz instability

For definiteness, we consider a 2D domain of extension $[0, L] \times [0, L]$ with periodic boundaries on the left and right sides, and reflecting boundaries on the top and bottom. Let the upper half of the box be filled with gas ($\gamma = 5/3$) at density $\rho_1 = 1.0$, pressure $P_1 = 1.0$, and velocity $u_1 = 0.3$ in the x-direction (i.e. to the right). The lower half has density $\rho_2 = 2.0$, the same pressure $P_2 = P_1$, and moves with velocity $u_2 = -0.3$ to the left. In order to avoid a perfectly sharp boundary in the initial conditions between these two phases (which is prone to triggering secondary instabilities at grid corners) we introduce a small transition region that smoothly connects them:

$$\rho(x, y) = \rho_1 + \frac{\rho_2 - \rho_1}{1 + \exp\left[(y - 0.5)/\sigma\right]}, \tag{1}$$

and similarly

$$u(x, y) = u_1 + \frac{u_2 - u_1}{1 + \exp\left[(y - 0.5)/\sigma\right]}, \tag{2}$$

with $\sigma = 0.01$. In these unperturbed initial conditions, we now impose a seed perturbation in the velocity in the y-direction of the form

$$v(x, y) = A\,\cos(kx)\,\exp(-k|y - 0.5|), \tag{3}$$

with wavenumber $k = 2 \times (2\pi/L)$ and perturbation amplitude $A = 0.05$. For simplicity, we refrain from imparting a perturbation in ρ and u as well that would be consistent with the velocity perturbation in the y-direction at the linear theory level, kind of hoping that we get away with this on the grounds that the perturbation should anyway grow (which it expected if the shear flow is indeed unstable against arbitrarily small transverse perturbations).

(a) We want to simulate this problem with the ATHENA mesh code developed by the group of Jim Stone (Princeton University). You can download version 4.1 of this code from `http://www.astro.princeton.edu/~jstone/downloads/athena/athena4.1.tar.gz` Then, unpack the code with the command: `tar -zxvf athena4.1.tar.gz`

We want to run the problem with ATHENA until time $t = 3.0$ and create an image of the resulting density field at the end. To this end, you need to implement appropriate initial conditions in a problem generator according to the design of this code, and then compile the code appropriately. For the problem generator, you can use the file `kelvin.c` provided on moodle and place it into the subdirectory `src/prob` of ATHENA. Edit the file to finish off the implementation of the initial conditions (there are primarily three lines to fill out – see the comments in the file). Then configure the code as
`./configure --with-problem=kelvin --with-gas=hydro --with-eos=adiabatic --with-flux=roe` followed by the compilation step with `make all`.

Next, you also need to setup a parameterfile that is passed to ATHENA at run time. This sets things such as the resolution you want to use, the number and times of outputs you want to have, the desired simulation time span, etc. You can try the parameterfile `kelvin.param` supplied on moodle to get started, which you may modify as you see fit (for example to change the resolution or the parameters of the initial conditions generator). Then run the code with
`./bin/Athena -i <parameterfile>`,
where you replace the name of the parameterfile with your file `kelvin.param`.

At the final time, you should get a ".ppm" image file displaying a slice of the density field, e.g. `kh.0060.d.ppm`. Load this into an image view program of your choice. Carry out a series of simulations with different resolutions, equal to 64×64, 128×128 and 256×256 mesh cells, and produce images for them at the same nominal pixel resolution, for example 512×512 pixels, by enlarging the images accordingly. Compare them visually and discuss.

(b) We now want to check whether we can verify the linear growth rate of the perturbation. As discussed in the lecture, the growth rate of a single mode k is given by $\propto \exp(\omega t)$, with

$$\omega = k|u_1 - u_2|\sqrt{\rho_1 \rho_2}/(\rho_1 + \rho_2). \tag{4}$$

Make a plot of the log of the mean kinetic energy in the y-direction as a function of time (you can get this quantity from the history output in `kh.hst`, column 1 has the time, column 9 the kinetic energy in the y-direction), and overplot a growth line reflecting the above timescale. Why is the growth initially slower than expected based on equation (4)? What could be the reason that there is a large slow-down at late times?

(c) Now repeat the Kelvin-Helmholtz simulation of (a) but add a constant velocity of $\Delta u = 5.0$ everywhere to the initial conditions. At time $t = 3$, would you expect the result to look different than in (a)? Compare with what you actually obtain when doing this test, and discuss the result.

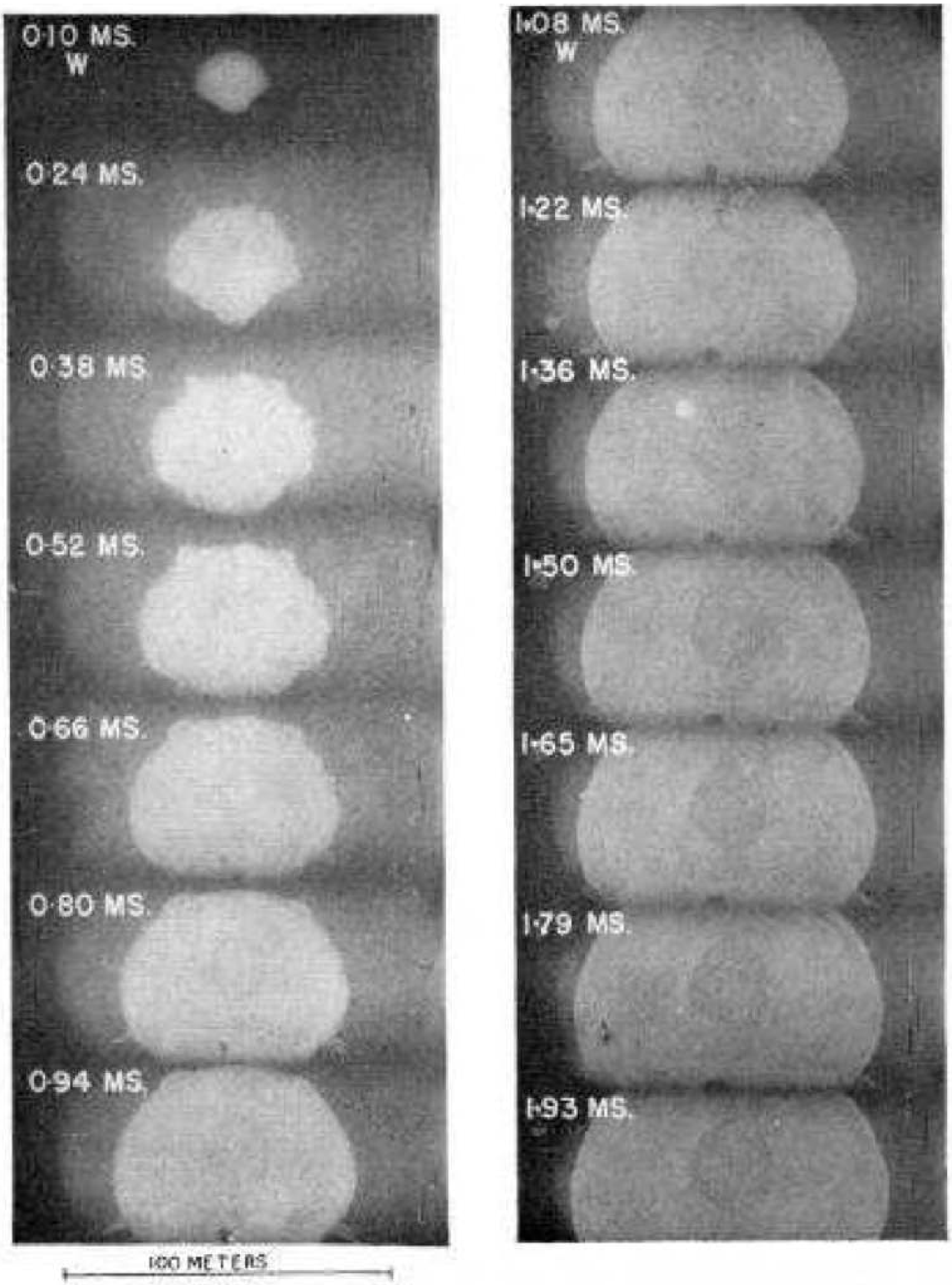

Figure 1: First exploding atomic bomb in New Mexico in 1945.