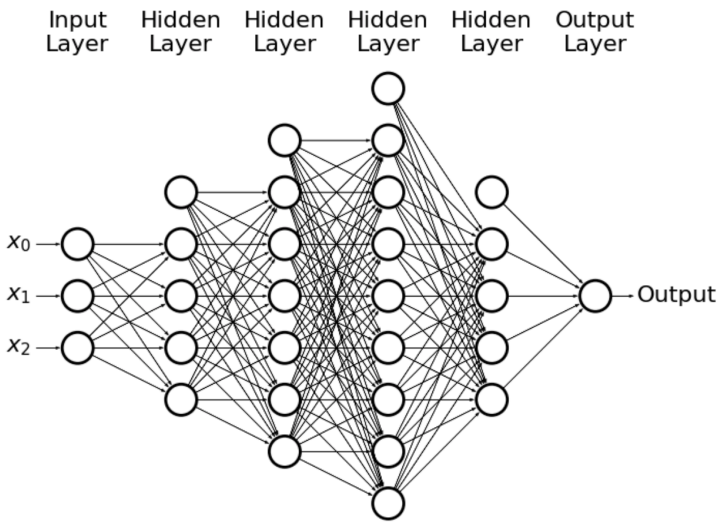
# Convolutional neural networks

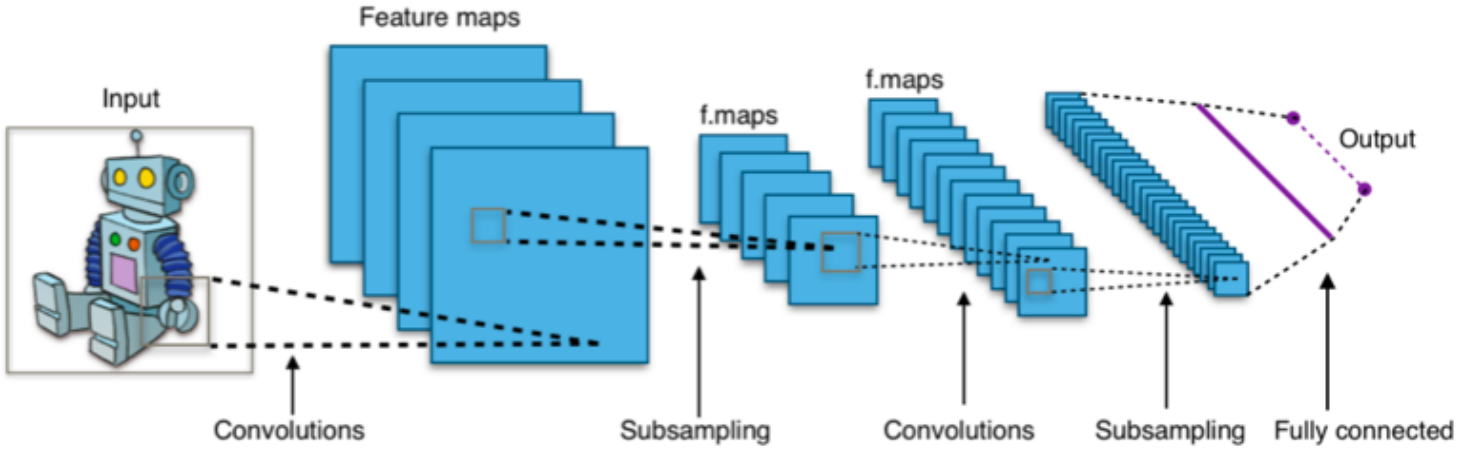
Materials for this section:

- https://cs231n.github.io/convolutional-networks
- simple review: https://arxiv.org/abs/1901.06032
- https://arxiv.org/pdf/1603.07285.pdf

# Convolutional neural networks




vs.




- fully-connected multi-layer perceptron uses (weights) matrix **multiplication** when passing between layers

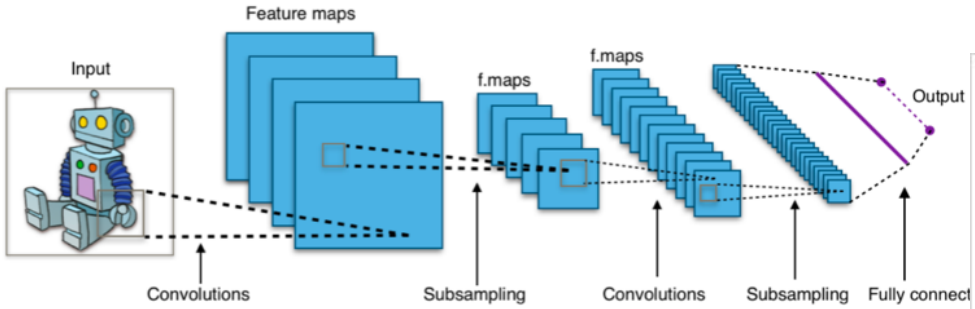
- *convolutional networks* use **convolution** operation in place of matrix multiplication in at least one of their layers
- other operation used: pooling (subsampling), dropout (regularization)

Convolutional neural network = ConvNet = CNN

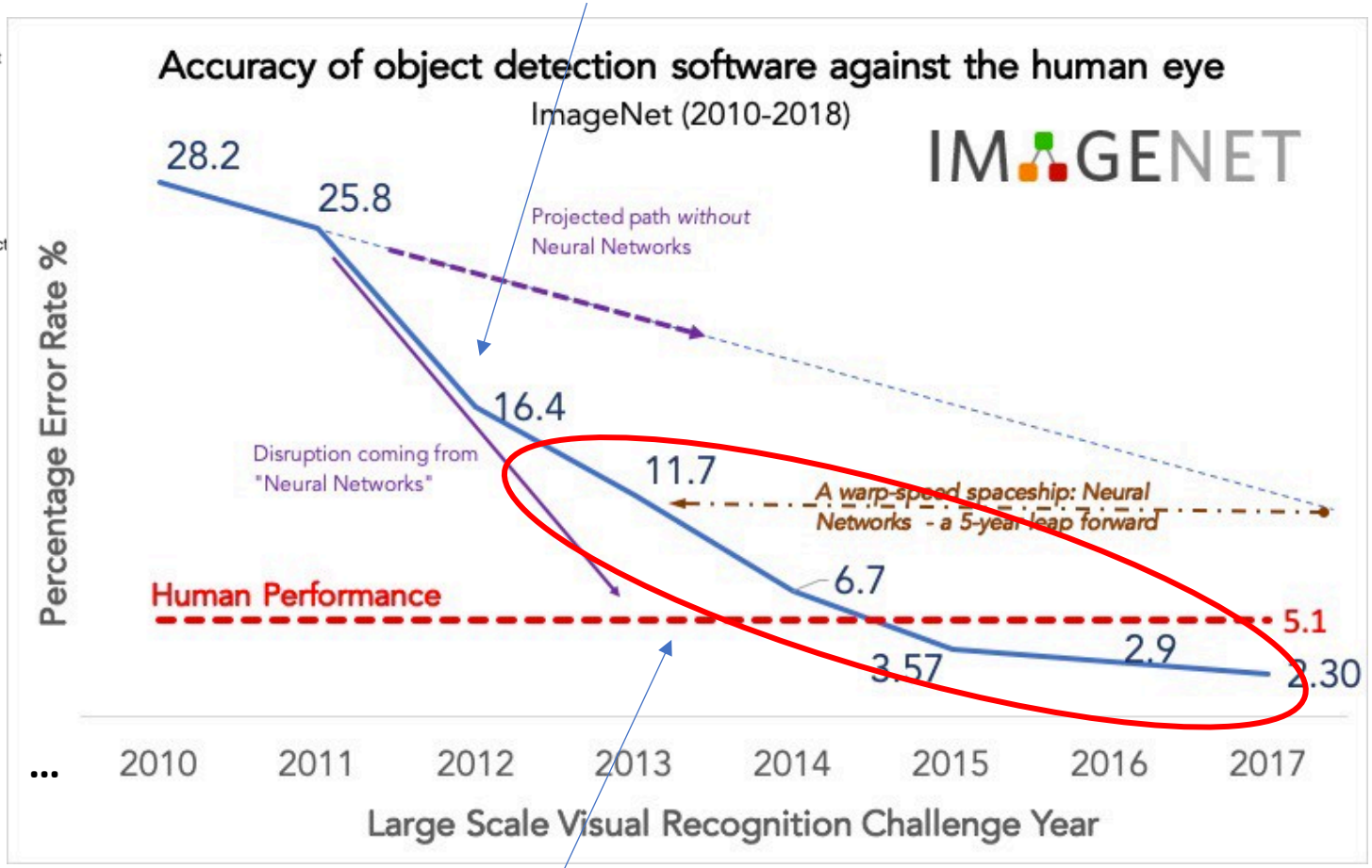
- CNNs start deep-learning revolution (AlexNet, 2012)
- widely used in *computer-vision*
- for processing data that has 1D, 2D, … grid topology

https://en.wikipedia.org/wiki/Convolutional_neural_network

# Convolutional neural networks




AlexNet CNN




- CNNs start deep-learning revolution: ImageNet competition
- are the most significant NN architecture type nowadays, also being a part of much complicated architectures

Yann LeCun (1989) used back-propagation to learn **shared** convolution kernel

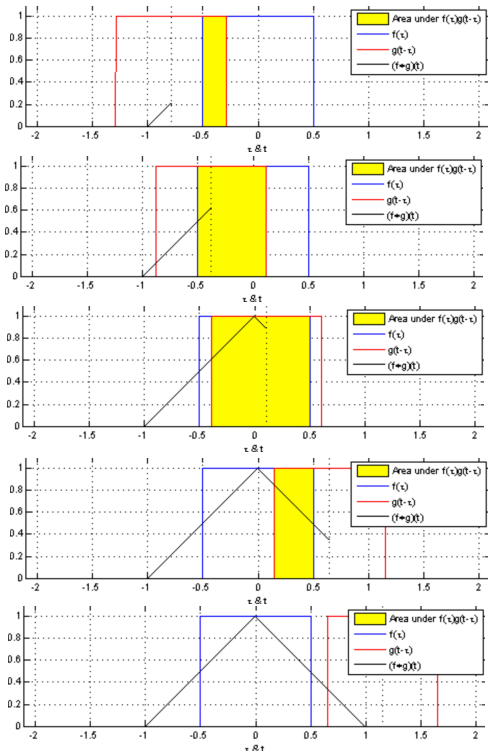
deep-learning era

http://image-net.org/

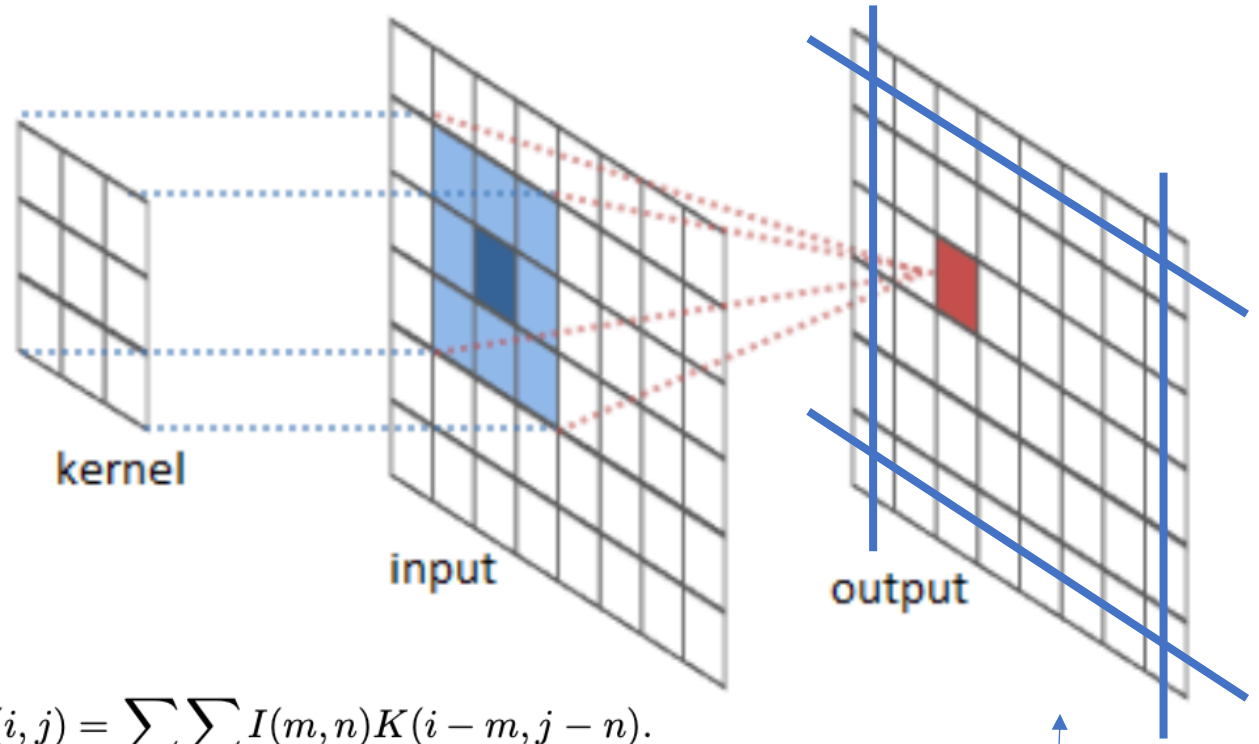
# Convolutional neural networks

Convolution in an operation where we multiply input by moving weight (kernel) function an obtaining subsequent outputs

$$s(t) = (x * w)(t) = \sum_{a=-\infty}^{\infty} x(a)w(t-a).$$



https://en.wikipedia.org/wiki/Convolution



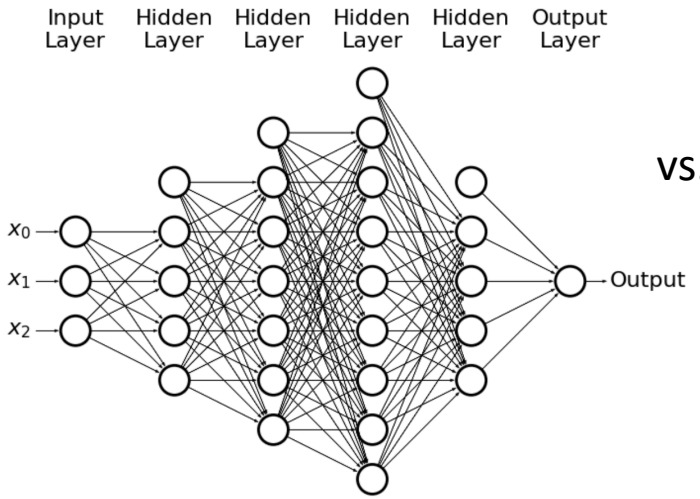

$$S(i,j) = (I * K)(i,j) = \sum_m \sum_n I(m,n)K(i-m, j-n).$$

- To get 1d or 2d **discrete** convolutions we move kernel over input grid, e.g. image,
- kernel is usually much smaller, e.g. 3x3, than input image, e.g. 512x512 pixels,
- and obtain reduced side output (510x510 in our example).

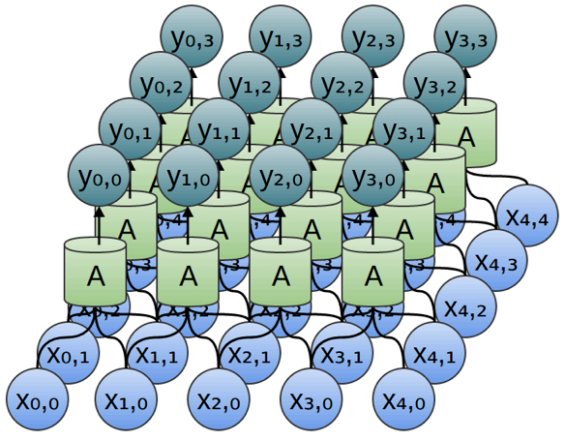
# Convolutional neural networks

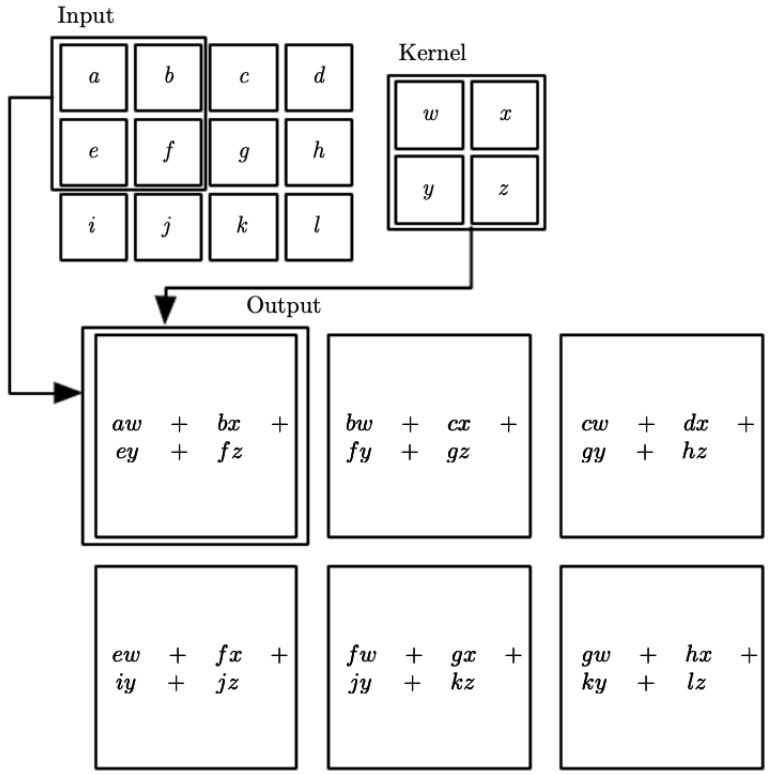
**Idea why to use convolutions: Sparse-connectivity**

- CNNs are regularized version of fully-connected MLPs
- A single element in the feature map is connected to only a small patch of pixels.
- This is very different from connecting to the whole input image, in the case of multi-layer perceptrons.




vs.




- CNNS are also much faster to train than fully MLPs.

# Convolutional neural networks

- Please note, that the output size ($W_{out}$ x $H_{out}$) is determined by the input size ($W_{in}$ x $H_{in}$) and the kernel size ($W_f$ x $H_f$):

$$W_{out} = W_{in} - W_f + 1$$
$$H_{out} = H_{in} - H_f + 1$$

**Padding**

- Padding (zero padding) - adding zeros around an image

```
                | 0 0 0 0 0 0 0 |
| 0 1 0 1 1 |   | 0 0 1 0 1 1 0 |
| 1 1 0 1 0 |   | 0 1 1 0 1 0 0 |
| 1 0 1 0 1 | -> | 0 1 0 1 0 1 0 |
| 1 1 0 1 0 |   | 0 1 1 0 1 0 0 |
| 0 0 1 1 0 |   | 0 0 0 1 1 0 0 |
                | 0 0 0 0 0 0 0 |
```

- The output size ($W_{out}$ x $H_{out}$) is determined by the input size ($W_{in}$ x $H_{in}$), the kernel size ( $W_f$ x $H_f$), and the padding size ($P$)

$

$$W_{out} = W_{in} - W_f + 2P + 1$$
$$H_{out} = H_{in} - H_f + 2P + 1$$

Sometimes downsizing is unwanted, and information on the edges is of the same importance

But sometimes downsampling is desired (e.g. too high resolution)

**Stride**

- Stride controls the movement of the filter

```
Stride = 1

| [0 1 0] 1 1 |   | 0 [1 0 1] 1 |
| [1 1 0] 1 0 |   | 1 [1 0 1] 0 |
| [1 0 1] 0 1 | -> | 1 [0 1 0] 1 | -> ...
|  1 1 0  1 0 |   | 1  1 0 1  0 |
|  0 0 1  1 0 |   | 0  0 1 1  0 |

Stride = 2

| [0 1 0] 1 1 |   | 0 1 [0 1 1] |
| [1 1 0] 1 0 |   | 1 1 [0 1 0] |
| [1 0 1] 0 1 | -> | 1 0 [1 0 1] | -> ...
|  1 1 0  1 0 |   | 1 1  0 1 0  |
|  0 0 1  1 0 |   | 0 0  1 1 0  |
```

- The output size ($W_{out}$ x $H_{out}$) is determined by the input size ($W_{in}$ x $H_{in}$), the kernel size ($W_f$ x $H_f$), the padding size ($P$), and the stride size ($W_s$ x $H_s$)

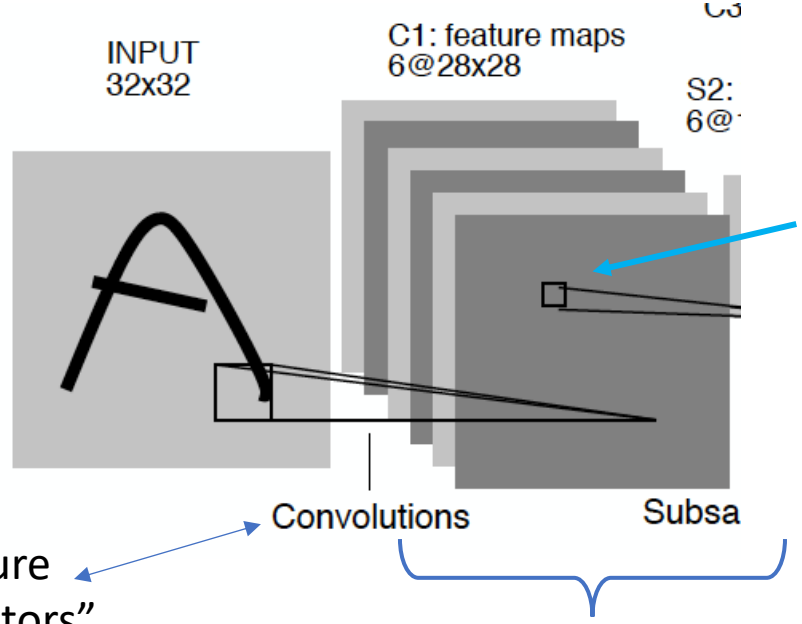
$$W_{out} = \frac{W_{in} - W_f + 2P}{W_s} + 1$$
$$H_{out} = \frac{H_{in} - H_f + 2P}{H_s} + 1$$

source: tomaszgolan.github.io/introduction_to_machine_learning

# CNNs: feature maps

The idea is to build hierarchy of concepts using multiple layers each consisting feature maps:

- we need to extract **features** from images, time series, etc.
- each kernel extract different feature, producing so-called *feature map*




Points on given map shares the **same** kernel

feature "extractors"

multiple feature maps, each with its own kernel forms single layer in CNN

example kernels

| | | |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

Sobel filter

| | | |
|---|---|---|
| 3 | 0 | -3 |
| 10 | 0 | -10 |
| 3 | 0 | -3 |

Scharr filter

| | | |
|---|---|---|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

Vertical

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

Horizontal

| | | | | | |
|---|---|---|---|---|---|
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |

6 x 6

*

| | | |
|---|---|---|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

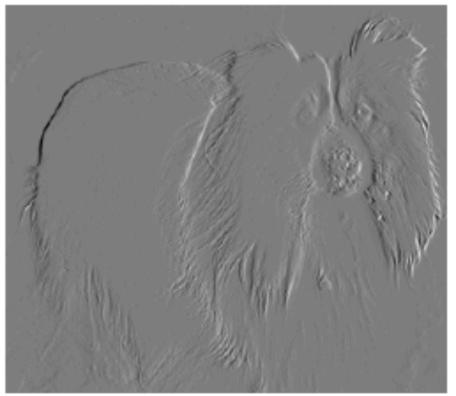
3 x 3

=

| | | | |
|---|---|---|---|
| -0 | 30 | 30 | 0 |
| 0 | 30 | 30 | 0 |
| 0 | 30 | 30 | 0 |
| 0 | 30 | 30 | 0 |

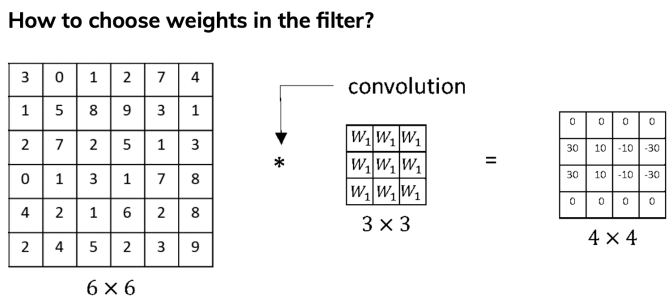
4 x 4

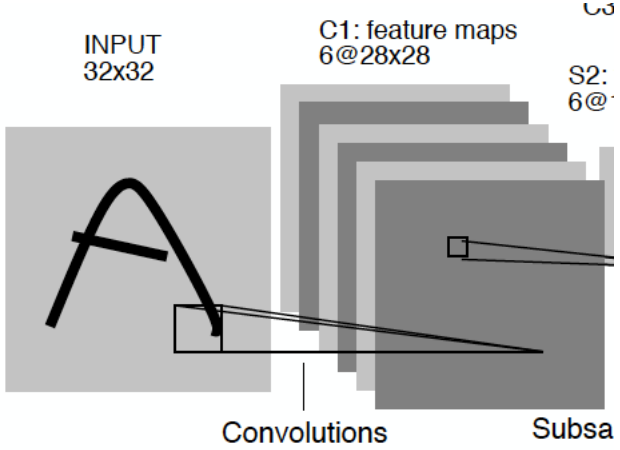
e.g. vertical edge detection kernel



# CNNs: learning kernels

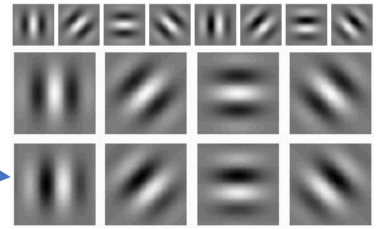
The coolest thing is that network learns kernel types automatically by itself!

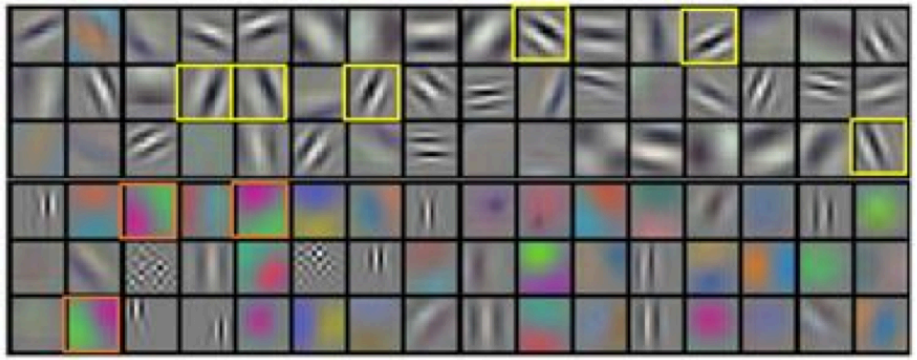


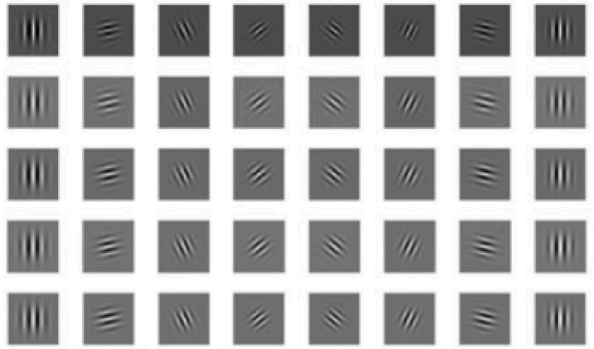



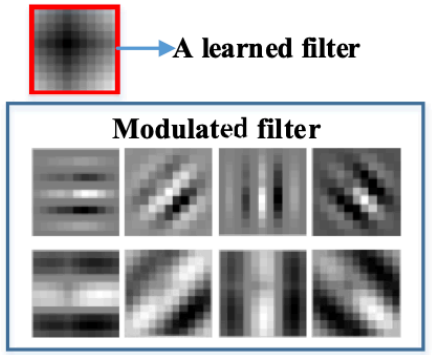

CNNs: Feature extractors using kernels learned by itself

AlexNet learnt by itself filters that resemble *Gabor filters* known from classical image processing, https://arxiv.org/pdf/1705.01450.pdf





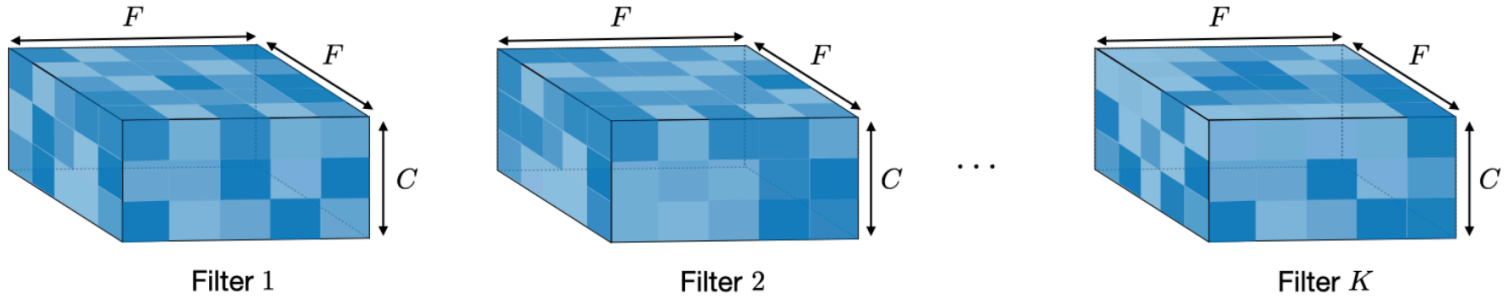




# Deep Convolutional neural networks

stanford.edu/~shervine/teaching/cs-230/cheatsheet-convolutional-neural-networks

❒ **Dimensions of a filter** — A filter of size $F \times F$ applied to an input containing $C$ channels is a $F \times F \times C$ volume that performs convolutions on an input of size $I \times I \times C$ and produces an output feature map (also called activation map) of size $O \times O \times 1$.




*Remark: the application of $K$ filters of size $F \times F$ results in an output feature map of size $O \times O \times K$.*

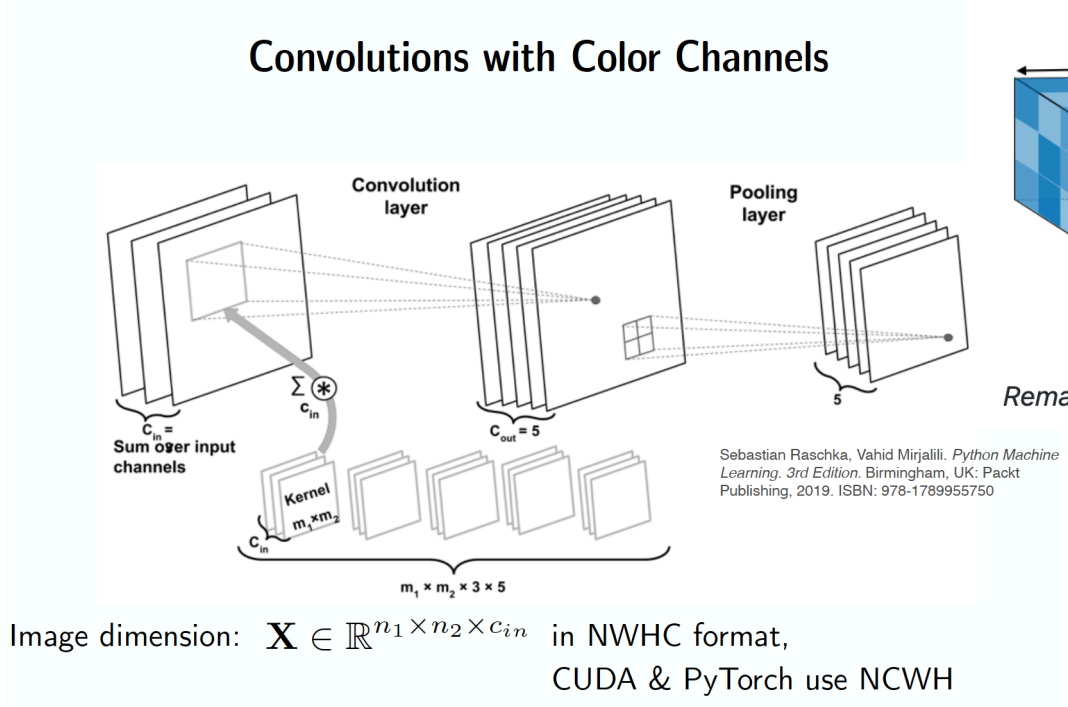



Image dimension: $\mathbf{X} \in \mathbb{R}^{n_1 \times n_2 \times c_{in}}$ in NWHC format,
CUDA & PyTorch use NCWH

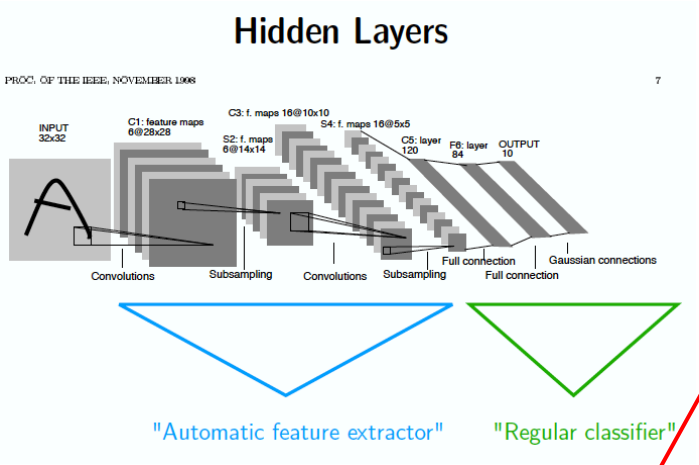
In cases where 2-d data have multiple channels kernel might be 3-dimensional

**Summary**

- Convolutional layer consist of $N$ filters, so the hyperparameters are:
  - the number of filters
  - the size of the filters
  - padding
  - stride

# CNN full architecture

Let's go deeper into full CNN




- feature maps in between neighbor convolutional layers are usually fully connected
- feature map is a basic unit of CNNs

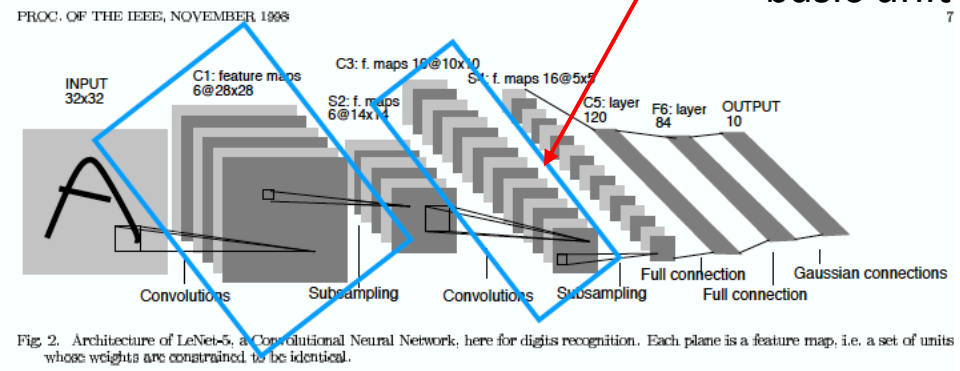



Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

Each "bunch" of feature maps represents one hidden layer in the neural network.

## Convolutional Neural Networks

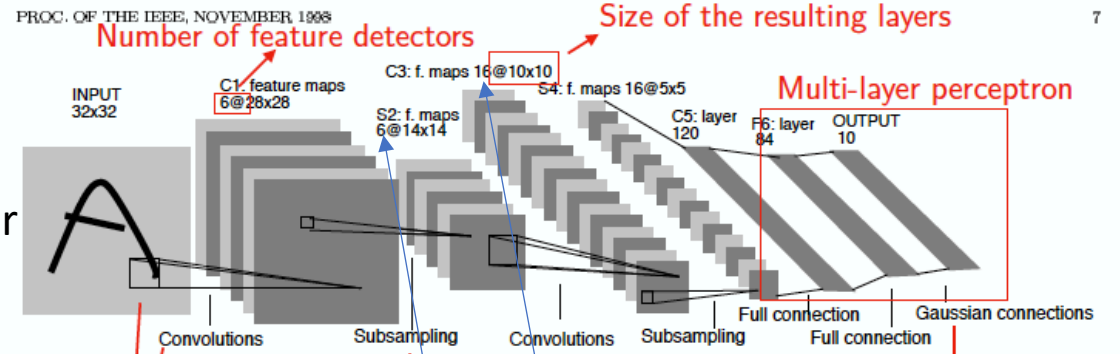



Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

nowadays called "pooling"

basically a fully-connected layer + MSE loss (nowadays better to use fc-layer + softmax + cross entropy

"Feature detectors" (weight matrices) that are being reused ("weight sharing") => also called "kernel" or "filter"

CONV2D

*N* *M*

CLASS `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')` [SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size $(N, C_{\text{in}}, H, W)$ and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

*N*M* kernels

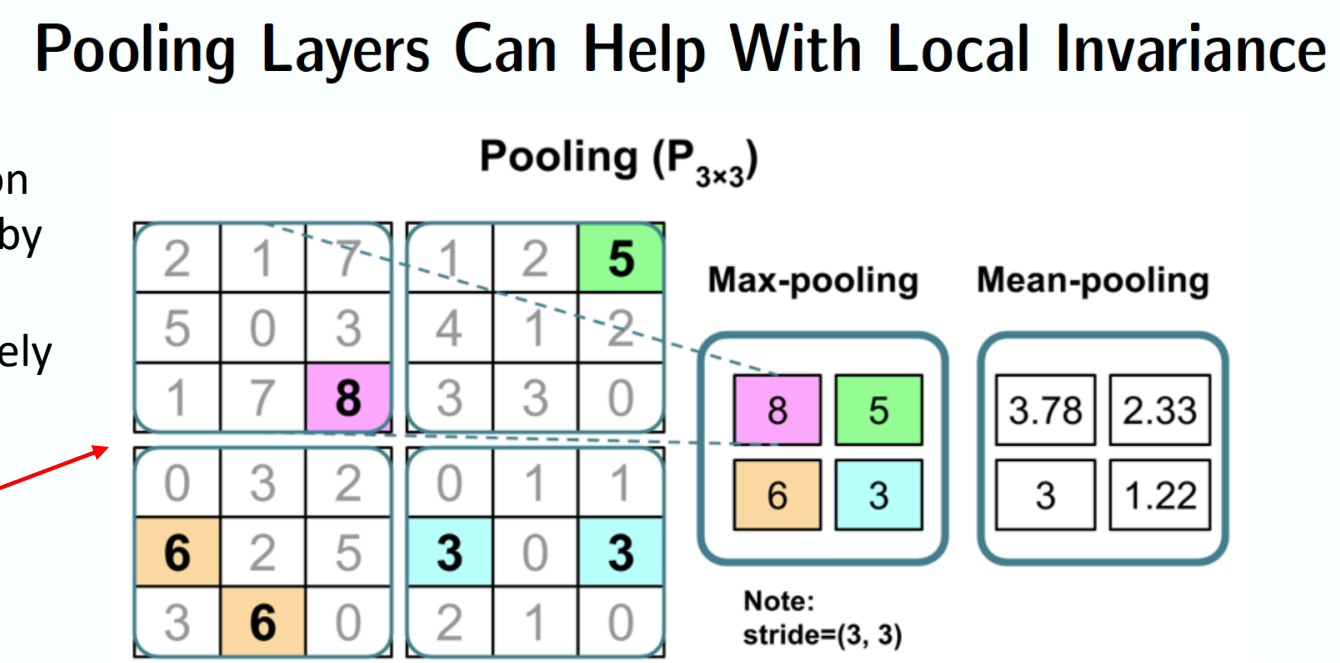
$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) \star \text{input}(N_i, k)$$

https://github.com/rasbt/stat453-deep-learning-ss20

# CNN components: pooling

Pooling operation

- A pooling function replaces the given output activation map with map composed of **local** statistic of the nearby values
- Pooling helps to make the representation approximately **invariant** to small translations of the input.




e.g. here pool size is of 3x3 and stride between pools equals 3

The pooling layer reduces the number of parameters and calculations in the network. This improves the efficiency of the network and avoids overfitting.

Note that typically pooling layers do not have any learnable parameters

# CNN components: dropout and batch-normalization

Another type of regularizations used in CNNs is **dropout**:

- frequently we apply dropout to fully-connected-layers part
- but sometimes also after pooling layers

Moreover to stabilize and speed-up the training process we apply **batch-normalization** layers:

- the idea is to make activations units (in the given layer) gaussian-like by scaling their input $x_i$ (from earlier layer)
- normalization is performed using averages for entire mini-batch: mini-batch mean and mini-batch variance (N is a batch size):

The mean and the variance are calculated:
- for given mini-batch (when training)
- for the whole training dataset (inference)

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

- and normalize inputs

$$\hat{x}_i = \frac{x_i - \mu}{\sqrt{\sigma^2 + \varepsilon}}$$

each input $x_i$ contains
- activations (MLP)
- or complete set of feature maps (conv layers)

from previous layer

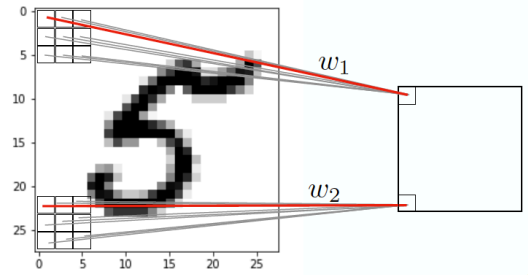
Note1: SGD (training) will still work since $\hat{x}_i$ is a differentiable function
Note2: when training CNNs we process whole mini-batch at once
(input tensor is of size $N \times C \times W \times H$, for batch of $N$ color images each of $W \times H$ size)

# CNNs: training and activations

## Backpropagation in CNNs

Same overall concept as before: Multivariable chain rule, but now with an additional weight sharing constraint

Due to weight sharing: $w_1 = w_2$




Optional averaging

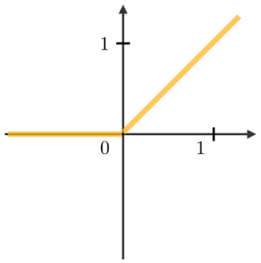
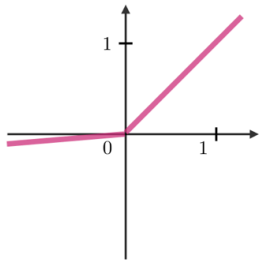
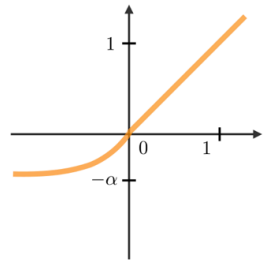
weight update:

$$w_1 := w_2 := w_1 - \eta \cdot \frac{1}{2}\left(\frac{\partial \mathcal{L}}{\partial w_1} + \frac{\partial \mathcal{L}}{\partial w_2}\right)$$

**Parameter-sharing**: the same weights are used for different patches of the input image.

### Commonly used activation functions

□ **Rectified Linear Unit** — The rectified linear unit layer (ReLU) is an activation function $g$ that is used on all elements of the volume. It aims at introducing non-linearities to the network. Its variants are summarized in the table below:

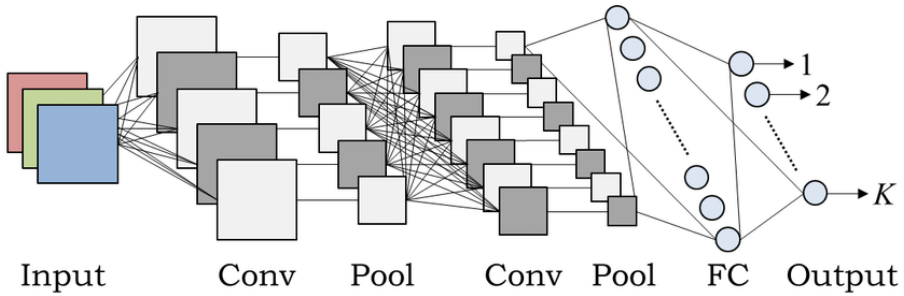
| ReLU | Leaky ReLU | ELU |
|---|---|---|
| $g(z) = \max(0, z)$ | $g(z) = \max(\epsilon z, z)$ with $\epsilon \ll 1$ | $g(z) = \max(\alpha(e^z - 1), z)$ with $\alpha \ll 1$ |
|  |  |  |
| • Non-linearity complexities biologically interpretable | • Addresses dying ReLU issue for negative values | • Differentiable everywhere |

Vanishing gradient problem – solutions:

- ReLUs activations
- batch normalization
- *residual* networks

# CNNs: common architectures

Typical architecture but in real tasks we may go *de*




## Main Breakthrough for CNNs: AlexNet & ImageNet

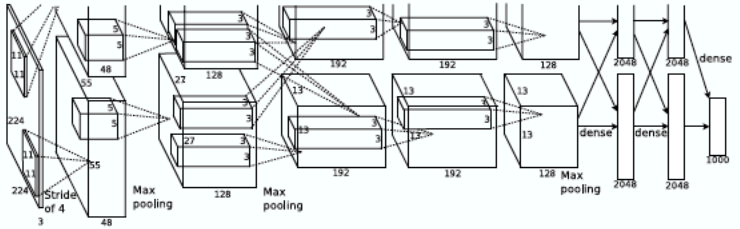


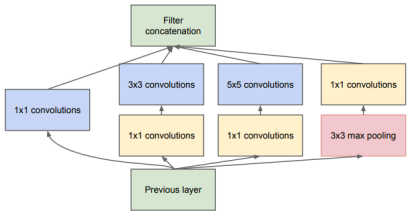

Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network's input is 150,528-dimensional, and the number of neurons in the network's remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

zhevsky, A., Sutskever, I., & Hinton, G. E. (2012). Imagenet classification with deep convolutional neural works. In *Advances in neural information processing systems* (pp. 1097-1105).

**AlexNet** details:

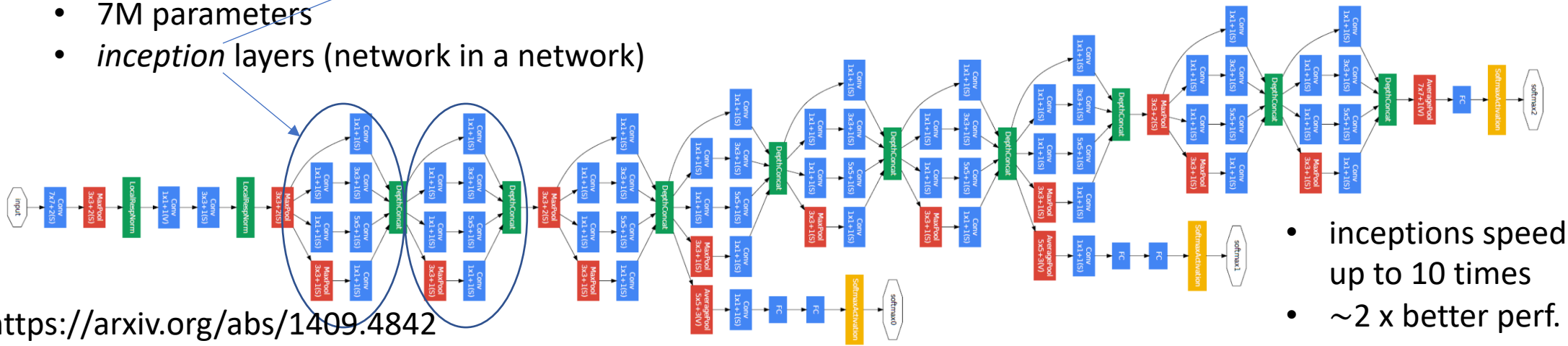
parameters:
- 3.7M conv
- 58.6M fully-conn

forward computations:
- 1.08G conv
- 58.6M f-c




(b) Inception module with dimensionality reduction

**GoogLeNet** details:
- 22 layers + 5 polling
- 7M parameters
- *inception* layers (network in a network)




https://arxiv.org/abs/1409.4842

- inceptions speeds up training up to 10 times
- ~2 x better perf. than AlexNet

https://medium.com/@smallfishbigsea/a-walk-through-of-alexnet

# CNNs: common architectures

ImageNet accuracy

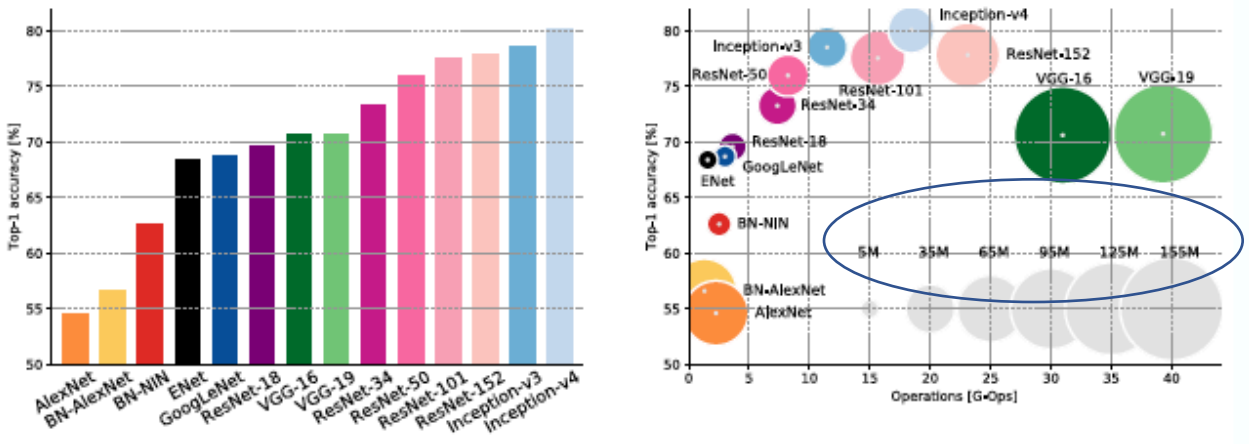
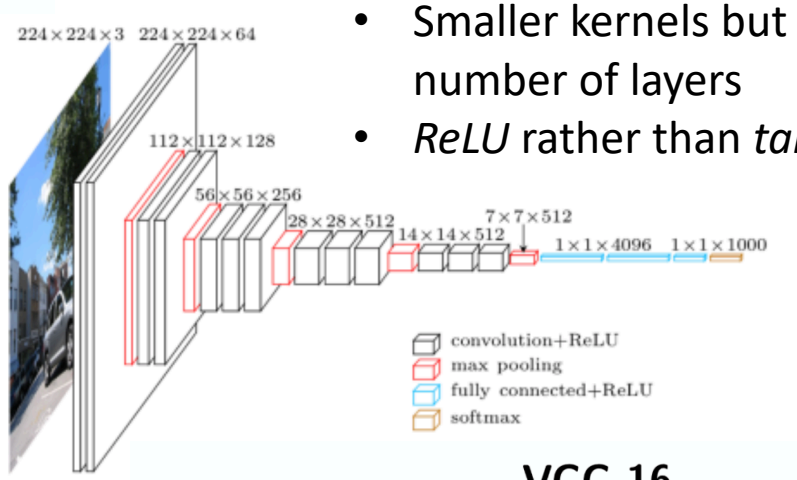



Figure 1: **Top1 *vs.* network.** Single-crop top-1 validation accuracies for top scoring single-model architectures. We introduce with this chart our choice of colour scheme, which will be used throughout this publication to distinguish effectively different architectures and their correspondent authors. Notice that networks of the same group share the same hue, for example ResNet are all variations of pink.

Figure 2: **Top1 *vs.* operations, size $\propto$ parameters.** Top-1 one-crop accuracy versus amount of operations required for a single forward pass. The size of the blobs is proportional to the number of network parameters; a legend is reported in the bottom right corner, spanning from $5\times10^6$ to $155\times10^6$ params. Both these figures share the same $y$-axis, and the grey dots highlight the centre of the blobs.

Canziani, A., Paszke, A., & Culurciello, E. (2016). An analysis of deep neural network models for practical applications. *arXiv preprint arXiv:1605.07678*.

## VGGNet




Differences from AlexNet

- Smaller kernels but with larger number of layers
- *ReLU* rather than *tanh* activation

## VGG-16

PyTorch implementation: https://github.com/rasbt/stat453-deep-learning-ss20/blob/master/L13-cnns-part2/code/vgg16.ipynb

| ConvNet Configuration | | | | | |
|---|---|---|---|---|---|
| A | A-LRN | B | C | D | E |
| 11 weight layers | 11 weight layers | 13 weight layers | 16 weight layers | 16 weight layers | 19 weight layers |
| input (224 × 224 RGB image) | | | | | |
| conv3-64 | conv3-64 **LRN** | conv3-64 **conv3-64** | conv3-64 conv3-64 | conv3-64 conv3-64 | conv3-64 conv3-64 |
| maxpool | | | | | |
| conv3-128 | conv3-128 | conv3-128 **conv3-128** | conv3-128 conv3-128 | conv3-128 conv3-128 | conv3-128 conv3-128 |
| maxpool | | | | | |
| conv3-256 conv3-256 | conv3-256 conv3-256 | conv3-256 conv3-256 | conv3-256 conv3-256 **conv1-256** | conv3-256 conv3-256 **conv3-256** | conv3-256 conv3-256 conv3-256 **conv3-256** |
| maxpool | | | | | |
| conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 **conv1-512** | conv3-512 conv3-512 **conv3-512** | conv3-512 conv3-512 conv3-512 **conv3-512** |
| maxpool | | | | | |
| conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 **conv1-512** | conv3-512 conv3-512 **conv3-512** | conv3-512 conv3-512 conv3-512 **conv3-512** |
| maxpool | | | | | |
| FC-4096 | | | | | |
| FC-4096 | | | | | |
| FC-1000 | | | | | |
| soft-max | | | | | |

Advantages:
very simple architecture,
3x3 convs, stride=1,
"same" padding, 2x2 max pooling

Disadvantage:
very large number of parameters and slow
(see previous slide)

Simonyan, Karen, and Andrew Zisserman. "Very deep convolutional networks for large-scale image recognition." *arXiv preprint arXiv:1409.1556* (2014).

# Very deep CNNs: degradation problem

Is learning better networks as easy as stacking more layers?
-- very deep CNNs, problems:

- Vanishing/exploding gradients (solutions: ReLU, batch normalization)
- **degradation** problem

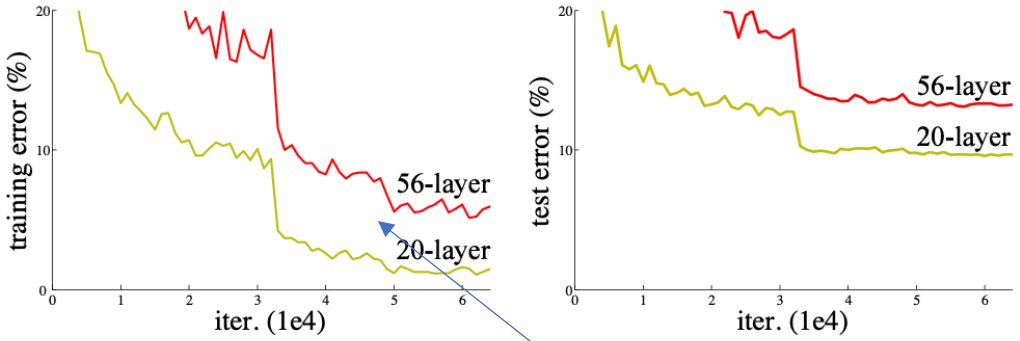



Figure 1. Training error (left) and test error (right) on CIFAR-10 with 20-layer and 56-layer "plain" networks. The deeper network has higher training error, and thus test error. Similar phenomena on ImageNet is presented in Fig. 4.

When deeper networks are able to start converging, a *degradation* problem has been exposed: with the network depth increasing, accuracy gets saturated (which might be unsurprising) and then degrades rapidly. Unexpectedly, such degradation is *not caused by overfitting*, and adding more layers to a suitably deep model leads to *higher training error*, as reported in [11, 42] and thoroughly verified by our experiments. Fig. 1 shows a typical example.

https://arxiv.org/pdf/1512.03385.pdf

deeper network has higher **training** error
(this is not just due to overfitting)

# Residual networks

Solution: **residual networks** – add identity shortcut to enable skipping, while existing layers are able to fit resting (*residual*) mapping

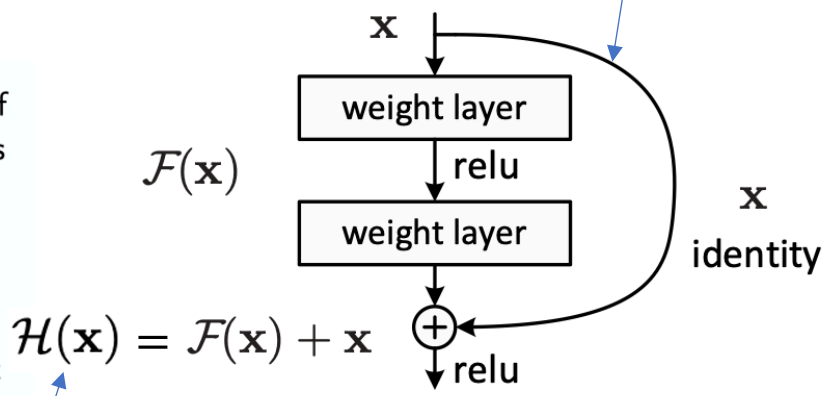
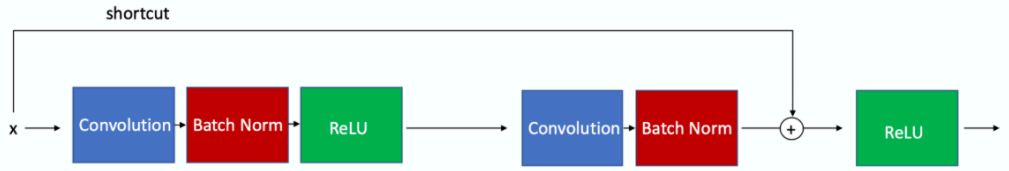



Figure 2. Residual learning: a building block.

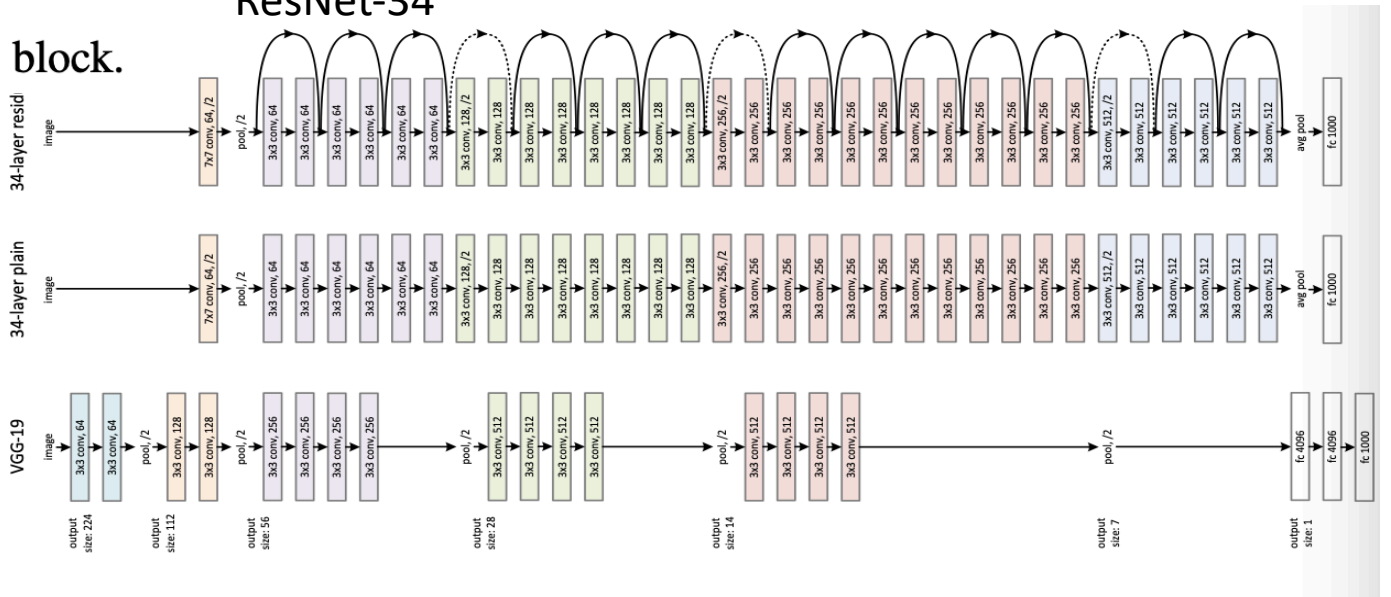
With their simple trick of allowing skip connections (the possibility to learn identity functions and skip layers that are not useful), ResNets allow us to to implement very, very deep architectures

optimum mapping




In general: $a^{(l+2)} = \sigma\left(z^{(l+2)} + a^{(l)}\right)$

ResNet-34




https://arxiv.org/pdf/1512.03385.pdf

# Residual networks

Results for various types (depths) of ResNets

- degradation problem disappears
- much faster training

| layer name | output size | 18-layer | 34-layer | 50-layer | 101-layer | 152-layer |
|---|---|---|---|---|---|---|
| conv1 | 112×112 | 7×7, 64, stride 2 | | | | |
| | | 3×3 max pool, stride 2 | | | | |
| conv2_x | 56×56 | [3×3, 64; 3×3, 64] ×2 | [3×3, 64; 3×3, 64] ×3 | [1×1, 64; 3×3, 64; 1×1, 256] ×3 | [1×1, 64; 3×3, 64; 1×1, 256] ×3 | [1×1, 64; 3×3, 64; 1×1, 256] ×3 |
| conv3_x | 28×28 | [3×3, 128; 3×3, 128] ×2 | [3×3, 128; 3×3, 128] ×4 | [1×1, 128; 3×3, 128; 1×1, 512] ×4 | [1×1, 128; 3×3, 128; 1×1, 512] ×4 | [1×1, 128; 3×3, 128; 1×1, 512] ×8 |
| conv4_x | 14×14 | [3×3, 256; 3×3, 256] ×2 | [3×3, 256; 3×3, 256] ×6 | [1×1, 256; 3×3, 256; 1×1, 1024] ×6 | [1×1, 256; 3×3, 256; 1×1, 1024] ×23 | [1×1, 256; 3×3, 256; 1×1, 1024] ×36 |
| conv5_x | 7×7 | [3×3, 512; 3×3, 512] ×2 | [3×3, 512; 3×3, 512] ×3 | [1×1, 512; 3×3, 512; 1×1, 2048] ×3 | [1×1, 512; 3×3, 512; 1×1, 2048] ×3 | [1×1, 512; 3×3, 512; 1×1, 2048] ×3 |
| | 1×1 | average pool, 1000-d fc, softmax | | | | |
| FLOPs | | $1.8\times10^9$ | $3.6\times10^9$ | $3.8\times10^9$ | $7.6\times10^9$ | $11.3\times10^9$ |

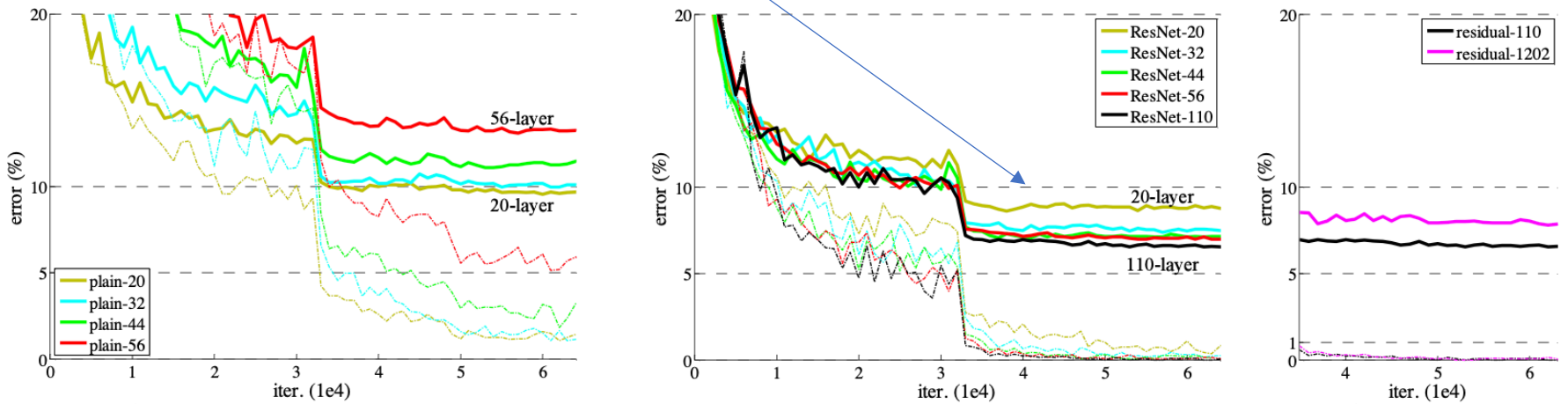
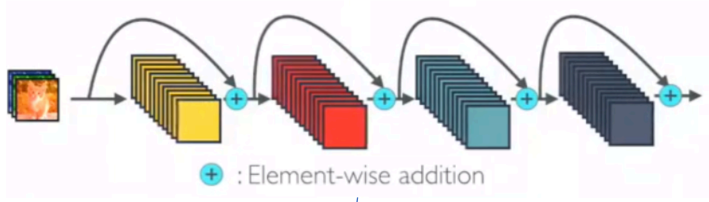
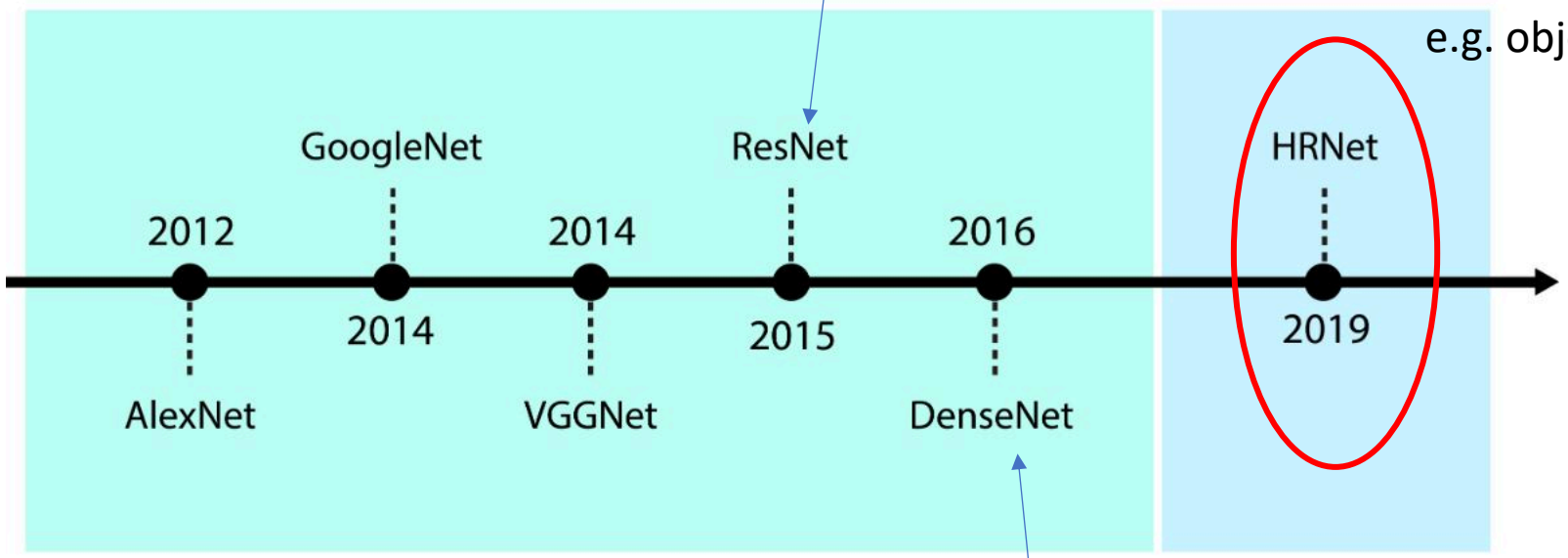



Figure 6. Training on **CIFAR-10**. Dashed lines denote training error, and bold lines denote testing error. **Left**: plain networks. The error of plain-110 is higher than 60% and not displayed. **Middle**: ResNets. **Right**: ResNets with 110 and 1202 layers.

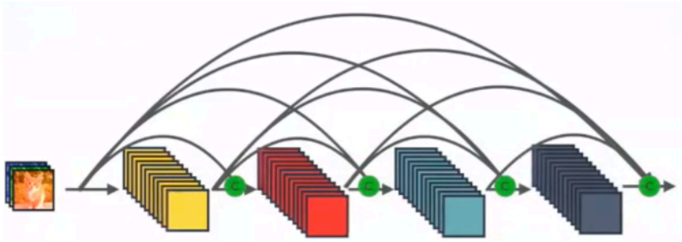
https://arxiv.org/pdf/1512.03385.pdf

# CNNs: milestone architectures




SOTA in computer vision,
e.g. objects recognition



https://www.microsoft.com/en-us/research/blog/high-resolution-network-a-universal-neural-architecture-for-visual-recognition/

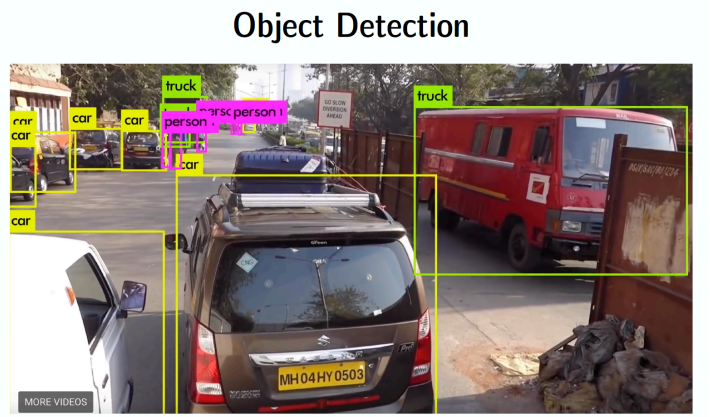
towardsdatascience.com/review-densenet-image-classification

# High-resolution representations

High resolution is essential for tasks beyond classification

- object *detection* on images
- *Segmentation* of detected objects

require **spatially fine** representations




Redmon, J., Divvala, S., Girshick, R., & Farhadi, A. (2016). You only look once: Unified, real-time object detection. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition* (pp. 779-788).

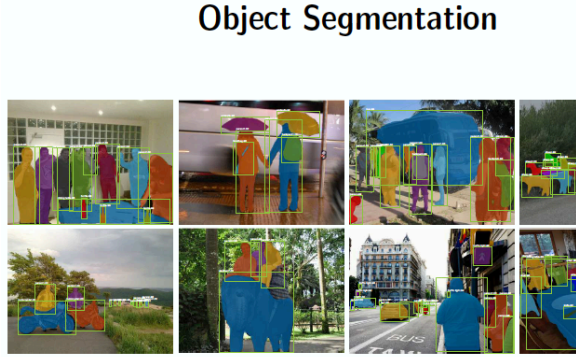



Figure 2. **Mask R-CNN** results on the COCO test set. These results are based on ResNet-101 [15], achieving a running at 5 fps. Masks are shown in color, and bounding box, category, and confidences are also shown.

, Kaiming, Georgia Gkioxari, Piotr Dollár, and Ross Girshick. "Mask R-CNN." In *Proceedings of the Conference on Computer Vision*, pp. 2961-2969. 2017.

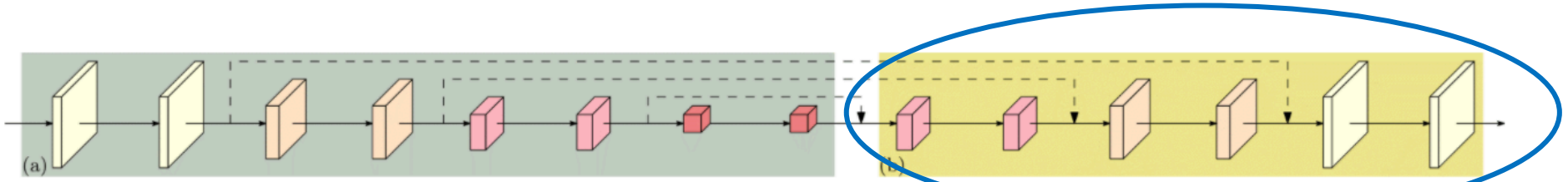



Figure 2: The structure of recovering high resolution from low resolution. (a) A low-resolution representation learning subnetwork (such as AlexNet, GoogleNet, VGGNet, ResNet, DenseNet), which is formed by connecting high-to-low convolutions in series. (b) A high-resolution representation recovering subnetwork, which is formed by connecting low-to-high convolutions in series. Representative examples include SegNet, DeconvNet, U-Net and Hourglass, encoder-decoder, and SimpleBaseline.

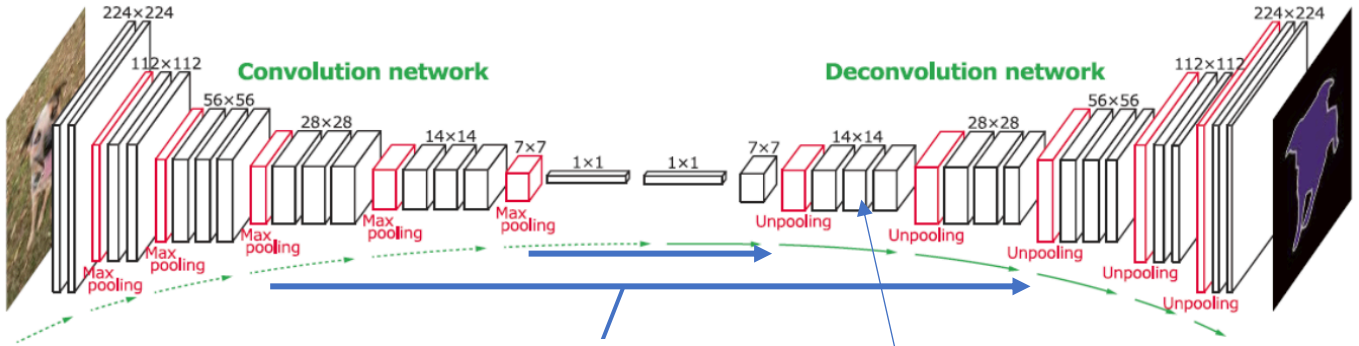
**Hourglass structure:**

added subnetwork recovering high-resolution representation

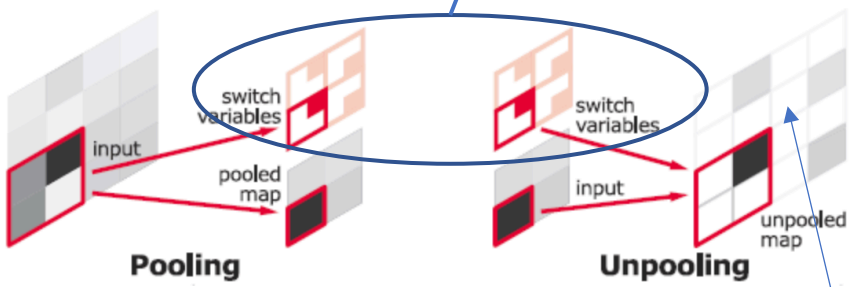
https://arxiv.org/pdf/1908.07919.pdf

# High-resolution representations: deconvolution layers

https://arxiv.org/abs/1505.04366




the locations of maximum activations selected during pooling operation

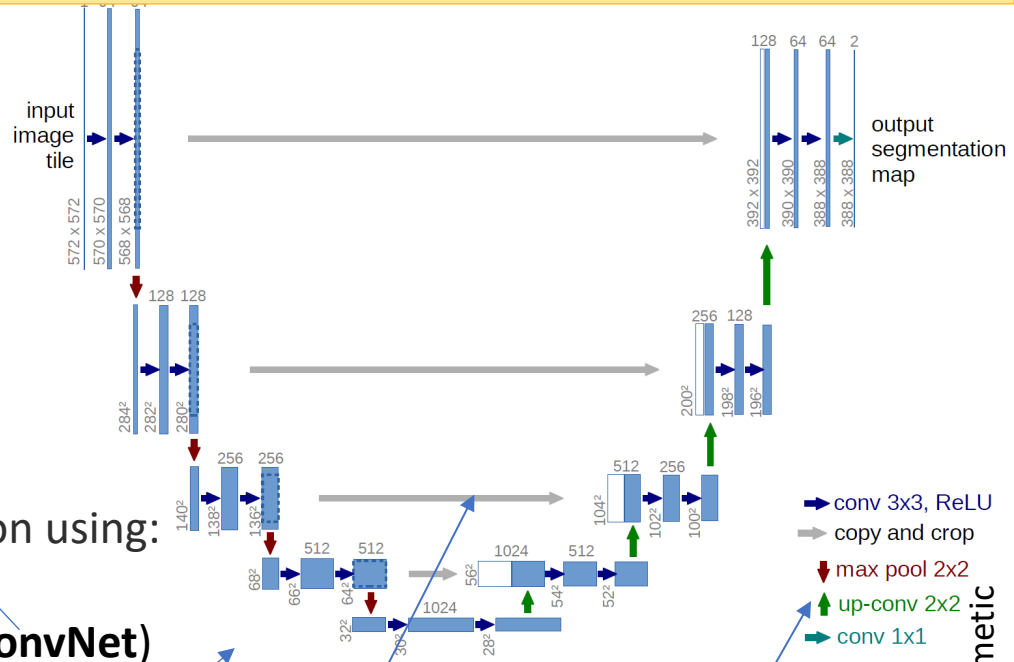



Remember positions when Pooling (Left), Reuse the position information during Unpooling (right)

We restore high resolution using:

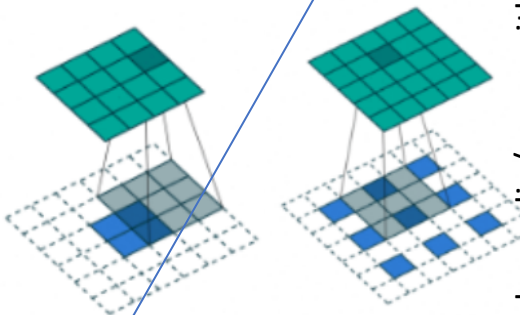
- *unpooling* layers (**DeconvNet**)
- *upsampling* layers (**U-Net**)

sparse map, additional convolutions will densify it




Upsampling:

1. *transposed* convolutions
2. *concatenation* with respective feature maps from “down” part



github.com/vdumoulin/conv_arithmetic

# Transposed convolution

## 4.1 Convolution as a matrix operation

Take for example the convolution represented in Figure 2.1. If the input and output were to be unrolled into vectors from left to right, top to bottom, the convolution could be represented as a sparse matrix $\mathbf{C}$ where the non-zero elements are the elements $w_{i,j}$ of the kernel (with $i$ and $j$ being the row and column of the kernel respectively):

$$\begin{pmatrix} w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} \end{pmatrix}^T = C^T$$

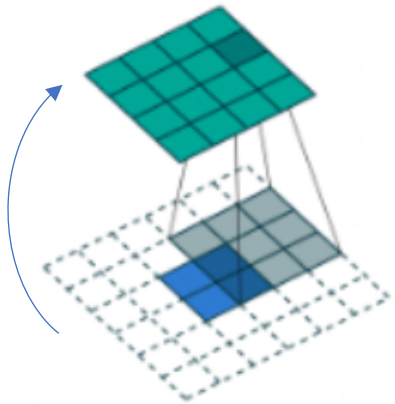
This linear operation takes the input matrix flattened as a 16-dimensional vector and produces a 4-dimensional vector that is later reshaped as the $2 \times 2$ output matrix.

Using this representation, the backward pass is easily obtained by transposing $\mathbf{C}$; in other words, the error is backpropagated by multiplying the loss with $\mathbf{C}^T$. This operation takes a 4-dimensional vector as input and produces a 16-dimensional vector as output, and its connectivity pattern is compatible with $\mathbf{C}$ by construction.

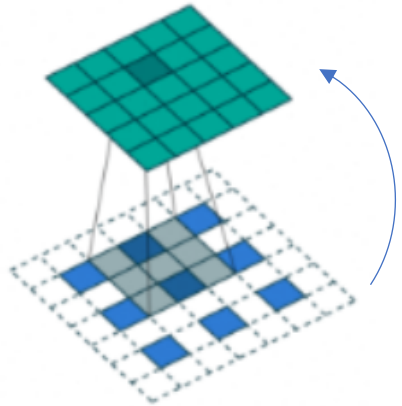
Notably, the kernel $\mathbf{w}$ defines both the matrices $\mathbf{C}$ and $\mathbf{C}^T$ used for the forward and backward passes.

## 4.2 Transposed convolution

https://arxiv.org/pdf/1603.07285.pdf

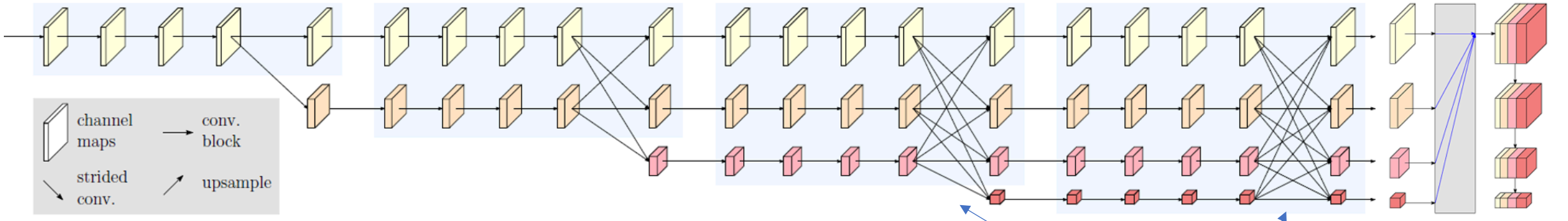


lower to higher resolution



github.com/vdumoulin/conv_arithmetic

# HRNet: a high-resolution network




4 resolutions here

Quite different approach: HRNet

- connecting multi resolution convolution *streams* **in parallel**
- maintains (not just recovers!) high-resolution representations through the whole process

*Fusion* module (just summation)
with appropriate (down-) up-samplings

https://github.com/HRNet

https://arxiv.org/pdf/1908.07919.pdf

# Convolutional neural networks

Hope you now understand this meme ☺