Definitions and synopsis. *Let $\mathcal{H}_n$ be a Hilbert space of vectors ψ that is spanned by a preferred basis that is given as an orthonormal set of n-factor tensor products (in physical terms, such a basis is variously called an n-particle, n-spin, or n-qudit basis). Allow the component vectors of the n-factor product to have various dimensions (in physical terms, allow the individual particles to carry various quantum numbers). Let $I_{\mathcal{H}}$ be any linear Hilbert operator that when given in the n-factor basis acts solely upon one particle (for example, $I_{\mathcal{H}}$ could be a single-particle raising or lowering operator) and associate to $I_{\mathcal{H}}$ the bilinear symbol function $i_{\mathcal{H}}(\bar{\psi}, \psi) = \langle \bar{\psi} | I_{\mathcal{H}} | \psi \rangle$. Similarly, let $H_{\mathcal{H}}$ be any general linear Hilbert operator (for example, $H_{\mathcal{H}}$ could be a multi-particle Hamiltonian operator) and associate to $H_{\mathcal{H}}$ the bilinear symbol function $h_{\mathcal{H}}(\bar{\psi}, \psi) = \langle \bar{\psi} | H_{\mathcal{H}} | \psi \rangle$. Let $\mathcal{K}_1$ be the subset of vectors in $\mathcal{H}_n$ that are n-factor products in the n-particle basis (algebraic geometers call $\mathcal{K}_1$ an n-factor Segre variety). Then by definition the rank-r Kronecker join $\mathcal{K}_r$ is the immersed join of r copies of $\mathcal{K}_1$ (algebraic geometers call $\mathcal{K}_r$ an $(r-1)$'th secant variety). Thus from the initial (non-entangled) state-space $\mathcal{K}_1$ there arises a rank-indexed stratification of Kronecker joins $\mathcal{K}_1 \subseteq \mathcal{K}_2 \subseteq \ldots \subseteq \mathcal{K}_r \subseteq \ldots \subseteq \mathcal{K}_\infty = \mathcal{H}_n$ (in physical terms, the rank r is a measure of quantum entanglement). Finally, let $\omega_{\mathcal{H}}$ be $\mathcal{H}_n$'s canonical (Kählerian) symplectic form, let $\iota_r \colon \mathcal{K}_r \hookrightarrow \mathcal{H}_n$ be $\mathcal{K}_r$'s inclusion map, and let ι_r^* be the pullback induced by ι_r. In summary, immersed in any n-particle Hilbert space $\mathcal{H}_n$ is a geometrically and algebraically natural stratification of Kronecker joins $\mathcal{K}_r$, that in physical terms is a rank-indexed classical-to-quantum succession of state-spaces $\mathcal{K}_1, \mathcal{K}_2, \ldots, \mathcal{K}_r, \ldots$, each of which supports the natural pullback $\iota_r^* \colon (\omega_{\mathcal{H}}, i_{\mathcal{H}}, h_{\mathcal{H}}) \to (\omega_{\mathcal{K}}, i_{\mathcal{K}}, h_{\mathcal{K}})$ of the symplectic forms and symbol functions that specify Hamiltonian dynamical flows.*

Theorem 1 (QUANTUM PULLBACK THEOREM). *For all n-particle Hilbert spaces $\mathcal{H}_n$, at all points of all ranks of $\mathcal{H}_n$'s natural stratification of Kronecker joins $\mathcal{K}_r$, this diagram commutes:*

Hilbert state-space $\mathcal{H}_n$

$$\begin{tikzcd}
(\omega_{\mathcal{H}}, i_{\mathcal{H}}, h_{\mathcal{H}}) \arrow[rr, "\langle di_{\mathcal{H}}, dh_{\mathcal{H}} \rangle_{\omega_{\mathcal{H}}^{-1}}", "\text{Poisson bracket}"'] \arrow[d, "\iota_r^*"'] & & c_{\mathcal{H}} \arrow[d, "\iota_r^*"] \\
(\omega_{\mathcal{K}}, i_{\mathcal{K}}, h_{\mathcal{K}}) \arrow[rr, "\langle di_{\mathcal{K}}, dh_{\mathcal{K}} \rangle_{\omega_{\mathcal{K}}^{-1}}", "\text{Poisson bracket}"'] & & c_{\mathcal{K}}
\end{tikzcd}$$

Kronecker join state-space $\mathcal{K}_r$