ENERGY223 Reservoir Simulation Phase 4

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1 Introduction

For phase 3b of the project we will update on the existing 3a phase. We will develop the correct treatment of wells and describe how to handle producers with either oil rate of bottomhole pressure specified.

2 Governing Equations

In phase 3 we are constructing a simulator for the two-phase oil-gas simulator. The reservoir domain is horizontal without gravity effect. The reservoir is heterogeneous with $\Delta x \neq \Delta y$. Capillary pressure effect is included in the system.

The general mass balance for all phase can be written as:

$$\sum_p [\frac{\partial(\phi s_p \rho_p y_{c,p})}{\partial t} - \nabla \rho_p y_{c,p} k \lambda_p (\nabla p_p - \gamma_p \nabla D) + \hat{m}^w_{c,p}] = 0 \tag{1}$$

Here in our problem we have 4 unknowns including p_o, p_g, s_o, s_g and two phase mass balance equations, we do not have the effect of gravity in this model. We will introduce two more relationships to help us solve the systems of equations:

$$s_g + s_o = 1 \tag{2}$$

$$p_c(s_g) = p_g - p_o \tag{3}$$

3 Numerical Discretization of the 2D oil-gas systems

3.1 Oil Conservation Equation

From Eq. 1 we have the oil phase mass balance equation in 2-D without gravity effect:

$$\frac{\partial}{\partial x}[k\frac{\lambda_o}{B_o}(\frac{\partial p_o}{\partial x})] + \frac{\partial}{\partial y}[k\frac{\lambda_o}{B_o}(\frac{\partial P_o}{\partial y})] = \frac{\partial}{\partial t}(\phi\frac{S_o}{B_o}) + \hat{q_o} \tag{4}$$

Spatial discretization of the LHS of Eq. 4 for block i,j:

$$\begin{aligned} &\frac{\partial}{\partial x}[k\frac{\lambda_o}{B_o}(\frac{\partial p_o}{\partial x})] + \frac{\partial}{\partial y}[k\frac{\lambda_o}{B_o}(\frac{\partial P_o}{\partial y})] \approx \\ &(\gamma_{ox})_{i-\frac{1}{2},j}[(p_o)_{i-1,j} - (p_o)_{i,j}] + (\gamma_{ox})_{i+\frac{1}{2},j}[(p_o)_{i+1,j} - (p_o)_{i,j}] \\ &+(\gamma_{oy})_{i,j-\frac{1}{2}}[(p_o)_{i,j-1} - (p_o)_{i,j}] + (\gamma_{oy})_{i,j+\frac{1}{2}}[(p_o)_{i,j+1} - (p_o)_{i,j}] \end{aligned} \tag{5}$$

$$(\gamma_{ox})_{i-\frac{1}{2},j} = (\frac{kk_{ro}}{B_o\mu_o}\frac{\Delta y\Delta z}{\delta x})_{i-\frac{1}{2},j} \tag{6}$$

$$(\gamma_{oy})_{i,j-\frac{1}{2}} = (\frac{kk_{ro}}{B_o\mu_o}\frac{\Delta x\Delta z}{\delta y})_{i,j-\frac{1}{2}} \tag{7}$$

Time discretization of the Eq. 4 RHS accumulation term:

$$\frac{\partial}{\partial t}(\phi\frac{s_o}{B_o}) \approx \frac{1}{\Delta t}\Delta_t(\phi\frac{s_o}{B_o}) \tag{8}$$

Let's have $b_o \equiv \frac{1}{B_o}$,

$$\frac{1}{\Delta t}\Delta t(\phi\frac{s_o}{B_o}) = (\phi b_o)^{n+1}\Delta_t s_o + s_o^n(b_o^{n+1}\Delta_t\phi + \phi^n\Delta_t b_o) \tag{9}$$

We can express $\Delta_t b_o$ and $\Delta_t\phi$ as:

$$\Delta_t b_o = \frac{\partial p_o}{\partial t}\frac{db_o}{dp_o} = b_o^{'}\Delta_t p_o \tag{10}$$

$$\Delta_t\phi = \frac{\partial p_o}{\partial t}\frac{d\phi}{dp_o} = \phi^{'}\Delta_t p_o \tag{11}$$

In addition, $s_o = 1 - s_g$. Then the accumulation term becomes:

$$\frac{\partial}{\partial t}(\phi\frac{s_o}{B_o}) \approx -(\phi b_o)^{n+1}\Delta_t s_g + (1-s_g)^n(b_o^{n+1}\phi^{'} + \phi^n b_o^{'})\Delta_t p_o \tag{12}$$

The residual for block i,j is:

$$\begin{aligned} (R_o)_{i,j} = &(\gamma_{ox})_{i-\frac{1}{2},j}[(p_o)_{i-1,j} - (p_o)_{i,j}] + (\gamma_{ox})_{i+\frac{1}{2},j}[(p_o)_{i+1,j} - (p_o)_{i,j}] \\ &+(\gamma_{oy})_{i,j-\frac{1}{2}}[(p_o)_{i,j-1} - (p_o)_{i,j}] + (\gamma_{oy})_{i,j+\frac{1}{2}}[(p_o)_{i,j+1} - (p_o)_{i,j}] \\ &-[-(\phi b_o)^{n+1}\Delta_t s_g + (1-s_g)^n(b_o^{n+1}\phi^{'} + \phi^n b_o^{'})\Delta_t p_o] - \hat{q_o} \end{aligned} \tag{13}$$

3.2 Gas Conservation Equation

The differential form of gas phase mass balance equation in 2-D without gravity effect:

$$\begin{aligned}[kR_{g,o}\frac{\lambda_o}{B_o}(\frac{\partial p_o}{\partial x})+kR_{g,g}\frac{\lambda_g}{B_g}(\frac{\partial p_g}{\partial x})]+[kR_{g,o}\frac{\lambda_o}{B_o}(\frac{\partial p_o}{\partial y})+kR_{g,g}\frac{\lambda_g}{B_g}(\frac{\partial p_g}{\partial y})]\\ =\frac{\partial}{\partial t}[\phi(s_o\frac{R_{g,o}}{B_o}+s_g\frac{R_{g,g}}{B_g})]+\frac{R_{g,o}}{B_o}\hat{q_o^w}+\frac{R_{g,g}}{B_g}\hat{q_g^w}\end{aligned} \tag{14}$$

We know $R_{g,g}=1$ and let $R_s=R_{g,o}$,

$$\begin{aligned}[kR_s\frac{\lambda_o}{B_o}(\frac{\partial p_o}{\partial x})+k\frac{\lambda_g}{B_g}(\frac{\partial p_g}{\partial x})]+[kR_s\frac{\lambda_o}{B_o}(\frac{\partial p_o}{\partial y})+k\frac{\lambda_g}{B_g}(\frac{\partial p_g}{\partial y})]\\ =\frac{\partial}{\partial t}[\phi(s_o\frac{R_s}{B_o}+s_g\frac{1}{B_g})]+\frac{R_s}{B_o}\hat{q_o^w}+\frac{1}{B_g}\hat{q_g^w}\end{aligned} \tag{15}$$

Now substitute in $p_c(s_g)=p_g-p_o$, and $\frac{\partial p_c}{\partial x}=\frac{\partial s_g}{\partial x}\frac{dp_c}{ds_g}=p_c^{'}\frac{\partial s_g}{\partial x}$, $\frac{\partial p_c}{\partial y}=\frac{\partial s_g}{\partial y}\frac{dp_c}{ds_g}=p_c^{'}\frac{\partial s_g}{\partial y}$. LHS of eq. 15 for block i,j becomes:

$$\begin{aligned}&[kR_s\frac{\lambda_o}{B_o}(\frac{\partial p_o}{\partial x})+k\frac{\lambda_g}{B_g}(\frac{\partial p_o}{\partial x})+k\frac{\lambda_g}{B_g}(\frac{\partial p_c}{\partial x})]+[kR_s\frac{\lambda_o}{B_o}(\frac{\partial p_o}{\partial y})+k\frac{\lambda_g}{B_g}(\frac{\partial p_o}{\partial y})+k\frac{\lambda_g}{B_g}(\frac{\partial p_c}{\partial y})]\approx\\ &(R_s\gamma_{ox})_{i-\frac{1}{2},j}[(p_o)_{i-1,j}-(p_o)_{i,j}]+(R_s\gamma_{ox})_{i+\frac{1}{2},j}[(p_o)_{i+1,j}-(p_o)_{i,j}]\\ &+(R_s\gamma_{oy})_{i,j-\frac{1}{2}}[(p_o)_{i,j-1}-(p_o)_{i,j}]+(R_s\gamma_{oy})_{i,j+\frac{1}{2}}[(p_o)_{i,j+1}-(p_o)_{i,j}]\\ &+(\gamma_{gx})_{i-\frac{1}{2},j}[(p_o)_{i-1,j}-(p_o)_{i,j}]+(\gamma_{gx})_{i+\frac{1}{2},j}[(p_o)_{i+1,j}-(p_o)_{i,j}]\\ &+(\gamma_{gy})_{i,j-\frac{1}{2}}[(p_o)_{i,j-1}-(p_o)_{i,j}]+(\gamma_{gy})_{i,j+\frac{1}{2}}[(p_o)_{i,j+1}-(p_o)_{i,j}]\\ &+(\gamma_{gx})_{i-\frac{1}{2},j}(p_c^{'})_{i-\frac{1}{2},j}[(s_g)_{i-1,j}-(s_g)_{i,j}]+(\gamma_{gx})_{i+\frac{1}{2},j}(p_c^{'})_{i+\frac{1}{2},j}[(s_g)_{i+1,j}-(s_g)_{i,j}]\\ &+(\gamma_{gy})_{i,j-\frac{1}{2}}(p_c^{'})_{i,j-\frac{1}{2}}[(s_g)_{i,j-1}-(s_g)_{i,j}]+(\gamma_{gy})_{i,j+\frac{1}{2}}(p_c^{'})_{i,j+\frac{1}{2}}[(s_g)_{i,j+1}-(s_g)_{i,j}]\\ &=(R_s\gamma_{ox}+\gamma_{gx})_{i-\frac{1}{2},j}[(p_o)_{i-1,j}-(p_o)_{i,j}]+(R_s\gamma_{ox}+\gamma_{gx})_{i+\frac{1}{2},j}+[(p_o)_{i+1,j}-(p_o)_{i,j}]\\ &+(R_s\gamma_{oy}+\gamma_{gy})_{i,j-\frac{1}{2}}[(p_o)_{i,j-1}-(p_o)_{i,j}]+(R_s\gamma_{oy}+\gamma_{gy})_{i,j+\frac{1}{2}}[(p_o)_{i,j+1}-(p_o)_{i,j}]\\ &+(\gamma_{gx})_{i-\frac{1}{2},j}(p_c^{'})_{i-\frac{1}{2},j}[(s_g)_{i-1,j}-(s_g)_{i,j}]+(\gamma_{gx})_{i+\frac{1}{2},j}(p_c^{'})_{i+\frac{1}{2},j}[(s_g)_{i+1,j}-(s_g)_{i,j}]\\ &+(\gamma_{gy})_{i,j-\frac{1}{2}}(p_c^{'})_{i,j-\frac{1}{2}}[(s_g)_{i,j-1}-(s_g)_{i,j}]+(\gamma_{gy})_{i,j+\frac{1}{2}}(p_c^{'})_{i,j+\frac{1}{2}}[(s_g)_{i,j+1}-(s_g)_{i,j}]\end{aligned} \tag{16}$$

The oil transimissibility terms are the same as the ones in oil conservation equations, the gas transmissibility terms are defined as:

$$(\gamma_{gx})_{i-\frac{1}{2},j}=(\frac{kk_{rg}}{B_g\mu_g}\frac{\Delta y\Delta z}{\delta x})_{i-\frac{1}{2},j} \tag{17}$$

$$(\gamma_{gy})_{i,j-\frac{1}{2}}=(\frac{kk_{rg}}{B_g\mu_g}\frac{\Delta x\Delta z}{\delta y})_{i,j-\frac{1}{2}} \tag{18}$$

The discretization of the accumulation term on the RHS of eq. 15 can be shown:

$$\begin{aligned}&\frac{\partial}{\partial t}[\phi(s_o\frac{R_s}{B_o}+s_g\frac{1}{B_g})]=\frac{\partial}{\partial t}(\phi R_s s_o b_o)+\frac{\partial}{\partial t}(\phi s_g b_g)=\\ &(\phi R_s s_o b_o)^{n+1}-(\phi R_s s_o b_o)^n+(\phi s_g b_g)^{n+1}-(\phi s_g b_g)^n\end{aligned} \tag{19}$$

So the residual for the gas at block i,j is:

$$\begin{aligned}(R_g)_{i,j} = &(R_s\gamma_{ox}+\gamma_{gx})_{i-\frac{1}{2},j}[(p_o)_{i-1,j}-(p_o)_{i,j}] + (R_s\gamma_{ox}+\gamma_{gx})_{i+\frac{1}{2},j} + [(p_o)_{i+1,j}-(p_o)_{i,j}] \\ &+(R_s\gamma_{oy}+\gamma_{gy})_{i,j-\frac{1}{2}}[(p_o)_{i,j-1}-(p_o)_{i,j}] + (R_s\gamma_{oy}+\gamma_{gy})_{i,j+\frac{1}{2}}[(p_o)_{i,j+1}-(p_o)_{i,j}] \\ &+(\gamma_{gx})_{i-\frac{1}{2},j}(p_c^{'})_{i-\frac{1}{2},j}[(s_g)_{i-1,j}-(s_g)_{i,j}] + (\gamma_{gx})_{i+\frac{1}{2},j}(p_c^{'})_{i+\frac{1}{2},j}[(s_g)_{i+1,j}-(s_g)_{i,j}] \\ &+(\gamma_{gy})_{i,j-\frac{1}{2}}(p_c^{'})_{i,j-\frac{1}{2}}[(s_g)_{i,j-1}-(s_g)_{i,j}] + (\gamma_{gy})_{i,j+\frac{1}{2}}(p_c^{'})_{i,j+\frac{1}{2}}[(s_g)_{i,j+1}-(s_g)_{i,j}] \\ &-[(\phi R_s s_o b_o)^{n+1}-(\phi R_s s_o b_o)^n + (\phi s_g b_g)^{n+1}-(\phi s_g b_g)^n] - [\frac{R_{g,o}}{B_o}\hat{q_o^w} + \frac{R_{g,g}}{B_g}\hat{q_g^w}]\end{aligned} \tag{20}$$

3.3 System of Equations for Oil-Gas system

Now after we discretized oil conservation and gas conservation equation, we can write down the system of equations as:

$$\boldsymbol{T}\boldsymbol{p}^{n+1} - \boldsymbol{D}(\boldsymbol{p}^{n+1}-\boldsymbol{p}^n) - \boldsymbol{G} - \boldsymbol{Q} = 0 = \boldsymbol{R} \tag{21}$$

Where $\boldsymbol{p}$ represents the vector of unknowns. $\boldsymbol{T}$ is the matrix of transimissbilities, $\boldsymbol{D}$ contains the accumulation terms, $\boldsymbol{G}$ represents gravitational effects, and $\boldsymbol{Q}$ contains the $sink\backslash source$ term. $\boldsymbol{R}$ is the residual vector. If we consider a system of N grid blocks, with Nx blocks in the x direction and Ny blocks in the y direction, the unknowns written as:

$$\boldsymbol{p} = [(p_o)_1, (s_g)_1, (p_o)_2, (s_g)_2, (p_o)_3, (s_g)_3, ...(p_o)_N, (s_g)_N,]^t \tag{22}$$

The vector $\boldsymbol{p}$ is of dimension 2N. The $\boldsymbol{T}$ matrix, of dimension $2N \times 2N$ is block pentadiagonal, with each element a 2×2 matrix. The $\boldsymbol{D}$ matrix is of dimension $2N \times 2N$as block diagonal matrix, with each element a 2×2 matrix as well.

A row of 2×2 blocks in the $\boldsymbol{T}$ matrix can be written as:

$$\boldsymbol{T}_i = [\boldsymbol{T}_{i,i-Nx} \quad \boldsymbol{T}_{i,i-1} \quad \boldsymbol{T}_{i,i} \quad \boldsymbol{T}_{i,i+1} \quad \boldsymbol{T}_{i,i+Nx}] \tag{23}$$

$\boldsymbol{T}_{i,j} = 0$ for $j \neq i-Nx, i-1, i, i+1, i+Nx$

$$\boldsymbol{T}_{i,i-Nx} = \begin{bmatrix}(R_s\gamma_{oy}+\gamma_{gy})_{i,j-\frac{1}{2}} & (\gamma_{gy}P_c^{'})_{i,j-\frac{1}{2}} \\ (\gamma_{oy})_{i,j-\frac{1}{2}} & 0\end{bmatrix} \tag{24}$$

$$\boldsymbol{T}_{i,i-1} = \begin{bmatrix}(R_s\gamma_{ox}+\gamma_{gx})_{i-\frac{1}{2},j} & (\gamma_{gx}P_c^{'})_{i-\frac{1}{2},j} \\ (\gamma_{ox})_{i-\frac{1}{2},j} & 0\end{bmatrix} \tag{25}$$

$$\boldsymbol{T}_{i,i} = -(\boldsymbol{T}_{i,i-Nx} + \boldsymbol{T}_{i,i-1} + \boldsymbol{T}_{i,i+1} + \boldsymbol{T}_{i,i+Nx}) \tag{26}$$

$$\boldsymbol{T}_{i,i+1} = \begin{bmatrix}(R_s\gamma_{ox}+\gamma_{gx})_{i+\frac{1}{2},j} & (\gamma_{gx}P_c^{'})_{i+\frac{1}{2},j} \\ (\gamma_{ox})_{i+\frac{1}{2},j} & 0\end{bmatrix} \tag{27}$$

$$\boldsymbol{T}_{i,i+Nx} = \begin{bmatrix} (R_s\gamma_{oy} + \gamma_{gy})_{i,j+\frac{1}{2}} & (\gamma_{gy}P_c')_{i,j+\frac{1}{2}} \\ (\gamma_{oy})_{i,j+\frac{1}{2}} & 0 \end{bmatrix} \tag{28}$$

These elements correspond to:

$$[(p_o)_{i-Nx}, (s_g)_{i-Nx}, (p_o)_{i-1}, (s_g)_{i-1}, (p_o)_i, (s_g)_i, (p_o)_{i+1}, (s_g)_{i+1}, (p_o)_{i+Nx}, (s_g)_{i+Nx}]^t \tag{29}$$

The accumulation term $\boldsymbol{D}$, the block row of i in $\boldsymbol{D}$ term can be represented as:

$$\boldsymbol{D}_i = \begin{bmatrix} \frac{V_i}{\Delta t}\Delta_t(\phi R_s s_o b_o) + \frac{V_i}{\Delta t}\Delta_t(\phi s_g b_g) \\ \frac{V_i}{\Delta t}\Delta_t(\phi s_o b_o) \end{bmatrix} \tag{30}$$

3.4 Fully Implicit Solution

For the fully implicit solution of the oil-gas system, we write the system of equations as:

$$\boldsymbol{T}^{n+1}\boldsymbol{p}^{n+1} - \boldsymbol{D}(\boldsymbol{p}^{n+1} - \boldsymbol{p}^n) - \boldsymbol{G}^{n+1} - \boldsymbol{Q} = 0 = \boldsymbol{R} \tag{31}$$

In our case we don't consider the effect of gravity, so the G term goes away, the equation above becomes:

$$\boldsymbol{T}^{n+1}\boldsymbol{p}^{n+1} - \boldsymbol{D}(\boldsymbol{p}^{n+1} - \boldsymbol{p}^n) - \boldsymbol{Q} = \boldsymbol{R} \tag{32}$$

We then apply Newton's method to drive the residual to 0 by solving the system of equations $\boldsymbol{J}\delta = -\boldsymbol{R}$, where $\boldsymbol{J}_{i,j} = \frac{\partial R_i}{\partial p_j}$, and $\delta_i = \boldsymbol{p}_i^{n+1,k+1} - \boldsymbol{p}_i^{n+1,k}$, with k + 1 designating the next iterate for $\boldsymbol{p}$. Nonzero derivatives appear only for $j = i - Nx, i-1, i, i+1, i+Nx$. Thus the structure of Jacobian is similar to the structure of $\boldsymbol{T}$.

3.5 Relevant Code

Code for transmissbility matrix:

`rock_geo_transmissbility.m`
`fluid_trans_upwind.m`
`transmissbility_mat_construct.m`

Code used to construct accumulation matrix:

`accumulation_construct.m`

Code used to put the residual vector together:

`residual_calc.m`

4 Jacobian

The Jacobian of the simulation can be broken into 3 different parts:

$$\boldsymbol{J} = \boldsymbol{J}^f + \boldsymbol{J}^a + \boldsymbol{J}^g + \boldsymbol{J}^w \tag{33}$$

$\boldsymbol{J}^T$ is the Jacobian of the flow term, $\boldsymbol{J}^a$ is the Jacobian of the accumulation term, $\boldsymbol{J}^g$ is the Jacobian of the gravity term, and $\boldsymbol{J}^w$ is the Jacobian of the well term. In phase 3 of this project we ignore the gravitational effect in our model, and we will discuss the well treatment in the phase 3b.

4.1 Jacobian of the flow term

The Jacobian of the flow term $\frac{\partial \boldsymbol{T}_i}{\partial \boldsymbol{p}_j^{n+1}}$ has similar structure as the transmissbility matrix. The Jacobian is a pentadiagonal matrix with non-zeros entries at j equals to i-Nx, i-1, i, i+1, i+Nx for row i of the Jacobian matrix. Each element of the Jacobian is a 2×2 matrix, it can be expressed as:

$$\boldsymbol{J}^f_{i,j} = \begin{bmatrix} (\boldsymbol{J}^f_{g,p})_{i,j} & (\boldsymbol{J}^f_{g,s_g})_{i,j} \\ (\boldsymbol{J}^f_{o,p})_{i,j} & (\boldsymbol{J}^f_{o,s_g})_{i,j} \end{bmatrix} \tag{34}$$

$(\boldsymbol{J}^f_{o,p})_{i,j}$ is the derivative of the oil conservation equation with respect to primary variable p_o. For row i of the flow term Jacobian:

$$(\boldsymbol{J}^f_{o,p})_{i,i-Nx} = (\gamma_{oy})_{i,j-\frac{1}{2}} + \frac{\partial (\gamma_{oy})_{i,j-\frac{1}{2}}}{\partial (p_o)^{n+1}_{i,j-1}} [(p_o)^{n+1,k}_{i,j-1} - (p_o)^{n+1,k}_{i,j}] \tag{35}$$

$$(\boldsymbol{J}^f_{o,p})_{i,i-1} = (\gamma_{ox})_{i-\frac{1}{2},j} + \frac{\partial (\gamma_{ox})_{i-\frac{1}{2},j}}{\partial (p_o)^{n+1}_{i-1,j}} [(p_o)^{n+1,k}_{i-1,j} - (p_o)^{n+1,k}_{i,j}] \tag{36}$$

$$(\boldsymbol{J}^f_{o,p})_{i,i+1} = (\gamma_{ox})_{i+\frac{1}{2},j} + \frac{\partial (\gamma_{ox})_{i+\frac{1}{2},j}}{\partial (p_o)^{n+1}_{i+1,j}} [(p_o)^{n+1,k}_{i+1,j} - (p_o)^{n+1,k}_{i,j}] \tag{37}$$

$$(\boldsymbol{J}^f_{o,p})_{i,i+Nx} = (\gamma_{oy})_{i,j+\frac{1}{2}} + \frac{\partial (\gamma_{oy})_{i,j+\frac{1}{2}}}{\partial (p_o)^{n+1}_{i,j+1}} [(p_o)^{n+1,k}_{i,j+1} - (p_o)^{n+1,k}_{i,j}] \tag{38}$$

$$(\boldsymbol{J}^f_{o,p})_{i,i} = -((\boldsymbol{J}^f_{o,p})_{i,i-Nx} + (\boldsymbol{J}^f_{o,p})_{i,i-1} + (\boldsymbol{J}^f_{o,p})_{i,i+1} + (\boldsymbol{J}^f_{o,p})_{i,i+Nx}) \tag{39}$$

$(\boldsymbol{J}^f_{o,s})_{i,j}$ is the derivative of the oil conservation equation with respect to primary variable s_g. For row i of the flow term Jacobian:

$$(\boldsymbol{J}^f_{o,s})_{i,i-Nx} = \frac{\partial (\gamma_{oy})_{i,j-\frac{1}{2}}}{\partial (s_g)^{n+1}_{i,j-1}} [(p_o)^{n+1,k}_{i,j-1} - (p_o)^{n+1,k}_{i,j}] \tag{40}$$

$$(\boldsymbol{J}^f_{o,s})_{i,i-1} = \frac{\partial (\gamma_{ox})_{i-\frac{1}{2},j}}{\partial (s_g)^{n+1}_{i-1,j}} [(p_o)^{n+1,k}_{i-1,j} - (p_o)^{n+1,k}_{i,j}] \tag{41}$$

$$(\boldsymbol{J}^f_{o,s})_{i,i+1} = \frac{\partial(\gamma_{ox})_{i+\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i+1,j}}[(p_o)^{n+1,k}_{i+1,j} - (p_o)^{n+1,k}_{i,j}] \tag{42}$$

$$(\boldsymbol{J}^f_{o,s})_{i,i+Nx} = \frac{\partial(\gamma_{oy})_{i,j+\frac{1}{2}}}{\partial(s_g)^{n+1}_{i,j+1}}[(p_o)^{n+1,k}_{i,j+1} - (p_o)^{n+1,k}_{i,j}] \tag{43}$$

$$(\boldsymbol{J}^f_{o,s})_{i,i} = -((\boldsymbol{J}^f_{o,s})_{i,i-Nx} + (\boldsymbol{J}^f_{o,s})_{i,i-1} + (\boldsymbol{J}^f_{o,s})_{i,i+1} + (\boldsymbol{J}^f_{o,s})_{i,i+Nx}) \tag{44}$$

$(\boldsymbol{J}^f_{g,p})_{i,j}$ is the derivative of the gas conservation equation with respect to primary variable p_o. For row i of the flow term Jacobian:

$$(\boldsymbol{J}^f_{g,p})_{i,i-Nx} = (R_s\gamma_{oy} + \gamma_{gy})_{i,j-\frac{1}{2}} + \frac{\partial(R_s\gamma_{oy} + \gamma_{gy})_{i,j-\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j-1}}[(p_o)^{n+1,k}_{i,j-1} - (p_o)^{n+1,k}_{i,j}] \tag{45}$$

$$(\boldsymbol{J}^f_{g,p})_{i,i-1} = (R_s\gamma_{ox} + \gamma_{gx})_{i-\frac{1}{2},j} + \frac{\partial(R_s\gamma_{ox} + \gamma_{gx})_{i-\frac{1}{2},j}}{\partial(p_o)^{n+1}_{i-1,j}}[(p_o)^{n+1,k}_{i-1,j} - (p_o)^{n+1,k}_{i,j}] \tag{46}$$

$$(\boldsymbol{J}^f_{g,p})_{i,i+1} = (R_s\gamma_{ox} + \gamma_{gx})_{i+\frac{1}{2},j} + \frac{\partial(R_s\gamma_{ox} + \gamma_{gx})_{i+\frac{1}{2},j}}{\partial(p_o)^{n+1}_{i+1,j}}[(p_o)^{n+1,k}_{i+1,j} - (p_o)^{n+1,k}_{i,j}] \tag{47}$$

$$(\boldsymbol{J}^f_{g,p})_{i,i+Nx} = (R_s\gamma_{oy} + \gamma_{gy})_{i,j+\frac{1}{2}} + \frac{\partial(R_s\gamma_{oy} + \gamma_{gy})_{i,j+\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j+1}}[(p_o)^{n+1,k}_{i,j+1} - (p_o)^{n+1,k}_{i,j}] \tag{48}$$

$$\begin{aligned}(\boldsymbol{J}^f_{g,p})_{i,i} = &-(R_s\gamma_{oy} + \gamma_{gy})_{i,j-\frac{1}{2}} + \frac{\partial(R_s\gamma_{oy} + \gamma_{gy})_{i,j-\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j-1}}[(p_o)^{n+1,k}_{i,j-1} - (p_o)^{n+1,k}_{i,j}] \\ &-(R_s\gamma_{ox} + \gamma_{gx})_{i-\frac{1}{2},j} + \frac{\partial(R_s\gamma_{ox} + \gamma_{gx})_{i-\frac{1}{2},j}}{\partial(p_o)^{n+1}_{i-1,j}}[(p_o)^{n+1,k}_{i-1,j} - (p_o)^{n+1,k}_{i,j}] \\ &-(R_s\gamma_{ox} + \gamma_{gx})_{i+\frac{1}{2},j} + \frac{\partial(R_s\gamma_{ox} + \gamma_{gx})_{i+\frac{1}{2},j}}{\partial(p_o)^{n+1}_{i+1,j}}[(p_o)^{n+1,k}_{i+1,j} - (p_o)^{n+1,k}_{i,j}] \\ &-(R_s\gamma_{oy} + \gamma_{gy})_{i,j+\frac{1}{2}} + \frac{\partial(R_s\gamma_{oy} + \gamma_{gy})_{i,j+\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j+1}}[(p_o)^{n+1,k}_{i,j+1} - (p_o)^{n+1,k}_{i,j}]\end{aligned} \tag{49}$$

$(\boldsymbol{J}^f_{g,s})_{i,j}$ is the derivative of the gas conservation equation with respect to primary variable s_g. For row i of the flow term Jacobian:

$$(\boldsymbol{J}^f_{g,s})_{i,i-Nx} = -(\gamma_{gy}p_c^{'})_{i,j-\frac{1}{2}} - \frac{\partial(\gamma_{gy}p_c^{'})_{i,j-\frac{1}{2}}}{\partial(s_g)^{n+1}_{i,j-1}}[(s_g)^{n+1,k}_{i,j-1} - (s_g)^{n+1,k}_{i,j}] \tag{50}$$

$$(\boldsymbol{J}^f_{g,s})_{i,i-1} = -(\gamma_{gx}p_c^{'})_{i-\frac{1}{2},j} - \frac{\partial(\gamma_{gx}p_c^{'})_{i-\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i-1,j}}[(s_g)^{n+1,k}_{i-1,j} - (s_g)^{n+1,k}_{i,j}] \tag{51}$$

$$(\boldsymbol{J}^f_{g,s})_{i,i+1} = -(\gamma_{gx}p_c^{'})_{i+\frac{1}{2},j} - \frac{\partial(\gamma_{gx}p_c^{'})_{i+\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i+1,j}}[(s_g)^{n+1,k}_{i+1,j} - (s_g)^{n+1,k}_{i,j}] \tag{52}$$

$$(\boldsymbol{J}^f_{g,s})_{i,i+Nx} = -(\gamma_{gy}p_c^{'})_{i,j+\frac{1}{2}} - \frac{\partial(\gamma_{gy}p_c^{'})_{i,j+\frac{1}{2}}}{\partial(s_g)^{n+1}_{i,j+1}}[(s_g)^{n+1,k}_{i,j+1} - (s_g)^{n+1,k}_{i,j}] \tag{53}$$

$$\begin{aligned}(\boldsymbol{J}^f_{g,s})_{i,i} = (\gamma_{gy}p_c^{'})_{i,j-\frac{1}{2}} &- \frac{\partial(\gamma_{gy}p_c^{'})_{i,j-\frac{1}{2}}}{\partial(s_g)^{n+1}_{i,j-1}}[(s_g)^{n+1,k}_{i,j-1} - (s_g)^{n+1,k}_{i,j}] \\ +(\gamma_{gx}p_c^{'})_{i-\frac{1}{2},j} &- \frac{\partial(\gamma_{gx}p_c^{'})_{i-\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i-1,j}}[(s_g)^{n+1,k}_{i-1,j} - (s_g)^{n+1,k}_{i,j}] \\ +(\gamma_{gx}p_c^{'})_{i+\frac{1}{2},j} &- \frac{\partial(\gamma_{gx}p_c^{'})_{i+\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i+1,j}}[(s_g)^{n+1,k}_{i+1,j} - (s_g)^{n+1,k}_{i,j}] \\ +(\gamma_{gy}p_c^{'})_{i,j+\frac{1}{2}} &- \frac{\partial(\gamma_{gy}p_c^{'})_{i,j+\frac{1}{2}}}{\partial(s_g)^{n+1}_{i,j-1}}[(s_g)^{n+1,k}_{i,j+1} - (s_g)^{n+1,k}_{i,j}]\end{aligned} \tag{54}$$

4.2 Jacobian of the accumulation term

The Jacobian of the accumulation term $\frac{\partial \boldsymbol{D}_i}{\partial \boldsymbol{p}_j^{n+1}}$ is a diagonal matrix with non-zeros entries at main diagonal only, since accumulation term depend on the pressure and gas saturation of the local cell only, not on the neighboring cells. Each element of the Jacobian is a 2×2 matrix, it can be expressed as:

$$\boldsymbol{J}^a_{i,j} = \begin{bmatrix}(\boldsymbol{J}^a_{g,p})_{i,j} & (\boldsymbol{J}^a_{g,s_g})_{i,j} \\ (\boldsymbol{J}^a_{o,p})_{i,j} & (\boldsymbol{J}^a_{o,s_g})_{i,j}\end{bmatrix} \tag{55}$$

$(\boldsymbol{J}^a_{o,p})_{i,j}$ is the derivative of accumulation term of the oil conservation equation with respect to primary variable p_o. For row i of the accumulation term Jacobian:

$$\begin{aligned}(\boldsymbol{J}^a_{o,p})_{i,j} &= \frac{\partial(\frac{V_{i,j}}{\Delta t}\Delta_t(\phi s_o b_o)_{i,j})}{\partial(p_o)^{n+1}_{i,j}} = \frac{V_{i,j}}{\Delta t}\frac{\partial((\phi s_o b_o)^{n+1,k}_{i,j} - (\phi s_o b_o)^{n+1,k}_{i,j})}{\partial(p_o)^{n+1,k}_{i,j}} \\ &= \frac{V_{i,j}}{\Delta t}\frac{\partial(\phi s_o b_o)^{n+1,k}_{i,j}}{\partial(p_o)^{n+1,k}_{i,j}} = \frac{V_{i,j}}{\Delta t}(1-(s_g)^{n+1,k}_{i,j})[(b_o)^{n+1,k}_{i,j}\frac{\mathrm{d}(\phi)^{n+1,k}_{i,j}}{\mathrm{d}(p_o)^{n+1,k}_{i,j}} + (\phi)^{n+1,k}_{i,j}\frac{\mathrm{d}(b_o)^{n+1,k}_{i,j}}{\mathrm{d}(p_o)^{n+1,k}_{i,j}}]\end{aligned} \tag{56}$$

$(\boldsymbol{J}^a_{o,s})_{i,j}$ is the derivative of accumulation term of the oil conservation equation with respect to primary variable s_g. For row i of the accumulation term Jacobian:

$$(\boldsymbol{J}^a_{o,s})_{i,j} = \frac{\partial(\frac{V_{i,j}}{\Delta t}\Delta_t(\phi s_o b_o)_{i,j})}{\partial(s_g)^{n+1}_{i,j}} = \frac{V_{i,j}}{\Delta t}\frac{\partial((\phi s_o b_o)^{n+1,k}_{i,j} - (\phi s_o b_o)^{n,k}_{i,j})}{\partial(s_g)^{n+1,k}_{i,j}} = -\frac{V_{i,j}}{\Delta t}(\phi b_o)^{n+1,k}_{i,j} \tag{57}$$

$(\boldsymbol{J}^a_{g,s})_{i,j}$ is the derivative of accumulation term of the gas conservation equation with respect to primary variable s_g. For row i of the accumulation term Jacobian:

$$\begin{aligned}(\boldsymbol{J}^a_{g,s})_{i,j} &= \frac{\partial(\frac{V_{i,j}}{\Delta t}\Delta_t(\phi R_s s_o b_o)_{i,j} + \frac{V_{i,j}}{\Delta t}\Delta_t(\phi s_g b_g)_{i,j})}{\partial(s_g)^{n+1}_{i,j}} \\ &= \frac{V_{i,j}}{\Delta t}[\frac{\partial(\phi R_s s_o b_o)^{n+1,k}_{i,j}}{\partial(s_g)^{n+1}_{i,j}} + \frac{\partial(\phi s_g b_g)^{n+1,k}_{i,j}}{\partial(s_g)^{n+1}_{i,j}}] \\ &= \frac{V_{i,j}}{\Delta t}[-(\phi R_s b_o)^{n+1,k}_{i,j} + (\phi b_g + \phi s_g\frac{\mathrm{d}b_g}{\mathrm{d}p_g}\frac{\mathrm{d}p_c}{\mathrm{d}s_g})^{n+1,k}_{i,j}]\end{aligned} \tag{58}$$

$(\boldsymbol{J}^a_{g,p})_{i,j}$ is the derivative of accumulation term of the gas conservation equation with respect to primary variable p_o. For row i of the accumulation term Jacobian:

$$
\begin{aligned}
(\boldsymbol{J}^a_{g,p})_{i,j} &= \frac{\partial(\frac{V_{i,j}}{\Delta t}\Delta_t(\phi R_s s_o b_o)_{i,j} + \frac{V_{i,j}}{\Delta t}\Delta_t(\phi s_g b_g)_{i,j})}{\partial(p_o)^{n+1}_{i,j}} \\
&= \frac{V_{i,j}}{\Delta t}[\frac{\partial(\phi R_s s_o b_o)^{n+1,k}_{i,j}}{\partial(p_o)^{n+1}_{i,j}} + \frac{\partial(\phi s_g b_g)^{n+1,k}_{i,j}}{\partial(p_o)^{n+1}_{i,j}}] \\
&= \frac{V_{i,j}}{\Delta t}[\frac{\partial(\phi R_s b_o)^{n+1,k}_{i,j}}{\partial(p_o)^{n+1}_{i,j}} - \frac{\partial(\phi R_s s_g b_o)^{n+1,k}_{i,j}}{\partial(p_o)^{n+1}_{i,j}} + \frac{\partial(\phi s_g b_g)^{n+1,k}_{i,j}}{\partial(p_o)^{n+1}_{i,j}}] \\
&= \frac{V_{i,j}}{\Delta t}[(\phi^{'})^{n+1,k}_{i,j}(R_s)^{n+1,k}_{i,j}(b_o)^{n+1,k}_{i,j} + (R^{'}_s)^{n+1,k}_{i,j}(\phi)^{n+1,k}_{i,j}(b_o)^{n+1,k}_{i,j} + (b^{'}_o)^{n+1,k}_{i,j}(R_s)^{n+1,k}_{i,j}(b_o)^{n+1,k}_{i,j} \\
&\quad -(\phi^{'})^{n+1,k}_{i,j}(R_s)^{n+1,k}_{i,j}(b_o)^{n+1,k}_{i,j}(s_g)^{n+1,k}_{i,j} - (R^{'}_s)^{n+1,k}_{i,j}(\phi)^{n+1,k}_{i,j}(b_o)^{n+1,k}_{i,j}(s_g)^{n+1,k}_{i,j} \\
&\quad -(b^{'}_o)^{n+1,k}_{i,j}(R_s)^{n+1,k}_{i,j}(\phi)^{n+1,k}_{i,j}(s_g)^{n+1,k}_{i,j} + (\phi^{'})^{n+1,k}_{i,j}(s_g)^{n+1,k}_{i,j}(b_g)^{n+1,k}_{i,j} + (b^{'}_g)^{n+1,k}_{i,j}(s_g)^{n+1,k}_{i,j}(\phi)^{n+1,k}_{i,j}]
\end{aligned}
\tag{59}
$$

Here $(\phi^{'})^{n+1,k}_{i,j} = (\frac{\mathrm{d}\phi}{\mathrm{d}p_o})^{n+1,k}_{i,j}$, $(b^{'}_o)^{n+1,k}_{i,j} = (\frac{\mathrm{d}b_o}{\mathrm{d}p_o})^{n+1,k}_{i,j}$, $(R^{'}_s)^{n+1,k}_{i,j} = (\frac{\mathrm{d}R_s}{\mathrm{d}p_o})^{n+1,k}_{i,j}$, $(b^{'}_g)^{n+1,k}_{i,j} = (\frac{\mathrm{d}b_g}{\mathrm{d}p_o})^{n+1,k}_{i,j}$

4.3 Relevant Code

Solve derivatives of bo, bg, Rs, kro, krg, μ_o, μ_g analytically:

`derivatives_calc.m`

`derivatives_matrix_calc.m`

`Dp_matrix_construct.m`

Flow Jacobian:

`flow_jacobian_calc.m`

Accumulation Jacobian:

`accumulation_jacobian_calc.m`

Code used to put the residual vector together:

`jacobian_matrix_construct.m`

5 Calculation of Transmissibility

As described above, the transmissibility for oil is:

$$(\gamma_{ox})_{i-\frac{1}{2},j} = (\frac{kk_{ro}}{B_o\mu_o}\frac{\Delta y\Delta z}{\Delta x})_{i-\frac{1}{2},j} \tag{60}$$

$$(\gamma_{oy})_{i,j-\frac{1}{2}} = (\frac{kk_{ro}}{B_o\mu_o}\frac{\Delta x\Delta z}{\Delta y})_{i,j-\frac{1}{2}} \tag{61}$$

And the transmissbility for gas is:

$$(\gamma_{gx})_{i-\frac{1}{2},j} = (\frac{kk_{rg}}{B_g\mu_g}\frac{\Delta y\Delta z}{\Delta x})_{i-\frac{1}{2},j} \tag{62}$$

$$(\gamma_{gy})_{i,j-\frac{1}{2}} = (\frac{kk_{rg}}{B_g\mu_g}\frac{\Delta x\Delta z}{\Delta y})_{i,j-\frac{1}{2}} \tag{63}$$

The geometric part of the transmissibility is $\frac{k\Delta y\Delta z}{\Delta x}$ and the fluid part of the transmissibility is $\frac{k_{rp}}{B_p\mu_p}$, $p = o, g$. We can generalize the transmissbility for each phase in the notation below, where p indicates oil, or gas:

$$(\gamma_{px})_{i-\frac{1}{2},j} = (\frac{kk_{rp}}{B_p\mu_p}\frac{\Delta y\Delta z}{\Delta x})_{i-\frac{1}{2},j} \tag{64}$$

$$(\gamma_{py})_{i,j-\frac{1}{2}} = (\frac{kk_{rp}}{B_p\mu_p}\frac{\Delta x\Delta z}{\Delta y})_{i,j-\frac{1}{2}} \tag{65}$$

5.1 Treatment of Geometric Term

For the geometric term we will treat them by taking the weighted harmonic average between the block and the neighboring block where the fluid is flowing through. For example, let us take a look at a 2-D grid. Block i,j has permeability of $k_{i,j}$ and block i+1,j has permeability $k_{i+1,j}$, the interface between the two blocks $i + \frac{1}{2}, j$ has the following permeability:

$$k_{i+\frac{1}{2},j} = \frac{\Delta x_{i,j} + \Delta x_{i+1,j}}{\Delta x_{i,j}/k_{i,j} + \Delta x_{i+1,j}/ki + 1} \tag{66}$$

5.2 Treatment of Fluid Term

The treatment for the fluid term is different than it for the rock and geometric term, we designate $(H_w)_{1+\frac{1}{2},j}$ to represent the fluid term:

$$(H_w)_{1+\frac{1}{2},j} = (\frac{k_{rp}}{B_p\mu_p})_{1+\frac{1}{2},j} = (\frac{k_{rp}b_p}{\mu_p})_{1+\frac{1}{2},j} \tag{67}$$

Because the hyperbolic characteristics of the governing equation, we will introduce a treatment with direction dependency. Specifically, the treatment of this term depend on the direction of the follow:

$$(H_w)_{i+\frac{1}{2},j} = \begin{cases} (H_w)_{i,j}, & \text{if } (u_p)_{i+\frac{1}{2},j} > 0. \\ (H_w)_{i+1,j}, & \text{if } (u_p)_{i+\frac{1}{2},j} < 0. \end{cases} \tag{68}$$

Because we do not have gravity here, the direction of the phase velocity can be determined by computing the sign of $(p_p)_{i,j} - (p_p)_{i+1,j}$. If $(p_p)_{i,j} - (p_p)_{i+1,j} > 0$ then $(u_p)_{i+\frac{1}{2},j} > 0$, if $(p_p)_{i,j} - (p_p)_{i+1,j} < 0$ then $(u_p)_{i+\frac{1}{2},j} < 0$. Because of the upwinding treatment, the derivatives of the transmissibility with respect to oil pressure and water saturation will be different than the transmissibility without treatment. The derivatives of one phase's transmissibility with respect to oil pressure are:

$$\frac{\partial(\gamma_{px})_{i-\frac{1}{2},j}}{\partial(p_o)^{n+1}_{i,j}} = \begin{cases} 0, & \text{if } (u_p)_{i-\frac{1}{2},j} > 0. \\ (\frac{k\Delta y\Delta z}{\Delta x})_{i-\frac{1}{2},j}(\frac{k_{rp}}{\mu_p}\frac{\mathrm{d}b_p}{\mathrm{d}p_o} - \frac{k_{rp}b_p}{\mu_p^2}\frac{\partial\mu_p}{\partial p_o})_{i,j}, & \text{if } (u_p)_{i-\frac{1}{2},j} < 0. \end{cases} \tag{69}$$

$$\frac{\partial(\gamma_{px})_{i-\frac{1}{2},j}}{\partial(p_o)^{n+1}_{i-1,j}} = \begin{cases} (\frac{k\Delta y\Delta z}{\Delta x})_{i-\frac{1}{2},j}(\frac{k_{rp}}{\mu_p}\frac{\mathrm{d}b_p}{\mathrm{d}p_o} - \frac{k_{rp}b_p}{\mu_p^2}\frac{\partial\mu_p}{\partial p_o})_{i-1,j}, & \text{if } (u_p)_{i-\frac{1}{2},j} > 0. \\ 0, & \text{if } (u_p)_{i-\frac{1}{2},j} < 0. \end{cases} \tag{70}$$

$$\frac{\partial(\gamma_{py})_{i,j-\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j}} = \begin{cases} 0, & \text{if } (u_p)_{i,j-\frac{1}{2}} > 0. \\ (\frac{k\Delta x\Delta z}{\Delta y})_{i,j-\frac{1}{2}}(\frac{k_{rp}}{\mu_p}\frac{\mathrm{d}b_p}{\mathrm{d}p_o} - \frac{k_{rp}b_p}{\mu_p^2}\frac{\partial\mu_p}{\partial p_o})_{i,j}, & \text{if } (u_p)_{i,j-\frac{1}{2}} < 0. \end{cases} \tag{71}$$

$$\frac{\partial(\gamma_{py})_{i,j-\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j-\frac{1}{2}}} = \begin{cases} (\frac{k\Delta x\Delta z}{\Delta y})_{i,j-\frac{1}{2}}(\frac{k_{rp}}{\mu_p}\frac{\mathrm{d}b_p}{\mathrm{d}p_o} - \frac{k_{rp}b_p}{\mu_p^2}\frac{\partial\mu_p}{\partial p_o})_{i,j-1}, & \text{if } (u_p)_{i,j-\frac{1}{2}} > 0. \\ 0, & \text{if } (u_p)_{i,j-\frac{1}{2}} < 0. \end{cases} \tag{72}$$

Because $p_c^{'}$ is a function of gas saturation, $\frac{\mathrm{d}b_g}{\mathrm{d}s_g} = \frac{\mathrm{d}b_g}{\mathrm{d}p_g}\frac{\mathrm{d}p_g}{\mathrm{d}s_g} = \frac{\mathrm{d}b_g}{\mathrm{d}p_g}\frac{\mathrm{d}(p_o+p_c)}{\mathrm{d}s_g} = \frac{\mathrm{d}b_g}{\mathrm{d}p_g}\frac{\mathrm{d}p_c}{\mathrm{d}s_g}$. We have to treat oil and gas transmissbility differently in this case. The derivatives of oil phase transmissibility with respect to gas saturation are:

$$\frac{\partial(\gamma_{ox})_{i-\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i,j}} = \begin{cases} 0, & \text{if } (u_o)_{i-\frac{1}{2},j} > 0. \\ (\frac{k\Delta y\Delta z}{\Delta x})_{i-\frac{1}{2},j}(\frac{b_o}{\mu_o})_{i,j}(\frac{\mathrm{d}k_{ro}}{\mathrm{d}s_g})_{i,j}, & \text{if } (u_o)_{i-\frac{1}{2},j} < 0. \end{cases} \tag{73}$$

$$\frac{\partial(\gamma_{ox})_{i-\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i-1,j}} = \begin{cases} (\frac{k\Delta y\Delta z}{\Delta x})_{i-\frac{1}{2},j}(\frac{b_o}{\mu_o})_{i-1,j}(\frac{\mathrm{d}k_{ro}}{\mathrm{d}s_g})_{i-1,j}, & \text{if } (u_o)_{i-\frac{1}{2},j} > 0. \\ 0, & \text{if } (u_o)_{i-\frac{1}{2},j} < 0. \end{cases} \tag{74}$$

$$\frac{\partial(\gamma_{oy})_{i,j-\frac{1}{2}}}{\partial(s_g)^{n+1}_{i,j}} = \begin{cases} 0, & \text{if } (u_o)_{i,j-\frac{1}{2}} > 0. \\ (\frac{k\Delta x\Delta z}{\Delta y})_{i,j-\frac{1}{2}}(\frac{b_o}{\mu_o})_{i,j}(\frac{\mathrm{d}k_{ro}}{\mathrm{d}s_g})_{i,j}, & \text{if } (u_o)_{i,j-\frac{1}{2}} < 0. \end{cases} \tag{75}$$

$$\frac{\partial(\gamma_{py})_{i,j-\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j-\frac{1}{2}}} = \begin{cases} (\frac{k\Delta x\Delta z}{\Delta y})_{i,j-\frac{1}{2}}(\frac{b_o}{\mu_o})_{i,j-\frac{1}{2}}(\frac{\mathrm{d}k_{ro}}{\mathrm{d}s_g})_{i,j-\frac{1}{2}}, & \text{if } (u_o)_{i,j-\frac{1}{2}} > 0. \\ 0, & \text{if } (u_o)_{i,j-\frac{1}{2}} < 0. \end{cases} \tag{76}$$

The derivatives of gas phase transmissibility with respect to gas saturation are:

$$\frac{\partial(\gamma_{gx})_{i-\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i,j}} = \begin{cases} 0, & \text{if } (u_g)_{i-\frac{1}{2},j} > 0. \\ (\frac{k\Delta y\Delta z}{\Delta x})_{i-\frac{1}{2},j}(\frac{\mu_g}{b_g}\frac{\mathrm{d}k_{rg}}{\mathrm{d}s_g} + \frac{k_{rg}}{\mu_g}\frac{\mathrm{d}b_g}{\mathrm{d}s_g} - \frac{k_{rg}b_g}{\mu_g^2}\frac{\mathrm{d}\mu_g}{\mathrm{d}s_g})_{i,j}, & \text{if } (u_g)_{i-\frac{1}{2},j} < 0. \end{cases} \tag{77}$$

$$\frac{\partial(\gamma_{gx})_{i-\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i-1,j}} = \begin{cases} (\frac{k\Delta y\Delta z}{\Delta x})_{i-\frac{1}{2},j}(\frac{\mu_g}{b_g}\frac{\mathrm{d}k_{rg}}{\mathrm{d}s_g} + \frac{k_{rg}}{\mu_g}\frac{\mathrm{d}b_g}{\mathrm{d}s_g} - \frac{k_{rg}b_g}{\mu_g^2}\frac{\mathrm{d}\mu_g}{\mathrm{d}s_g})_{i-1,j}, & \text{if } (u_o)_{i-\frac{1}{2},j} > 0. \\ 0, & \text{if } (u_o)_{i-\frac{1}{2},j} < 0. \end{cases} \tag{78}$$

$$\frac{\partial(\gamma_{gy})_{i,j-\frac{1}{2}}}{\partial(s_g)^{n+1}_{i,j}} = \begin{cases} 0, & \text{if } (u_g)_{i,j-\frac{1}{2}} > 0. \\ (\frac{k\Delta x\Delta z}{\Delta y})_{i,j-\frac{1}{2}}(\frac{\mu_g}{b_g}\frac{\mathrm{d}k_{rg}}{\mathrm{d}s_g} + \frac{k_{rg}}{\mu_g}\frac{\mathrm{d}b_g}{\mathrm{d}s_g} - \frac{k_{rg}b_g}{\mu_g^2}\frac{\mathrm{d}\mu_g}{\mathrm{d}s_g})_{i,j}, & \text{if } (u_g)_{i,j-\frac{1}{2}} < 0. \end{cases} \tag{79}$$

$$\frac{\partial(\gamma_{gy})_{i,j-\frac{1}{2}}}{\partial(s_g)^{n+1}_{i,j-\frac{1}{2}}} = \begin{cases} (\frac{k\Delta x\Delta z}{\Delta y})_{i,j-\frac{1}{2}}(\frac{\mu_g}{b_g}\frac{\mathrm{d}k_{rg}}{\mathrm{d}s_g} + \frac{k_{rg}}{\mu_g}\frac{\mathrm{d}b_g}{\mathrm{d}s_g} - \frac{k_{rg}b_g}{\mu_g^2}\frac{\mathrm{d}\mu_g}{\mathrm{d}s_g})_{i,j-\frac{1}{2}}, & \text{if } (u_g)_{i,j-\frac{1}{2}} > 0. \\ 0, & \text{if } (u_g)_{i,j-\frac{1}{2}} < 0. \end{cases} \tag{80}$$

So when we calculate the Jacobian of the gas transmissbility:

$$\frac{\partial(R_s\gamma_{ox} + \gamma_{gx})_{i-\frac{1}{2},j}}{\partial(p_o)^{n+1}_{i-1,j}} = \begin{cases} 0, & \text{if } (u_g)_{i-\frac{1}{2},j} > 0. \\ (\gamma_{ox})_{i-\frac{1}{2},j}\frac{\partial(R_s)_{i-\frac{1}{2},j}}{\partial(p_o)^{n+1}_{i-1,j}} + (R_s)_{i-\frac{1}{2},j}\frac{\partial(\gamma_{ox})_{i-\frac{1}{2},j}}{\partial(p_o)^{n+1}_{i-1,j}} + \frac{\partial(\gamma_{gx})_{i-\frac{1}{2},j}}{\partial(p_o)^{n+1}_{i-1,j}}, & \text{if } (u_g)_{i-\frac{1}{2},j} < 0. \end{cases} \tag{81}$$

$$\frac{\partial(R_s\gamma_{oy} + \gamma_{gy})_{i,j-\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j-1}} = \begin{cases} 0, & \text{if } (u_g)_{i,j-\frac{1}{2}} > 0. \\ (\gamma_{oy})_{i,j-\frac{1}{2}}\frac{\partial(R_s)_{i,j-\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j-1}} + (R_s)_{i,j-\frac{1}{2}}\frac{\partial(\gamma_{oy})_{i,j-\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j-1}} + \frac{\partial(\gamma_{gy})_{i,j-\frac{1}{2}}}{\partial(p_o)^{n+1}_{i,j-1}}, & \text{if } (u_g)_{i,j-\frac{1}{2}} < 0. \end{cases} \tag{82}$$

$$\frac{\partial(\gamma_{gy}p_c^{'})_{i,j-\frac{1}{2}}}{\partial(s_g)^{n+1}_{i,j-1}} = \begin{cases} 0, & \text{if } (u_g)_{i,j-\frac{1}{2}} > 0. \\ \frac{\partial(\gamma_{gy})_{i,j-\frac{1}{2}}}{\partial(s_g)^{n+1}_{i,j-1}}(p_c^{'})_{i,j-\frac{1}{2}} + (\gamma_{gy})_{i,j-\frac{1}{2}}(p_c^{''})_{i,j-\frac{1}{2}}, & \text{if } (u_g)_{i,j-\frac{1}{2}} < 0. \end{cases} \tag{83}$$

$$\frac{\partial(\gamma_{gx}p_c^{'})_{i-\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i-1,j}} = \begin{cases} 0, & \text{if } (u_g)_{i-\frac{1}{2},j} > 0. \\ \frac{\partial(\gamma_{gy})_{i-\frac{1}{2},j}}{\partial(s_g)^{n+1}_{i-1,j}}(p_c^{'})_{i-\frac{1}{2},j} + (\gamma_{gy})_{i-\frac{1}{2},j}(p_c^{''})_{i-\frac{1}{2},j}, & \text{if } (u_g)_{i-\frac{1}{2},j} < 0. \end{cases} \tag{84}$$

6 Treatment of Wells

6.1 Single Phase Well Model

For a single-phase, steady state, incompressible flow in a uniform square grid, we have:

$$\sum_{j=1}^{4} q_j = q^w \tag{85}$$

Where j = 1, 2, 3, 4 is the index of the four blocks surrounding block 0 and

$$q_j = \frac{kA}{\mu}(\frac{p_j - p_o}{a}) \tag{86}$$

$a = \Delta x = \Delta y$ and $A = ha$ is the cross-sectional area of each block.

Assuming radial flow from the surrounding blocks to the well block, we can write the analytical solution:

$$p_i = p_0 + \frac{q^w \mu}{2\pi kh} \ln\left(\frac{a}{r_0}\right), i = 1, 2, 3, 4 \tag{87}$$

Where p_i are the pressures for grid blocks surrounding block 0(the well block). r_0 can be obtained by:

$$\begin{aligned} \frac{r_0}{a} = \exp\left(-\frac{\pi}{2}\right) &\approx 0.208 \\ r_0 &\approx 0.208a \end{aligned} \tag{88}$$

We can write the well equation as:

$$q^w = T^w(p_0 - p^w) \tag{89}$$

where T^w is defined as:

$$T^w = \frac{2\pi kh}{\mu}[\frac{1}{\ln\left(\frac{r_0}{r_w}\right) + s}] \tag{90}$$

We can define well index as:

$$WI = \frac{2\pi kh}{\ln\left(\frac{r_0}{r_w}\right) + s} \tag{91}$$

s is the skin factor, which indicates the damage of the formation caused by drilling and completion near the wellbore. Then, for single phase well flow we have:

$$T^w = WI \cdot \frac{1}{\mu} \tag{92}$$

6.2 Multiphase Flow Well Model

For multiphase flow from block i into a well in that block, the rate for a given phase p is:

$$q_{p_i}^w = (WI)_i \lambda_{p_i}(p_{p_i} - p_{p_i}^w) \tag{93}$$

The phase mobility is defined as:

$$\lambda_{p_i} = (\frac{k_{rp}}{\mu_p}\frac{1}{B_p})_i \tag{94}$$

r_0 is defined differently than it in the single phase, uniform grid case:

$$r_0 = 0.28\frac{[(\frac{k_y}{k_x})^{\frac{1}{2}}\Delta x^2 + (\frac{k_x}{k_y})^{\frac{1}{2}}\Delta y^2]^{\frac{1}{2}}}{(\frac{k_y}{k_x})^{\frac{1}{4}} + (\frac{k_x}{k_y})^{\frac{1}{4}}} \tag{95}$$

In multiphase with anisotropy we can approximate k by:

$$k = \sqrt{k_x k_y} \tag{96}$$

6.3 Well Production Rate Control

6.3.1 Oil Rate Control

If the oil rate q_o^w is specified, then the gas flow rate at the well can be computed as:

$$q_g^w = q_o^w(\frac{T_g^w}{T_o^w} + R_s) = q_o^w(\frac{k_{rg}B_o\mu_o}{k_{ro}B_g\mu_g} + R_s) \tag{97}$$

If grid block i is the block that contains the well, then the residual of that block will be modified in the following manner after well treatment:

$$R_i = R_i - \begin{bmatrix} q_o^w(\frac{k_{rg}b_g\mu_o}{k_{ro}b_o\mu_g} + R_s) \\ q_o^w \end{bmatrix} \tag{98}$$

The modifications to the Jacobian after well treatment is only local to the grid block that contains the well. Because we control production of oil at constant rate, Jacobian changes only exist on the gas equations. The modifications are shown below:

$$\frac{\partial(R_g)_i}{\partial(p_o)_{i,j}} = q_o^w\frac{\partial(\frac{T_g^w}{T_o^w} + R_s)}{\partial p_o} = q_o^w[\frac{k_{rg}}{k_{ro}}\frac{b_o\mu_g(\mu_o b_g^{'} + b_g\mu_o^{'}) - b_g\mu_o(\mu_g b_o^{'} + b_o\mu_g^{'})}{(\mu_g b_o)^2} + \frac{\mathrm{d}R_s}{\mathrm{d}p_o}] \tag{99}$$

$$\frac{\partial(R_g)_i}{\partial(s_g)_{i,j}} = q_o^w\frac{\partial(\frac{T_g^w}{T_o^w} + R_s)}{\partial s_w} = q_o^w(\frac{b_g\mu_o}{b_o\mu_g})(\frac{k_{ro}k_{rg}^{'} - k_{rg}k_{ro}^{'})}{k_{ro}^2}) \tag{100}$$

6.3.2 Gas Rate Control

If the gas rate q_g^w is specified, then the oil flow rate at the well can be computed as:

$$q_o^w = q_g^w(\frac{T_o^w}{T_g^w}) = (q_g^w - R_s q_o^w)(\frac{k_{ro}B_g\mu_g}{k_{rg}B_o\mu_o}) \tag{101}$$

$$q_o^w = \frac{q_g^w(\frac{T_o^w}{T_g^w})}{1 + R_s\frac{T_o^w}{T_g^w}} = q_g^w[\frac{T_o^w}{T_g^w + R_s T_o^w}] \tag{102}$$

The residual of the well block will be modified in the following manner after well treatment:

$$R_i = R_i - \begin{bmatrix} q_g^w \\ q_g^w[\frac{T_o^w}{T_g^w + R_s T_o^w}] \end{bmatrix} \tag{103}$$

The modifications to the Jacobian after well treatment is only local to the grid block that contains the well. Because we control production of gas at constant rate, Jacobian changes only exist on the oil equations. The modifications are shown below:

$$\frac{\partial(R_o)_i}{\partial(p_o)_{i,j}} = q_g^w \frac{\partial[\frac{T_o^w}{T_g^w + R_s T_o^w}]}{\partial p_o} \tag{104}$$

$$\frac{\partial(R_o)_i}{\partial(s_g)_{i,j}} = q_g^w \frac{\partial[\frac{T_o^w}{T_g^w + R_s T_o^w}]}{\partial s_w} \tag{105}$$

6.3.3 Total Rate Control

If the total flow rate q_t^w is specified, then the oil flow rate and the gas flow rate at the well can be computed as:

$$q_o^w = q_t^w(\frac{T_o^w}{T_g^w + T_o^w}) \tag{106}$$

$$q_g^w = q_t^w(\frac{T_g^w}{T_g^w + T_o^w}) + R_s q_t^w(\frac{T_o^w}{T_g^w + T_o^w}) \tag{107}$$

The residual and the Jacobian of the well block will be modified accordingly in the same fashion as the last two examples.

6.4 Bottom Hole Pressure Control

If the bottom hole pressure of the well is specified instead of the flow rate, then the oil and gas flow rate at the well block can be calculated:

$$\begin{aligned} q_o^w &= T_o^w[(p_o)_i - (p_o)_i^w] \\ q_g^w &= T_g^w[(p_g)_i - (p_g)_i^w] + R_s T_o^w[(p_o)_i - (p_o)_i^w] \end{aligned} \tag{108}$$

Then the change to residual at block i after well treatment is:

$$R_i = R_i - \begin{bmatrix} T_o^w[(p_o)_i - (p_o)_i^w] \\ T_g^w[(p_g)_i - (p_g)_i^w] + R_s T_o^w[(p_o)_i - (p_o)_i^w] \end{bmatrix} \tag{109}$$

The change to Jacobian after well treatment is:

$$\frac{\partial(R_o)_i}{\partial(p_o)_{i,j}} = \frac{\partial T_o^w[(p_o)_i - (p_o)_i^w]}{\partial p_o} = WI\lambda_o + WI[(p_o)_i - (p_o)_i^w]\frac{\partial \lambda_o}{\partial p_o} \tag{110}$$

$$\frac{\partial(R_o)_i}{\partial(s_g)_{i,j}} = \frac{\partial T_o^w[(p_o)_i - (p_o)_i^w]}{\partial s_g} = WI[(p_o)_i - (p_o)_i^w]\frac{\partial \lambda_o}{\partial s_g} \tag{111}$$

$$\begin{aligned} \frac{\partial(R_g)_i}{\partial(p_o)_{i,j}} &= \frac{\partial[T_g^w[(p_g)_i - (p_g)_i^w] + R_s T_o^w[(p_o)_i - (p_o)_i^w]]}{\partial p_o} \\ &= WI\lambda_g + WI[(p_o)_i - (p_o)_i^w]\frac{\partial \lambda_g}{\partial p_o} + R_s' T_o^w[(p_o)_i - (p_o)_i^w] + R_s WI[\lambda_o + [(p_o)_i - (p_o)_i^w]\frac{\partial \lambda_o}{\partial p_o}] \end{aligned} \tag{112}$$

$$\begin{aligned}\frac{\partial(R_g)_i}{\partial(s_g)_{i,j}} &= \frac{\partial[T_g^w[(p_g)_i - (p_g)_i^w] + R_s T_o^w[(p_o)_i - (p_o)_i^w]]}{\partial s_g} \\ &= WI[(p_o)_i - (p_o)_i^w]\frac{\partial \lambda_g}{\partial s_g} + R_s WI[(p_o)_i - (p_o)_i^w]\frac{\partial \lambda_o}{\partial s_g}\end{aligned} \tag{113}$$

$$\begin{aligned}\frac{\partial \lambda_o}{\partial p_o} &= k_{ro}[\frac{b_o^{'}\mu_o - \mu_o^{'}b_o}{(\mu_o)^2}] \\ \frac{\partial \lambda_o}{\partial s_g} &= \frac{b_o}{\mu_o}\frac{\partial k_{ro}}{\partial s_g} \\ \frac{\partial \lambda_g}{\partial p_o} &= k_{rg}[\frac{b_g^{'}\mu_g - \mu_g^{'}b_g}{(\mu_g)^2}] \\ \frac{\partial \lambda_g}{\partial s_g} &= \frac{b_g}{\mu_g}\frac{\partial k_{rg}}{\partial s_g}\end{aligned} \tag{114}$$

6.5 Relevant Code

To construct the well term for the residual:

`well_index_calc.m`

`well_construct.m`

For well Jacobian:

`well_jacobian_calc.m`

7 Convergence Criteria

There are two types of convergence checks for Newton's method:

1. Inifinity norm of the residual:

$$\|R_l^{normalized}\|_\infty = \max_N |5.616 \times B_l \times \Delta t \times \frac{(R_l)_N}{(PV)_N} <= \epsilon_1 for l = o, g \tag{115}$$

2. Max change in pressure and saturation

$$\begin{aligned} \max_N |(s_g)_N^{n+1,v} - (s_g)_N^{n+1,v-1}| < \epsilon_2 \\ \max_N |\frac{(p_o)_N^{n+1,v} - (p_o)_N^{n+1,v-1}}{(p_o)_a v g^{n+1,v}}| < \epsilon_3 \end{aligned} \tag{116}$$

Final tolerance criterias chosen for the simulator are : $\epsilon_1 = 10^{-3}, \epsilon_2 = 0.01\, and\, \epsilon_3 = 0.001$.

7.1 Relevant Code

`convergence_check.m`

8 Automatic Time Step Control

We begin the simulation with a time step of 0.01 days because flow between cell blocks are usually very large at beginning of simulation. Short time step is required to make sure convergence criteria is met. After the first step, automatic time stepping is used to determine the step size for the next set of Newton's iterations.

The formula for calculating the next time step:

$$\Delta t^{n+1} = max[\Delta t^n \times \min_x [\min_N \frac{(1+\omega)\eta_x}{\delta x_N + \omega \eta_x}], (\Delta t)_{max}] \tag{117}$$

- δx_N is the observed change in variable x in gridblock N during timestep n
- η_x is a maximum desired change of the variable x at any gridblock during a timestep
- ω is a tunning parameter

In the simulator we built here $\omega = 0.5$, $\eta_s = 0.05$ and $\eta_p = 50 psi$.

8.1 Relevant Code

`time_stepping.m`

9 Flow Chart of Simulation

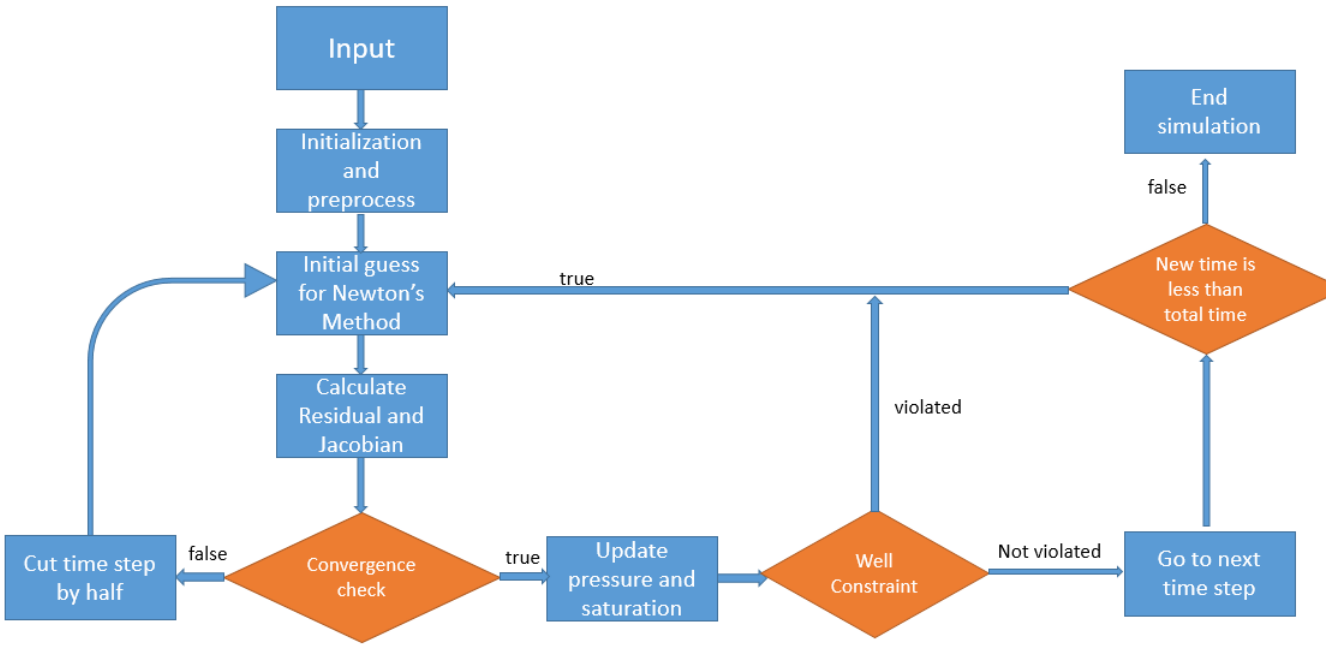


Figure 1: Flow chart of reservoir simulation process

10 Pseudocode for the Overall Simulation

Algorithm 1 Simulation Algorithm

procedure INPUT

grid dimensions, initial reservoir conditions

rock and fluid properties

well parameters, total run time t

end procedure

procedure INITIALIZATION

calculate rock and geometric part of transmissibility for all flow interface

initial time step Δt*,initial guess of* $\boldsymbol{p}$ *for Newton's Method*

end procedure

for $time = 0 : \Delta t : t$ **do**

$k = 1$

while $k <$ *max iterations* **do**

calculate the fluid part of transmissbility

calculate the residual $\boldsymbol{R}$ *and the Jacobian matrix* $\boldsymbol{J}$

$\delta = \boldsymbol{J} \backslash \boldsymbol{R}$

$\boldsymbol{p}^{n+1,k+1} = \boldsymbol{p}^{n+1,k} + \delta$

if $\boldsymbol{R} <$ *error tolerance* **then**

Newton's method converged. Break.

else

$k = k + 1$, continue

end if

if $k < k_{max}$ **then**

Cut time step by half: $\Delta t = \frac{\Delta t}{2}$

end if

end while

$\boldsymbol{p}^{n+1,k} = \boldsymbol{p}^{n,k}$

$t = t + \Delta t$

timestep control

end for

11 Results and Discussion

In this section we will talk about the performance of the built simulator as well as the results acquired from our simulations. We will begin by comparing built simulator to industry standard ECLIPSE 300. Then sensitivity analysis is done to investigate how grid refinement as well as temporal step refinement changes the performance of our simulations. Last, we will introduce two additional cases ran by our built simulator. First additional case runs on the same parameters as the base case used in model validation but includes two production wells. The second additional case is a stepwise injection into a fractured reservoir.

11.1 Base Simulation Parameters

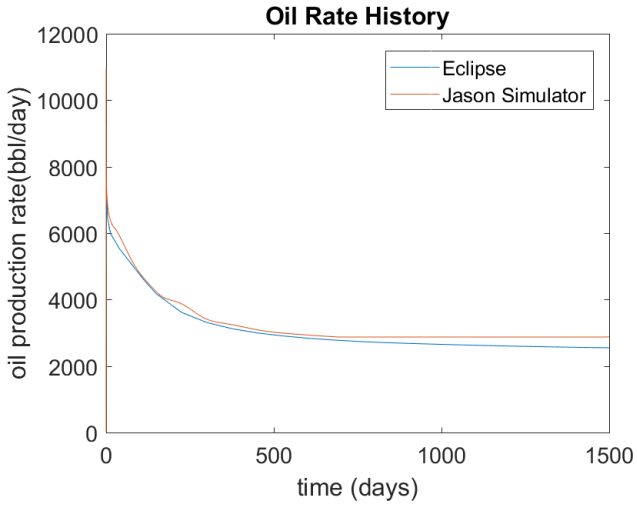
- 2D horizontal reservoir with dimension $6900ft \times 6900ft \times 100ft$
- Top depth of the reservoir is 1200ft
- Homogeneous isotropic permeability kx = 80md and ky = 120md
- Constant porosity of 0.22 throughout the reservoir
- Initial reservoir pressure is 4500psi at depth of 1250ft
- Initial gas saturation is 0, pay zone is below the oil - gas contact and above the water-oil contact
- Bubble point pressure is 3400 psi
- Well diameter is 0.5ft

11.2 Model Validation

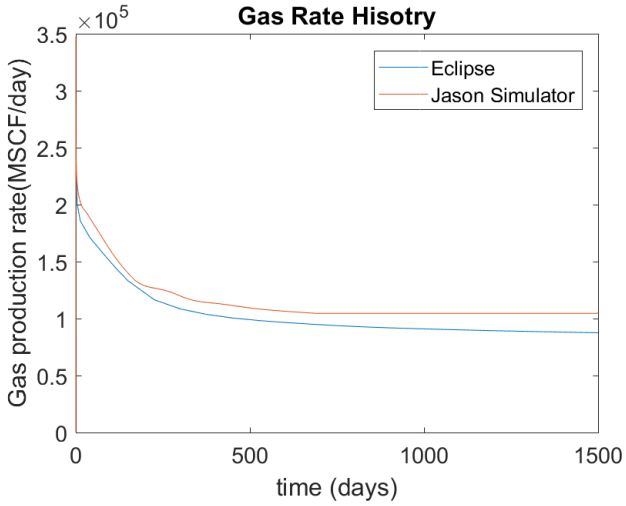
Here all simulation are ran on the base simulation parameters. The grid dimensions are $23 \times 23 \times 1$, and the total simulation run time is 1500 days in all model comparisons .

11.2.1 Case 1 - BHP Controlled Production

In the first case we have one production well located at grid block [12,12,1]. This well is controlled based on a fixed bottom hole pressure of 2000 psi for the entire duration of the simulation. Oil production rate, gas production rate, bottom hole pressure, average reservoir pressure, and bottom hole pressure are all plotted against the results from ECLIPSE 300 for validation. As we can see, most of the results from our simulator is quite similar to ECLIPSE result with some minor discrepancies in oil rate history and gas rate history. The reason could be a more loosened tolerance which allowed bigger step size and thus coarser accuracies.

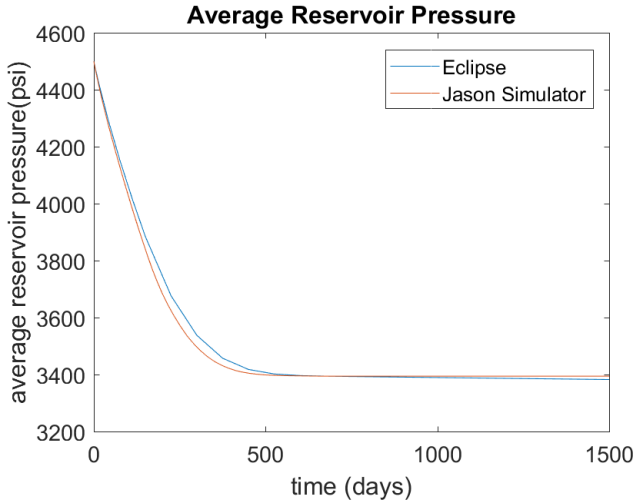


(a) oil rate history of case 1

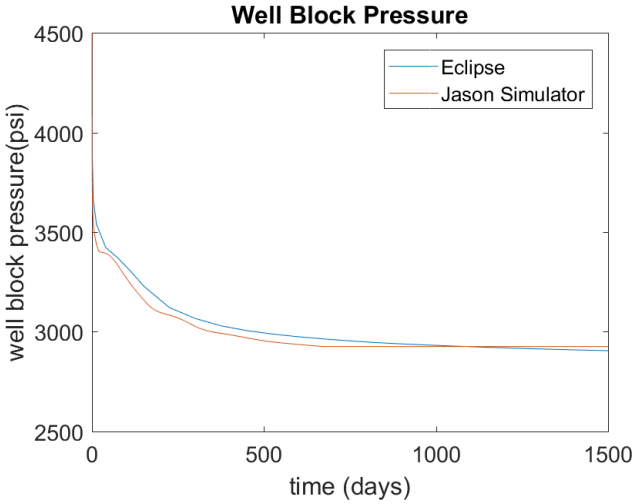


(b) gas rate history of case 1

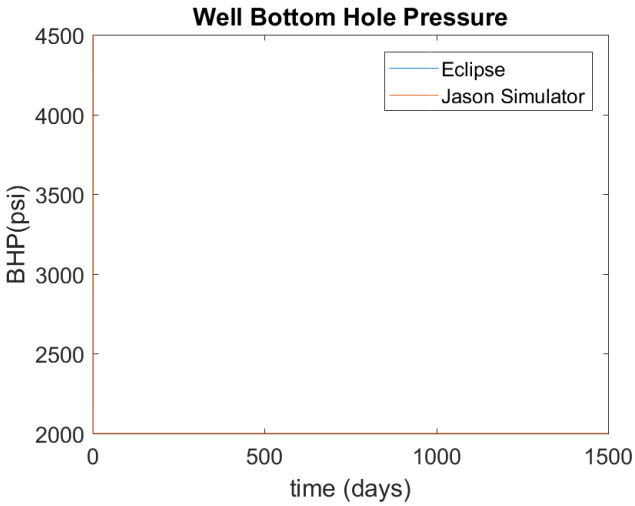
Figure 2: Oil and gas production rate history



(a) average reservoir pressure history of case 1



(b) average block pressure history of case 1

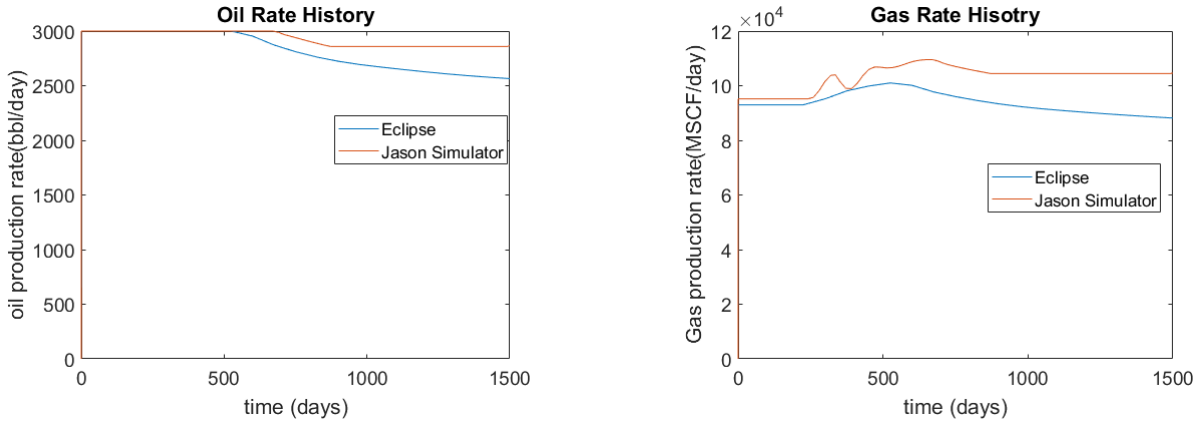


(c) bottom hole pressure history of case 1

Figure 3: Well block pressure, bottom hole pressure and average reservoir pressure of the duration of the simulation

11.2.2 Case 2 - Rate controlled Production

In the second case we still have one production well located at grid block [12,12,1]. This well is controlled based on a fixed oil production of 3000 bbl/day at the beginning of the simulation. When bottom hole pressure is declined to 2000 psi we switch well control from oil production rate controlled regime to fixed BHP well controlled regime. Results of simulation in this case is plotted against results from ECLIPSE 300. We can see in fig. 5a and fig. 5b the difference between our simulator and ECLIPSE 300 is bigger than the BHP controlled case, even though pressure history of both simulators are similar. The reason could be Eclipse take shorter time step size and strict convergence criteria to ensure the best performance.

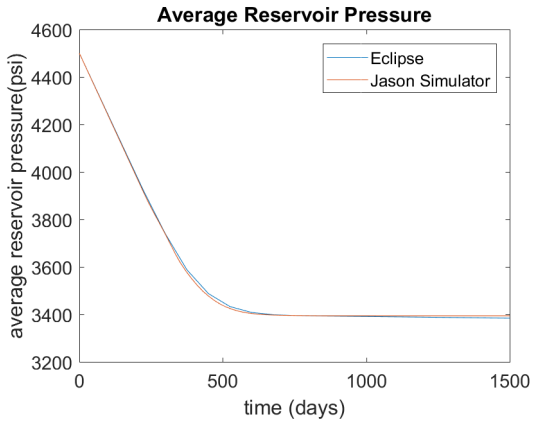


(a) oil rate history of case 2

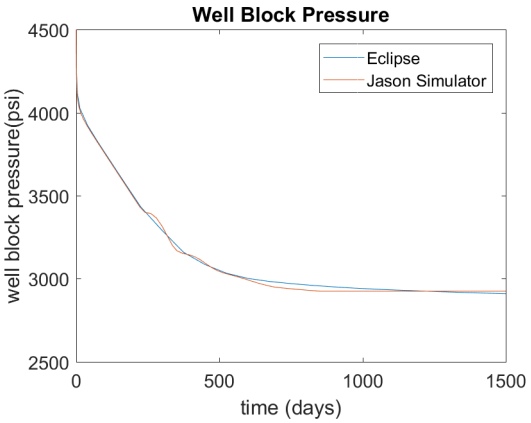
(b) gas rate history of case 2

Figure 4: Oil and gas production rate history

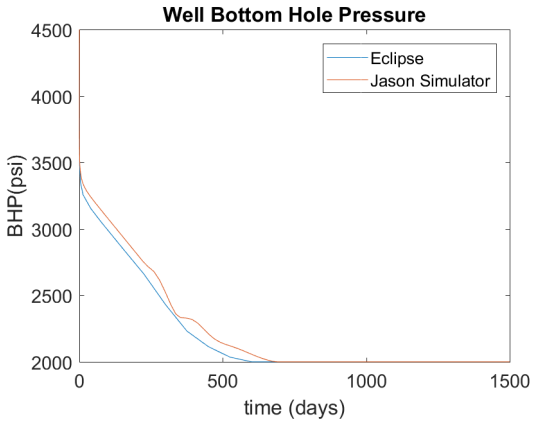
Overall the simulator built for this project performed well and could be considered as a valid simulator going forward for other applications and tasks.



(a) average reservoir pressure history of case 2



(b) average block pressure history of case 2



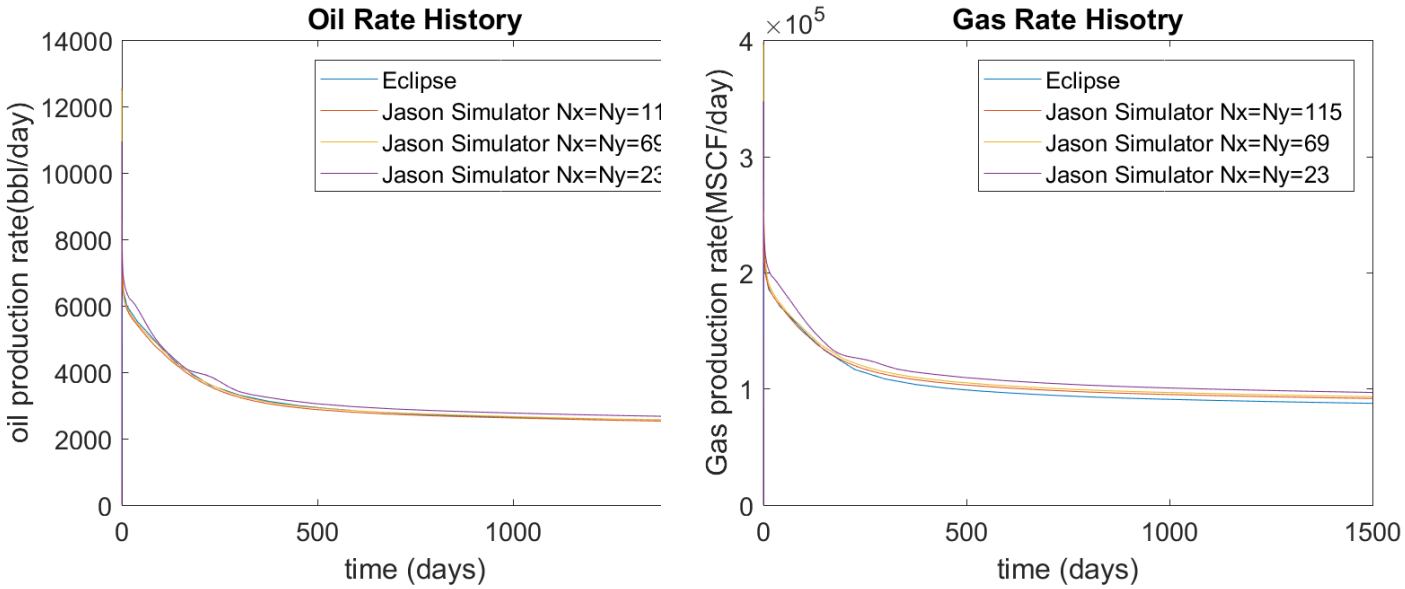
(c) bottom hole pressure history of case 2

Figure 5: Well block pressure, bottom hole pressure and average reservoir pressure of the duration of the simulation

11.3 Sensitivity Analysis

To understand the performance of the simulator better, we conducted sensitivity analysis on our simulation. Simulation is ran on grid dimension 23×23, 69×69, 115×115 separately to check how results correspond to these changes.

We can see from fig. 7a and fig. 6b that simulation results between grid size 69×69 are fairly close and this confirms the consistency and accuracy of our model. The simulator after grid refinement performs closer to the ECLIPSE simulation results, which proves more strict tolerance criteria should be used to achieve better results in case 1 and 2.

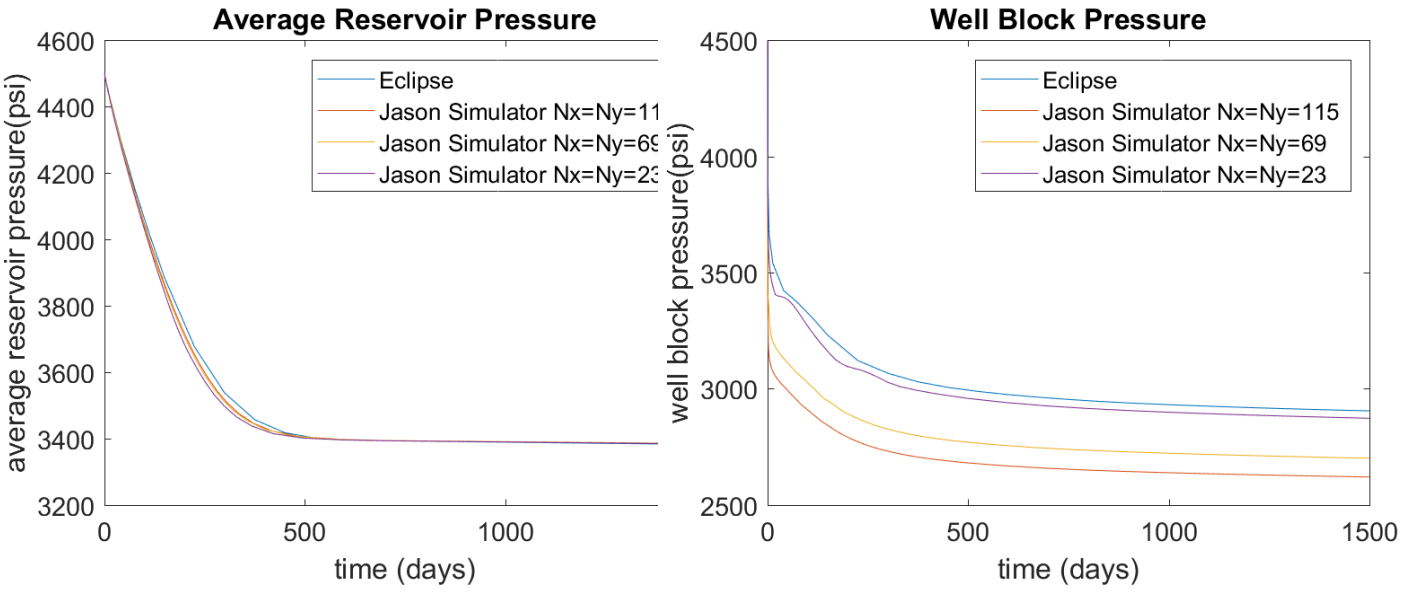


(a) oil rate history of case 3

(b) gas rate history of case 3

Figure 6: Oil and gas production rate history

Checking Newton's iterations number at each step provides us insights to the time steps simulator has to take enable to comply with the convergence criteria. It's clear from fig. 8 that with finer grids, the simulator has to revert back to smaller time steps in order to satisfy the convergence. CFL numbers gives us insights into the flow condition of a particular block. From fig. 9 we can see CFL number increases with finer grids. So in IMPES regime we should avoid use grids that are too fine, in order to achieve the stability required for simulation.



(a) average reservoir pressure history of case 3 (b) well block pressure history of case 3

Figure 7: Well block pressure, average reservoir pressure of the duration of the simulation

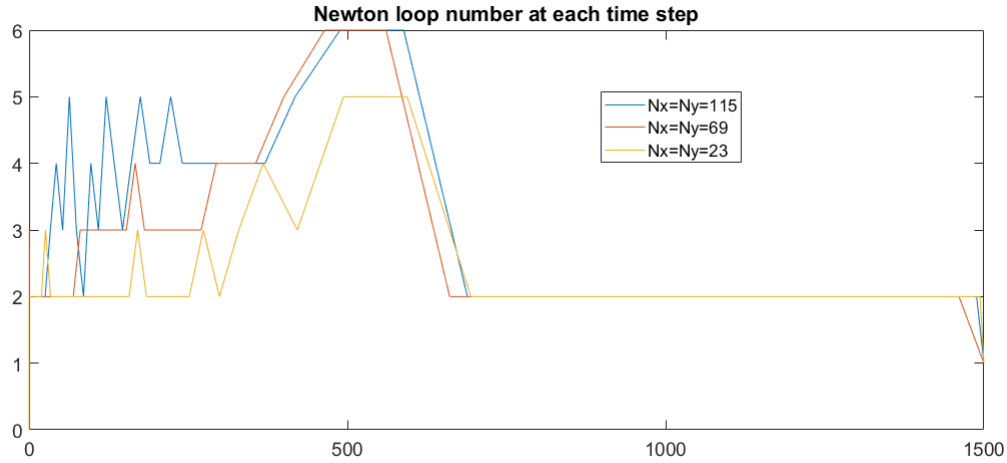


Figure 8: Newton steps of case 3

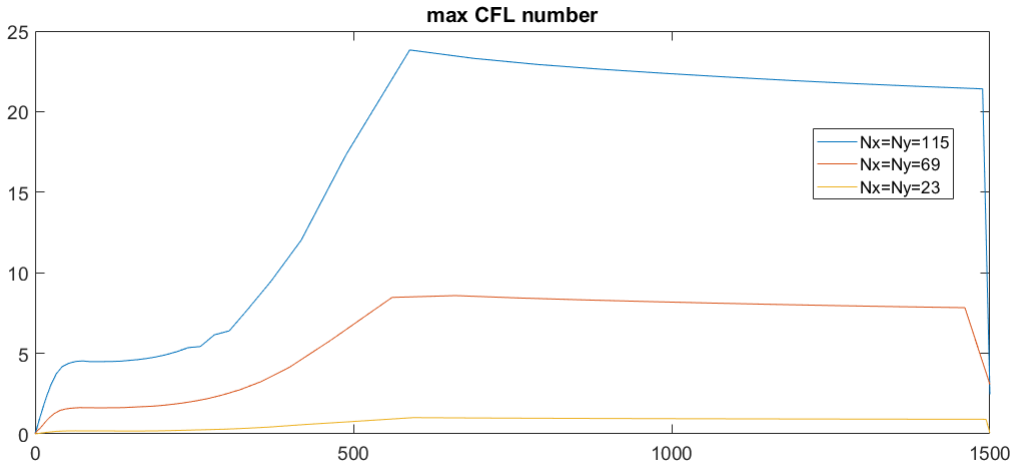


Figure 9: Max CFL number of case 3

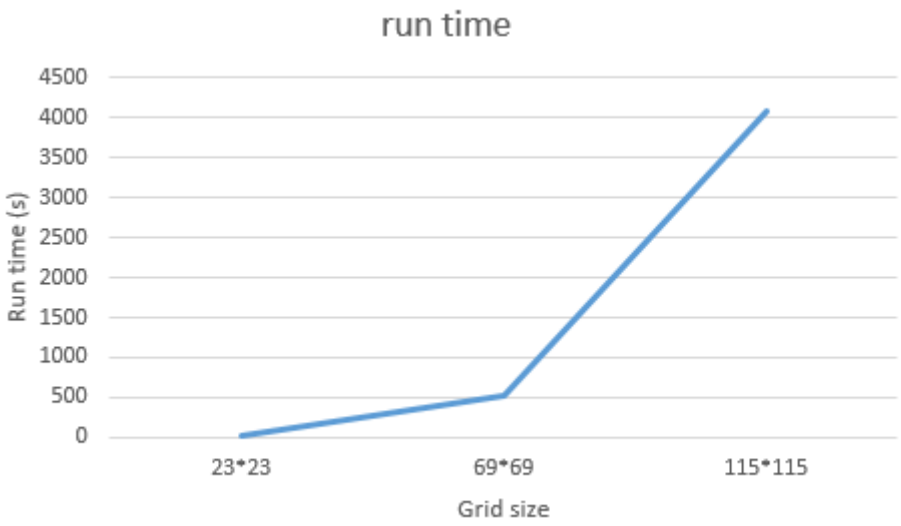
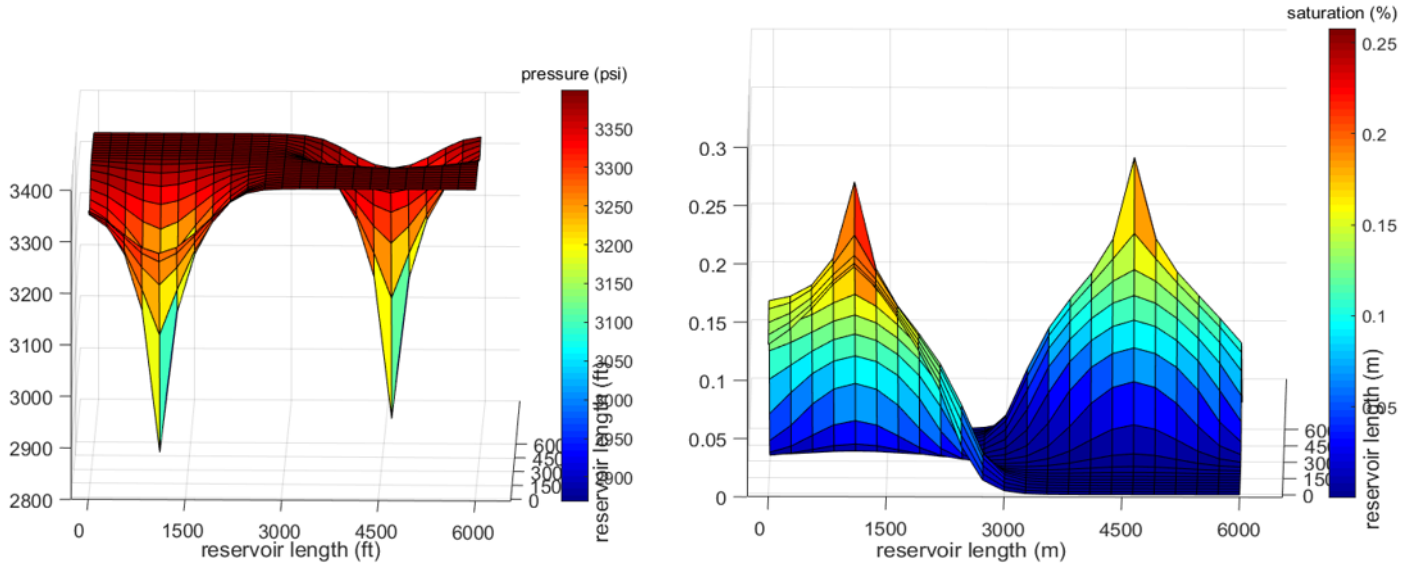


Figure 10: run time for the simulations

11.4 Two Production Wells

The first additional case is a two production well model on the base simulation parameters we used for case 1 and case 2. Both wells are BHP controlled at 2000 psi. Creating just two production wells on the original model is generic and boring, so in this section two different cases are set up to understand the effect of well interference. In the case the two wells are well-separated and radius of investigation from each well never reach the other well, and this can be observed from the pressure field at the end of the simulation in fig. 11a and the gas saturation field at the end of the simulation fig. 11b. In the second case the two wells are fairly close to each other and the pressure effect of one well interferes with the pressure field of the other, the interference can be observed visually through fig. 12a and fig. 12b.

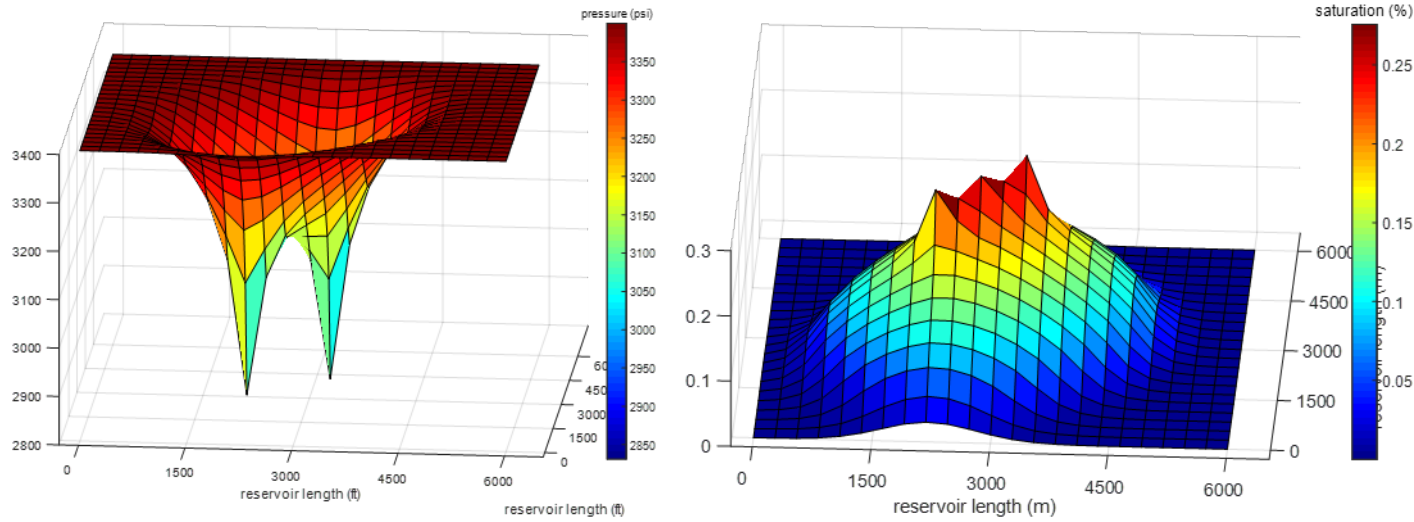


(a) pressure field

(b) saturation field

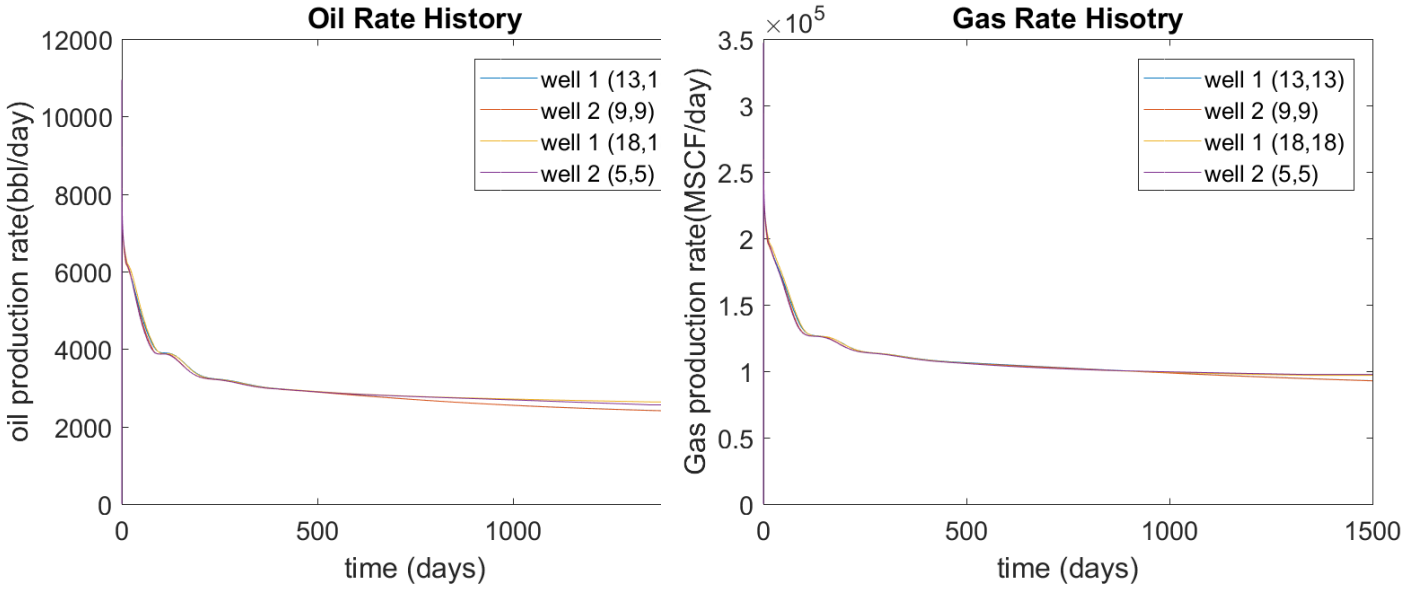
Figure 11: pressure and saturation field of distanced wells model at end of simulation

Oil rate history fig. 13a and gas rate history fig. 13b are subsequently plotted to understand the effect of interference. We can observe oil and gas rate decline in the distanced well case is slower than the oil and gas rate decline in the closer well case. The same effect can be observed in the average reservoir pressure fig. 14a and well block pressure fig. 14b. In the distanced well case reservoir pressure decline is slower than the closer well case. Thus we can conclude well interference is detrimental to well performance. Well interference causes lower production and faster reservoir pressure depletion and should be avoided as much as possible. It is recommended to perform well location optimization to minimize the effect of well interference before operations take place.



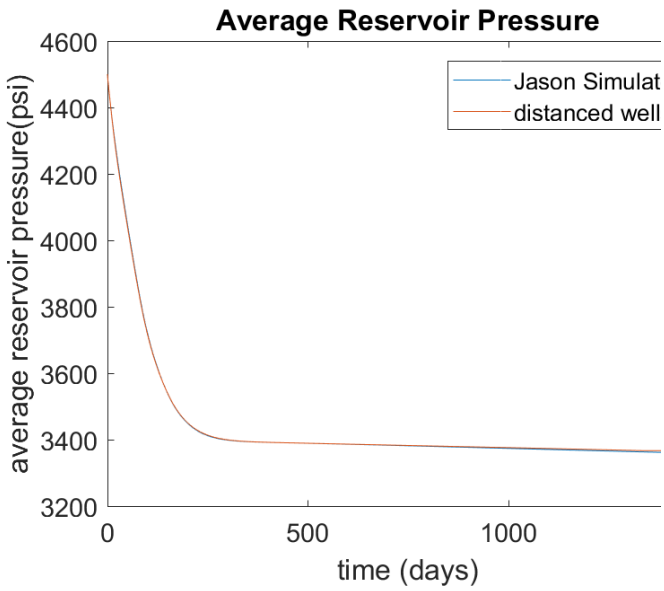
(a) pressure field (b) saturation field

Figure 12: pressure and saturation field of close wells model at end of simulation

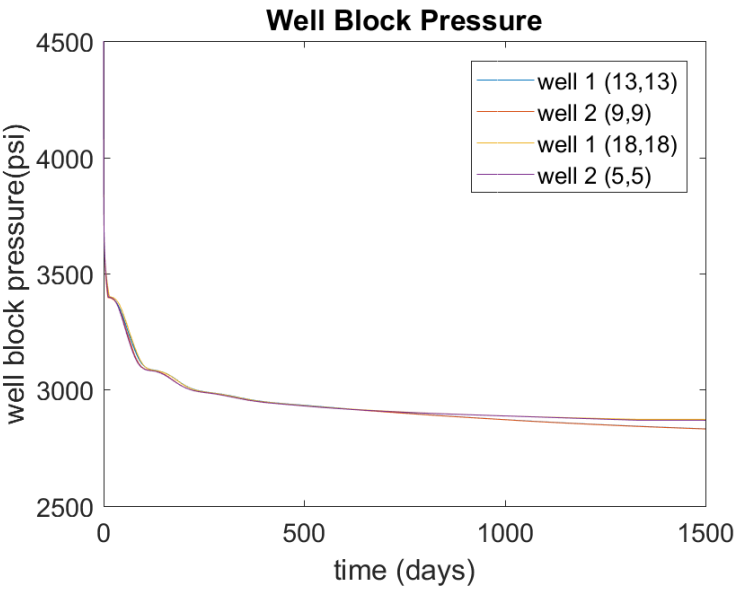


(a) oil rate history of case 4.1 (b) gas rate history of case 4.1

Figure 13: Oil and gas production rate history

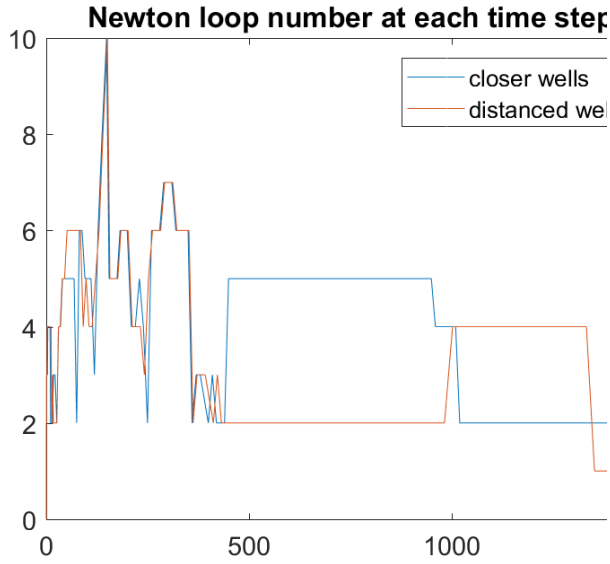


(a) average reservoir pressure history of case 4

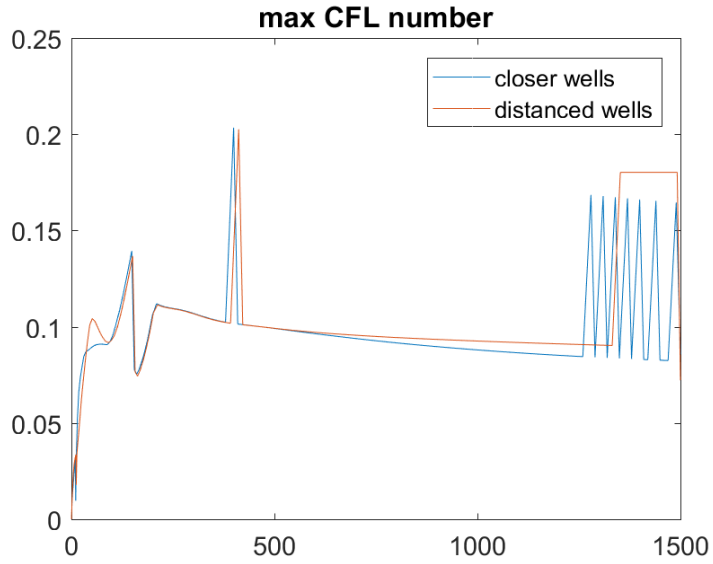


(b) well block pressure history of case 4

Figure 14: Well block pressure, bottom hole pressure and average reservoir pressure of the duration of the simulation



(a) Newton steps of case 4

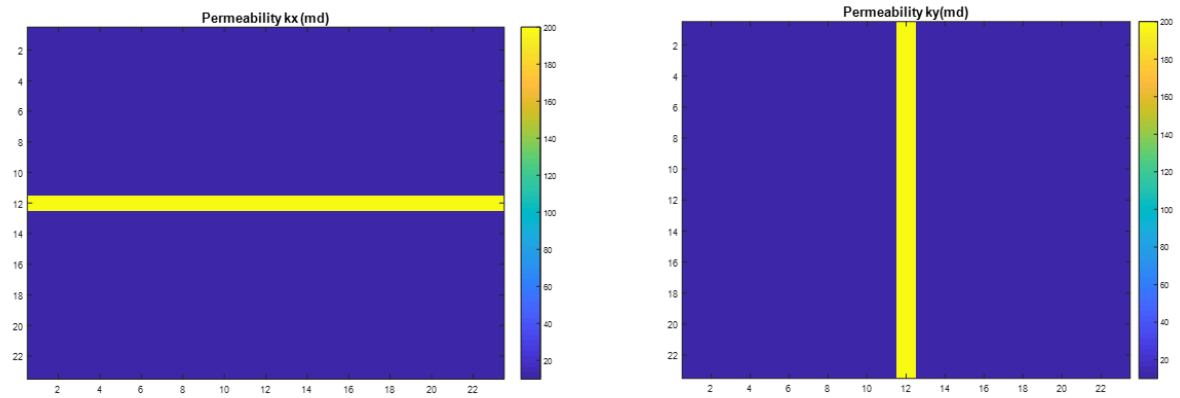


(b) Max CFL number of case 4

Figure 15: Newton and CFL

11.5 Stepwise Injection in Horizontal Fractures

The second additional case considers heterogeneity in reservoir permeability field and well injection scheme. The reservoir is considered to be an "unconventional reservoir" with permeability kx and ky equal to 10 md except for the location of the fractures. Two fractures located at row 12 and column 12 of the grid blocks respectively and can be visualized through fig. 16a and fig. 16b. We set up an injection well in the middle of the reservoir at location [12,12] and the injection scheme is constant injection rate from day 0 to day 100, then switch the injection on and off every 200 days for the rest duration of the simulation.

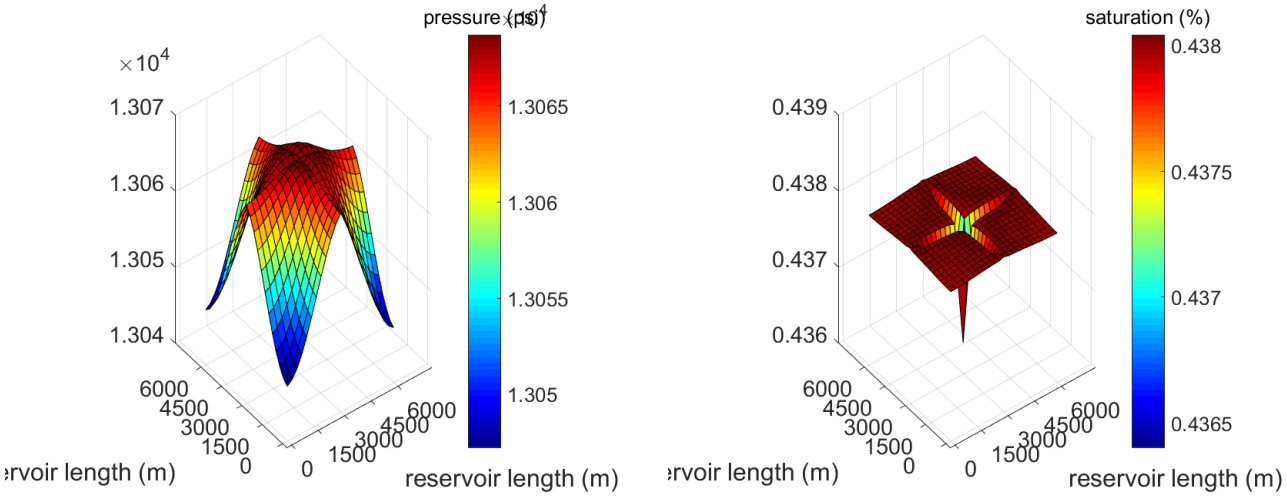


(a) pressure field (b) saturation field

Figure 16: permeability field for the fractured case

Pressure field and saturation field at the end of the simulation can be visualized on fig. 17a and fig. 17b. We can see fractured channels have significantly higher pressure than the rest of the reservoir because most of the injected fluid end up being stored in the fractures. The gas saturation in the fractures are lower than the rest of the reservoir mainly because we are injecting oil into the reservoir and the migration of injected oil to the fractures cause less fraction of pore volume occupied by the gas.

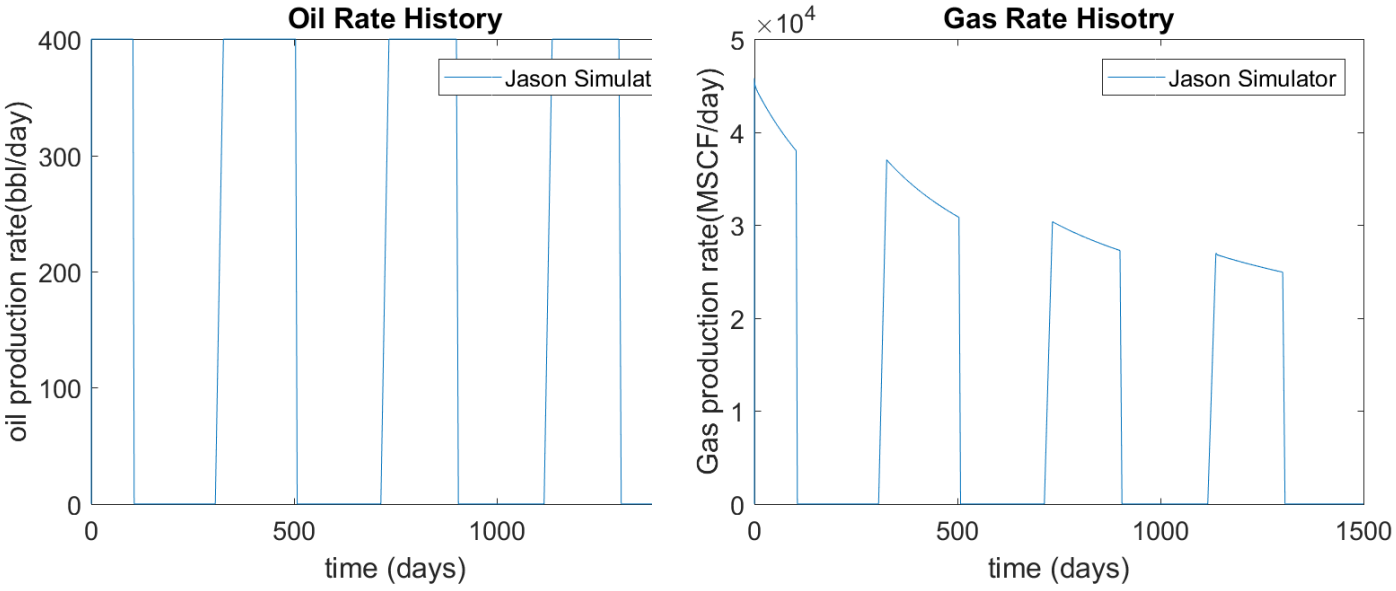
Oil and gas injection rate, bottom hole pressure, average reservoir pressure and well block pressure are plotted in fig.18a, fig.18b, fig.19a, fig.19b, fig.19c. We can see that the simulator fully captured the stepwise injection regime, flow rate as well as pressure change all respond to injection and non-injection time intervals.



(a) pressure field

(b) saturation field

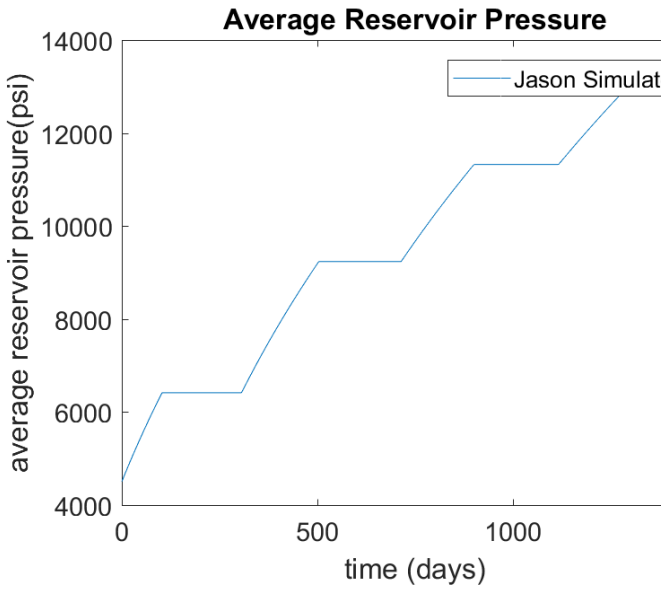
Figure 17: pressure and saturation field of fractured model at end of simulation



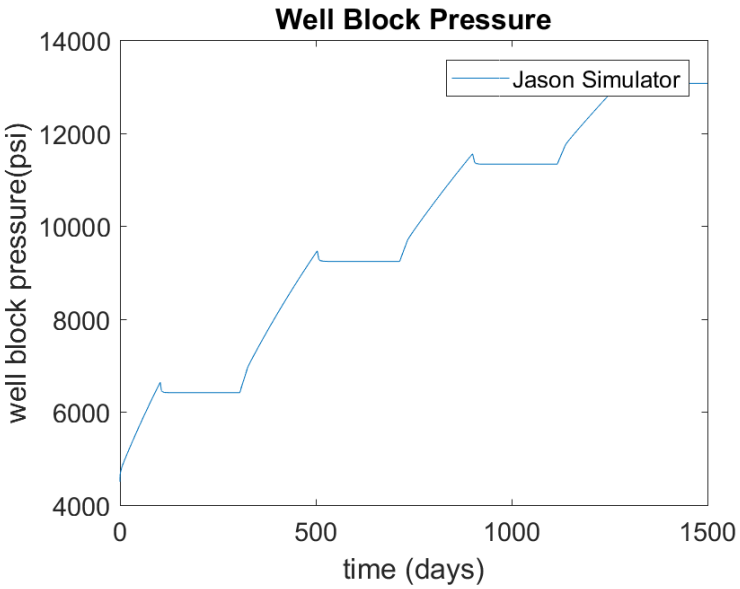
(a) oil rate history of case 4.2

(b) gas rate history of case 4.2

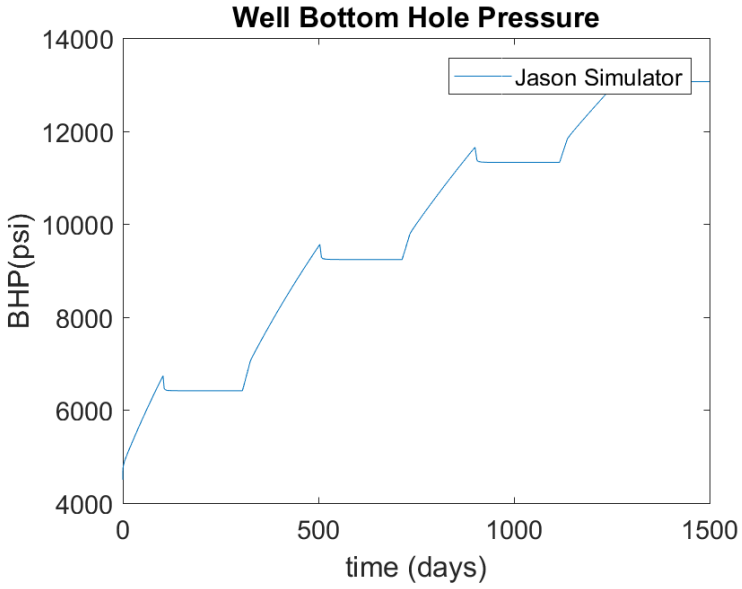
Figure 18: Oil and gas production rate history



(a) average reservoir pressure history of case 4.2

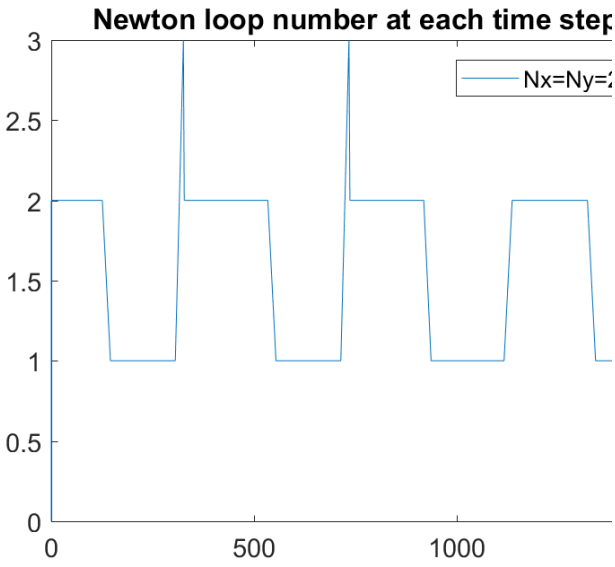


(b) average block pressure history of case 4.2

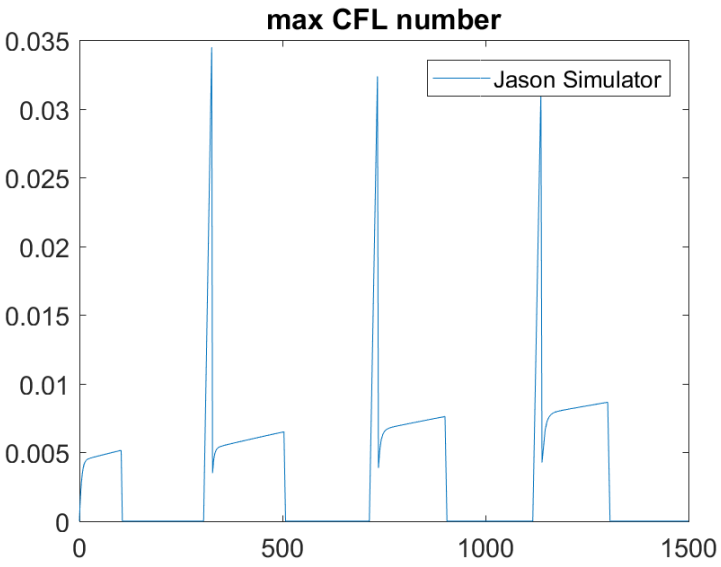


(c) bottom hole pressure history of case 4.2

Figure 19: Well block pressure, bottom hole pressure and average reservoir pressure of the duration of the simulation



(a) Newton steps of case 4.2



(b) Max CFL number of case 4.2

Figure 20: Newton and CFL